

# Pricing Energy in the Presence of Renewables

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**Abstract**—The intermittent nature of renewable energy generation implies that renewable producers rely on non-renewable producers to ensure the aggregate power delivered meets the promised quality of service. Therefore, the intermittent nature of renewable energy generation affects the committed power and market price of energy. We consider an electricity market where renewable and non-renewable generators bid by proposing their asking price per unit of energy to an independent system operator (ISO). The ISO solves a dispatch optimization problem to minimize the cost of purchased energy on behalf of the consumers. We incorporate the notion of net-load variance using the Conditional Value-at-Risk (CVAR) measure in the dispatch optimization problem to ensure that the generators are able to meet the load within a desired confidence level. We analytically derive the market clearing price of energy and dispatched powers as a function of CVAR and show that a higher penetration of renewable energies may increase the market clearing price of energy. Finally, we present descriptive simulations to illustrate the impact of renewable energy penetration on the market price of energy.

## I. INTRODUCTION

Growing concern over the impact of climate change has led to a noticeable shift from non-renewable resources in many parts of the world. As a result, renewable energy resources are anticipated to play a crucial role in power systems of the near-future. However, integration of renewable resources into the electricity market, specifically high levels of penetration, requires several modifications in the electricity market [1]. The primary reason for this modification is that production from renewable energy sources is highly uncertain and variable. Thus, the electricity market should be modified to take this uncertainty and variability into account and to mitigate the impact on other entities involved in the market.

In this paper, we consider an electric grid that delivers demanded electricity to consumers. The grid consists of a non-renewable generator unit, a renewable generator unit with stochastic production, a transmission line, and a consumer (load).

We assume that the grid takes all resultant renewable energy and the non-renewable generator is dispatched to meet the net-load, which is the consumer load minus the renewable generator output (Figure 1).

The renewable and non-renewable generators propose their asking price per unit of energy and their generation constraints to an independent system operator (ISO) [2]–[4]. The ISO, after receiving the price of non-renewable and renewable energies, minimizes the total cost of energy and determines an output power for each non-renewable

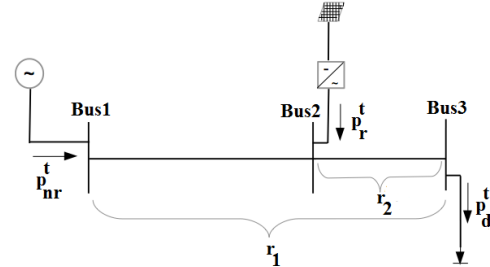


Fig. 1. Electric grid model;  $r_1$  is the resistance from bus 1 to the bus 3;  $r_2$  is the resistance from bus 2 to the bus 3;  $p_{nr}^t$  is the non-renewable power;  $p_r^t$  is the renewable power;  $p_d^t$  is the load power.

generator that will ensure a given level of reliability to meet the load. The ISO then determines the corresponding market-clearing prices through the next 24 hours.

The focus of this work is to quantify the required power and market price of energy in the market as a function of renewables and loads statistics while considering grid reliability. The main result is that the market clearing price of energy may increase with greater penetration of renewable energy into the grid because of uncertainty in renewables realizations. The market clearing price is shown to be a non-decreasing function of uncertainty of the net load, the reliability demanded by the consumer, and the loss in the grid. We provide numerical results obtained through simulations which consider these scenarios.

We use the notion of Conditional Value-at-Risk (CVAR) to quantify the effect of uncertainty of the net-load on the market price of energy. CVAR has been used in the electricity market and literature [5]–[6] to measure the risks of dispatch strategies. Several works that are representative of the direction of this study include [7]–[8], which analyze the problem under different market settings. The studies most related to our work are [7] and [9]. They develop optimal strategies to inject wind energy into the grid under a fixed market price of energy. Unlike these works, we develop the market clearing price of energy to quantify the effect of uncertainty of the load and renewable energies on the market clearing price of energy. Unlike our work, in [7] and [9] the market prices are not affected by the penetration of renewables.

