Transportation Research Part B 115 (2018) 1–16

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Dynamic bus substitution strategy for bunching intervention

Antoine Petit, Yanfeng Ouyang*, Chao Lei

Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA

ARTICLE INFO

Article history:
Received 16 February 2018
Revised 13 April 2018
Accepted 3 June 2018
Available online 4 July 2018

Keywords:
Bus bunching
Transit operations
Approximate dynamic programming
Autonomous vehicles

ABSTRACT

Bus headways are typically susceptible to external disturbances (e.g., due to traffic congestion, clustered passenger arrivals, and special passenger needs), which create gaps in the system that grow eventually into bunching. Although many control strategies, such as static and dynamic holding strategies, have been implemented to mitigate the effects of unreliable bus schedules, most of them would impose longer dwell times on the passengers. In this paper, we investigate the potential of an alternative bus substitution strategy that is currently implemented by some transit agencies in an ad-hoc manner. In this strategy, the agency deploys a fleet of standby buses to take over service from any early or late buses so as to contain deviations from schedule, and the intention is to impose minimum penalties on the onboard passengers. We develop a discrete-time infinite-horizon approximate dynamic programming approach to find the optimal policy to minimize the overall agency and passenger costs. It is shown through numerical examples that schedule deviations can be controlled by regularly inserting standby buses as substitutions. In some implementation scenarios, the proposed strategy holds the potential to achieve comparable performance with some of the most advanced strategies, and to outperform the conventional slack-based schedule control scheme. In light of the emerging opportunities associated with autonomous driving, the performance of the proposed strategy can become even stronger due to the reduction in costs for keeping the fleet of standby buses.

1. Introduction

Reliability of service is a key performance indicator for transit agencies. To ensure a satisfying level of service for public transportation passengers, agencies should maintain regularity and punctuality of service. However, since bus travel time is usually subject to randomness, e.g., buses have to travel within mixed traffic which is subject to congestion and the dwell time at a stop depends on the random number of boarding/alighting passengers, bus headways are likely to be irregular and unreliable. For example, when a bus falls behind schedule, it would serve an increased number of passengers, which in turn delays it even further. In symmetry, the following bus may pick up fewer passengers and speed up. Inevitably, buses end up bunching into pairs instead of being evenly spaced. This phenomenon is well-known in the industry as an illustration of the instability of uncontrolled transit systems (Newell and Potts, 1964). Schedule unreliability from bus bunching first affects passengers. As more passengers are served by late buses than by early buses, the expected waiting time for public transit passengers increases as the variance of the headways increases (Osuna and Newell, 1972; Daganzo, 2008). In addition, passengers on the late buses have to travel in more crowded vehicles, which leads to additional discomfort and inconvenience.

* Corresponding author.
E-mail address: yfouyang@illinois.edu (Y. Ouyang).

https://doi.org/10.1016/j.trb.2018.06.001

To mitigate bunching, various control strategies have been proposed. The conventional strategy used by most agencies consists of adding slacks into the published schedule, in order to hold early transit vehicles at designated stops (Osuna and Newell, 1972; Newell, 1974). However, those holding strategies trade system stability for extra passenger dwell/travel time. While this may ensure consistent headways, passengers still experience extra delays at those stops, which potentially causes confusion and dissatisfaction. More recently, holding strategies have been proposed to take advantage of real-time information so as to reduce the waiting time at control points (Abkowitz and Lepofsky, 1990; Dessouky et al., 1999; Hickman, 2001; Eberlein et al., 2001). Some literature on headway-based dynamic holding strategies also used real-time information to develop adaptive control schemes (Daganzo, 2009; Bartholdi and Eisenstein, 2012). Yet, some of those holding strategies, especially those earlier ones, may not be able to prevent very large schedule disruptions. Typically, if the headway or the spacing between two consecutive buses becomes too large, the holding strategy could not help the following bus to catch up. Daganzo and Pilachowski (2011) filled this gap using a two-way-looking speed control strategy, where buses cooperate with each other to eliminate bus bunching, even in case of large disruptions. This approach was generalized in Xuan et al. (2011), and later expanded into robust versions that can deal with more complex systems; e.g., multiple interacting lines (Argote-Cabanero et al., 2015).

Stop-skipping strategies have also been proposed to help transit systems recover from severe schedule disruptions (Fu et al., 2003; Sun and Hickman, 2005; Liu et al., 2013). Similar “limited-boarding” strategies, in which some passengers waiting for a late bus could be requested to wait for the next bus, have been developed as well (Delgado et al., 2009; 2012). However, these types of strategies downgrade services to a large fraction of passengers whose demand is either skipped or delayed. This is usually not desirable as it often creates frustration among the affected passengers, who have to either walk a longer distance or resort to additional transfers. Another set of strategies, called transit signal priority strategies, consists in optimizing the traffic flow (e.g., using controlled traffic lights) to help late buses to catch up (Liu et al., 2003; Ling and Shalaby, 2004; Estrada et al., 2016). But those strategies could be difficult to implement in many cities as transit agencies have limited control over traffic signals or roadway infrastructures.

The transit agency in Champaign-Urbana area in Illinois, United States, Champaign-Urbana Mass Transit District (CUMTD), uses a bus substitution strategy to deal with bunching. The agency is able to monitor the locations of all running buses in real time with Global Positioning System (GPS) units. Once a running bus is detected to be significantly late at a control point, or when bunching is about to form, CUMTD dispatches a standby bus from a small reserve pool to take over the schedule of the late bus. Then, the late bus goes to *“not-in-service”* mode and it keeps running along the line but only drops off current onboard passengers. Once the not-in-service bus is empty, it is either positioned at a standby location or directly inserted at another location. As of now, there are no precise guidelines from CUMTD on the operational details of the strategy, such as the exact triggering condition to substitute a bus (e.g., a bus being “too late”), and the bus substitution decision is typically left to the discretion of the dispatcher.

This bus substitution strategy is appealing to transit agencies because it requires minimal hardware (e.g., GPS units and communications devices are already in place nowadays), it is extremely simple to implement, and it does not affect bus drivers’ driving behavior. It is particularly friendly to the onboard passengers as they do not get disrupted by any extra dwell time or transfers. The obvious shortcoming, nevertheless, is the extra resources (vehicles and drivers) needed in the standby fleet. Hence, it is important for the transit industry to not only have a clear understanding of such benefit-cost trade-offs, but also seek systematic implementation policies to maximize the overall benefits. To the authors’ best knowledge, however, this substitution strategy has not yet been studied systematically; its effectiveness has not been quantified and compared with other state-of-art bunching mitigation strategies. In addition, the optimal implementation policy for substituting transit vehicles (e.g., timing and location) and the associated resource planning and management decisions (e.g., the size of standby bus fleet) are lacking.

This paper aims at filling these gaps by developing a systematic modeling framework to reveal the optimal policy structure and implementation guidelines. Our model allows the transit agency to make dynamic substitution decisions at regular time intervals to minimize the sum of passenger and agency costs. We develop a non-myopic approximate dynamic programming (ADP) algorithm, combined with offline simulations and estimation modules, to solve the proposed model. Numerical examples with realistic parameters show that bus schedule deviations as well as bunching can be effectively controlled. If the agency cannot make extra investments in the standby bus fleet, the ransom for the substitution strategy is the reallocation of some available resources (buses and drivers) to the reserve pool, which could be otherwise used in operation. Even though this may increase the waiting time experienced by passengers due to larger headways, the system costs can be noticeably reduced if the resources are optimally allocated between the operating and the standby fleets. If the agency can afford to operate additional standby buses, the substitution strategy could yield significant passenger cost reductions at the expense of increased agency costs (as expected, due to standby buses). Our analysis also considers a number of alternative implementation scenarios such as provision of real-time schedule information to passengers, and the use of emerging self-driving vehicles. It is shown that, in the very near future when transit systems become more intelligent and people’s needs for high-quality transit services become more significant, the substitution strategy would be a more promising approach to improving the reliability and the overall performance of transit systems.

The remainder of the paper is organized as follows. Section 2 describes the system characteristics and operations as well as the formulation of the bus substitution problem. Section 3 describes the solution algorithm based on ADP. Then, Section 4 presents multiple numerical examples. Finally, Section 5 provides concluding remarks.

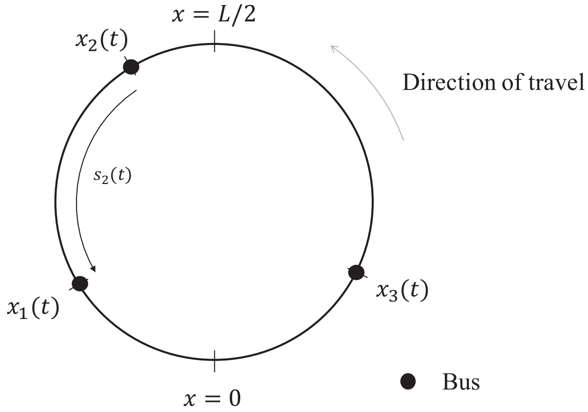


Fig. 1. Typical bus line characteristics ($N = 3$).

