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Dynamic Intermittent Suboptimal Control: Performance Quantification and Comparisons

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Abstract: This paper presents a novel intermittent suboptimal event-triggered controller design for continuous-time nonlinear systems. The stability of the equilibrium point of the closed-loop system, and the performances are analyzed and quantified theoretically. It is proven that the static and the dynamic event-triggered suboptimal controllers have a known degree of sub-optimality compared to the conventional optimal control policy. In order to generate dynamic event-triggering framework, we introduce an internal dynamical system. Moreover, the Zeno behavior is excluded. Finally, a simulation example is conducted to show the effectiveness of the proposed intermittent mechanisms.

Key Words: Event-triggered control, performance analysis, optimal control, dynamic triggering condition.

1 Introduction

In real-world applications, the majority of the feedback controllers is implemented on digital platforms, such as embedded micro-processor and/or on-board modules of communication and actuation. Traditional digital control techniques depend on periodic sampling, computation, and actuation. But limited communication and computation resources along with energy saving targets require that every information through a network should be carefully decided when to transmit. Therefore, it is necessary to design aperiodic-sampling based controllers that can function in event-driven environments and update their values only when it is necessary to guarantee stability and a level of optimality.

The event-triggered control design is a framework that can efficiently deal with applications of limited bandwidth. Such framework requires a sampled-state component, and an event-triggering mechanism that determines when the control signal has to be updated [1–7]. The work of [8] proposed a dynamic framework to further reduce the communication burden. However, in the above results, the performance of the event-triggered controller was not discussed and quantified. This work provides such a quantification of sub-optimality with respect to the time-triggered optimal controller.

Reinforcement learning (RL) is a branch of machine learning which aims to find an optimal action to minimize or maximize a long term reward. Recently, intermittent design, i.e., event-triggered control, has been combined with RL to reduce the communication load and computation burden in feedback controller design for multi-agent systems in a model-free manner [9] and single-agent system with linear dynamics [10] and nonlinear dynamics [11–15]. However, the event-triggered mechanisms in above results are static in essence because the parameters for the event-triggered condition is time-invariant. To further reduce the communication burden, a novel dynamic event-triggered mechanism with time-varying parameters is developed in this paper.

A common issue for optimal control problem is that the analytical solutions for systems with general nonlinear dynamics is not available. A future design may equip the presented dynamic event-triggered mechanism with RL algorithm [16–20], which approximates the optimal controller in an iterative and/or online manner. Due to the page limitation, RL-based dynamic event-triggered mechanism design will be introduced in a future paper.

The remainder of this paper is structured as follows. Section 2 formulates the problem. In Section 3, a static event-triggered mechanism is introduced. Section 4 extends the static mechanism to the dynamic case. A simulation example is shown in Section 5. Finally, Section 6 concludes the whole paper and talks about future work.

Notation: For the ease of readers, we provide here a partial notation list which will be also explained in more details later throughout this article. $\mathbb{R}^+$ denotes the set $\{x \in \mathbb{R} : x > 0\}$. $\mathbb{N}^+$ denotes the set of positive integers. $\underline{\lambda}_M$ and $\bar{\lambda}_M$ denote the minimum and maximum eigenvalues of the matrix M, respectively. $\{t_k\}_{k=0}^{\infty}$ is a monotonically increasing sequence of sampling instants with t_k the k-th consecutive sampling instant satisfying $\lim\limits_{k\to\infty} t_k = \infty$.

2 Problem Formulation

Consider the following stabilizable nonlinear system

$$\dot{x} = f(x) + g(x)u(t), x(t_0) = x_0, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector and $u \in \mathbb{R}^m$ is the control input.

We shall find a controller u that minimizes a cost functional similar to the one with an infinite bandwidth con-

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troller,

$$V(x_0) = \int_{t_0}^{\infty} U(x(t), u(t))\, dt,\ \forall x_0, \tag{2}$$

where the utility function $U(x(t), u(t))$ is differentiable and satisfies $U(x, u) \geq 0$, for $\forall x, u$.