There exists a rich literature which studies various aspects of renewable energy resources in the power grid that includes forecasting methods, energy storage, frequency regulation, and technological challenges (see, e.g., [10], [11] and the references therein). In [10] an overview of the current and future

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trends in power electronics as well as appropriate storage technology implemented for the integration of intermittent renewable energy sources, like wind and photovoltaic power generators, are presented. [11] discusses the challenges of power fluctuation and frequency regulation imposed by the integration of variable renewable energy into the power network. In addition, they present an updated load frequency control (LFC) model, and analyze the system frequency response to the new model.

Further, there is a trend of papers which focuses on the integration of renewable energy producers into electricity markets, their competition, and their impact on the market (see [12]–[14]). Of particular relevance, [12] investigates the integration of renewable energy resources, specifically wind power in Germany, Spain, and the UK. [13] focuses on the efficiency of incentive schemes for the development of renewable energy sources and their integration into the electricity market. Authors in [14] study the strategic behaviors of renewable power producers in electricity markets.

Another line of related literature answers the question of how to allocate the cost generated by the uncertainty and variability of the renewable energy producers or the benefit produced by their aggregation among them, satisfying certain properties in the electricity market (see e.g., [15]–[17]).

Despite available methods for mitigating uncertainty, e.g. energy storage systems, demand response etc, the uncertainty of renewable generation and the load remain as challenges for the optimization of generator dispatch in the market.

The rest of the paper is structured as follows. In Section II, we formulate and solve the dispatch optimization problem of the ISO with the goal of minimizing the cost of energy while ensuring committed generators in the market are able to meet the load within a desired confidence level. In Section III we show the numerical results obtained through simulations. Finally, concluding remarks are provided in Section IV.

## II. ISO PROBLEM

### A. Notation

The following notations are used in this work.

- $\mathfrak{R}$  and  $\mathfrak{R}^+$  denote the sets of real and non-negative real numbers respectively
- $p_\ell^t$ : Total active power loss at time  $t$
- $p_d^t$ : Active power requested by consumer
- $p_i^t$ : Output power of the  $i$ th non-renewable generator at time  $t$
- $p_{nr}^t$ : Total active power generated by non-renewable generator unit
- $p_r^t$ : Active power generated by renewable source
- $r_1$ : Resistance from bus 1 to bus 3
- $r_2$ : Resistance from bus 2 to bus 3
- $(\cdot)^+ = \max(\cdot, 0)$
- $s^t := p_d^t + r_2(p_r^t)^2 - p_r^t$
- $\pi_i$ : Asking price per unit of energy of the  $i$ th generating unit
- $\pi_r$ : Asking price per unit of renewable energy.

### B. Load and Renewable Energy

It is assumed that the load and renewable resource output are independent, random variables with known distributions. Let  $\mathfrak{T} = \{1, 2, \dots, T\}$  be the index set. Define  $(P_D, P_r)$  as the random processes on the probability space  $(\Omega, \mathfrak{F}, \mathbb{P})$ .  $P_D = \{P_D^t, t \in \mathfrak{T}\}$  and  $P_r = \{P_r^t, t \in \mathfrak{T}\}$  represent the load and renewable power respectively. For a fixed  $t$ , and for all  $\omega \in \Omega$ ,  $P_D^t(\omega)$  and  $P_r^t(\omega)$  are non-negative random variables with known continuous probability density functions. For the given  $\omega \in \Omega$ ,  $P_D^t$  and  $P_r^t$  are deterministic functions of  $t$  that denote the load realization and renewable resource power realization respectively, at time  $t$ , denoted by  $p_d^t$ , and  $p_r^t$ .