2. Mathematical modeling

A bus route provides bidirectional service between two termini along a corridor, which can be seen as a bidirectional closed loop with the two centers being diametrically opposed to each other. Without loss of generality, we consider such a loop of length L, and we focus on one direction of travel, as shown in Fig. 1. Let $\mathcal{N} = \{1, \ldots, N\}$ denote the set of buses in operation along one direction, where N is the number of operating buses in this direction. In a representative homogeneous case, the expected spacing between any two consecutive buses, denoted by S, can thus be given as $S = \frac{L}{N}$.

In the remainder of this paper, the transit system is modeled as a discrete-time dynamical system. The time t is expressed in unit of time steps, $t \in \{0, 1, 2, \ldots\}$. We consider an infinite horizon, since we focus on the stationary state of the system and assume that all parameters defined in this paper are invariant over time. Let $x_n(t) \in [0, L)$ denote the position of bus $n \in \mathcal{N}$ along the route at time t as shown in Fig. 1, whereas $x = 0$ is an arbitrary location along the route (e.g., one terminus).

We define the spacing of bus n at time t, denoted by $s_n(t)$, as the distance between bus n and its preceding bus $n-1$, i.e., for all $n \in \mathcal{N}, t \in \{0, 1, 2, \ldots\}$,

$$s_n(t) = \begin{cases} x_{n-1}(t) - x_n(t) & \text{if } x_{n-1}(t) - x_n(t) \geq 0, \\ x_{n-1}(t) - x_n(t) + L & \text{otherwise.} \end{cases} \quad (1)$$

Note that indices are modular, i.e., index “0” refers to bus N. We assume that buses are not allowed to leapfrog (i.e., pass each other), therefore the spacing is always non-negative.[1]

We can define the trajectory of bus n as a continuous set $\{y_n(t)\}_{t \geq 0}$, where $y_n(t)$ represents the absolute distance at time t that bus n has traveled from location $x = 0$. The relationship between $x_n(t)$ and $y_n(t)$ can be written as

$$x_n(t) = y_n(t) - \left\lfloor \frac{y_n(t)}{L} \right\rfloor \cdot L. \quad (2)$$

Formulations for $y_n(t)$ will be provided in the consequent sections.

2.1. Bus operation and dynamics

Consider first the ideal case that buses are all evenly spaced and have the same constant cruising speed E, while passengers arrive with a steady rate over time and space, λ. In the remainder of this section, bus speed will be expressed in distance traveled per time step, unless stated otherwise. In this case, all buses would experience the same number of pickups and drop-offs within an arbitrary time range. We let A and B denote the dwell time needed for the alighting and boarding of a passenger, respectively. A is typically less than B according to the reported data from many sources such as Transportation Research Board (2000). We further assume that when a bus is in service, passengers are picked up and dropped off simultaneously, and therefore, the dwell times due to alighting can be ignored whenever some passengers are boarding the bus.[2] The commercial speed of each bus, denoted by V, is its long-term average speed and can be defined as

$$V = \left(\frac{1}{E} + B \cdot \frac{\lambda\, S}{V} \right)^{-1}.$$

[1] This is not overly restrictive. If leapfrog does occur, we only need to swap the involved bus indices and the spacing will remain non-negative.

[2] This is not a critical assumption, however. The proposed framework can also accommodate other cases; e.g., when boarding and alighting occur sequentially such that the dwell time is measured by $A + B$.

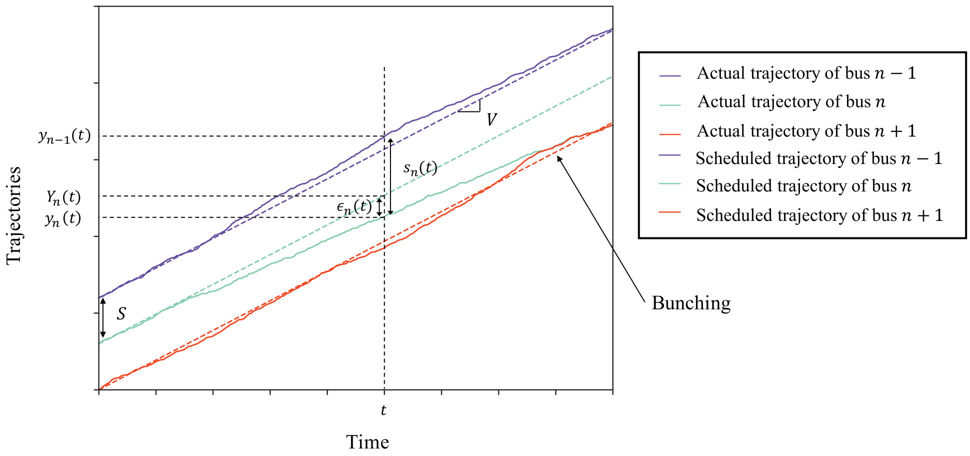


Fig. 2. Uncontrolled bus trajectories.

The terms in the parentheses define the total time that a bus spends in traveling one unit of distance, which is the summation of the cruising time $\frac{1}{E}$ and the dwell time to serve boarding passengers $\left(B \cdot \frac{\lambda S}{V}\right)$. Therefore, similar to Daganzo and Pilachowski (2011), the commercial speed can be derived as

$$V = E(1 - \lambda BS). \tag{3}$$

From Eq. (3), it is easy to observe that the average commercial speed is lower than the cruising speed since it accounts for the times spent in serving passengers.

Given the average spacing S and commercial speed V, one can derive the "scheduled" bus trajectories, as shown by dashed lines in Fig. 2. The dynamics of the "scheduled" bus trajectories, denoted by $\{Y_n(t)\}_{t\geq 0}$, can be given as

$$\begin{aligned} &Y_n(t+1) = Y_n(t) + V, \ \forall n \in \mathcal{N}, t \in \{0, 1, 2, \dots\}, \\ &Y_n(0) = (N - n) \cdot S, \ \forall n \in \mathcal{N}. \end{aligned} \tag{4}$$

As such, the equilibrium occupancy (i.e., the number of onboard passengers in steady state) can also be derived based on the occupancy dynamics defined in the Appendix A, as follows:

$$O_{eq} = \frac{\lambda S}{(2 - 2V/L)2V/L}. \tag{5}$$

In the real world, there usually exist multiple sources of randomness in bus operations. Hence, it is actually very rare for bus trajectories to stick to their schedules. As shown in Fig. 2, the buses may deviate from their scheduled trajectories and the spacing between them may vary over time (i.e., $s_n(t)$ deviates from S for all $n \in \mathcal{N}$). After some time, these buses are almost always found to bunch into pairs. To model the randomness, we let $\tilde{\omega}_n(t)$ denote an additive disturbance to the trajectory of bus n at time t, which follows an independent and identically distributed (i.i.d.) normal distribution with mean 0 and variance σ^2.

To describe the discrete-time dynamical system, we need to estimate the instantaneous commercial speed of each bus n, denoted by $v_n(t)$, which can be approximated by calculating the average speed of a bus for a time step assuming that the demand generated by the spacing, during that time step, is held constant. The time that a bus requires to travel a unit distance, i.e., $1/v_n(t)$, should include the time spent on cruising, $1/E$, and the time spent on serving the passengers, i.e., $\frac{\lambda B s_n(t)}{v_n(t)}$. For simplicity, we assume that the dwell time to serve passengers is continuously distributed along the route instead of being concentrated at the stops, similar to Daganzo and Pilachowski (2011), as if passengers are allowed to flag down buses at any locations along the route (e.g., by simply waving their hands). In this way, the trajectories of the buses can be smoothened out for a good estimation of their commercial speed. Based on Daganzo and Pilachowski (2011), the instantaneous commercial speed of bus n at time t can be expressed as

$$v_n(t) \approx E(1 - \lambda B s_n(t)), \ \forall n \in \mathcal{N}, t \in \{0, 1, 2, \dots\}. \tag{6}$$

The uncontrolled motion of the bus system as in Pilachowski (2009) can be expressed as follows:

$$y_n(t+1) = y_n(t) + v_n(t) + \tilde{\omega}_n(t), \ \forall n \in \mathcal{N}, t \in \{0, 1, 2, \dots\}. \tag{7}$$

We define the deviation of bus n from schedule at time t, denoted by $\epsilon_n(t)$, as the difference between its "scheduled" position and its "actual" position (as shown in Fig. 2), i.e.,

$$\epsilon_n(t) = Y_n(t) - y_n(t), \ \forall n \in \mathcal{N}, t \in \{0, 1, 2, \dots\}. \tag{8}$$

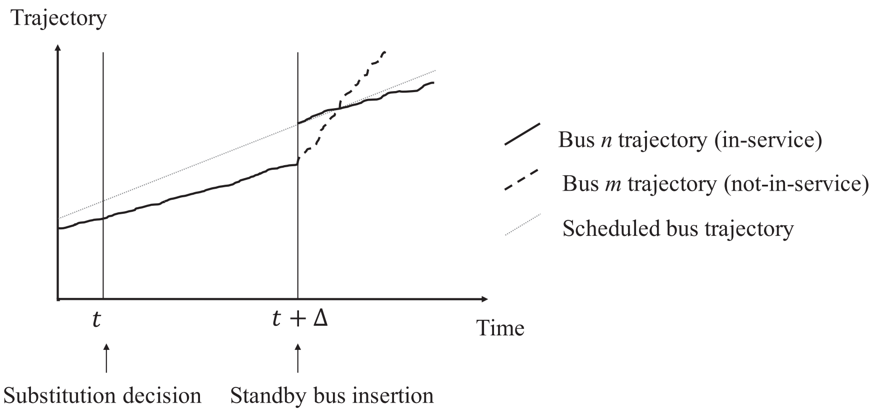


Fig. 3. Bus trajectory with substitution.