For this paper we will choose,

$$U(x, u) := Q(x) + \|u\|_R, \tag{3}$$

where $Q(x) := x^T Q x$ and $\|u\|_R := u^T R u$, with $Q \succeq 0$ and $R \succ 0$, represent the trade-off between driving the state to the origin and saving the energy applied to the system respectively.

Based on [21], a necessary condition for optimality is provided by

$$\begin{aligned} 0 &= H(u^*(t); x(t), V^*(x(t))) \\ &= \min_{u(t)} H(u(t); x(t), V^*(x(t))), \end{aligned} \tag{4}$$

with a boundary condition $V^*(x(\infty)) = 0$ and is termed as a Hamilton-Jacobi-Bellman (HJB) equation. Assuming that the minimum on the right hand side of (4) exists and is unique, then the relationship between the solution of the HJB equation (4) and the optimal control problem of system (1) with respect to the cost (2) can be described as,

$$u^*(x) = -\frac{1}{2} R^{-1} g^T(x) \frac{\partial V^*(x)}{\partial x}. \tag{5}$$

The control (5) satisfies the following assumption.

Assumption 1. *The controller $u^*(x)$ is Lipschitz continuous in the following sense,*

$$\|u^*(x(t)) - u^*(x(t) + y(t))\| \leq L \|y(t)\|,\ \forall y \tag{6}$$

where L is a positive constant.

Remark 1. *Note that the optimal control policy $u(\cdot)$ depends on the optimal value gradient $\frac{\partial V^*(x)}{\partial x}$. However, solving the HJB equation* (4) *is challenging since it is a nonlinear partial differential equation and does not have an analytical solution for general nonlinear systems.*

The controller (5) requires the signal to be updated continuously and is therefore referred to as time-triggered controller. In contrast, event-triggered controller will be introduced in the next section.

3 Suboptimal Static Event-Triggered Control

Consider an aperiodic sampling component that yields,

$$\hat{x}(t) = x(t_k), \forall t \in [t_k, t_{k+1}). \tag{7}$$

Define the gap or difference between the current state $x(t)$ and the sampled state $\hat{x}$ as,

$$e(t) = \hat{x}(t) - x(t). \tag{8}$$

Then, based on the time-triggered optimal control (5), the event-triggered controller can be determined by following [22] as,

$$u_e(x) = u^*(\hat{x}) = -\frac{1}{2} R^{-1} g^T(\hat{x}) \frac{\partial V^*(\hat{x})}{\partial \hat{x}}. \tag{9}$$

Based on Assumption 1, the event-triggered policy $u_e(\cdot)$ will satisfy

$$\|u^*(x(t)) - u_e(x(t))\| \quad \leq \quad L \|e(t)\| \tag{10}$$

The dynamics of the system (1) by using (9) can be written as,

$$\begin{aligned} \dot{x} &= f(x) + g(x) u_e(x) \\ &= f(x) + g(x) u^*(\hat{x}). \end{aligned} \tag{11}$$

Taking into account (11), the Hamiltonian of the event-triggered control policy $u_s(\cdot)$ parametrized by the optimal time-triggered value function $V^*(\cdot)$ can be further expressed as

$$\begin{aligned} & H(u_e(\cdot); V^*(\cdot), x) \\ = & \left[\frac{\partial V^*(x)}{\partial x}\right]^T \left[f(x) - \frac{1}{2} g(x) R^{-1} g^T(\hat{x}) \frac{\partial V^*(\hat{x})}{\partial \hat{x}}\right] \\ + & Q(x) + \frac{1}{4} \left[\frac{\partial V^*(\hat{x})}{\partial \hat{x}}\right]^T g(\hat{x}) R^{-1} g^T(\hat{x}) \frac{\partial V^*(\hat{x})}{\partial \hat{x}} \end{aligned} \tag{12}$$

The following lemmas provide a relationship between $u^*(\cdot)$ and $u_e(\cdot)$.