### C. Power Flow Constraints

We assume the non-renewable generator unit is composed of  $N$  generators. Let  $p_i^t$  be the output power of the  $i$ th generator at time  $t$ . Rapid changes in output power, which cause rapid changes in the generator temperature or physical design, may increase maintenance costs. The output of the non-renewable generators can be limited by the generator capacity, or constraints on the quantity of fuel and  $CO_2$  emissions. Each unit must obey an output limit such that for all  $i = 1, \dots, N$

$$p_i^{\min} \leq p_i^t \leq p_i^{\max}. \quad (1)$$

The total output of non-renewable generator units is given as

$$p_{nr}^t = \sum_{i=1}^N p_i^t. \quad (2)$$

We assume that the voltage at each bus is equal to one. The total active power loss ( $p_\ell^t$ ) is approximated as

$$p_\ell^t \approx r_1(p_{nr}^t)^2 + r_2(p_r^t)^2. \quad (3)$$

The power balance equation is given as

$$p_{nr}^t + p_r^t - p_\ell^t = p_d^t, \quad (4)$$

with a desired level of reliability  $\omega \in \Omega$ .

### D. Dispatch Problem

Let  $\pi_r$  be the asking price per unit of energy of the renewable generator unit. Let  $\pi_i$  be the asking price per unit of energy of the  $i$ th non-renewable generator. Without loss of generality, we assume that  $0 < \pi_r < \pi_1 < \dots < \pi_N$ . Let  $p_{nr} := \{(p_1^t, \dots, p_N^t)\}_{t=1}^{24}$  and  $p_r := \{p_r^t\}_{t=1}^{24}$ . The ISO decides on a  $p_{nr}$  that minimizes the total cost of energy through the next 24 hours as follows

$$\min_{p_{nr}} \sum_{t=1}^{24} E_{P_D, P_r} \left[ \sum_{i=1}^N \pi_i p_i^t + \pi_r p_r^t \right], \quad (5)$$

with respect to (1)–(4). In our model, the ISO dispatches the generators based on their asking price of energy. The multi-stage optimization problem (5) is disjointed through stages (time). Let  $n^t = (p_\ell^t + p_d^t - p_{nr}^t - p_r^t)^+$  and  $F_{n^t}$  be the cumulative distribution function of the  $n^t$ , we use the

concept of Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), [18].  $VaR_\alpha(n^t)$  determines the worst possible  $n^t$  that may occur within a given confidence level  $\alpha$ . For a given  $0 < \alpha < 1$ , the amount of  $n^t$  will not exceed  $VaR_\alpha(n^t)$  with probability  $\alpha$ ,

$$VaR_\alpha(n^t) = \min\{z | F_{n^t}(z) \geq \alpha\}. \quad (6)$$

CVaR is defined as the conditional expectation of  $n^t$  above the amount  $VaR_\alpha$ . Let  $E$  denote the expectation over  $n^t$ .

$$CVaR_\alpha(n^t) = E[n^t | n^t > VaR_\alpha(n^t)], \quad (7)$$

$$CVaR_\alpha(n^t) = \int_{-\infty}^{\infty} z dF_{n^t}^\alpha(z), \quad (8)$$

where

$$F_{n^t}^\alpha(z) = \begin{cases} 0, & \text{if } z < VaR_\alpha(n^t) \\ \frac{F_{n^t}(z) - \alpha}{1 - \alpha}, & \text{otherwise} \end{cases}.$$

The objective is to plan for the generators such that they are capable of meeting the load within the confidence level. We write condition (4) as

$$CVaR_\alpha(n^t) = 0. \quad (9)$$

Let  $p_{nr}^t = (p_1^t, \dots, p_N^t)$ . We define  $(\tilde{\mu}_i, \mu_i)$  and  $\lambda^t$  as the lagrange multipliers corresponding to (1) and (9). The lagrange function for the ISO problem is given as

$$\begin{aligned} L^t(p_{nr}) = & E_{P_D, P_r} \left[ \sum_{i=1}^N \pi_i p_i^t + \pi_r p_r^t \right] + \lambda^t [CVaR_\alpha(n^t)] \\ & + \sum_{i=1}^N \mu_i [p_i^t - p_i^{\max}] + \sum_{i=1}^N \tilde{\mu}_i [p_i^{\min} - p_i^t]. \end{aligned} \quad (10)$$

**Theorem 1:** Let  $s^t := p_d^t + r_2(p_r^t)^2 - p_r^t$ , it is claimed that

$$CVaR_\alpha(n^t) = r_1 \left( \sum_{i=1}^N p_i^t \right)^2 - \sum_{i=1}^N p_i^t + CVaR_\alpha(s^t). \quad (11)$$

Proof. The proof is given in the Appendix.