Note that $\epsilon_n(t) > 0$ means that bus n is late, and $\epsilon_n(t) < 0$ means that it is early. Moreover, we can estimate the actual bus occupancy, denoted by $O_n(t)$, as follows:

$$O_n(t+1) \approx \max\left\{O_n(t)\left[1-\frac{v_n(t)}{L/2}\left(2-\frac{v_n(t)}{L/2}\right)\right],0\right\}+\lambda s_n(t),\ \forall n\in\mathcal{N}, t\in\{0,1,2,\dots\}. \tag{9}$$

Details on the derivation of (9) are provided in the Appendix A.

2.2. Substitution procedure

To avoid bunching, we consider a bus substitution process where a so-called *standby* bus takes over service from a late or early bus. The set of standby buses, denoted by $\mathcal{M}=\{1,\dots,M\}$, are positioned at certain locations along the bus line.

Fig. 3 gives a simple illustration of a bus trajectory involving a substitution for a late bus. At a decision epoch t, the transit agency would decide if it is necessary to substitute a late or early bus $n\in\mathcal{N}$, by a standby bus $m\in\mathcal{M}$. However, due to the lead time needed to move the standby bus from its parking location to a proper insertion location, the substitution cannot be done instantly. We assume that the time interval of two consecutive decision epochs, denoted by Δ, is no less than the maximum value of all needed lead times, and is a multiple of the system time step (i.e., $\Delta\in\{1,2,3,\dots\}$). Starting from time $t+\Delta$, the bus that has been retired changes to the not-in-service mode, i.e., it only drops off onboard passengers but does not pick up new passengers. Eventually, the retired bus stops and stays at the place where it becomes empty (or a nearby parking location that is available). As shown by the dashed curve in Fig. 3, the expected commercial speed of a retiring bus is greater than that of an operating bus. That is because (i) retiring buses are operated in an alighting-only mode and dwell time for an alighting passenger is shorter than that for a boarding passenger, and (ii) the dwell time keeps decreasing as the bus becomes empty during the retiring process.

The agency's substitution decision at t is denoted by $z_{m,n}(t)\in\{0,1\}$, $\forall n\in\mathcal{N}$, $m\in\mathcal{M}$, where $z_{m,n}(t)=1$ if the operating bus n is substituted by the standby bus m at time $t'=t+\Delta$; or $z_{m,n}(t)=0$ otherwise. It shall be noted that if $z_{m,n}(t)=1$, the indices of the inserted bus and the retired bus are also swapped, i.e., n and m still refer to the buses in service and in reserve (or just retired), respectively. In other words, after the substitution, bus n remains in the dynamical system (but receives an instantaneous location change).

Suppose that a bus goes into not-in-service mode at $t+\Delta$ and becomes retired bus m, then its trajectory, denoted by y^R_m, can be given as

$$y^R_m(t'+1)=y^R_m(t')+v^R_m(t'),\ \forall t'\in\{t+\Delta,t+\Delta+1,\dots\}, \tag{10}$$

with initial conditions $y^R_m(t+\Delta)=y_n(t+\Delta)$ and $O^R_m(t+\Delta)=O_n(t+\Delta)$. The associated instantaneous commercial speed, v^R_m, is defined as

$$v^R_m(t')\approx\begin{cases}\left[E^{-1}+\max\left\{\frac{A}{L/2+y^R_m(t+\Delta)-y^R_m(t')}\left(2-\frac{1}{L/2+y^R_m(t+\Delta)-y^R_m(t')}\right)O^R_m(t'),0\right\}\right]^{-1} & \text{if } O^R_m(t')>0,\\ 0 & \text{otherwise,}\end{cases}$$
$$\forall\ t'\in\{t+\Delta,t+\Delta+1,\dots\}, \tag{11}$$

and its occupancy O^R_m can be given as

$$O^R_m(t'+1)\approx\max\left\{O^R_m(t')\left[1-\frac{v^R_m(t')}{L/2+y^R_m(t+\Delta)-y^R_m(t')}\left(2-\frac{v^R_m(t')}{L/2+y^R_m(t+\Delta)-y^R_m(t')}\right)\right],0\right\},$$
$$\forall\ t'\in\{t+\Delta,t+\Delta+1,\dots\}. \tag{12}$$

Even though retired buses continue to be exposed to disturbances (e.g., from traffic congestion), we omitted the disturbance term in Eq. (10) because: (i) the retiring bus does not interact with other operating buses, and therefore does not influence the future evolution of spacings between the operating buses; (ii) while the delay of alighting passengers from this retired bus is still subject to disturbances, the expected total delay across all passengers shall converge to zero. Eq. (11) pertains to the approximated commercial speed of retired buses, where the dwell time term is proportional to the number of onboard passengers alighting. Since bus service is bidirectional, the retired bus would travel a maximum distance of $L/2$ before it becomes empty. Once the retired bus is empty then it stays where it is, i.e., $v_m^R(t') = 0$. Eq. (12) is similar to Eq. (9), except that no passenger boards the retiring not-in-service bus.

Hence, similar to Eq. (8), the deviation of retired bus m from schedule, $\epsilon_{m,n}^R$, can be expressed as

$$\epsilon_{m,n}^R(t') = Y_n(t') - y_m^R(t'),\ \forall t' \in \{t+\Delta, t+\Delta+1, \dots\}, \tag{13}$$

where, again, n denotes the index of the bus that is swapped with bus m. To keep track of the standby buses, we introduce a term $\tau_m(t)$ to denote the remaining length of time from time t to the time that bus $m \in \mathcal{M}$ becomes empty. We assume conservatively that the retired bus travels a distance of $L/2$ before it becomes empty,[3] the associated running time τ_m, at the time of the insertion, can be given as

$$\tau_m(t+\Delta) = \frac{L}{2E} + A \cdot O_m^R(t+\Delta). \tag{14}$$

Note that at t, $O_m^R(t+\Delta)$ is a random variable, therefore $\tau_m(t+\Delta)$ is also random. Similarly, we define $\tau_m^R(t')$ as the time left before retired bus m can be reassigned:

$$\begin{aligned} &\tau_m^R(t+\Delta) = \tau_m(t+\Delta), \\ &\tau_m^R(t'+1) = \max\{\tau_m^R(t') - 1, 0\},\ \forall t' \in \{t+\Delta, t+\Delta+1, \dots\}. \end{aligned} \tag{15}$$

Note that bus m is idling and ready to be inserted whenever $\tau_m^R(t) = 0$. In the consequent sections, we denote the availability of bus m, with the binary variable, $u_m(t)$, such that

$$u_m(t') = \begin{cases} 1 & \text{if } \tau_m^R(t') = 0, \\ 0 & \text{if } \tau_m^R(t') > 0, \end{cases} \forall t' \in \{t+\Delta, t+\Delta+1, \dots\}. \tag{16}$$

2.3. Costs formulation

The goal of operating the bus system is to minimize both the passenger cost incurred by passengers being delayed from schedule and the agency cost for performing substitution operations.

Passenger costs consist of the total delay that passengers experience when they alight the buses (as compared to the scheduled arrival time), and the out-of-vehicle waiting time. We use an average value of time, μ, to convert time into an equivalent monetary value. As a result, the passenger costs at time t for an operating bus n, C_n^{pax}, in dollar per unit of time, is formulated as

$$C_n^{\text{pax}}(t) = \mu\left[\max\left\{\frac{\epsilon_n(t)}{V}, 0\right\} \cdot \frac{O_n(t)v_n(t)}{L/2}\left(2 - \frac{v_n(t)}{L/2}\right) + \frac{1}{2}\lambda\frac{s_n^2(t)}{v_n(t)}\right],\ \forall t, \forall n \in \mathcal{N}. \tag{17}$$

The first term in the bracket is the product of the delay of bus n, $\frac{\epsilon_n(t)}{V}$, and the number of passengers alighting at t during one time step, $\frac{O_n(t)v_n(t)}{L/2}(2 - \frac{v_n(t)}{L/2})$. The second term is the expected waiting time of boarding passengers at t, which is approximated as half of the current headway for each passenger. Note that the early buses yield no delay cost as alighting passengers arrive early at their destinations to stay conservative, we also do not count the benefits due to early passenger arrivals. However, early buses remain a burden for the system as they create larger gaps in front of the following buses, as well as longer out-of-vehicle waiting times for passengers waiting for those buses.

