

Robust decentralized frequency control: A leaky integrator approach

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Abstract—We investigate the robustness of the so-called *leaky integral* frequency controller for the power network. In particular, using a strict Lyapunov function, we show the closed-loop system is robust in the input-to-state stability sense to measurement noise in the controller. Moreover, an interesting and explicit trade-off between controller performance and robustness is discussed and illustrated using a benchmark study of the 39-bus New England reference network.

I. INTRODUCTION

The core operation principle of an AC power system is to balance supply and demand in nearly real time. Any instantaneous imbalance results in a deviation of the global system frequency from its nominal value. Thus, a central control task is to regulate the system frequency in an economically efficient way and despite fluctuating loads, variable generation, and possibly faults. Traditionally, this is achieved using a hierarchical scheme consisting of the generators' rotational inertia providing an instantaneous frequency response and primary (droop control), secondary automatic generation (AGC), and tertiary (economic dispatch) control layers operating at different time scales [1], [2].

This conventional operational strategy is currently challenged by increasing volatility on all time scales (due to variable renewable generation and high penetration of low-inertia sources) as well as the ever-growing complexity of future power systems integrating distributed generation, demand response, microgrids, and HVDC systems, among others. Motivated by these paradigm shifts and recent advances in distributed control and optimization methods, an active research has emerged aiming at developing more flexible distributed schemes to replace, or complement, the traditional frequency control layers.

In this article, we focus on the secondary and tertiary control layer. We refer to [3, Section IV.C] for a survey

covering recent approaches. Although existing schemes, such as semi-centralized broadcast-based schemes [4]–[6], consensus-based averaging [7]–[12], primal-dual methods [13]–[16] and fully decentralized integral control [5], [7], [17], ensure nominal closed-loop stability at a correct steady-state frequency, in practice they suffer from poor robustness to measurement bias and clock drifts [4], [5], [11], [18].

A conventional remedy to overcome performance and robustness issues of integral controllers is to implement them as lag elements with finite DC gain [19]. Indeed, such approaches have been investigated for fully decentralized frequency control: [17] provides insights on the closed-loop steady states and transient dynamics mainly based on numerical analysis and asymptotic arguments; [20] provides a numerically computable certificate for ultimate boundedness; and [21] analyses a similar controller augmented with further lead-lag filter filters and provides a numerical small-signal analysis.

Here we follow the latter approach and propose a fully decentralized *leaky integral controller* derived from a standard lag element. We consider this controller in feedback with a nonlinear multi-machine power system model and provide a formal analysis of the closed-loop system concerning (i) steady-state frequency regulation, power sharing, and dispatch properties, and (ii) the transient dynamics in terms of nominal exponential stability and input-to-state stability with respect to disturbances. A trade-off between dynamic and steady-state performance can be tuned by the DC gain of our controller. We illustrate our analytical findings with a detailed simulation study. We find that our fully decentralized controller is able to strike an acceptable trade-off between dynamic and steady-state performance and can compete with other communication-based distributed controllers.

In the interest of space, all proofs have been omitted. They can be found in [22].

II. POWER SYSTEM FREQUENCY CONTROL

A. System Description

Consider a network-reduced power system with n generators modeled by interconnected swing equations [1]

$$\dot{\theta} = \omega \quad (1a)$$

$$M\dot{\omega} = -D\omega + P^* - \nabla U(\theta) + u, \quad (1b)$$

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where $\theta \in \mathbb{T}^n$, the n -dimensional torus, and $\omega \in \mathbb{R}^n$ collect the generator rotor angles and frequencies relative to the utility frequency, usually given by $2\pi 50$ Hz or $2\pi 60$ Hz. The diagonal matrices $M, D \in \mathbb{R}^{n \times n}$ collect the generator inertia and damping coefficients $M_i, D_i > 0$, respectively. The generator primary (droop) control is integrated in the damping coefficient D_i , $P^* \in \mathbb{R}^n$ is vector of nominal power injections, and $u \in \mathbb{R}^n$ is a control input to be designed later on. The offset of the magnetic energy stored in the purely inductive (lossless) power transmission lines, with respect to a constant reference, is

$$U(\theta) = -\frac{1}{2} \sum_{i,j=1}^n B_{ij} V_i V_j \cos(\theta_i - \theta_j),$$

where $B_{ij} \geq 0$ is the inductance of the line connecting generators i and j with terminal voltage magnitudes $V_i, V_j > 0$, which we assume to be constant. Observe that the vector of power injections with components

$$(\nabla U(\theta))_i = \sum_{j=1}^n B_{ij} V_i V_j \sin(\theta_i - \theta_j) \quad (2)$$

satisfies a zero net flow balance: $\mathbf{1}_n^T \nabla U(\theta) = 0$, where $\mathbf{1}_n \in \mathbb{R}^n$ is the vector of unit entries.