It is evident from (11) that  $CVaR_\alpha(n^t)$  is convex in  $p_{nr}^t$ , therefore the lagrange function (10) is convex in  $p_{nr}^t$ . By substituting (11) in (10)

$$\begin{aligned} L^t(p_{nr}^t) = & E_{P_D, P_r} \left[ \sum_{i=1}^N \pi_i p_i^t + \pi_r p_r^t \right] \\ & + \lambda^t \left[ r_1 \left( \sum_{i=1}^N p_i^t \right)^2 - \sum_{i=1}^N p_i^t + CVaR_\alpha(s^t) \right] \\ & + \sum_{i=1}^N \mu_i [p_i^t - p_i^{\max}] + \sum_{i=1}^N \tilde{\mu}_i [p_i^{\min} - p_i^t]. \end{aligned} \quad (12)$$

The necessary Karush-Kuhn-Tucker (KKT) conditions for the ISO's problem are

$$\pi_i + 2\lambda^t r_1 \sum_{i=1}^N p_i^t - \lambda^t + \mu_i - \tilde{\mu}_i = 0, \quad (13)$$

$$r_1 \left( \sum_{i=1}^N p_i^t \right)^2 + CVaR_\alpha(s^t) = \sum_{i=1}^N p_i^t, \quad (14)$$

$$p_i^{\min} \leq p_i^t \leq p_i^{\max}, \quad (15)$$

$$\mu_i (p_i^t - p_i^{\max}) = 0, \quad (16)$$

$$\tilde{\mu}_i (p_i^{\min} - p_i^t) = 0, \quad (17)$$

$$\mu_i \geq 0, \tilde{\mu}_i \geq 0. \quad (18)$$

To ensure a feasible solution for the ISO's problem (13)-(18), Assumption 1 is considered. It is worth noting that this assumption must be modified for a more complex topology of the power grid e.g. with a higher number of generator buses and congestion constraints.

**Assumption 1:** Let  $p^{\min} = \min_{1 \leq k \leq N} p_k^{\min}$ . It is assumed that

- $1 - 4r_1 CVaR_\alpha(s^t) \geq 0$ .
- For all  $t = 1, \dots, T$

$$\begin{cases} p^{\min} \leq CVaR_\alpha(s^t) \leq \sum_{i=1}^N p_i^{\max}, & \text{if } r_1 = 0 \\ p^{\min} \leq \frac{1 \pm \sqrt{1 - 4r_1 CVaR_\alpha(s^t)}}{2r_1} \leq \sum_{i=1}^N p_i^{\max}, & \text{if } r_1 > 0 \end{cases} \quad (19)$$

- $\max_{i \in \{1, \dots, N\}} p_i^{\min} < \min_{i \in \{1, \dots, N\}} \{p_i^{\max} - p_i^{\min}\}$ .

Let  $p^t$  be the solution of  $r_1(p^t)^2 - p^t + CVaR_\alpha(s^t) = 0$ . Because of part a) of Assumption 1 the value of  $p^t$  is real and because of part b) of Assumption 1, there exists a unique  $1 \leq k \leq N$  such that  $\sum_{i=1}^{k-1} p_i^{\max} < p^t \leq \sum_{i=1}^k p_i^{\max}$ . The value of  $p^t$  based on the value of  $r_1$  is given as follows.

$$\begin{cases} p^t = \frac{1 \pm \sqrt{1 - 4r_1 CVaR_\alpha(s^t)}}{2r_1}, & \text{if } r_1 > 0 \\ p^t = CVaR_\alpha(s^t), & \text{if } r_1 = 0 \end{cases} \quad (20)$$

Part c) of Assumption 1 ensures that if the first  $(k-1)$ th generators are operating at their maximum power and additional power is needed to meet the load, when it is less than  $p_k^{\min}$ , then generator  $(k-1)$ th can lower its output power without violating its constraints, such that the  $k$ th generator operates at its minimum power ( $p_k^{\min}$ ). This is proved in Lemma 1, shown below, and is drawn from part c) of Assumption 1.