The substitution related costs include (i) the cost to requisition a standby bus at the targeted insertion location, (ii) the costs to operate the not-in-service bus, and (iii) the costs from onboard passengers alighting from the not-in-service bus. We let coefficient c_M denote the agency's operating costs associated with times of operation. Drivers' wages are assumed to be paid for all involved buses regardless of the substitution operations such that they are not included in the substitution costs. Thus, the costs for substituting bus n with standby bus m at time t, in dollar per unit of time, can be formulated as follows:

$$\begin{aligned} C_{m,n}^R(t) = \Bigg\{ &\frac{c_M}{E} \cdot \delta\Big(y_m^R(t), Y_n(t+\Delta)\Big) + \mathbb{E}[c_M \cdot \tau_m(t+\Delta) \\ &+ \int_{t+\Delta}^{t+\Delta+\tau_m(t+\Delta)} C_{m,n}^{\text{pax}}(t)dt'\Big]\Bigg\} \cdot z_{m,n}(t),\ \forall t, n \in \mathcal{N}, m \in \mathcal{M}, \end{aligned} \tag{18}$$

[3] The expected length of the "retiring process" and the associate variance could be estimated from a inhomogeneous geometric process; we choose to be conservative on the operations of the substitution strategy.

where $\delta(y_1, y_2)$ defines the shortest deadheading distance along the bus route between location y_1 and y_2, and $C^{\text{pax}}_{m,n}$ denotes the expected delay costs associated with the passengers alighting from bus m that initially substituted bus n. Again, to stay conservative, we assume the deadheadings occur along the bus route (i.e., we ignore favorable options such as taking short-cut routes). The term $C^{\text{pax}}_{m,n}$ can be similarly formulated as C^{pax}_{n} in Eq. (17) except that the waiting term is ignored for $C^{\text{pax}}_{m,n}$ since no boarding is allowed for retiring buses.

$$C^{\text{pax}}_{m,n}(t) = \mu \max\left\{\frac{\epsilon^R_{m,n}(t)}{V}, 0\right\} \cdot \frac{O^R_m(t) v^R_m(t)}{L/2 + \epsilon^R_{m,n}(t) - Y_n(t) + y^R_m(t+\Delta)} \cdot \left(2 - \frac{v^R_m(t)}{L/2 + \epsilon^R_{m,n}(t) - Y_n(t) + y^R_m(t+\Delta)}\right), \forall t, n \in \mathcal{N}, m \in \mathcal{M}. \tag{19}$$

The first term in the curly braces of Eq. (18) refers to the cost of moving standby bus m from its location $y^R_m(t)$ to the insertion location $Y_n(t+\Delta)$ at cruising speed E. The second term in the curly braces pertains to the expected costs for running the retired bus along the route to drop off the remaining onboard passengers, as well as the expected delay costs experienced by the alighting passengers.

At this point, the dynamic bus substitution problem over the infinite horizon can be formulated as follows:

$$\min \sum_{t=0}^{\infty} \sum_{n=1}^{N} \left\{ \mathbb{E}\left[\int_t^{t+\Delta} C^{\text{pax}}_n(t) dt\right] + \sum_{m=1}^{M} C^R_{m,n}(t) \right\} \tag{20}$$

s.t. (7)-(14), and

$$\epsilon_n(t+\Delta) = \left(1 - \sum_{m=1}^{M} z_{m,n}(t)\right) \cdot (Y_n(t+\Delta) - y_n(t+\Delta)), \ \forall n \in \mathcal{N}, t \in \{0,1,2,\ldots\}, \tag{21}$$

$$\tau^R_m(t+\Delta) = \left(1 - \sum_{n=1}^{N} z_{m,n}(t)\right) \cdot \max\left\{\tau^R_m(t) - \Delta, 0\right\} + \sum_{n=1}^{N} z_{m,n}(t) \cdot \tau_m(t+\Delta), \ \forall m \in \mathcal{M}, t \in \{0,1,2,\ldots\}, \tag{22}$$

$$y^R_m(t+\Delta) = \sum_{n=1}^{N} z_{m,n}(t) \cdot y_n(t+\Delta), \ \forall m \in \mathcal{M}, t \in \{0,1,2,\ldots\}, \tag{23}$$

$$\sum_{m=1}^{M} z_{m,n}(t) \le 1, \ \forall n \in \mathcal{N}, t \in \{0,1,2,\ldots\}, \tag{24}$$

$$\sum_{n=1}^{N} z_{m,n}(t) \le u_m(t), \ \forall m \in \mathcal{M}, t \in \{0,1,2,\ldots\}, \tag{25}$$

$$-u_m(t) < \tau^R_m(t), \ \forall m \in \mathcal{M}, t \in \{0,1,2,\ldots\}, \tag{26}$$

$$\tau^R_m(t) \le \left(\tau^R_m(t) + \Delta\right) \cdot (1 - u_m(t)), \ \forall m \in \mathcal{M}, t \in \{0,1,2,\ldots\}, \tag{27}$$

$$u_m(t) \in \{0,1\}, \ \forall m \in \mathcal{M}, t \in \{0,1,2,\ldots\}, \tag{28}$$

$$z_{m,n}(t) \in \{0,1\}, \ \forall n \in \mathcal{N}, m \in \mathcal{M}, t \in \{0,1,2,\ldots\}. \tag{29}$$

The objective function (20) is to minimize the total costs, which include the passengers' total costs and the agency's operation and substitution-related costs. The dynamics of the system between t and $t+\Delta$ are described by (7)–(14). Eqs. (21)–(23) are equivalent to the transition functions. If an operating bus n is substituted by standby bus m, i.e., $z_{m,n}(t) = 1$, the deviation of bus n is reset to 0 at $t+\Delta$, the position of bus m at $t+\Delta$ corresponds to the position of bus n at $t+\Delta$ just before the substitution, $y_n(t+\Delta)$, and the time left before bus m becomes empty corresponds to $\tau_m(t+\Delta)$. Constraints (24) ensure that an operating bus can be substituted by at most one standby bus, and constraints (25) state that only an available standby bus can be inserted at $t+\Delta$. Constraints (26)–(28) cast $u_m(t)$ as a boolean variable to indicate availability of bus m at time $t+\Delta$. Constraints (29) pertains to the binary decision of replacing bus n by bus m at time t.

3. Solution approach

3.1. Approximate dynamic programming

The bus substitution problem defined in Eqs. (20)–(29) is a stochastic infinite-horizon mixed integer program with multidimensional continuous state variables. We propose to address this problem using dynamic programming. We consider each decision epoch as a stage. The state of the system can be described by the occupancies of all operating buses $\boldsymbol{O} = \{O_n(0)\}_{n\in\mathcal{N}}$, the deviations from schedule $\boldsymbol{\epsilon} = \{\epsilon_n(0)\}_{n\in\mathcal{N}}$, the time left before the retired buses can be reassigned $\boldsymbol{\tau^R} = \{\tau_m^R(0)\}_{m\in\mathcal{M}}$, and their positions $\boldsymbol{y^R} = \{y_m^R(0)\}_{m\in\mathcal{M}}$. Therefore, the state space is $\Omega = \mathbb{R}^N \times \mathbb{R}^N \times \mathbb{R}^M \times \mathbb{R}^M$ and Eqs. (21)–(23) are equivalent to transition functions. We write the Bellman equation as:

$$J(\boldsymbol{\epsilon}, \boldsymbol{O}, \boldsymbol{\tau^R}, \boldsymbol{y^R}) = \min_{\boldsymbol{z}\in\{0,1\}^{N\times M}} \sum_{n=1}^{N} \left(\mathbb{E}\left[\int_0^{\Delta} C_n^{\text{pax}}(t)dt \right] + \sum_{m=1}^{M} C_{m,n}^R(\Delta) \right) + \mathbb{E}\left[J(\boldsymbol{\epsilon}', \boldsymbol{O}', \boldsymbol{\tau^R}', \boldsymbol{y^R}') \right],$$
$$\forall \left(\boldsymbol{\epsilon}, \boldsymbol{O}, \boldsymbol{\tau^R}, \boldsymbol{y^R}\right) \in \Omega, \tag{30}$$

where the second expectation $\mathbb{E}$ is taken with respect to all transitioned states $(\boldsymbol{\epsilon}', \boldsymbol{O}', \boldsymbol{\tau^R}', \boldsymbol{y^R}') \in \Omega$.

In this problem, state variables are continuous while control variables are binary variables. The large size of the state space Ω makes regular solution approaches inappropriate. Therefore, we adopt an approximate dynamic programming (ADP) approach (Powell, 2011). The basic framework of the ADP is presented in pseudocode Algorithm 1 below. The major steps of this approach include the approximation of the value function, a forward simulation that solves successive one-stage problems, and an update process to improve the approximation of the value function at each step. Some of these steps are rather standard in the ADP literature, and hence will only be explained briefly below.