In the following, we will also write these quantities in vector notation

$$U(\theta) = -\mathbf{1}^T \Gamma \cos(\mathcal{B}^T \theta), \quad \nabla U(\theta) = \mathcal{B} \Gamma \sin(\mathcal{B}^T \theta).$$

In the identities above $\Gamma \in \mathbb{R}^{m \times m}$ is the diagonal matrix with its diagonal entries being all the nonzero $V_i V_j B_{ij}$'s corresponding to the inductance and voltage magnitudes of the transmission lines, and $\mathcal{B} \in \mathbb{R}^{n \times m}$ is the incidence matrix of the power network.

We remark that all of our subsequent developments can also be extended to more detailed structure-preserving power system models with first-order dynamics, algebraic load flow equations, and variable voltages by using the techniques in [7], [9]. In the interest of clarity, we will present our ideas for the more concise model (1).

B. Secondary Frequency Control

In the following, we will refer to a solution $(\theta(t), \omega(t))$ of (1) as a *synchronous solution* if it is of the form $\dot{\theta}(t) = \omega(t) = \omega_{\text{sync}} \mathbf{1}_n$, where ω_{sync} is the synchronous frequency.

Lemma 1 (Synchronization frequency) *If there is a synchronous solution to the power system model (1), then the synchronous frequency is given by*

$$\omega_{\text{sync}} = \frac{\sum_{i=1}^n P_i^* + \sum_{i=1}^n u_i^*}{\sum_{i=1}^n D_i}, \quad (3)$$

where u_i^* denotes the steady-state control action.

Equation (3) implies that $\omega_{\text{sync}} = 0$ if and only if all power injections are balanced across the entire network, i.e., $\sum_{i=1}^n P_i^* + u_i^* = 0$. In this case, a synchronous solution coincides with an equilibrium $(\theta^*, \omega^*, u^*) \in$

$\mathbb{T}^n \times \{0_n\} \times \mathbb{R}^n$ of (1). Our first objective is frequency regulation, also referred to as secondary control.

Problem 1 (Frequency regulation) *Given an unknown constant vector P^* , design a control strategy $u = u(\omega)$ to stabilize the power system model (1) to an equilibrium $(\theta^*, \omega^*, u^*) \in \mathbb{T}^n \times \{0_n\} \times \mathbb{R}^n$.*

Here, we denote by 0_n the zero vector in \mathbb{R}^n . Observe that there are manifold choices of u^* to achieve this task. Thus, a further objective is the most economic allocation of steady-state control inputs u^* given by a solution to the following *economic dispatch problem* [23]:

$$\text{minimize}_{u \in \mathbb{R}^n} \sum_{i=1}^n a_i u_i^2 \quad (4a)$$

$$\text{subject to} \quad \sum_{i=1}^n P_i^* + \sum_{i=1}^n u_i = 0. \quad (4b)$$

The term $a_i u_i^2$ with $a_i > 0$ is the quadratic generation cost for generator i . Observe that the unique (due to strict convexity of the objective and linearity of the constraint) optimal solution u^* of the economic dispatch must satisfy the identical marginal cost requirement [8], [10]:

$$a_i u_i^* = a_j u_j^* \quad \forall i, j \in \{1, \dots, n\}. \quad (5)$$

We remark that a special case of the identical marginal cost criterion (5) is fair proportional power sharing [24] when the coefficients a_i are chosen inversely proportional to a reference power $\bar{P}_i > 0$ for every generator i :

$$u_i^* / \bar{P}_i = u_j^* / \bar{P}_j \quad \forall i, j \in \{1, \dots, n\}. \quad (6)$$

Problem 2 (Optimal frequency regulation) *Given an unknown constant vector P^* , design a control strategy $u = u(\omega)$ to stabilize the power system model (1) to an equilibrium $(\theta^*, \omega^*, u^*) \in \mathbb{T}^n \times \{0_n\} \times \mathbb{R}^n$ where u^* minimizes the economic dispatch problem (4).*

Aside from frequency regulation with steady-state optimality, we are interested in fully decentralized controllers with $u_i = u_i(\omega_i)$ making use only of local frequency measurements.

