2017 IEEE 56th Annual Conference on Decision and Control (CDC)
December 12-15, 2017, Melbourne, Australia

A Near-Optimal Decoupling Principle for Nonlinear Stochastic Systems Arising in Robotic Path Planning and Control*

Mohammadhussein Rafieisakhaei[1], Suman Chakravorty[2] and P. R. Kumar[1]

***Abstract*— We consider nonlinear stochastic systems that arise in path planning and control of mobile robots. As is typical of almost all nonlinear stochastic systems, optimally solving the problem is intractable. Moreover, even if obtained it would require centralized control, while the path planning problem for mobile robots requires decentralized solutions. We provide a design approach which yields a tractable design that is quantifiably near-optimal. We exhibit a decoupling result under a small noise assumption consisting of the optimal open-loop design of nominal trajectory followed by decentralized feedback law to track this trajectory. As a corollary, we obtain a trajectory-optimized linear quadratic regulator design for stochastic nonlinear systems with Gaussian noise.**

I. Introduction

Many robotic systems, in particular, mobile aerial and ground robots, are equipped with noisy actuators that require feedback compensation or planning ahead in a policy that accounts for the random perturbations. Simply ignoring the noise and planning for the unperturbed equivalent of the stochastic system can yield crucial errors leading to failure in reaching the end-goal, or result in the system falling into unsafe states. Moreover, the solution should not require a fully centralized control since that would require pervasive constant communication among all robots.

In a stochastic setting, the general problem of sequential decision-making can be formulated as a Markov Decision Problem (MDP) [1], [2]. The optimal solution of the stochastic control problem can be obtained iteratively by value or policy iteration methods to solve the Hamilton-Jacobi-Bellman equations [2]. Except in special cases, such as in a linear Gaussian environment, this involves discretization of the underlying spaces [3]; an approach whose scalability faces the curse of dimensionality [4]. As a result, the solutions require a computation time that is provably exponential in the state dimension, in a real number based model of complexity, without any assumption that $P \neq NP$ [5].

Many approaches have been proposed based on their tractability. Model Predictive Control (MPC)-based methods [6], [7], robust formulations [8], [9], and other designs that relate to the Pontryagin's Maximum Principle [10] are some of the methods that have been successfully used as surrogate design approaches. Another popular approach utilizes Differential Dynamic Programing (DDP) [11] and DDP-based variations such as the Stochastic DDP [12], iLQR and iLQG [13]–Stochastic DDP relies on second order approximation of the dynamics and cost, whereas iLQR and iLQG uses second order approximation of the cost but first order linearization of the dynamics. These methods propose iterative methods that attempt to find "locally-optimal" solutions in a tube around a nominal trajectory [13] by coupling the design of feedback policy and the nominal trajectory of the system.

In this paper, we address the nonlinear stochastic control problem and propose an architecture under which the decoupled design of an optimal open-loop control sequence and a decentralized feedback policy is both tractable and near-optimal. In particular, we show that under a small noise assumption, the decoupling into globally-optimal trajectory design and a decentralized feedback control law holds for fully-observed nonlinear stochastic systems of the type of interest in mobile robotic systems.

The design can be broken into two parts: *i)* an open-loop optimal control problem that designs the nominal trajectory of the LQR controller, which respects the nonlinearities as well as state and control constraints; *ii)* the design of a decentralized LQR policy around the optimized nominal trajectory. The quality of the design is rigorously provided by the main results of the paper. We quantify the first order stochastic error for small-noise levels based on large deviations theory. We thereby arrive at a Trajectory-optimized decoupled Linear Quadratic Regulator (T-LQR) design for fully-observed nonlinear stochastic systems under Gaussian small-noise perturbations.

The organization of the paper is as follows. Section II states a simple large deviations result for a linear Gaussian system. Section III defines a general stochastic control problem for a fully-observed system. Section IV provides the main results by first analyzing the effect of feedback compensation on the linearization error, and then providing the state and control error propagations along with probabilistic bounds based on the large deviations results developed in Section II. It also describes the T-LQR approach and proves its near-optimality. Finally, Section V provides a design based on T-LQR for a non-holonomic car-like robot and provides numerical results illustrating the proposed approach to design.

*This material is based upon work partially supported by NSF under Contract Nos. CNS-1646449 and Science & Technology Center Grant CCF-0939370, the U.S. Army Research Office under Contract No. W911NF-15-1-0279, and NPRP grant NPRP 8-1531-2-651 from the Qatar National Research Fund, a member of Qatar Foundation.

[1]M. Rafieisakhaei and P. R. Kumar are with the Department of Electrical and Computer Engineering, and [2]S. Chakravorty is with the Department of Aerospace Engineering, Texas A&M University, College Station, Texas, 77840 USA. {mrafieis, schakrav, prk@tamu.edu}