Lemma 1. *Under Assumption 1, the relationship between the event-triggered Hamiltonian $H(u_e; x, V^*(x))$ in* (12) *and the time-triggered Hamiltonian $H(u^*; x, V^*(x))$ in* (4) *can be found to be*

$$\begin{aligned} & H(u_e; x, V^*(x)) - H(u^*; x, V^*(x)) \\ = & (u_e - u^*)^T R (u_e - u^*). \end{aligned} \tag{13}$$

Proof. The proof is similar to [22]. □

Lemma 2. *Consider the system given by* (1) *with the event-triggered control given by* (9)*. Then, one has,*

$$\dot{x} \leq A_e \|x\| + B_e \|e\| \tag{14}$$

where A_e and B_e are positive constants.

Proof. The proof is omitted due to page limitation and will be provided in a future paper. □

Subtracting the time-triggered HJB equation (4) from (12) yields,

$$\begin{aligned} & \|H(u_e(\cdot); V^*(\cdot), x) - H(u^*(\cdot); V^*(\cdot), x)\| \\ \leq & \bar{\lambda}_R L^2 \|e(t)\|^2, \end{aligned} \tag{15}$$

where $\bar{\lambda}_R$ denotes the maximum eigenvalue of matrix R.

Before we proceed, we require the following assumption.

Assumption 2. *At the event-triggering instant, t_k, $\forall k \in \mathbb{N}^+$, finite-time stabilization is not achieved, i.e., $x(t_k) \neq 0$.*

Lemma 3. *Consider the event-triggered control,*

$$u_s(x) = u^*(\hat{x}) = -\frac{1}{2} R^{-1} g^T(\hat{x}) \frac{\partial V^*(\hat{x})}{\partial \hat{x}}, \tag{16}$$

with a triggering condition given as,

$$\|e\|^2 \leq \frac{\underline{\lambda}_Q \left(1 - \sigma^2\right) \|x\|^2 + \bar{\lambda}_R \|u^*(\hat{x})\|^2}{\bar{\lambda}_R L^2} \tag{17}$$

where $\sigma \in \left(\max\left\{0, 1 - \frac{\bar{\lambda}_R L^2}{\underline{\lambda}_Q}\right\}, 1\right) \in \mathbb{R}^+$. Then, the origin of the closed-loop system is asymptotically stable.

Proof. The proof follows [22]. □

The triggering condition (17) can be equivalently written as,

$$\begin{aligned} h &= \underline{\lambda}_Q \left(1-\sigma^2\right) \|x\|^2 + \bar{\lambda}_R \|u^*(\hat{x})\|^2 - \bar{\lambda}_R L^2 \|e\|^2 \\ h &\geq 0. \end{aligned} \quad (18)$$

The event triggering instants determined by the condition (17) are

$$\begin{aligned} t_0 &= 0, \\ t_{k+1} &= \inf_{t\in\mathbb{R}_0^+} \left\{ t > t_k \wedge h \leq 0 \right\}. \end{aligned} \quad (19)$$

Thus, one requires $h \geq 0$ at all times. The triggering condition (17) is denoted as static, whereas the dynamic will be considered in the next section.

4 Suboptimal Dynamic Event-Triggered Control

The static event-triggered condition (17) needs to be satisfied $\forall t$. In order to further reduce the communication load, this section will relax such requirement by introducing a dynamic intermittent framework.

Following [8], we will require the condition to be nonnegative in an average sense over an interval. To formulate the dynamic event-triggered mechanism, the following internal dynamical system is required

$$\dot{\eta} = -\mu\eta + h, \ \forall \eta(t_0) = \eta_0, t \in R_0^+ \quad (20)$$

where $\mu \in \mathbb{R}^+$ is a parameter to be designed later.

We are now ready to present the following dynamic event-triggering mechanism,

$$\eta(t) + \theta h(t) \leq 0, \quad (21)$$

where $\theta \in \mathbb{R}^+$ is a parameter to be designed later.