**Lemma 1:** If  $0 < p^t - \sum_{i=1}^{k-1} p_i^{\max} < p_k^{\min}$  then

$$p_{k-1}^{\min} < p^t - \sum_{i=1}^{k-2} p_i^{\max} - p_k^{\min} < p_{k-1}^{\max}. \quad (21)$$

Proof. The proof is given in the Appendix.

By solving (13)-(18), the values of  $\{\mu_i\}_{i=1}^N$ ,  $\{\tilde{\mu}_i\}_{i=1}^N$  and  $\lambda^t$  are given below based on the value of  $p^t - \sum_{i=1}^{k-1} p_i^{\max}$ .

- $p_k^{\min} \leq p^t - \sum_{i=1}^{k-1} p_i^{\max} \leq p_k^{\max}$   
 $\mu_i > 0, \tilde{\mu}_i = 0, p_i^t = p_i^{\max}$  for  $i = 1, \dots, k-1$ , (22)

$$\mu_k = 0, \tilde{\mu}_k = 0, p_k^t = p^t - \sum_{i=1}^{k-1} p_i^{\max}, \quad (23)$$

$$\mu_i = 0, \tilde{\mu}_i = 0, p_i^t = 0, \text{ for all } i = k+1, \dots, N, \quad (24)$$

$$\lambda^t = \frac{\pi_i + \mu_i - \tilde{\mu}_i}{1 - 2r_1 p^t} \quad (25)$$

$$= \frac{\pi_i + \mu_i - \tilde{\mu}_i}{\sqrt{1 - 4r_1 CVaR_\alpha(s^t)}}, \text{ for all } i = 1, \dots, k$$

$$\lambda^t = \frac{\pi_k}{\sqrt{1 - 4r_1 CVaR_\alpha(s^t)}}. \quad (26)$$

- $0 < p^t - \sum_{i=1}^{k-1} p_i^{\max} < p_k^{\min}$

$$\mu_i > 0, \tilde{\mu}_i = 0, p_i^t = p_i^{\max} \text{ for } i = 1, \dots, k-2, \quad (27)$$

$$\mu_{k-1} = 0, \tilde{\mu}_{k-1} = 0, p_{k-1}^t = p^t - \sum_{i=1}^{k-2} p_i^{\max} - p_k^{\min}, \quad (28)$$

$$\mu_k = 0, \tilde{\mu}_k > 0, p_k^t = p_k^{\min}, \quad (29)$$

$$\mu_i = 0, \tilde{\mu}_i = 0, \quad (30)$$

$$p_i^t = 0, \text{ for all } i = k+1, \dots, N, \quad (31)$$

$$\lambda^t = \frac{\pi_i + \mu_i - \tilde{\mu}_i}{1 - 2r_1 p^t} \quad (32)$$

$$= \frac{\pi_i + \mu_i - \tilde{\mu}_i}{\sqrt{1 - 4r_1 CVaR_\alpha(s^t)}}, \text{ for all } i = 1, \dots, k$$

$$\lambda^t = \frac{\pi_{k-1}}{\sqrt{1 - 4r_1 CVaR_\alpha(s^t)}}. \quad (33)$$

- $0 < p^t < p_1^{\min}$

From part b) of Assumption 1, there exists an  $1 \leq \bar{k} \leq N$  such that  $p_{\bar{k}}^{\min} \leq p^t < p_{\bar{k}}^{\max}$ . Let  $\bar{k}$  be the smallest  $k$  that satisfies this condition, then

$$\mu_i = 0, \tilde{\mu}_i = 0, p_i^t = 0, \text{ for all } i \neq \bar{k}, \quad (34)$$

$$\mu_{\bar{k}} = 0, \tilde{\mu}_{\bar{k}} = 0, p_{\bar{k}}^t = p^t, \quad (35)$$

$$\lambda^t = \frac{\pi_{\bar{k}}}{\sqrt{1 - 4r_1 CVaR_\alpha(s^t)}}. \quad (36)$$