First of all, in order to simplify the estimation of J, we conduct a series of simplifications to reduce the dimension of the state space. Instead of using $\boldsymbol{\tau^R}$ to describe the state of the retired buses, we use U to denote the number of standby buses ready to be inserted at the next decision epoch. It allows to significantly reduce the state space, without losing the essential information on the standby buses availability:

$$U = \sum_{m=1}^{M} u_m(0). \tag{31}$$

We also ignore the position of the standby buses, as the value function turns out to be relatively insensitive to them, and the occupancies $\boldsymbol{O}$, as they are highly correlated with the deviations $\boldsymbol{\epsilon}$. We now write the control variable $\boldsymbol{z}$ as a policy denoted by $\boldsymbol{z}(\boldsymbol{\epsilon}, U)$ and the value function approximation (VFA) by $\widetilde{J}(\boldsymbol{\epsilon}, U)$. Since the VFA $\widetilde{J}$ should be a smooth function of the deviations $\boldsymbol{\epsilon}$ and the number of standby buses available U, we approximate J with a finite set of separable piecewise linear functions, $\{\phi_n\}_{n\in 0\ldots N}$, i.e.,

$$\widetilde{J}(\boldsymbol{\epsilon}, U) = \phi_0(U) + \sum_{n=1}^{N} \phi_n(\epsilon_n). \tag{32}$$

The piecewise linear function can provide a more stable approximation than a purely linear function or higher order polynomials. We assume that the breakpoints of the piecewise linear function ϕ_0 only occur at integer values of U.

Given $\boldsymbol{\epsilon}$, the integral of the passenger cost function C_n^{pax} between 0 and Δ in Eq. (30) is independent of the decision $\boldsymbol{z}$, therefore the objective function boils down to that presented in Problem (33). We use a Monte Carlo method to simulate and estimate the value of the objective function. Then, this combinatorial optimization problem can be solved with well-known techniques (e.g., branch-and-bound). Given the small number of decision variables in the reduced form, simple enumeration can also be used to solve Problem (33).

Then, to update the VFA, we numerically estimate the marginal value information at the current system state. We slightly perturb each element of the state independently and solve Problem (33) to obtain the numerical gradient. A simple harmonic step size is used to update the corresponding slopes of the VFA, $\alpha_p = \frac{1}{1+p}$.

3.2. Implementation options

One of the most important features of the proposed algorithm is that the quality of its decision policy can be improved continuously, while the solution efficiency can be maintained in a real-time decision-making environment. That means, in the “offline” sense, decision-makers can let the algorithm run for an arbitrary length of time over a large static dataset (either historical or simulated), aiming at reaching a high-quality VFA and policy structure. In fact, this training process can be continuously performed since it does not involve any real-time decisions. Meanwhile, as part of the “online” decision-making process, the optimal policy can be implemented in real time by simply solving Problem (33) in a few seconds, solely based on the current state of the system and the most updated VFA. Considering the fact that the agency’s decision intervals are typically on the order of 10 to 15 min, the proposed approach can be easily implemented to make decisions in real time.

At each decision epoch, dispatchers should have the exact positions and status (e.g., late, early or on-time) of all the buses (e.g., operating, standby, not-in-service) by using automatic vehicle location devices such as GPS. Given this

Algorithm 1 ADP algorithm.

Generate a random set of states $\boldsymbol{\epsilon}^{(1)}, \boldsymbol{\tau}^{\boldsymbol{R}^{(1)}}, \boldsymbol{y}^{\boldsymbol{R}^{(1)}}, \boldsymbol{\epsilon}^{(2)}, \boldsymbol{\tau}^{\boldsymbol{R}^{(2)}}, \boldsymbol{y}^{\boldsymbol{R}^{(2)}}$ ...
Initialize $\widetilde{J}$ for all states $(\boldsymbol{\epsilon}, U)$.
for $l = 1$ to ... **do**
  Initialize the state of the system $\boldsymbol{\epsilon}^{(l)}, \boldsymbol{\tau}^{\boldsymbol{R}^{(l)}}, \boldsymbol{y}^{\boldsymbol{R}^{(l)}}, \boldsymbol{O}^{(l)} = \{O_{eq}\}_{n\in N}$.
  Sample the set of noise vectors $\left\{\{\boldsymbol{\omega}^{(l,p)}(t)\}_{t=0,1,\dots,\Delta}\right\}_{p\in 1..P}$.
  for $p = 1$ to P **do**
    Solve the one-stage problem:

$$\min_{\boldsymbol{z}\in\{0,1\}^{N\times M}} \sum_{n=1}^{N}\sum_{m=1}^{M}\left(C^R_{m,n}(\Delta)\right) + \mathbb{E}\left[\widetilde{J}(\boldsymbol{\epsilon}', U')\right], \tag{33}$$

    where $\widetilde{J}$ is the VFA obtained so far.
    Update system state using Eqs. (7-12), $\{\boldsymbol{\omega}^{(l,p)}(t)\}_{t=0,1,\dots\Delta}$, and solution from the previous step.
    Update the VFA.
  end for
  if VFA converged within a specified tolerance **then**
    break
  end if
end for
return the current value function $\widetilde{J}$.

information, dispatchers can thus apply the optimal policy and make decisions accordingly. Yet, they should ensure that the standby buses can reach their insertion locations on time so as to guarantee the efficiency of the bus substitution. In this regard, the transit agency may use historical data (e.g., areas with high traffic congestion or high passenger demand) to preposition the standby buses to the neighborhoods that the bus bunching is more likely to happen.

So far, in this paper, we have been implicitly assuming that the buses must be operated by human drivers and passengers have no access to real-time bus schedules. Therefore, the extra operating costs for additional drivers of the standby bus fleet, and the waiting costs of passengers are considered. However, given the rapid development of information technologies and transportation autonomy, these two aspects may no longer be a concern to transit agencies in the very near future. For example, public transportation passengers can already easily access real-time information on bus schedules through smartphone applications or websites. They can then adjust their departure times such that the waiting time at the bus stops can be greatly reduced. Moreover, self-driving transit vehicles are either being tested or will soon be in commercial operations; e.g., in cities such as Shenzhen, China (Tao, 2017), and Las Vegas, USA (Jones, 2018). Without drivers, the agency can expect a greater reduction in operating costs when self-driving vehicles are widely used (especially as standby buses).

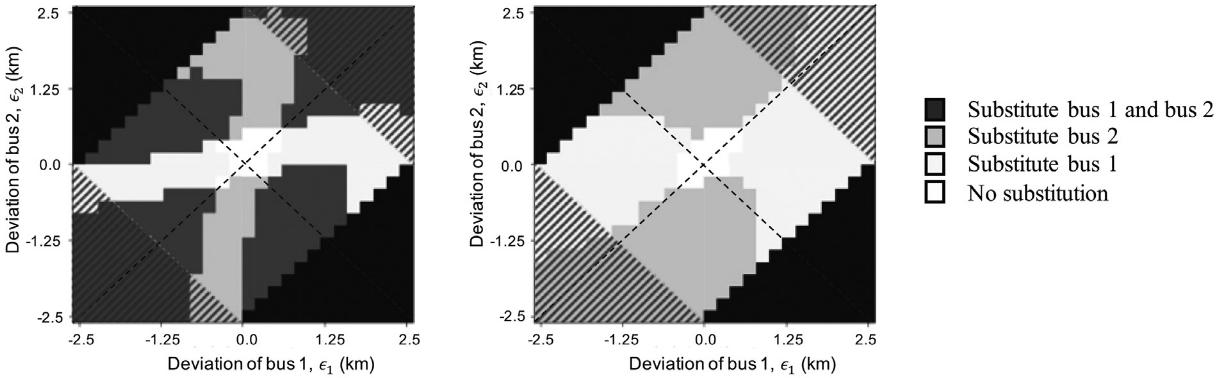
4. Numerical experiments

In this section, we present a few numerical experiments to illustrate the optimal policy of the proposed strategy (in a very simple setting), and to compare the proposed substitution strategy with existing holding methods. The transit agency decides whether to perform a substitution action every $\Delta = 10$ min within a four-hour time horizon.[4]

The passenger's value of time $\mu = \$10$/h (about 50% of the average hourly wage in Champaign-Urbana area (Bureau of Labor Statistics, 2017) while the agency's operating cost factor $c_M = \$37$/veh-h (assuming a total hourly operating cost of \$100/veh-h and about 63% for employee wages and benefits Neff and Dickens, 2015). In addition to the operating costs, we assume the same depreciation cost of \$11/h (based on Kay et al., 2011) for each bus regardless of the operation status. The trajectory time step is set to be 10 s. All tests are performed on a desktop computer with 3.1 GHz CPU and 16GB RAM.

