A NON-CONVEX APPROACH TO JOINT SENSOR CALIBRATION AND SPECTRUM ESTIMATION

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ABSTRACT

Blind sensor calibration for spectrum estimation is the problem of estimating the unknown sensor calibration parameters as well as the parameters-of-interest of the impinging signals simultaneously from snapshots of measurements obtained from an array of sensors. In this paper, we consider blind phase and gain calibration (BPGC) problem for direction-of-arrival estimation with multiple snapshots of measurements obtained from an uniform array of sensors, where each sensor is perturbed by an unknown gain and phase parameter. Due to the unknown sensor and signal parameters, BPGC problem is a highly nonlinear problem. Assuming that the sources are uncorrelated, the covariance matrix of the measurements in a perfectly calibrated array is a Toeplitz matrix. Leveraging this fact, we first change the nonlinear problem to a linear problem considering certain rank-one positive semidefinite matrix, and then suggest a non-convex optimization approach to find the factor of the rank-one matrix under a unit norm constraint to avoid trivial solutions. Numerical experiments demonstrate that our proposed non-convex optimization approach provides better or competitive recovery performance than existing methods in the literature, without requiring any tuning parameters.

Index Terms— sensor calibration, nonconvex, spectrum estimation, off-the-grid, direction-of-arrival estimation

1. INTRODUCTION

In many fields of science and engineering, improving the performance of measurement tools including radar, microscope, and crystallography, is of great interest. However, due to the variations such as temperature, pressure, and humidity in manufacture or even in measurement procedure, the performance or sensitivity of each sensor in a measurement tool may deviate from prescription or during measurement procedure. Therefore, it is required to calibrate the sensor parameters as well as recover the signal of interest from the collected measurements at the same time to have the accurate signal measurement performance in measurement tools, especially in an array of sensors.

Direction-of-Arrival (DoA) estimation [1, 2, 3] is a well-known problem in the field of signal processing. It aims to estimate the direction of impinging signals by using measurements collected from an array of sensors. Due to the relationship between beam patterns and excitation at the array introduced by the Fourier transform, DoA estimation can be treated as a spectrum estimation problem, where the spectrum corresponds to the direction-of-arrivals.

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Unlike the conventional DoA problem, Blind Phase and Gain Calibration (BPGC) with spectrum estimation is the problem of recovering unknown sensor calibration parameters as well as the location of frequencies of a signal of interest simultaneously [4, 5, 6, 7]. Since each sensor is assumed to suffer from an unknown gain and phase perturbation to be estimated, BPGC with spectrum estimation becomes much more challenging than the conventional DoA estimation problem. For this problem, the authors of [7] proposed an alternating method that iteratively updates sensor parameters by fixing frequencies, and then the frequencies by fixing the sensor parameters until certain predetermined tolerance is met. However, due to the alternating nature of the method, it may suffer from error propagation. In [5, 6], the authors proposed algebraic methods under the assumption that the sources are uncorrelated; hence, the covariance matrix of the measurements in a perfectly calibrated array is a Toeplitz matrix. By leveraging the Teoplitz structure, they obtained a system of linear equations and solved it to recover the sensor parameters ignoring phase wrapping. Furthermore, the authors of [4] proposed to simultaneously estimate the sensor parameters as well as the Toeplitz covariance matrix by minimizing a nonlinear least-squares loss function with regularization terms via gradient descent using Wirtinger calculus. However, the algorithm contains many regularization parameters that need to be set carefully.

In this paper, we study BPGC with off-the-grid spectrum estimation, which is a nonlinear problem over unknown sensor parameters as well as signal parameters. Unlike prior works that directly try to estimate the sensor parameters, we aim to estimate the sensor parameters through an inverse vector that perfectly calibrates the measurements when it is multiplied to the sensor measurements. This allows us to change the nonlinear sensor calibration problem to a linear problem over certain rank-one positive semidefinite matrix and a positive semidefinite Toeplitz matrix. We, then, propose a non-convex minimization approach to find the factor of the rank-one matrix under a unit norm constraint to avoid trivial solutions. This problem is solved via the trust-region method [8, 9, 10]. Numerical experiments demonstrate that our proposed non-convex optimization approach outperforms other known methods [7, 6, 4] in the various aspects including frequency separation limit, sensitivity to noise, and scaling with the number of snapshots.