It is evident from (26), (33) and (36), that the market clearing price of energy ( $\lambda^t$ ) is higher at the times that  $s^t$  has a heavier tail distribution. A heavier tail distribution leads to a higher value of  $CVaR_\alpha(s^t)$  and larger index of  $k$  in (26) and (33). Similarly, a higher level of reliability (larger  $\alpha$ ) leads to a higher market clearing price of energy. The accuracy of market clearing price (26) and (33) is heavily dependent on the accuracy of the tail distribution of  $s^t$ . The tail distribution of  $s^t$  depends on the load and renewable energy distributions and model of loss function. In the next section, more descriptive simulations are presented.

### III. SIMULATIONS

**Setup:** We consider that the non-renewable generator is composed of 6 units. The asking price per unit of energy

and the maximum output power for each unit is given in Table I. It is assumed that  $p_i^{\min} = 0$  for all  $i = 1, \dots, 6$ . The level of reliability ( $\alpha$ ) demanded by the consumer is fixed at 0.9.

TABLE I  
MAXIMUM OUTPUT POWER  $p_i^{\max}$  AND ASKING PRICE  $\pi_i$

$p_i^{\max}$ (MW)	0.05	0.1	0.12	0.15	0.18	0.25
$\pi_i$ (\$/MW)	20	30	40	50	60	70

In the simulations, load and renewable energy are considered as the aggregates of the distributed loads and renewable generations. By the central limit theorem, the sum of the independent and identically distributed random variables tend to a Gaussian distribution as the number of random variables increases. We assume the load and renewable energies have Gaussian distributions truncated on a non-negative domain. The load has a mean of 0.7 and standard deviation of 0.1. We repeat the simulation analysis for different scenarios of renewable energy penetration.

**Case I:** We assume the standard deviation of the renewable energy is fixed at 0.1 and the mean of renewable energy takes values  $\{0, 0.15, 0.25, 0.3, 0.45, 0.5, 0.65, 0.75, 0.8, 0.9\}$ . This corresponds to the naive expectation that the renewable energy penetration increases, while the uncertainty does not increase. The market clearing price of energy is plotted in Figure 2.

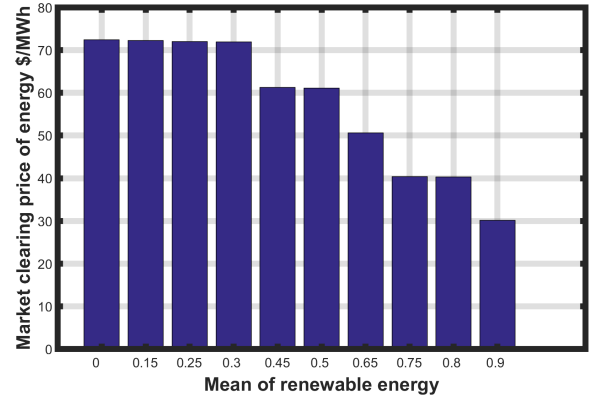


Fig. 2. Market clearing price versus the mean of renewable energy.

It is observable from Figure 2 that the market clearing price of energy is decreasing in renewable energy penetration, if the standard deviation (i.e., the uncertainty) of renewable energy is kept constant while increasing the penetration. In reality of course, as more renewable energy is integrated the uncertainty in the total renewable energy also increases. We consider that scenario in Case III and IV below.

**Case II:** We assume that the mean of renewable energy is fixed at 0.5 and the standard

deviation of renewable energy takes values  $\{0.01, 0.04, 0.1, 0.15, 0.2, 0.25, 0.3, 0.4, 0.45, 0.5\}$ . The market clearing price of energy is plotted in Figure 3.