The event triggering instants sequence can be determined by (21) as,

$$\begin{aligned} t_0 &= 0, \\ t_{k+1} &= \inf_{t\in\mathbb{R}^+} \left\{ (t > t_k) \wedge (\eta(t) + \theta h(t) \leq 0) \right\}. \end{aligned} \quad (22)$$

Consequently, the control for this dynamic event-triggered case can be found to be

$$u_d(x) = u^*(\hat{x}) = -\frac{1}{2} R^{-1} g^T(\hat{x}) \frac{\partial V^*(\hat{x})}{\partial \hat{x}}. \quad (23)$$

Lemma 4. *Let μ be a positive constant, $\eta_0, \theta \in \mathbb{R}_0^+$, and h be defined as in* (18). *Then,*

1) $\eta(t) + \theta h(t) \geq 0$, $\forall t \in \mathbb{R}_0^+$;
2) $\eta \geq 0$, $\forall t \in \mathbb{R}_0^+$.

Proof. 1) According to (22), the following condition is true

$$\eta(t) + \theta h(t) \geq 0. \quad (24)$$

2) Consider the case that $\theta = 0$. Then the first proposition $\eta(t) + \theta h(t) \geq 0$ implies that $\eta \geq 0$, $\forall t \in R_0^+$.
For the case when $\theta \neq 0$, the first proposition implies

$$h(t) \geq -\frac{1}{\theta}\eta(t). \quad (25)$$

Considering now (20) and (25), allow us to write,

$$\dot{\eta}(t) \geq -\left(\mu + \frac{1}{\theta}\right)\eta(t), \forall \eta_0, t \in R_0^+. \quad (26)$$

Now let $y(y_0, t)$ be a solution of the differential equation

$$\dot{y}(t) = -\left(\mu + \frac{1}{\theta}\right) y(t), \forall y_0 = \eta_0, t \in R_0^+. \quad (27)$$

Then, we can conclude according to the comparison principle [23] that $\eta(t) \geq y(t) \geq 0$. □

The following theorem guarantees the stability of the equilibrium point of the closed-loop system with the dynamic event-triggering mechanism provided by (20) and (21).

Theorem 1. *Let σ be selected as in Lemma 3 and $\mu, \eta_0, \theta \in \mathbb{R}^+$. Suppose that the controller (23) with the dynamic event-triggered mechanism given in (20) and (21) is applied to the system* (1). *Then, the following properties hold.*

1) *The origin of the closed-loop system is asymptotically stable.*
2) *The event-triggered condition* (21) *is Zeno-free. Moreover, let $q = 2A_e - \mu$, $\mu \in (0, 2A_e)$, $\theta \in \left(\frac{1}{2q}, \frac{1}{q}\right]$, the inter-event interval $\underline{\tau}_d = \min_k (t_{k+1} - t_k)$ determined implicitly by* (22) *is lower-bounded by a positive constant $\underline{\tau}_d$, which is given by*

$$\underline{\tau}_d = \int_0^1 \frac{1}{A_e\sqrt{\frac{a}{b}} + \left(B_e + \frac{\mu}{2}\right)s + B_e\sqrt{\frac{b}{a}}s^2 + \frac{1}{2\theta}s^3} ds, \quad (28)$$

where a, b are given as

$$\begin{aligned} a &= \bar{\lambda}_R L^2, \\ b &= \underline{\lambda}_Q \left(1-\sigma^2\right). \end{aligned} \quad (29)$$

3) *The cost of applying $u_d(\cdot)$ is quantified as,*

$$\begin{aligned} &J(u_d(\cdot); x_0) = J(u^*(\cdot); x_0) \\ &+ \int_{t_0}^{\infty} \|u_d(x(\tau)) - u^*(x(\tau))\|_R d\tau. \end{aligned} \quad (30)$$

Proof. 1) Given now the augmented system of (11) and (20), consider the following Lyapunov candidate function $W : \mathbb{R}^n \times \mathbb{R}_0^+ \to \mathbb{R}_0^+$,

$$W(x, \eta) = V^*(x) + \eta. \quad (40)$$

We can show that $W(x, \eta)$ is a positive definite and radially unbounded function, since

$$W(x, \eta) \geq V^*(x), \forall (x, \eta) \in \mathbb{R}^n \times \mathbb{R}_0^+. \quad (41)$$