III. DECENTRALIZED LAG AND LEAKY INTEGRAL CONTROL

The secondary frequency regulation Problems 1 and 2 have seen many centralized and distributed control approaches. Since P^* is generally unknown, all of these approaches explicitly or implicitly rely on integral control of the frequency error. In the following we focus on a fully decentralized integral control approach.

In standard frequency-domain control design, a stable and finite DC-gain implementation of a proportional-integral (PI) controller is given by a *lag element* [19]

$$\alpha \frac{Ts + 1}{\alpha Ts + 1} = \underbrace{1}_{\text{proportional control}} + \underbrace{\frac{\alpha - 1}{\alpha Ts + 1}}_{\text{leaky integral control}},$$

where $T > 0$ and $\alpha \gg 1$ is the finite DC gain of the lag element. The lag element consists of a proportional channel as well as a first-order lag often referred to as a

leaky integrator. In our context, a state-space realization of a decentralized lag element for frequency control is

$$u = -\omega - (\alpha - 1)p \quad (7a)$$

$$\alpha T \dot{p} = \omega - p, \quad (7b)$$

where T is again a diagonal matrix of positive time constants $T_i > 0$, and $\alpha \gg 1$ is a scalar time constant. We disregard the proportional channel (that would add damping akin to droop control) and focus solely on the leaky integrator to remedy some of the shortcomings of decentralized integral control (9).

Consider a *leaky integral controller* of the form

$$u = -p \quad (8a)$$

$$T \dot{p} = \omega - K p, \quad (8b)$$

where $K, T \in \mathbb{R}^{n \times n}$ are diagonal matrices of positive time constants $K_i, T_i > 0$.

Note that for $K = 0$, the controller dynamics (8b) reduces to a simple decentralized integral controller

$$T \dot{p} = \omega. \quad (9)$$

In the next section, we will analyze the leaky integrator (8) in closed loop with the power system (1) and highlight its merits and trade-offs.

IV. PROPERTIES OF THE LEAKY INTEGRAL CONTROLLER

The power system model (1) controlled by the leaky integrator (8) gives rise to the closed-loop system

$$\dot{\theta} = \omega \quad (10a)$$

$$M \dot{\omega} = -D\omega + P^* - \nabla U(\theta) - p \quad (10b)$$

$$T \dot{p} = \omega - K p. \quad (10c)$$

We will analyze the closed-loop system (10) under the following standing assumption:

Assumption 1 (Existence of synchronous solution) *Assume that the closed-loop (10) admits a synchronous solution $(\theta^*, \omega^*, p^*)$ of the form*

$$\dot{\theta}^* = \omega^* \quad (11a)$$

$$0_n = -D\omega^* + P^* - \nabla U(\theta^*) - p^* \quad (11b)$$

$$0_n = \omega^* - K p^*. \quad (11c)$$

where $\omega^* = \omega_{\text{sync}} \mathbf{1}_n$ for some $\omega_{\text{sync}} \in \mathbb{R}$. \square

By eliminating the variable p^* from (11), we arrive at

$$P^* - (D + K^{-1}) \omega_{\text{sync}} \mathbf{1}_n = \nabla U(\theta^*). \quad (12)$$

Equations (12) take the form of lossless active power flow equations with injections $P^* - (D + K^{-1}) \omega_{\text{sync}} \mathbf{1}_n$. Hence, Assumption 1 is equivalent to saying that the power flow equations (12) admit a solution. This is always satisfied provided that $\|P^*\|$ is sufficiently small that the power transmission lines' inductance and voltages permit transporting the required power across the network. In what follows we show the properties of the leaky integrator and its trade-offs as displayed by the gain K .