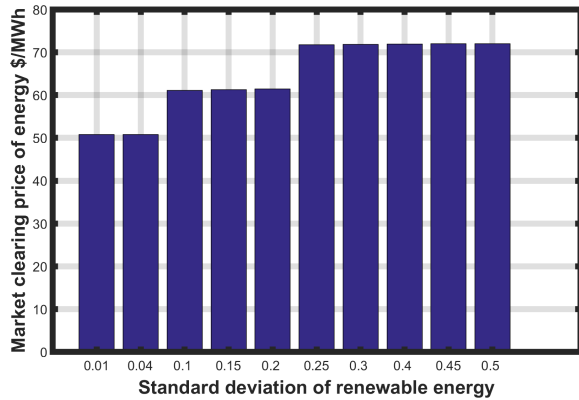


Fig. 3. Market clearing price versus the variance of renewable energy penetration.

It can be seen from Fig. 3 that higher variability in renewable energy production increases the uncertainty in the net load. This increases the risk of unnecessary high commitment of non-renewable generators in the market, and leads to a higher market clearing price of energy.

**Case III:** Let the mean and standard deviation of renewable energy correspond to values  $\{0.05, 0.15, 0.25, 0.3, 0.45, 0.5, 0.65, 0.75, 0.8, 0.9\}$  and  $\{0.06, 0.1, 0.12, 0.15, 0.32, 0.2, 0.3, 0.4, 0.45, 0.5\}$  respectively. The market clearing price of energy is plotted in figure 4.

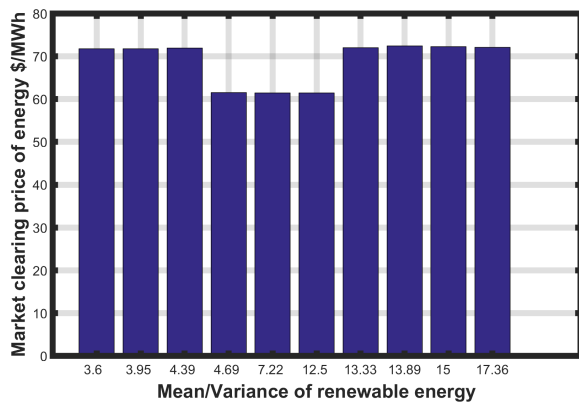


Fig. 4. Market clearing price versus the  $\frac{\text{mean}}{\text{variance}}$  of renewable energy penetration.

In Figure 4, the market clearing price of energy decreases until a certain level of the penetration is reached, after which the price increases. The price decreases in the beginning because of the lower marginal cost of renewable energy. However, after a certain level, the payment to the non-renewable generator to maintain the reliability constraints catches up and the market clearing price increases. The

plot shows that (i) if the consumer insists on the lowest possible market clearing price, then the penetration level of renewable energy is capped; and (ii) if the consumer insists on a given price for the energy it may become important for the renewable producer to pay the non-renewable to compensate the latter.

**Case IV:** We assume the mean and standard deviation of renewable energy are fixed at 0.5 and 0.1 respectively. The line resistance  $r_1$  takes values  $\{0.04, 0.06, 0.08, 0.1, 0.12, 0.14, 0.16, 0.18, 0.2, 0.22\}$ . The market clearing price of energy is plotted in figure 5.

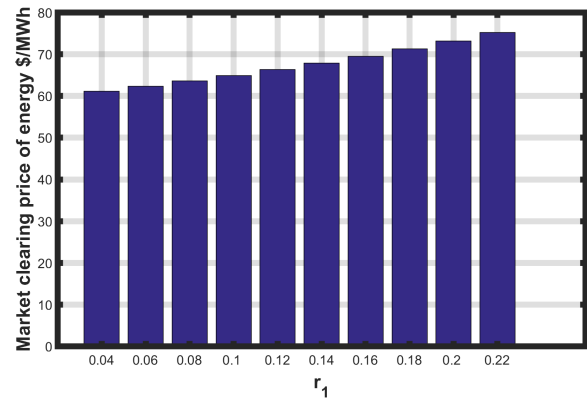


Fig. 5. Market clearing price versus the value of ( $r_1$ ).