In a broader context, the BPGC problem has been studied extensively in the recent literature under different assumptions. Our paper considers multiple snapshots of measurements and exploits the covariance structure. In comparison, several recent works consider only a single measurement vector by imposing additional structural assumptions on the sensor parameters, such as a sparsity or subspace prior [11, 12, 13, 14]. Also, the multiple snapshot setting without spectrum estimation is considered in [13, 15, 16].

Notations: We denote the set of complex numbers as $\mathbb{C}$. We denote a scalar, a vector, and a matrix by a, $\boldsymbol{a}$, and $\boldsymbol{A}$ respectively. We

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reserve the superscripts H, T, and $\bar{\cdot}$ for conjugate transpose, transpose, and conjugate respectively. For the diagonal operator, denoted by diag$(\cdot)$, if it is used over a matrix, it represents the diagonal elements of the matrix. If the diagonal operator is used over a vector, it represents a matrix having the vector as diagonal elements of the matrix, and 0 elsewhere.

2. PROBLEM FORMULATION

The BPGC with off-the-grid spectrum estimation problem is an inverse problem with both unknown sensor calibration and signal parameters. In this problem, the measurement obtained from a uniform array of sensors is stated as

$$\boldsymbol{y}_e[t] = \mathrm{diag}(\boldsymbol{g})\boldsymbol{V}\boldsymbol{x}[t] + \boldsymbol{e}[t], \quad t = 0, 1, \ldots, L-1, \tag{1}$$

where $\boldsymbol{y}_e[t] \in \mathbb{C}^N$ is the noisy measurement from N sensors at time t, $\boldsymbol{g} \in \mathbb{C}^N$ is the vector containing unknown sensor calibration parameters. More specifically, we denote the n-th gain parameter and the phase parameter as $|g_n|$ and $\angle g_n$ respectively. $\boldsymbol{V} = [\boldsymbol{v}_0, \ldots, \boldsymbol{v}_{r-1}] \in \mathbb{C}^{N\times r}$ is an unknown Vandermonde matrix whose columns are given as

$$\boldsymbol{v}_s = \frac{1}{\sqrt{N}}[1, e^{j2\pi f_s}, \ldots, e^{j2\pi(N-1)f_s}]^T, \quad 0 \le f_s < 1.$$

We denote the set of distinct frequencies as $\mathcal{F} := \{f_s\}_{s=0}^{r-1}$. The coefficient at time t is given as $\boldsymbol{x}[t] \in \mathbb{C}^r$, which is assumed to be zero-mean. The noise vector is given as $\boldsymbol{e}[t] \in \mathbb{C}^N$, which is also zero-mean and satisfies $\mathbb{E}[\boldsymbol{e}[t]\boldsymbol{e}[t]^H] = \sigma^2\boldsymbol{I}$, where σ is the noise level. Here, L represents the number of snapshots. Throughout the paper, we assume that $\boldsymbol{x}[t]$ and $\boldsymbol{e}[t]$ are independent, and the sources are uncorrelated; namely, $\boldsymbol{R}^x = \mathbb{E}[\boldsymbol{x}[t]\boldsymbol{x}[t]^H] = \mathrm{diag}(\{\gamma_s^2\}_{s=0}^{r-1})$. Therefore, the covariance matrix of the calibrated sensor measurements $\boldsymbol{z}[t] = \boldsymbol{V}\boldsymbol{x}[t]$, is written as

$$\boldsymbol{R}^z = \mathbb{E}[\boldsymbol{z}[t]\boldsymbol{z}[t]^H] = \boldsymbol{V}\boldsymbol{R}^x\boldsymbol{V}^H = \mathrm{Toep}(\boldsymbol{u}), \tag{2}$$

where Toep$(\boldsymbol{u})$ is the Hermitian Topelitz matrix with the first column given by the vector $\boldsymbol{u} \in \mathbb{C}^N$. Clearly, the covariance matrix $\boldsymbol{R}^z$ can be used to recover the set of frequencies and their corresponding power via standard spectrum estimation methods such as MUSIC [17] and ESPRIT [18].