A. Steady-State Analysis

Observe that at steady state, the control input u^* of the leaky integrator (8) takes the value

$$u^* = -p^* = -K^{-1} \omega^* = -K^{-1} \omega_{\text{sync}} \mathbf{1}_n, \quad (13)$$

that is, it has a finite DC gain analogous to a primary control (or damping) with coefficient K^{-1} . Hence, analogous to Lemma 1, we arrive at the following result.

Lemma 2 (Steady-state frequency) *Consider the closed-loop system (10) and its equilibria (11). The explicit synchronization frequency is given by*

$$\omega_{\text{sync}} = \frac{\sum_{i=1}^n P_i^*}{\sum_{i=1}^n D_i + K_i^{-1}} \quad (14)$$

Our first (unsurprising) observation is that the leaky integral controller (8) does generally not regulate the synchronous frequency ω_{sync} to zero unless $\sum_i P_i^* = 0$. However, it can achieve *approximate frequency* regulation within a prespecified tolerance band.

Corollary 1 (Banded frequency regulation) *Consider the closed-loop system (10). The synchronous frequency ω_{sync} takes value in a band around zero that can be made arbitrarily small by choosing the gains $K_i > 0$ sufficiently small. In particular, for any $\varepsilon > 0$, if*

$$\sum_{i=1}^n K_i^{-1} \geq \frac{|\sum_{i=1}^n P_i^*|}{\varepsilon} - \sum_{i=1}^n D_i,$$

then $|\omega_{\text{sync}}| \leq \varepsilon$.

Though regulating the frequencies to a band around zero is sufficient in practice, the control input (8) may become ineffective due to a small bandwidth (gain K). We will repeatedly encounter this trade-off between choosing a small gain (for desirable steady-state properties) and large gain (for transient performance). A formal performance analysis is deferred to a future work.

Since the steady-state injections of the leaky integral controller (8) are given by (13), we also find that the leaky integrator can achieve arbitrary power sharing ratios by choosing the gains appropriately:

Corollary 2 (Steady-state power sharing) *Consider the closed-loop system (10). The steady-state injections u^* of the leaky integral controller achieve fair proportional power sharing as follows:*

$$K_i u_i^* = K_j u_j^* \quad \forall i, j \in \{1, \dots, n\}. \quad (15)$$

Hence, arbitrary power sharing as in (6) can be prescribed by choosing the control gains K_i inversely proportional to the reference powers $K_i \sim 1/\bar{P}_i$. Similarly, we have the following result on steady-state optimality:

Corollary 3 (Steady-state optimality) *Consider the closed-loop system (10). The steady-state injections u^* of the leaky integral controller minimize the economic*

dispatch problem

$$\text{minimize}_{u \in \mathbb{R}^n} \sum_{i=1}^n K_i u_i^2 \quad (16a)$$

$$\text{subject to } \sum_{i=1}^n P_i^* + \sum_{i=1}^n (1 + D_i K_i) u_i = 0. \quad (16b)$$

The steady-state injections of the leaky integrator are optimal for the modified dispatch problem (16) with appropriately chosen cost functions and whose constraint (16b) matches the constraint of (4b) arbitrarily well for K chosen sufficiently small.

B. Stability & Robustness Analysis

For ease of analysis, in this subsection we introduce a change of coordinates for the voltage phase angle θ . Let $\delta = \theta - \frac{1}{n} \mathbf{1}_n \mathbf{1}_n^\top \theta = \Pi \theta$ be the center-of-inertia coordinates (see e.g., [25], [9]), where $\Pi = I - \frac{1}{n} \mathbf{1}_n \mathbf{1}_n^\top$. In these coordinates, the open-loop system (1) becomes

$$\dot{\delta} = \Pi \omega \quad (17a)$$

$$M \dot{\omega} = -D \omega + P^* - \nabla U(\delta) + u, \quad (17b)$$

where by an abuse of notation we use the same symbol U for the potential function expressed in terms of δ ,

$$U(\delta) = -\mathbf{1}^\top \Gamma \cos(\mathcal{B}^\top \delta), \quad \nabla U(\delta) = \mathcal{B} \Gamma \sin(\mathcal{B}^\top \delta).$$

Note that $\mathcal{B}^\top \Pi = \mathcal{B}^\top$.