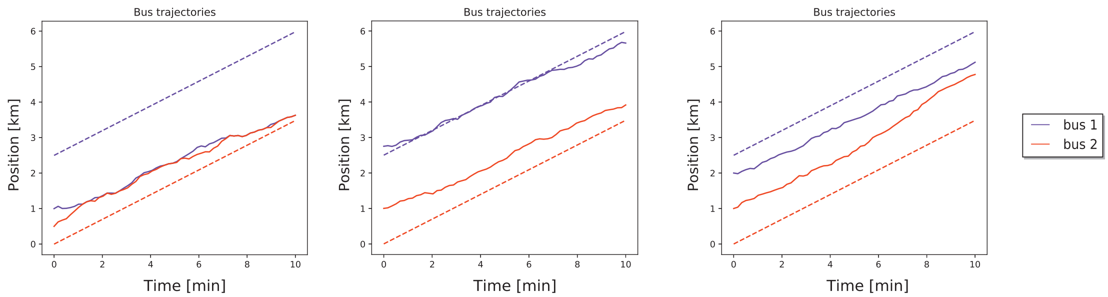
4.1. Illustration of optimal policy

To give a simple and intuitive illustration of the bus substitution policy, we first test a small case with two operating buses and two standby buses, i.e., $N = 2$, $M \in \{1, 2\}$. The system characteristics are set as follows: the length of the bus route $L = 5$ km, the demand density $\lambda = 27$ pax/km-h, the cruising speed $E = 25$ km/h, the alighting time $A = 4$ s/pax, the boarding time $B = 8$ s/pax, the schedule spacing $S = 2.5$ km, and the standard deviation of the noises σ is set to be 0.086 km/min (Pilachowski, 2009).



(a) 2 standby buses available (b) 1 standby bus available

Fig. 4. Bus substitution policy for the case $N = 2$ and $M \in \{1, 2\}$.



(a) $\boldsymbol{\epsilon} = [1.5, -0.5]^T$ (b) $\boldsymbol{\epsilon} = [-0.25, -1]^T$ (c) $\boldsymbol{\epsilon} = [0.5, -1]^T$

Fig. 5. Uncontrolled bus trajectories with different initial deviations.

Fig. 4 illustrates the optimal policy for a large set of deviations for two scenarios: (a) two standby buses are available, and (b) only one standby bus is available. Fig. 5 illustrates the trajectories of the two buses within a 10 min time horizon, for various initial deviations.

The policies presented in Fig. 4 are approximately symmetrical for two buses, and hence we choose to only explore the detail when $\epsilon_2 \leq \epsilon_1$. The regions in black refer to the infeasible sets of deviations, because buses have already bunched but they are not allowed to leapfrog. The shaded areas correspond to extreme scenarios that are unlikely to happen, as substitutions should have already occurred before the deviations reach the values in this area. We can observe that for both scenarios (a) and (b), there is a small region close to the center (0,0) where no substitution is needed, until the deviation of either bus reaches −0.75 km or +0.5 km approximately. As long as the deviation is small, the passenger delay does not yield a bus substitution. In (a), if the deviation of bus 2 is close to 0 and the deviation of bus 1 is larger than 0.5 km, only bus 1 should be substituted at the next decision epoch (see Fig. 5(a)). On the other hand, if bus 1 is slightly early and bus 2 is significantly ahead of schedule, only bus 2 should be substituted (see Fig. 5(b)). Otherwise, both buses should be substituted at the next decision epoch (see Fig. 5(c)). In Fig. 4(b), four feasible regions are roughly delimited by $\epsilon_2 = \epsilon_1$ and $\epsilon_2 = S - \epsilon_1$ (represented by the dashed lines). If $\epsilon_2 < \epsilon_1$ and $\epsilon_2 < S - \epsilon_1$, bus 2 should be substituted. If $\epsilon_2 < \epsilon_1$ and $\epsilon_2 > S - \epsilon_1$, bus 1 should be substituted. Near the edge of those regions, the decisions could be slightly different, especially if the deviations are small.

Fig. 6 illustrates the sample-path evolution of the deviation of the operating buses during a four-hour time horizon. The insertions can be observed when the deviation of a bus is reset to 0 instantaneously.

4.2. Comparison with existing control methods

We now compare the substitution strategy with several existing alternatives. We consider the substitution strategy under two scenarios: (i) the fleet size is fixed at $N + M = 7$ with a variety of splits between operating and standby buses, and (ii) the operating fleet size is fixed at $N = 7$ but the size of the standby bus fleet is variable, $M \in \{1, 2, 3, 4\}$. The former case tells us the passengers' experience if the agency has different ways of utilizing a fixed fleet; the latter informs us

[4] In the numerical examples, we consider a four-hour time horizon because it is typically the length of a rush-hour period (e.g., 6am–10am and 4pm–8pm). Moreover, since transit headways are usually on the orders of minutes, the system performance in a four-hour time horizon should be long enough for the purpose of the analysis.

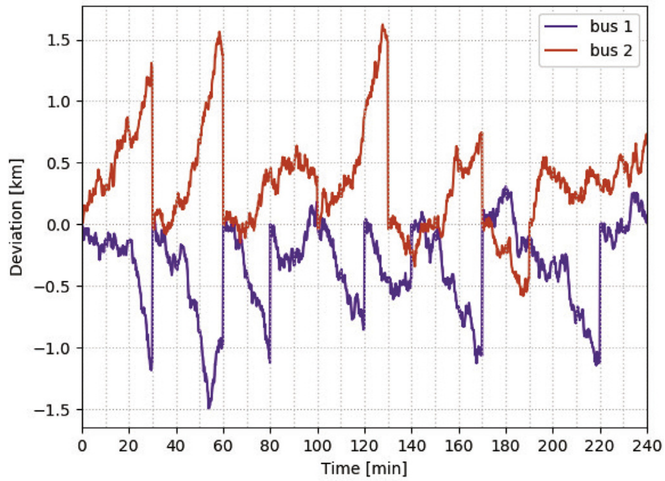


Fig. 6. Bus deviations from schedules with substitutions.

about the potential of providing better passenger experience at the cost of extra standby buses. The bus line parameters are set as follows: $L = 9$ km, $E = 30$ km/h, $B = 4$ s/pax, $A = 2$ s/pax. The standard deviation of noises is set to be 0.045 km/min (Daganzo and Pilachowski, 2011). When multiple buses are positioned along the route, the lead time needed by the agency to deploy a standby bus is typically shorter than 9 min. Therefore, we assume that standby buses can always be inserted within $\Delta = 10$ min. We perform 50 off-line runs of the system simulation over the four-hour time horizon and apply the estimated VFA and ADP policy for the real-time decisions.

For the sake of comparison, we also present the estimated costs for three alternatives while using all 7 buses in operation: (i) the benchmark strategy where fixed optimal slacks of 60 s/km are inserted into the schedule, as in Daganzo and Pilachowski (2011); and two dynamic speed control methods including (ii) the one-way looking strategy (Daganzo, 2009) and (iii) the two-way looking strategy (Daganzo and Pilachowski, 2011), which result in optimal reductions of 2 km/h and 1.6 km/h in commercial speed, respectively.[5] The riding time for those three methods is calculated based on the commercial speed resulting from the different control rules and the expected distance traveled by each passenger, i.e., $L/4$. As we mentioned in Section 1, we suspect that the slack-based methods could lead to higher riding times for onboard passengers. We are also interested in seeing if the dynamic speed control methods may increase the passengers' waiting times due to reduced commercial speeds. Other than these, we assume favorably that passengers do not experience any other delays under these alternative strategies.

Table 1 summarizes the average performance of the strategies in terms of different types of costs, including the passengers' waiting costs, their riding costs, the agency's operating costs associated with running the fleet and paying $N + M$ drivers' wages, and the costs related to substitutions.[6] The total costs under four different scenarios (i.e., with or without human drivers, and with or without considering passengers' waiting costs) are presented in the last four columns of Table 1.

Recall that the proposed substitution strategy is to serve the passengers well at the cost of agency investment. From Table 1, we observe that the substitution strategy yields a significant reduction in passenger costs (i.e., waiting plus riding costs) if the agency is able to invest more in standby resources. Compared with the fixed-slack strategy, the reduction of passenger costs under a fixed operating fleet size (i.e., $N = 7$) is quite significant for all $M \geq 2$, and it reaches a 28% reduction for $M = 4$. It can yield a lower passenger riding costs than the one-way looking method even with only 1 standby bus. However, it is not surprising that the proposed substitution strategy would result in higher total costs as compared with the alternatives, as the agency must pay extra drivers to operate these additional buses. In contrast, when the agency has a fixed fleet budget (i.e., $N + M = 7$), the substitution strategy generally outperforms the fixed-slack method in terms of the total costs. Because some of the vehicles are used as standby buses, the operating costs are reduced due to fuel savings. The drawback, however, is that the passengers may experience longer waiting times due to larger headways; as a result, the overall passenger costs slightly increase. Yet, in cities where the passenger value of time is higher, the total cost difference between the substitution strategy (under fixed operating fleet) and the fixed-slack method should be smaller, since the weight of the passenger costs in the total system costs would be larger.

[5] The corresponding reduction in commercial speed is given by $5\sigma \cdot (\lambda BE)^{1/2}$ and $6\sigma \cdot (\lambda BE)^{1/2}$ for the two-way looking and the one-way looking methods, respectively (see Daganzo and Pilachowski, 2011), where σ^2 is the variance of the disturbances per unit of time (km^2/h).