Based on Lemma 3, the orbital derivative of (41) is,

$$\begin{aligned} \dot{W}(x, \eta) &= \dot{V}^*(x) + \dot{\eta} \\ &\leq \left(-\underline{\lambda}_Q \sigma^2 \|x\|^2 - h\right) + (-\mu\eta + h) \\ &= -\underline{\lambda}_Q \sigma^2 \|x\|^2 - \mu\eta. \end{aligned} \quad (42)$$

$$\begin{aligned}\dot{\xi} &\le \frac{\sqrt{a\theta}}{\sqrt{\eta+b\theta\|x\|^2}}\left(A_e\|x\|+B_e\|e\|\right)+\frac{\sqrt{a\theta}\,\|e\|}{2\left(\eta+b\theta\|x\|^2\right)^{\frac{3}{2}}}\left(\mu\eta-b\|x\|^2+a\|e\|^2+2b\theta A_e\|x\|^2+2b\theta B_e\|x\|\,\|e\|\right)\\ &\le A_e\sqrt{\frac{a}{b}}+\left(B_e+\frac{\mu}{2}\right)\xi+B_e\sqrt{\frac{b}{a}}\xi^2+\frac{1}{2\theta}\xi^3 \end{aligned} \quad (31)$$

According to Lemma 3 and 4, one has $\dot{W}(x,\eta)<0$. Therefore, $W(x(t),\eta(t))$ decreases and both $x(t)$ and $\eta(t)$ converge to the origin asymptotically.

2) As shown in Lemma 3, $\frac{\|e\|}{\|x\|}$ evolves from 0 to L_x during the interval $[t_k, t_{k+1})$. Equivalently, the term $\frac{\sqrt{a}\|e\|}{\sqrt{b}\|x\|}$ evolves from 0 to 1, where $a=\bar{\lambda}_R L^2$ and $b=\underline{\lambda}_Q\left(1-\sigma^2\right)$.

For the dynamic event-triggered mechanism, according to (21) one has,

$$\eta(t)+\theta\left[b\|x(t)\|^2+\bar{\lambda}_R\|u_d(t)\|^2-a\|e(t)\|^2\right]\le 0, \quad (43)$$

which can further yield,

$$\begin{aligned} a\theta\|e(t)\|^2 &\ge \eta(t)+b\theta\|x(t)\|^2+\theta\bar{\lambda}_R\|u_d(t)\|^2\\ &\ge \eta(t)+b\theta\|x(t)\|^2. \end{aligned} \quad (44)$$

Therefore, when $\theta>0$, the interval of $[t_k,t_{k+1})$, $\frac{\sqrt{a\theta}\|e(t)\|}{\sqrt{\eta(t)+b\theta\|x(t)\|^2}}$, evolves from 0 to 1. Denote now,

$$\xi(t)=\frac{\sqrt{a\theta}\,\|e(t)\|}{\sqrt{\eta(t)+b\theta\|x(t)\|^2}}, \quad (45)$$

By investigating the dynamics of $\xi(t)$ we can bound the inter-event interval as

$$\begin{aligned}\dot{\xi} &= -\frac{\sqrt{a\theta}\,\|e\|}{2\left(\eta+b\theta\|x\|^2\right)^{\frac{3}{2}}}\left(\dot{\eta}+2b\theta x^T\dot{x}\right)\\ &\quad+\frac{\sqrt{a\theta}e^T\dot{e}}{\sqrt{\eta+b\theta}\cdot\|e\|\cdot\|x\|}, \end{aligned} \quad (46)$$

with initial condition $\xi_0=0$. Considering the fact that

$$\begin{aligned} \dot{e} &= -\dot{x}\\ \|\dot{x}\| &\le A_e\|x\|+B_e\|e\|\\ \dot{\eta} &= -\mu\eta+b\|x\|^2+\bar{\lambda}_R\|u^*(\hat{x})\|^2-a\|e\|^2\\ &\ge -\mu\eta+b\|x\|^2-a\|e\|^2 \end{aligned} \quad (47)$$

gives (31) (see top on next page), in which the last inequality holds if $\mu\in(0,2A_e)$ and $\theta\in\left(0,\frac{1}{2A_e-\mu}\right]$. By using the assumption that $\theta\in\left(\frac{1}{2q},\frac{1}{q}\right]$ and $q=2A_e-\mu$, then one can obtain $\theta\in\left(0,\frac{1}{2A_e-\mu}\right]$.