The synchronous solution $(\theta^*, \omega^*, p^*)$ defined in (11) is mapped into the point $(\delta^*, \omega^*, p^*)$, with $\delta^* = \Pi \theta^*$, satisfying

$$\dot{\delta}^* = \mathbb{0}_n \quad (18a)$$

$$\mathbb{0}_n = -D \omega^* + P^* - \nabla U(\delta^*) - p^* \quad (18b)$$

$$\mathbb{0}_n = \omega^* - K p^*. \quad (18c)$$

The existence of $(\delta^*, \omega^*, p^*)$ is guaranteed by Assumption 1. Additionally, we make the following standard assumption, derived from [8], constraining steady-state angle differences.

Assumption 2 (Security constraint) *The synchronous solution (18) is such that $\mathcal{B}^\top \delta^* \in \Theta := (-\frac{\pi}{2} + \rho, \frac{\pi}{2} - \rho)^m$, where $\delta^* = \Pi \theta^*$, for a constant scalar $\rho \in (0, \frac{\pi}{2})$.*

This Assumption is necessary for the theory to follow, and unremarkable from a practical point of view, as normally loaded systems usually encounter angle differences smaller than 15 degrees.

Theorem 1 (Exponential stability under leaky integral control) *Consider the closed-loop system (17), (8). Let Assumption 2 hold. The equilibrium $(\delta^*, \omega^*, p^*)$ is locally exponentially stable. In particular, given the incremental state vector*

$$x = x(\delta, \omega, p) = \text{col}(\delta - \delta^*, \omega - \omega^*, p - p^*), \quad (19)$$

the solutions $x(t)$ to (17), (8) that start sufficiently close to the origin satisfy for all $t \geq 0$,

$$\|x(t)\|^2 \leq \lambda e^{-\alpha t} \|x(0)\|^2, \quad (20)$$

where λ and α are positive constants.

Recall that one key disadvantage of fully decentralized integral control is its lack of robustness to biased measurement errors. We now show that leaky integral control (8) is indeed robust to such measurement errors. Instead of (8), consider

$$u = -p \quad (21a)$$

$$T \dot{p} = \omega - K p + \eta, \quad (21b)$$

where the measurement noise $\eta = \eta(t) \in \mathbb{R}^n$ is assumed to be an ∞ -norm bounded disturbance. In this case, the bias-induced instability does not occur. In particular, for a constant vector η , the equilibrium equation (11c) becomes $\mathbb{0}_n = \omega^* - K p^* + \eta$, so that the closed loop (1), (21) will indeed admit synchronous equilibria. The governing equations (12) determining the synchronous frequency ω_{sync} change to

$$(D + K^{-1}) \omega_{\text{sync}} \mathbf{1} = P^* - \nabla U(\theta^*) - K^{-1} \eta.$$

Observe that the bias term η plays now a similar role as the constant injections P^* , and their effect can be made arbitrarily small by increasing the gain K .

It is possible to derive also a robustness performance criterion. First, we define the specific robust stability criterion that we will use, adapted from [26].

Definition 1 (Input-to-state-stability with restrictions) *A system $\dot{x} = f(x, \eta)$ is said to be input-to-state stable (ISS) with restriction \mathcal{X} on $x(0) = x_0$ and restriction $\bar{\eta} \in \mathbb{R}_{>0}$ on $\eta(\cdot)$ if there exist a class \mathcal{KL} -function β , a class \mathcal{K}_∞ -function γ such that*

$$\|x(t)\| \leq \beta(\|x_0\|, t) + \gamma(\|\eta(\cdot)\|_\infty)$$

for all $t \in \mathbb{R}_{\geq 0}$, $x_0 \in \mathcal{X}$ and inputs $\eta(\cdot) \in L_\infty^{\mathbb{R}^n}$ satisfying

$$\|\eta(\cdot)\|_\infty := \text{ess sup}_{t \in \mathbb{R}_{\geq 0}} \|\eta(t)\| \leq \bar{\eta}.$$