[6] Opportunity costs could be derived by comparing the different substitution strategies with the fixed total fleet size of 7 buses (first five rows in Table 1). Similarly, the opportunity costs could be included in the case of fixed operating fleet (6th to 9th rows in Table 1) by reassigning some of the standby buses to the operating fleet. For the sake of brevity, we choose not to present them here.

Table 1
Average results for different standby fleet sizes and different control methods (4-hr time horizon).

	Operating fleet size	Standby fleet size	Average waiting cost	Average riding cost	Passenger cost	Operating cost		Substitution cost	Agency cost		Total costs			
						w/ driver	w/o driver		w/ driver	w/o driver	w/ driver + w/ waiting	w/ driver + w/o waiting	w/o driver + w/ waiting	w/o driver + w/o waiting
Substitution with fixed total fleet size	2	5	857	835	1692 (15.8%)	2368	604	78	2446 (−21.3%)	682 (−49.3%)	4138 (−9.4%)	3281 (−22.7%)	2374 (−15.4%)	1517 (−38.9%)
	3	4	548	796	1344 (−8.0%)	2516	752	79	2595 (−16.5%)	831 (−38.2%)	3939 (−13.8%)	3391 (−20.1%)	2175 (−22.5%)	1627 (−34.4%)
	4	3	416	782	1198 (−18.0%)	2664	900	88	2752 (−11.5%)	988 (−26.5%)	3950 (−13.5%)	3534 (−16.7%)	2186 (−22.1%)	1770 (−28.7%)
	5	2	382	777	1159 (−20.7%)	2812	1048	78	2890 (−7.0%)	1126 (−16.2%)	4049 (−11.4%)	3667 (−13.6%)	2285 (−18.5%)	1903 (−23.3%)
	6	1	402	770	1172 (−19.8%)	2960	1196	43	3003 (−3.4%)	1239 (−7.8%)	4175 (−8.6%)	3773 (−11.1%)	2411 (−14.0%)	2009 (−19.0%)
Substitution with fixed operating fleet size	7	4	281	765	1046 (−28.4%)	4292	1520	96	4388 (41.2%)	1616 (20.2%)	5434 (18.9%)	5153 (21.4%)	2662 (−5.1%)	2381 (−4.0%)
	7	3	309	769	1078 (−26.2%)	3996	1476	84	4080 (31.3%)	1560 (16.1%)	5158 (12.9%)	4849 (14.2%)	2638 (−6.0%)	2329 (−6.1%)
	7	2	325	771	1096 (−25.0%)	3700	1432	74	3774 (21.4%)	1506 (12.1%)	4870 (6.6%)	4545 (7.1%)	2602 (−7.2%)	2277 (−8.2%)
	7	1	340	774	1114 (−23.8%)	3404	1388	49	3453 (11.1%)	1437 (6.9%)	4567 (0.0%)	4227 (−0.4%)	2551 (−9.1%)	2211 (−10.9%)
One-way looking	7	0	251	814	1065 (−27.1%)	3108	1344	0	3108 (0%)	1344(0%)	4173 (−8.7%)	3922 (−7.6%)	2409 (−14.1%)	2158 (−13.0%)
Two-way looking	7	0	244	804	1048 (−28.3%)	3108	1344	0	3108 (0%)	1344(0%)	4156 (−9.0%)	3912 (−7.8%)	2392 (−14.7%)	2148 (−13.4%)
Fixed slack	7	0	324	1137	1461 (–)	3108	1344	0	3108 (–)	1344 (–)	4569 (–)	4245 (–)	2805 (–)	2481 (–)

Table 2
Cost comparison under correlated and heterogeneous disturbances.

Scenario	Average waiting cost	Average riding cost	Passenger cost	Substitution cost	Total cost
${\sigma_n}^2 = \sigma^2, cov = 0$	325	771	1096	74	4870
$\sigma_n^2 = \sigma^2, \quad cov = 0.2 \cdot \sigma^2$	323	770	1093	73	4866
$\sigma_n^2 = \sigma^2, \quad cov = 0.5 \cdot \sigma^2$	320	770	1090	67	4857
$\sigma_n^2 = \sigma^2, cov = 0.9 \cdot \sigma^2$	318	769	1087	66	4853
${\sigma_n}^2 = \sigma^2 \pm 0.5 \cdot \sigma^2, cov_{(n-1)n} = 0$	323	770	1093	69	4862

With modern information technologies, public transportation passengers often have easy access to real-time bus arrival times. In such cases, the waiting time may be reduced to a negligible level. The bus substitution strategy under a fixed fleet size (i.e., $N + M = 7$) benefits the most (e.g., up to 22% cost reduction as compared with the fixed-slack method), as the agency can keep the passenger costs at a very low level. The substitution strategy is most effective when the standby fleet size is small, since the cost increase from operating more standby buses dominates the passengers' waiting costs. Note that the performance of the dynamic speed control methods is also slightly improved under this scenario, because we no longer have to worry about the extra waiting times imposed by reduced commercial speeds.

When self-driving transit vehicles become available, the agency can afford to maintain a larger fleet (including both operating and standby buses) without worrying too much about increasing operating costs. As expected, the substitution strategy benefits the most from self-driving technologies when extra standby buses are acquired. Taking the instance $N = 7, M = 2$ as an example, the proposed substitution strategy yields a 7.2% reduction in total costs with self-driving buses, while for the system with human-operated buses, the same strategy may result in a 6.6% cost increase. If we further allow passengers to have access to real-time schedule, the substitution strategy is then able to outperform the fixed-slack method for all test instances. In fact, the agency can achieve a good performance by only investing in a small fleet size of standby buses, e.g., the total costs reduce by over 10% (compared with the fixed-slack method) with only one extra standby bus. In particular, the substitution strategy under fixed fleet size yields a cost reduction of more than 19% (and up to 39%), and outperforms both dynamic speed control methods. Therefore, providing the support from the advanced information technologies and transportation autonomy, the substitution strategy holds the promise to be a very attractive way to improve the reliability and overall performance of transit systems.

It shall be noted that, as a final remark, the substitution strategy holds even higher potential to improve the system performance when the standby fleet is shared across multiple bus lines. This feature shall improve the utilization of the standby buses, reduce the total standby fleet size, and make it very appealing to transit agencies in practice.

4.3. Effect of noise heterogeneity and correlation

In real-world settings, the disturbances $\widetilde{\omega}_n(t)$ (e.g., representing randomness in traffic congestion and passenger arrival processes) tend to be positively correlated over space. While it is intuitive that our system set-up (i.e., i.i.d. disturbances) is less favorable to the bus substitution strategy – e.g., there is less opportunity for "pooling" the disturbances and strategically positioning standby buses at most problematic neighborhoods. In the following, we quantitatively investigate the impact of heterogeneity and potential correlations in the disturbances $\widetilde{\omega}_n(t)$. Since it is very likely that the correlation between two buses is related to their relative position in the sequence and the spatial settings of the system (e.g., congestion, passenger demand), we assume that the noise affecting bus n is only correlated with those affecting the immediately nearby buses, i.e. buses $n-1$ and $n+1$. As such, the covariance matrix has the following form:

$$\begin{pmatrix} \sigma_1^2 & cov_{12} & 0 & \cdots & 0 & cov_{N1} \\ cov_{12} & \sigma_2^2 & cov_{23} & \ddots & \ddots & 0 \\ 0 & cov_{23} & \sigma_3^2 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \sigma_{N-1}^2 & cov_{(N-1)N} \\ cov_{N1} & 0 & \cdots & 0 & cov_{(N-1)N} & \sigma_N^2 \end{pmatrix}$$

We further focus on the following cases: (i) correlated disturbances, where $\sigma_n^2 = \sigma^2, \quad \forall n$, and $cov_{(n-1)n} = cov, \quad \forall n \in 1, \ldots, \quad N$, for $cov \in \{0.2\sigma^2, \ 0.5\sigma^2, \ 0.9\sigma^2\}$, and (ii) heterogeneous disturbances, where $\sigma_n^2 = \sigma^2 \pm 0.5 \cdot \sigma^2, \quad \forall n$ with $\sum_n \sigma_n^2 = N \cdot \sigma^2$ and $cov_{(n-1)n} = 0, \quad \forall n \in 1, \ldots, \quad N$. The results for $N = 7$, $M = 2$ (w/driver) are summarized in Table 2. We set $\sigma_n^2 = 1.5\sigma^2, \quad \forall n \in \{1, 2, 3\}$, $\sigma_n^2 = 0.5\sigma^2, \quad \forall n \in \{4, 5, 6\}$, and $\sigma_7^2 = \sigma^2$ to ensure $\sum_n \sigma_n^2 = 7 \cdot \sigma^2$.