From the signal model in (1), the covariance matrix of $\boldsymbol{y}_e[t]$ can be expressed as

$$\boldsymbol{R}_e^y = \mathbb{E}[\boldsymbol{y}_e[t]\boldsymbol{y}_e[t]^H] = \mathrm{diag}(\boldsymbol{g})\mathrm{Toep}(\boldsymbol{u})\mathrm{diag}(\bar{\boldsymbol{g}}) + \sigma^2\boldsymbol{I}. \tag{3}$$

In the noiseless setting, the above model reduces to

$$\boldsymbol{R}^y = \mathrm{diag}(\boldsymbol{g})\mathrm{Toep}(\boldsymbol{u})\mathrm{diag}(\bar{\boldsymbol{g}}). \tag{4}$$

Our goal in this paper is to simultaneously estimate $\boldsymbol{g}$ and $\boldsymbol{u}$ up to trivial ambiguities by using a finite number of snapshots. Clearly, it is not possible to uniquely determine $\boldsymbol{g}$ and $\boldsymbol{u}$ from (4), even assuming perfect estimation of the covariance matrix $\boldsymbol{R}^y$. This ambiguity, introduced in Definition 1, has been characterized in [4].

Definition 1 (Trivial ambiguity) *Let $\{\boldsymbol{g}, \boldsymbol{u}\}$ be a solution to* (4). *Then $\{\tilde{\boldsymbol{g}}, \tilde{\boldsymbol{u}}\}$ is equivalent to $\{\boldsymbol{g}, \boldsymbol{u}\}$ up to a trivial ambiguity if there exist $c_0 > 0, c_1, c_2$ such that*

$$\tilde{g}_n = c_0 e^{j(c_1+nc_2)}g_n, \;\; \tilde{u}_n = c_0^{-2}e^{-jnc_2}u_n, \;\; n = 0, 1, ..., N-1.$$

To be precise, this ambiguity suggests that we can only recover the set of frequencies up to circular shift, and their power up to scaling.

3. REVIEW OF EXISTING APPROACHES

We first review existing methods including the algebraic method [6] and the optimization method using Wirtinger Flow [4].

The algebraic method [6] solves a linear system of equations brought up from the ratios between entries of the covariance matrix $\boldsymbol{R}^y$. The m-th row and n-th column entry of $\boldsymbol{R}^y$ is specified as

$$\boldsymbol{R}_{m,n}^y = \frac{g_m\bar{g}_n}{N}\sum_{s=0}^{r-1}\gamma_s^2 e^{j2\pi f_s(m-n)}. \tag{5}$$

From (5), the amplitudes of the sensor parameters (gain parameters) are recovered by calculating $\sqrt{\boldsymbol{R}_{n,n}^y}$, for $n = 0, 1, ..., N-1$. Also, by computing the ratio between $\boldsymbol{R}_{l+k+1,l+1}^y$ and $\boldsymbol{R}_{l+k,l}^y$, we have the following equation for the phase information:

$$\beta_{l+k+1} - \beta_{l+1} - \beta_{l+k} + \beta_l = \angle\frac{\boldsymbol{R}_{l+k+1,l+1}^y}{\boldsymbol{R}_{l+k,l}^y} \bmod 2\pi, \tag{6}$$

where mod is the modulo operator and $\beta_l = \angle g_l$ represents the phase information of the l-th sensor parameter. In the same way, by comparing the entries of $\boldsymbol{R}^y$ for $l = 0, 1, ..., N-k-2$ and $k = 1$, and setting $\beta_0 = \beta_{N-1} = 0$, we can have N numbers of independent equations with N unknown variable β's including β_0 and β_{N-1}. By ignoring the modulo operator, this set of equations becomes linear, and the phase information of the sensor parameters are recovered by inverting the linear system. However, since N numbers of equations are obtained by setting $k = 1$, there are many disregarded equations in (6).

In [4], the authors consider the following non-convex optimization problem that simultaneously optimizes $\boldsymbol{g}$ and $\boldsymbol{u}$:

$$\min_{\boldsymbol{g},\boldsymbol{u}\in\mathbb{C}^N} \;\; \frac{1}{2}\|\boldsymbol{R}^y - \mathrm{diag}(\boldsymbol{g})\mathrm{Toep}(\boldsymbol{u})\mathrm{diag}(\bar{\boldsymbol{g}})\|_F^2 + \lambda\mathcal{G}(\boldsymbol{g},\boldsymbol{u}), \tag{7}$$

where $\mathcal{G}(\boldsymbol{g},\boldsymbol{u}) = (\max(c_0\|\boldsymbol{g}\|_2^2 - 1, 0))^2 + (\max(c_1\|\boldsymbol{u}\|_2^2 - 1, 0))^2$ is introduced to prevent $\boldsymbol{g}$ and $\boldsymbol{u}$ from going to zero with some constant c_0 and c_1, and λ is a regularization parameter. By solving (7) over $\boldsymbol{g}$ and $\boldsymbol{u}$ via gradient descent using Wirtinger derivatives, the sensor parameter $\boldsymbol{g}$ and Toep$(\boldsymbol{u})$ are recovered. This method is called Wirtinger Flow and requires a tuning parameter λ. In practice, the solution from the algebraic method is used as the initialization of the gradient descent.