Denote now $\psi(t,\psi_0)$ as the solution of,

$$\begin{aligned}\dot{\psi} &= A_e\sqrt{\frac{a}{b}}+\left(B_e+\frac{\mu}{2}\right)\psi+B_e\sqrt{\frac{b}{a}}\psi^2+\frac{1}{2\theta}\psi^3,\\ \psi_0 &= \xi_0. \end{aligned} \quad (48)$$

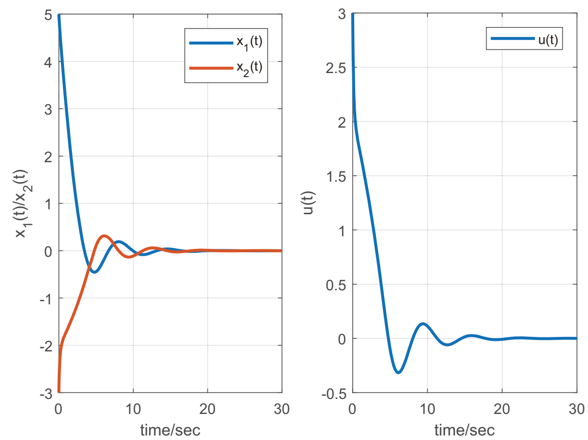

Fig. 1: Time-triggered control policy $u^*(x)$

Based on the comparison principle [23] and (31), $\xi(t)$ satisfies $\xi(t)\le\psi(t,\psi_0)$. Moreover, time needed by $\xi(t)$ to evolve from 0 to 1 is lower bounded by the positive constant $\underline{\tau}_d$ in (28). Therefore, the event-triggered condition (21) is Zeno-free;

3) This proof follows from [22]. □

5 Simulation

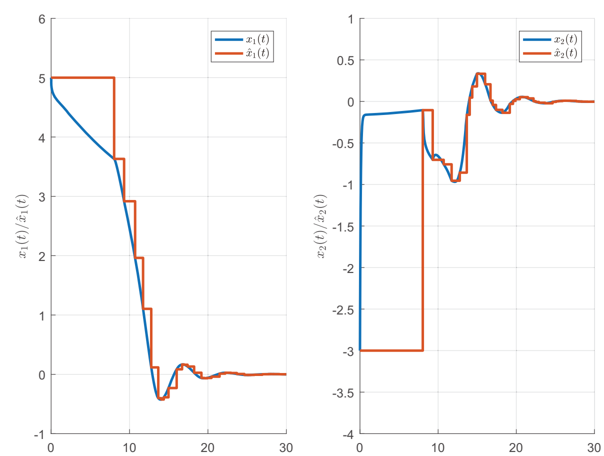

Fig. 2: The evolution of the state trajectories of the static event-triggered mechanism.

In order to validate the effectiveness of the presented Hamiltonian event-triggered control policies we will use a system adopted from [24].

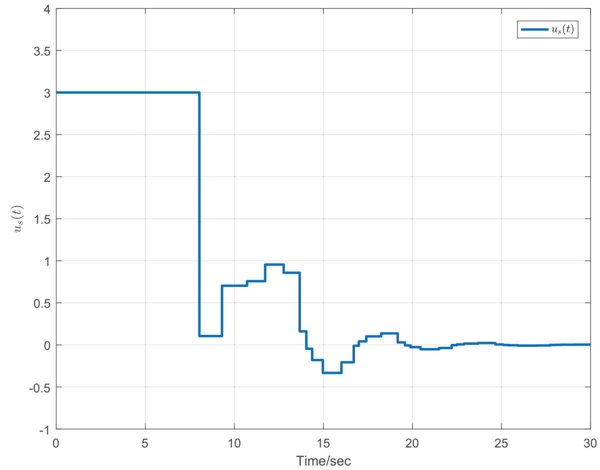

Fig. 3: The evolution of the control input of the static mechanism.