It is observable from Figure 5 that the market price of energy is increasing in the line resistance. A longer grid line (higher resistance) increases the loss of energy and increases the required capacity for non-renewable generators. Locating the renewable generators closer to the load leads to a lower market price of energy. This observation provides alternate methods to increase the renewable penetration.

#### IV. CONCLUSION

In this paper, we studied the impact of intermittent and uncertain renewable energy generation on the committed power and market price of energy of non-renewable generators. We quantified the market clearing price of energy in a market as a function of the desired level of reliability. Uncertainty in net load was shown as a possible reason for increases in the market price of energy. The expectation that increasing the penetration of renewable energy reduces the market price of energy would be untrue if uncertainty in renewable energy generation is increased by the higher penetration level. In a situation where the consumer insists on paying no more than a certain price for energy, the renewable producer must transfer funds to the non-renewable producer or consumer to compensate the cost of the uncertainty and intermittency of renewable generations. This cost is currently ignored, which implies a hidden subsidy from the non-renewables and consumers to the renewable producer. Understanding and resolving such frictions, to fully consider the effects

of uncertainty and fluctuations of renewable energies, are central to evaluating the benefits of renewable energy.

#### ACKNOWLEDGMENT

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#### V. APPENDIX

**Proof of Theorem 1:** From [18]

$$CVaR_\alpha(n^t) = \min_{\eta \in \mathbb{R}} \left\{ \eta + \frac{1}{1-\alpha} E[n^t - \eta]^+ \right\}. \quad (37)$$

We first prove that the minimizer of (37) is non-negative. Suppose the minimizer of (37) be negative ( $\eta \leq 0$ ). Let  $\eta = -\eta^+$ , where  $\eta^+ = |\eta|$ , then

$$\begin{aligned} & \arg \min_{\eta \in \mathbb{R}} \left\{ \eta + \frac{1}{1-\alpha} E[n^t - \eta]^+ \right\} \\ &= \arg \min_{\eta^+ \in \mathbb{R}^+} \left\{ \frac{\alpha \eta^+}{1-\alpha} + \frac{1}{1-\alpha} E[n^t] \right\}. \end{aligned} \quad (38)$$

The equality in (38) is because of  $n^t \geq 0$ , and linear property of the expectation. It is evident that the minimizer of (38) is  $\eta^+ = 0$ . Therefore, the minimizer of (37) is non-negative ( $\eta \geq 0$ ). Because of the convexity of  $CVaR$ , the minimizer of (37) is obtained by taking the derivative of  $\eta + \frac{1}{1-\alpha} E[n^t - \eta]^+$  with respect to  $\eta$  as

$$\eta^* = VaR_\alpha(s^t) + r_1 \left( \sum_{i=1}^N p_i^t \right)^2 - \sum_{i=1}^N p_i^t. \quad (39)$$

By substituting  $\eta^*$  in  $\eta + \frac{1}{1-\alpha} E[n^t - \eta]^+$

$$CVaR_\alpha(n^t) = r_1 \left( \sum_{i=1}^N p_i^t \right)^2 - \sum_{i=1}^N p_i^t + CVaR_\alpha(s^t). \quad (40)$$

**Proof of Lemma 1:** The right side inequality in (21) is obvious. Below the left side inequality is proven. Because of part c) of Assumption 1,

$$p_{k-1}^{\min} < p_{k-1}^{\max} - p_k^{\min}. \quad (41)$$

Because of  $0 < p^t - \sum_{i=1}^{k-1} p_i^{\max}$

$$p_{k-1}^{\max} < p^t - \sum_{i=1}^{k-2} p_i^{\max}. \quad (42)$$

It is concluded from (41) and (42)

$$p_{k-1}^{\min} < p_{k-1}^{\max} - p_k^{\min} < p^t - \sum_{i=1}^{k-2} p_i^{\max} - p_k^{\min}. \quad (43)$$

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