4. PROPOSED NON-CONVEX APPROACH

In this paper, we propose our non-convex approach to solve the BPGC with off-the-grid spectrum estimation problem. Different from existing approaches, we first reformulate (4) by introducing the entry-wise inverse vector of the sensor parameter, denoted by $\boldsymbol{h} \in \mathbb{C}^N$, such that

$$\boldsymbol{h} \odot \boldsymbol{g} = \boldsymbol{1}, \tag{8}$$

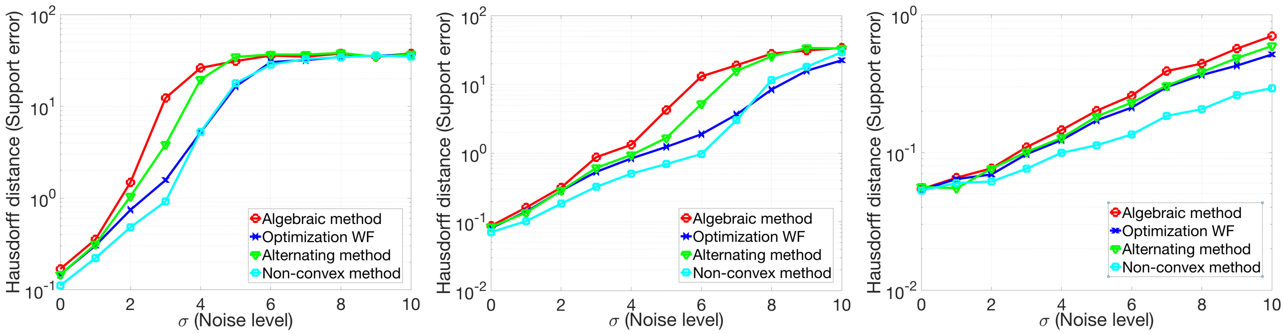
where $\odot$ denotes entry-wise product. The vector $\boldsymbol{h}$ is well-defined as long as all the entries in $\boldsymbol{g}$ are non-zero, which is a mild assumption. We can then equivalently rewrite (4) as

$$\mathrm{diag}(\boldsymbol{h}) \cdot \boldsymbol{R}^y \cdot \mathrm{diag}(\bar{\boldsymbol{h}}) = \mathrm{Toep}(\boldsymbol{u}). \tag{9}$$

Note that $\mathrm{diag}(\boldsymbol{h}) \cdot \boldsymbol{R}^y \cdot \mathrm{diag}(\bar{\boldsymbol{h}}) = \boldsymbol{R}^y \odot \boldsymbol{h}\boldsymbol{h}^H$.

By defining $\boldsymbol{H} = \boldsymbol{h}\boldsymbol{h}^H$, (9) is further rewritten as

$$\boldsymbol{R}^y \odot \boldsymbol{H} - \mathrm{Toep}(\boldsymbol{u}) = 0, \tag{10}$$



(a) $L = 10^3$ (b) $L = 10^5$ (c) $L = 10^6$

Fig. 1. Frequency support recovery error in Hausdorff distance (Unit: RL) by varying the noise level σ. The number of sensors N and the number of support r are 20 and 5 respectively. RL is set to $1/N$. (a) Number of snapshots $L = 10^3$. (b) $L = 10^5$. (c) $L = 10^6$.