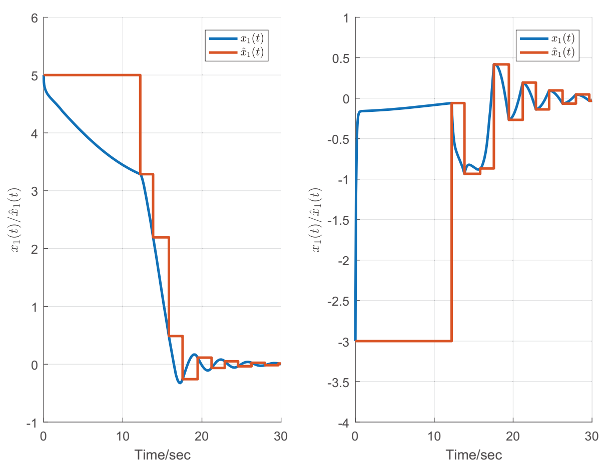

Fig. 4: The evolution of the state trajectories of the dynamic event-triggered mechanism.

Consider the following controlled Van der Pol oscillator,

$$\dot{x} = \begin{bmatrix} x_2 \\ -x_1 + 0.5\left(1 - x_2^2\right)x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u. \tag{49}$$

The user-defined matrices of (2) are picked as, $R = 1$ and $Q = I$, then the optimal value function for this system is $V^*(x) = x_1^2 + x_2^2$ and the optimal controller is $u^*(x) = -x_2$ [24].

When applying the optimal time-triggered control policy $u^*(x)$, the results are shown in Figure 1, where the continuous-time optimal control input is a continuously time-varying signal with infinite bandwidth.

The static event-triggered mechanism is designed based on Lemma 3. When the static event-triggered mechanism is applied to system (49), the evolution of the state trajectories and the control signal are presented in Figures 2 and 3, respectively. A comparison between the continuous-time optimal control and the static event-triggered control shows that the static event-triggered mechanism is able to reduce the communication bandwidth.

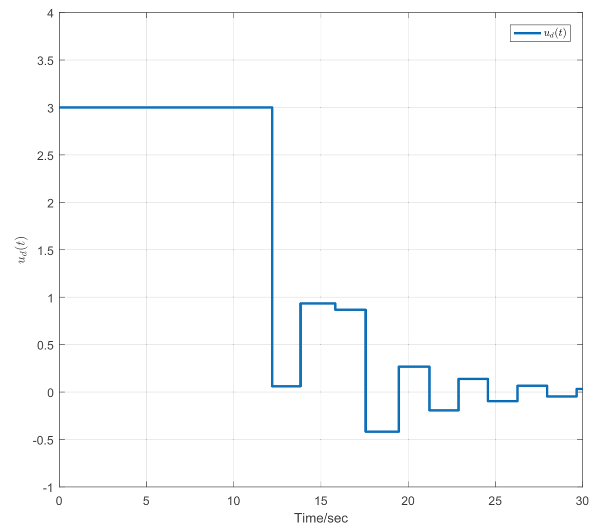

Fig. 5: The evolution of the control input of the dynamic mechanism.

The dynamic event-triggered mechanism is designed based on Theorem 1. When the dynamic event-triggered mechanism is applied to system (49), the evolution of the state trajectories and the control signal are presented in Figures 4 and 5, respectively. To compare the sampling frequency of the static and dynamic event-triggered controllers, from Figures 2 – 5, one can observe that the dynamic event-triggered mechanism is able to further reduce the communication bandwidth. Therefore, the dynamic event-triggered mechanism is superior to the static event-triggered mechanism with respect to the communication bandwidth.

6 Conclusion and Future Work

In this paper, a novel dynamic event-triggered suboptimal control policy is developed. The stability of the equilibrium point for the closed-loop system is discussed, while also guaranteeing exclusion of Zeno behavior about the proposed dynamic event-triggered mechanism. A simulation example is carried out to validate the dynamic event-triggered mechanism.

Future work will focus on combining the model-free reinforcement learning techniques with the presented dynamic event-triggered mechanism.

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