Theorem 2 (ISS under biased leaky integral control) *Consider system (17) in closed-loop with the biased leaky integral controller (21). Let Assumptions 1 and 2 hold. Given a matrix $K > 0$, there exist a positive constant $\bar{\eta}$ and a set \mathcal{X} such that the closed-loop system is ISS from the noise η to the state $x = \text{col}(\delta - \delta^*, \omega - \omega^*, p - p^*)$ with restrictions \mathcal{X} on $x(0)$ and $\bar{\eta}$ on $\eta(\cdot)$. In particular, the solutions $x(t) = \text{col}(\delta(t) - \delta^*, \omega(t) - \omega^*, p(t) - p^*)$, with $(\delta(t), \omega(t), p(t))$ a solution to (17), (21) for which $x(0) \in \mathcal{X}$ and $\|\eta(\cdot)\|_\infty \leq \bar{\eta}$ satisfy for all $t \in \mathbb{R}_{\geq 0}$,*

$$\|x(t)\|^2 \leq \lambda e^{-\hat{\alpha} t} \|x(0)\|^2 + \gamma \|\eta(\cdot)\|_\infty^2, \quad (22)$$

where $\hat{\alpha}$, λ and γ are positive constants. Furthermore, as a function of the controller gain K , γ is monotonically decreasing.

Discussion. A few remarks are in order.

– The above theorem tells us that a larger gain K reduces the effect of the noise η on the state x . This confirms the beneficial effect of a larger gain K on the robustness to noise of the leaky integral controller, which was observed at steady state before. Theorem 2 extends this observation to the dynamic response of the closed-

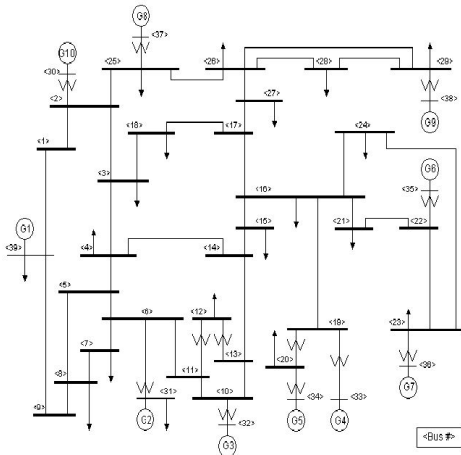


Fig. 1. The 39-bus New England system used in simulations.

loop system. Note, however, that the choice of K affects the safety region as well as the equilibrium of the system. We refer to [22] for a more thorough discussion.

- The \mathcal{KL} -function from the ISS property in (22) turns out to be an exponential function, so the stability property is in fact exponential ISS with restrictions.
- The need to include restrictions \mathcal{X} on the initial conditions and $\bar{\eta}$ on the noise magnitude is due to the requirement of maintaining the state response within the safety region Θ .

V. CASE STUDY

Simulations are performed with the 39-bus New England system in Figure 1. The power network is modeled as (1)-(2) with M_i , V_i , and B_{ij} taken from [27]. The constants M_i are zero for the 29 load buses. For every generator bus i , D_i is chosen as 20 per unit (pu) so that a 0.05pu (3Hz) change in frequency will cause a 1pu (1000MW) change in the generator output power. For every load bus i , D_i is chosen as 0.1pu.

At time $t = 5s$, a 300MW step increase in real-power load occurs at each of buses 15, 23, 39. We compare the following five controllers (located at the 10 generators) to stabilize the system after such an event:

- 1) Distributed-averaging based integral control (DAI):

$$u = -p \quad (23a)$$

$$T\dot{p} = A^{-1}\omega - LAp. \quad (23b)$$

Here A is a diagonal matrix with diagonal entries a_i corresponding to the cost coefficients in (4). The matrix L is the Laplacian matrix of a communication graph, which we chose as a ring graph connecting all the generators, with weight 0.1 on each link. Even though this controller is based on a reliable and fast communication environment, we include it here for comparison purposes.

- 2) Decentralized integral control (9).
- 3) Leaky integral control (8). The leaky proportional gain K_i is chosen as 1×10^{-4} for generators G3, G5, G6, G9, G10 and 5×10^{-5} for other generators.

- 4) Leaky integral control (8), with K_i 's increased to 50 times of those in case 3.

Note that the values of K_i in all leaky integral control implementation are proportional to a_i 's in DAI designed to solve the same dispatch problem (4). All controllers have the integral constant T_i set to 0.05 for every generator i .