which is *linear* in both $\boldsymbol{H}$ and $\boldsymbol{u}$. Furthermore, the matrix $\boldsymbol{H}$ is rank-one and positive semidefinite. Let us denote the projection onto a Toeplitz matrix and the null space of Toeplitz matrices as $\mathcal{P}_{\mathcal{T}}(\cdot)$ and $\mathcal{P}_{\mathcal{T}^{\perp}}(\cdot)$ respectively. Then, (9) suggests that

$$\mathcal{P}_{\mathcal{T}^{\perp}}(\boldsymbol{R}^y \odot \boldsymbol{H}) = 0, \tag{11}$$

which is a set of linear constraints on $\boldsymbol{H}$. Therefore, we can reformulate the problem as a rank-one matrix recovery problem. To avoid scaling ambiguity, we propose a constrained optimization problem for the recovery of $\boldsymbol{h}$ as

$$\min_{\|\boldsymbol{h}\|_2=1} F(\boldsymbol{h}) := \|\mathcal{P}_{\mathcal{T}^{\perp}}(\boldsymbol{R}^y \odot \boldsymbol{h}\boldsymbol{h}^H)\|_F^2, \tag{12}$$

where the constraint $\|\boldsymbol{h}\|_2 = 1$ is introduced to avoid scaling ambiguity. Note that this new formulation (12) is simpler than (7), since there is only one variable $\boldsymbol{h} \in \mathbb{C}^N$.

The problem (12) can be solved by the general non-convex optimization solvers such as the Riemannian trust-region algorithm [8, 9, 10], which provides convergence to a critical point from any initial point and linear local convergence to a local minimum with gradient information if the initial point is near a local minimum. By the Wirtinger derivative, the gradient of the objective function $F(\boldsymbol{h})$, denoted by $\nabla F(\boldsymbol{h}) := \frac{\partial F(\boldsymbol{h})}{\partial \boldsymbol{h}}$, is stated as

$$\nabla F(\boldsymbol{h}) = 2\text{diag}\left(\overline{\boldsymbol{R}^y \text{diag}(\bar{\boldsymbol{h}}) \mathcal{P}_{\mathcal{T}^{\perp}}\left(\text{diag}(\boldsymbol{h}) \boldsymbol{R}^y \text{diag}(\bar{\boldsymbol{h}})\right)}\right). \tag{13}$$

With this gradient information in (13), the Riemannian trust-region algorithm can solve our newly proposed non-convex optimization problem (12).

One interesting question is under what conditions we can uniquely identify $\{\boldsymbol{h}, \boldsymbol{u}\}$ up to trivial ambiguities defined in Definition 1. This is supplied in the proposition below.

Proposition 1 (Uniqueness) *Suppose $u_1 \neq 0$. Let $\{\boldsymbol{g}, \boldsymbol{u}\}$ be a solution to* (4) *and $\boldsymbol{h} \odot \boldsymbol{g} = \boldsymbol{1}$. If there is another solution $\{\tilde{\boldsymbol{h}}, \tilde{\boldsymbol{u}}\}$ to* (9) *satisfying $\tilde{\boldsymbol{h}} \neq 0$ and $\tilde{u}_1 \neq 0$, then $\{\tilde{\boldsymbol{h}}, \tilde{\boldsymbol{u}}\}$ is equivalent to $\{\boldsymbol{h}, \boldsymbol{u}\}$ up to a trivial ambiguity.*

The assumption on $u_1 \neq 0$, which is made in [4], prevents $\boldsymbol{R}^y_{l+1,l}$, $l = 0, 1, ..., N-1$, to be zero. Hence, from linear equations of (6), $\boldsymbol{g}$ satisfying (4) is uniquely determined up to trivial ambiguity. Also, as long as one of the entries in $\tilde{\boldsymbol{h}}$ is zero, Toep$(\tilde{\boldsymbol{u}})$ will have a zero row and a zero column, which implies $\tilde{u}_1 = 0$. Therefore, all entries in $\tilde{\boldsymbol{h}}$ should not vanish. The whole proof of Proposition 1 is similar to the proof of uniqueness in [4, Sec. 2.2]. Due to the space limitation, we omit the proof here.

5. NUMERICAL EXPERIMENTS

We compare our non-convex optimization approach against the algebraic method [6], the optimization method using Wirtinger Flow (WF) [4], and the alternating method [7]. We implement the trust-region algorithm on the Riemannian manifold [8] by using Manopt [19] - a Matlab toolbox for optimization on manifolds - to solve (12) with the initial value obtained from the algebraic method – similar to [4]. We expect the initial value obtained from the algebraic method to be close enough to a local minimum, which helps the trust-region algorithm to converge to the local minimum. We conduct 100 trials and measure the average Hausdorff distance of the support (frequency localization) error. The Hausdorff distance between the ground truth support $\mathcal{F}$ and the support of recovered frequencies $\hat{\mathcal{F}} = \{\hat{f}_s\}_{s=0}^{r-1}$, denoted by $HD(\mathcal{F}, \hat{\mathcal{F}})$, is calculated as

$$HD(\mathcal{F}, \hat{\mathcal{F}}) = \min_{c_2 \in [0,1)} \max\left(\max_{\hat{f} \in \hat{\mathcal{F}}} \min_{f \in \mathcal{F}} d(\hat{f} + c_2, f), \max_{f \in \mathcal{F}} \min_{\hat{f} \in \hat{\mathcal{F}}} d(\hat{f} + c_2, f)\right),$$

where $d(f_1, f_2)$ represents the closest distance between two frequencies f_1 and f_2 in a circular manner. For all simulations, we set $\gamma_s \in [1, 2]$, $s = 0, 1, ..., r-1$ and $|g_n| \in [1, 2]$, $n = 0, 1, ..., N-1$. We randomly choose the phases of $\boldsymbol{g}$ and $\boldsymbol{x}[t]$ in $[0, 2\pi)$ with holding $\mathbb{E}[\boldsymbol{x}[t]\boldsymbol{x}[t]^H] = \text{diag}(\{\gamma_s^2\}_{s=0}^{r-1})$. For the recovery of frequencies, we use the Root MUSIC algorithm in Matlab [20] for all algorithms.

We first conduct numerical experiments to examine the performance with respect to the noise level. For the noisy measurements, we add the i.i.d. Gaussian random noise following $\mathcal{CN}(0, \sigma^2\boldsymbol{I})$ to the ground truth signal $\boldsymbol{y}[t]$ generated from 5 frequencies, which are located in $[0, 1)$ and separated by the Rayleigh Limit (RL) $1/N$. In order to exploit the sensitivity to noise, we vary the noise level σ from 0 to 10 and measure the Hausdorff distance of support error with a fixed number of snapshots. Fig. 1(a), (b), and (c) show the average Hausdorff distance of support error obtained from 100 trials in the unit of RL for 10^3, 10^5 and 10^6 numbers of snapshots respectively. As shown in Fig. 1(c), when $L = 10^6$, our non-convex approach outperforms other known methods.

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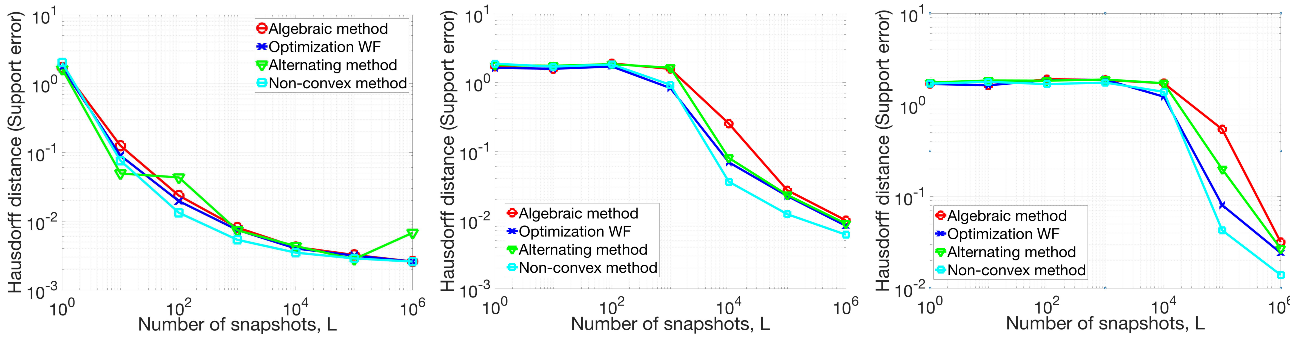
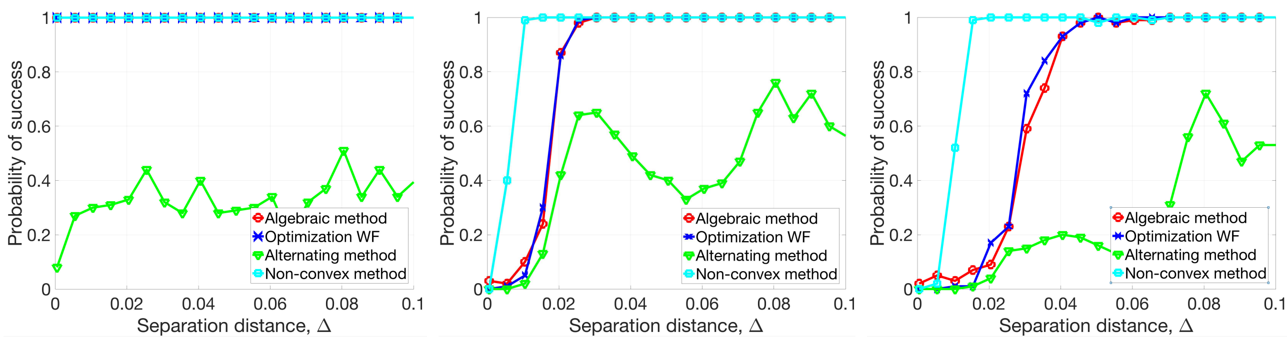