Figure 2 (dashed plots) shows the frequency at generator 1 (similar trends hold for frequencies at other generators). First, note that all controllers converge to stable steady values. Second, it is observed from Figure 2 that both the decentralized integral control and the DAI control can perfectly restore the frequencies to the nominal value, whereas the leaky integral control leads to a steady-state frequency error that increases when the leaky proportional gains K_i increase, as predicted in Corollary 1. We conclude that, depending on the system specifications, the fully decentralized leaky integral controller can achieve a performance similar to the communication-based DAI controller – though at the cost of either a slower convergence rate or a steady-state offset that can be adjusted by tuning the gain K .

Next, an input noise $\eta_i(t)$ is added to the right-hand-sides of (23b), (9), and (8b) for DAI control, decentralized integral control, and leaky integral control of generator i , respectively. Every second, $\eta_i(t)$ is generated subject to a uniform distribution on $[0, \bar{\eta}_i]$. It can be observed from Figures 2(b)–2(d) and Figures 3(b)–3(d) that the leaky integral control is more robust to input noise as K increases; for small values of the K_i , the noise used in the simulations in fact exceeds the tolerance of the leaky integral controller. By comparing to Figures 2(a) and 3(a), we see that DAI is more robust to input noise than the leaky integral control in terms of frequency (unless K is very high) and generator power outputs.

An important observation from the results above is that the selection of controller gain K can significantly impact the performance of the leaky integral control. A good trade-off between frequency restoration, fair power sharing, convergence rate, and robustness to input noise can be achieved under the case of $50 \times K$.

VI. CONCLUSIONS

We have discussed the steady-state behavior and robustness to disturbances of the leaky integral controller, when applied to the AC power grid. In particular, we have shown that the closed-loop system is input-to-state stable with restrictions, with respect to additive measurement noise. Moreover, given that the noise-to-state ISS gain is a decreasing function of the controller's integral gain K , a trade-off between frequency regulation on one side and robustness and controller performance on the other side, was discussed and illustrated using an extensive simulation study of an IEEE benchmark power network.

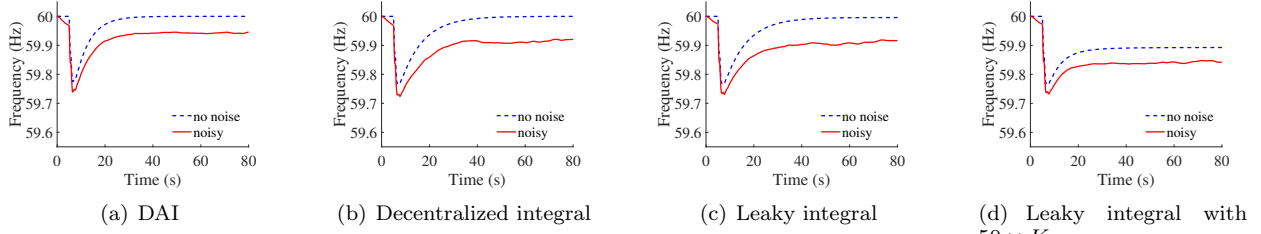


Fig. 2. Frequency at generator 1, under different control methods and different input noise levels.

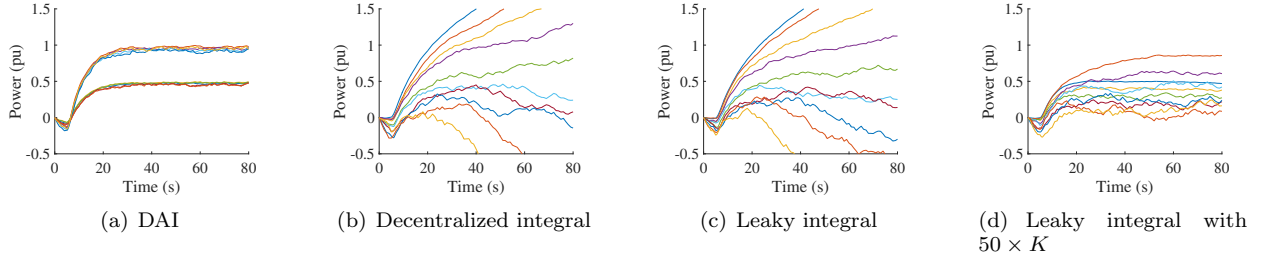


Fig. 3. Changes in real-power outputs of all the generators, with an input noise level $\bar{\eta} = 0.01$.

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