(a) $\sigma = 0$ (b) $\sigma = 5$ (c) $\sigma = 10$

Fig. 2. Frequency support recovery error in Hausdorff distance (Unit: RL) by varying the number of snapshots L. The number of sensors N and the number of support r are 20 and 5 respectively. RL is set to $1/N$. (a) Noise level $\sigma = 0$. (b) $\sigma = 5$. (c) $\sigma = 10$.



(a) $\sigma = 0$ (b) $\sigma = 5$ (c) $\sigma = 10$

Fig. 3. Probability of frequency support recovery by varying the separation distance between two frequencies. The number of sensors N, support r, and snapshots L are 20, 2, and 10^6 respectively. (a) Noise level $\sigma = 0$. (b) $\sigma = 5$. (c) $\sigma = 10$.

We next examine the support recovery error with respect to the number of snapshots. In this numerical experiment, we choose 5 frequencies separated by $1/N$, where the number of sensors $N = 20$. We compare our non-convex method to other methods in the noise-free setting with $\sigma = 0$ as well as the noisy setting with $\sigma = 5$ and $\sigma = 10$. Fig. 2(a), (b) and (c) show the average Hausdorff distance of support error in the unit of RL by varying the number of snapshots L from 1 to 10^6 for $\sigma = 0$, $\sigma = 5$, and $\sigma = 10$ respectively. As the number of snapshots increases in $\sigma = 0$ the Hausdorff distance of all methods decreases. However, when $\sigma = 5$ and $\sigma = 10$, it is shown that over 10^3 and 10^4 numbers of snapshots are required for the recovery respectively. Also, our non-convex approach shows better performance than other algorithms in these regions.

Finally, we examine the resolution limit of the proposed algorithm. In this experiment, we consider two close frequencies with randomly generated amplitudes and phases. We randomly choose the location of the first frequency in $[0, 1)$, and then, obtain the second frequency by adding the separation distance Δ in a circular manner; namely, $f_2 = (f_1 + \Delta)$ mod 1, where f_1 and f_2 are the first and the second frequencies respectively, and mod represents the modulo operator. We conduct 100 trials at each separation condition with a fixed number of snapshots and a fixed noise level, and measure the probability of success. We consider a recovery successful if $HD(\mathcal{F}, \hat{\mathcal{F}}) < \Delta/2$. Fig. 3(a), (b). and (c) show the probability of support recovery by varying the separation condition between two frequencies in the noise level $\sigma = 0$, $\sigma = 5$, and $\sigma = 10$ respectively. The number of snapshots L is set to 10^6. The simulation demonstrate that when noise exists, our non-convex method outperforms other known methods in the resolution limit.

6. CONCLUSIONS

In this paper, we studied the problem of blind phase and gain calibration with direction-of-arrival estimation. Under the assumption of uncorrelated sources, the covariance matrix of the measurements obtained from a uniform array of sensors is a Toeplitz matrix. By introducing an inverse calibration vector, we modified the blind phase and gain calibration problem into a linear problem in a rank-one positive semidefinite matrix. And then, we proposed a non-convex minimization approach to find the inverse calibration parameter by considering the null space of the Toeplitz matrix under the unit norm constraint to avoid trivial solutions. Numerical experiments demonstrate that our proposed non-convex approach outperforms existing methods including the algebraic method [6], the optimization WF [4], and the alternating method [7], with no tuning parameters.

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