It can be seen from Table 2 that, as expected, positive correlations among the disturbances tends to produce lower system costs. They tend to force adjacent buses to behave similarly, especially when their deviations from the schedule are small. In Eq. (7), if the deviation from the target spacing for two adjacent buses is small and they share similar noises, their spacing tends to remain stable, because their speeds are almost equal. As a result, fewer bus substitutions would be needed (saving in agency costs) and less schedule deviations would happen (savings in passengers' waiting costs). Similarly, as shown in the last row, heterogeneous disturbances would also lead to lower system costs. When disturbances concentrate around a subset of the buses, the standby resources can target those problematic ones more effectively such that the system can achieve a better performance.

5. Conclusion

This paper systematically studies the potential of a transit schedule control strategy that is easy to implement in practice. In carrying out the strategy, the transit agency keeps a fleet of standby buses and dynamically determines the optimal times and locations to substitute late/early buses with the standby ones. The goal is to maintain bus service punctuality, avoid bus bunching, and minimize the overall system-wide costs. A mathematical modeling framework with embedded optimization and simulation modules is proposed to reveal the optimal policy structure of bus substitution decisions, and to provide insights and guidelines for effective implementation. The optimal policy is obtained by formulating the decision problem as a stochastic infinite-horizon optimization program and solving it via an ADP-based algorithm. Numerical experiments with realistic parameters are conducted to test how bus bunching can be effectively controlled by the strategy, and to compare the proposed strategy with the traditional slack-based control method and dynamic speed control methods. Multiple scenarios are considered, e.g., when real-time schedules are available to passengers, and when self-driving buses are available for deployment. The results show that, even without making extra investments in the fleet, the system costs can be effectively reduced (i.e., up to 10% as compared with the fixed-slack method) from the substitution strategy if the resources are effectively allocated between the operating and the standby fleet. If the agency can invest in additional standby buses, the substitution strategy can further reduce passenger costs (e.g., more than 20% as compared with the fixed-slack method). Moreover, the benefit of the bus substitution strategy can be even more prominent if we provide real-time schedule information to passengers so they do not need to wait for service, as well as if self-driving buses become available for reducing costs. The substitution strategy can even outperform the dynamic speed control methods using the same fleet of vehicles. As such, we believe that the substitution strategy holds the promise to effectively improve reliability and efficiency transit systems in the near future.

To explore the full potential of the bus substitution strategy, we are very interested in studying more complex (and more realistic) systems with multiple bus routes. By sharing the fleet of standby buses across multiple routes, we expect the utilization and cost reduction potential to be further enhanced. The model for such multi-line systems would inevitably become more complex. Similar opportunities can be explored with regard to temporal and spatial heterogeneity. The model in this paper is built for a time-invariant and spatially-homogeneous bus route. Future research could consider the possibility of maintaining a time-varying stand-by fleet when the operating environment is heterogeneous over time and space. In such cases, stand-by buses could be used for regular service from time to time (as needed) so as to exercise their full potential (i.e., reducing their opportunity costs). In addition, it would be very interesting to investigate the possibility of mixing alternative bunching control strategies (e.g., dynamic holding together with dynamic substitution approach) to complement each other. In particular, we know that the dynamic holding strategies can effectively absorb small schedule deviations up to a certain magnitude. The bus substitution strategy (now with a smaller need for standby buses) could then be used to address only very large deviations. Such cooperation among multiple strategies shall be quite promising.

Acknowledgments

The authors thank Mr. Jay Rank and his colleagues at CUMTD for providing helpful information and insights on their business operations. Comments from Dr. Nicolas Chiabaut (IFSTTAR) at the 2018 TRB Annual Meeting helped improve the organization of the numerical experiments section. This work was financially supported in part by the U.S. National Science Foundation through grants CMMI-1234085 and CMMI-1662825. Very helpful comments from the two anonymous reviewers are also gratefully acknowledged.

Appendix A. Occupancy dynamics for operating and not-in-service buses

Since the origins and destinations of the passengers are uniformly distributed along the line, for an arbitrary passenger and some $dx \to 0^+$, we have

$$\Pr(\text{destination} \in [x, x + dx]) = \frac{dx}{L}. \tag{34}$$

With bidirectional service, the maximum passenger trip distance is $L/2$. For a passenger to be onboard at an arbitrary location y, its destination x can only be between y and $y + L/2$, and for a given destination x, the origin can only be between $-L/2 + y + x$ and y. Hence,

$$\Pr(\text{onboard at } y) = \int_{x=0}^{L/2} \int_{y=-L/2+x}^{0} \frac{dy}{L/2} \frac{dx}{L} = \frac{1}{4}. \tag{35}$$

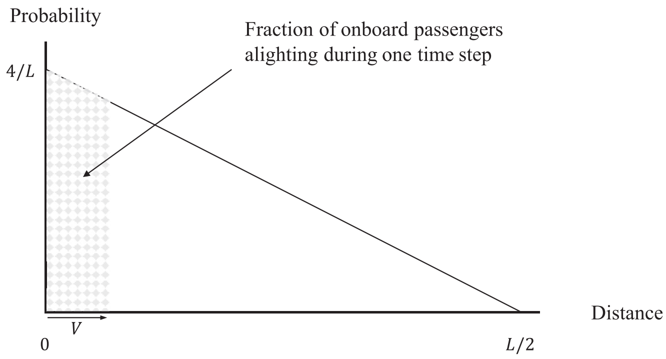


Fig. 7. Probability density function of onboard passenger destinations.

On the other hand, conditional on the destination, we have

$$\Pr(\text{onboard at } y| \text{ destination} \in [x, x+dx]) = \begin{cases} 1 - \frac{2(x-y)}{L} & \text{if } y \in [x - L/2, x], \\ 0 & \text{otherwise.} \end{cases} \tag{36}$$

Using Bayes' theorem, the conditional probability of a passenger's destination to be in $[x, x+dx]$ given that it is onboard the bus at y is (34) multiplied by (36) divided by (35); i.e.,

$$\Pr(\text{destination} \in [x, x+dx]| \text{ onboard at y}) = 2 \cdot (L/2 - x)\left(\frac{L}{2}\right)^{-2} dx. \tag{37}$$

Note that $2 \cdot (L/2 - x)(\frac{L}{2})^{-2}$ is the probability density function of a triangular distribution with lower limit 0, upper limit $L/2$ and mode 0 (as shown in Fig. 7).

Let $\bar{O}_n(t)$ denote the "scheduled" occupancy (i.e., the number of onboard passengers when there are no disturbances) of bus n at time t.

$$\bar{O}_n(t+1) \approx \bar{O}_n(t)(1-f) + \lambda S, \ \forall n \in \mathcal{N}, t \in \{0, 1, 2, \dots\}, \tag{38}$$

where f denotes the fraction of onboard passengers alighting during one time step. The number of boarding passengers is proportional to the spacing S. The fraction of passengers alighting during one time step corresponds to the shaded trapezoid in Fig. 7, whereas the height is given by the fraction of distance $L/2$ that the bus can cover within one time step, i.e.

$$f = \frac{V}{L/2}\left(2 - \frac{V}{L/2}\right), \tag{39}$$

Note that the occupancy dynamics for the operating buses, $O_n(t)$, are derived in a similar manner in Eq. (9), whereas the distance traveled within one time step is $v_n(t)$. Furthermore, given that $1 - \frac{V}{L/2}(2 - \frac{V}{L/2}) < 1$, the arithmetico-geometric sequence presented in Eq. (38) converges to

$$O_{eq} = \frac{\lambda S}{2V/L(2 - 2V/L)}. \tag{40}$$

To derive Eq. (9), simply replace V by the current speed $v_n(t)$, and S by the current spacing $s_n(t)$. Similarly, to derive Eq. (12), one has to replace V by the current speed $v'_n(t)$, and $L/2$ by $L/2 - y$, where y is the distance that the bus has traveled since the substitution.

References

Abkowitz, M.D., Lepofsky, M., 1990. Implementing headway-based reliability control on transit routes. J. Transp. Eng. 116 (1), 49–63.

Argote-Cabanero, J., Daganzo, C.F., Lynn, J.W., 2015. Dynamic control of complex transit systems. Transp. Res. Part B 81 (1), 146–160. doi:10.1016/j.trb.2015.09.003.

Bartholdi, J.J., Eisenstein, D.D., 2012. A self-coördinating bus route to resist bus bunching. Transp. Res. Part B 46 (4), 481–491. doi:10.1016/j.trb.2011.11.001.

Bureau of Labor Statistics, 2017. May 2016 metropolitan and nonmetropolitan area occupational employment and wage estimates Champaign-Urbana, IL. https://www.bls.gov/oes/current/oes_16580.htm.

Daganzo, C.F., 2008. Fundamentals of Transportation and Traffic Operations. Emerald, Inc., Bingley, UK.

Daganzo, C.F., 2009. A headway-based approach to eliminate bus bunching: systematic analysis and comparisons. Transp. Res. Part B 43 (10), 913–921. https://doi.org/10.1016/j.trb.2009.04.002.

Daganzo, C.F., Pilachowski, J., 2011. Reducing bunching with bus-to-bus cooperation. Transp. Rese. Part B 45 (1), 267–277. https://doi.org/10.1016/j.trb.2010.06.005.

Delgado, F., Munoz, J.C., Giesen, R., 2012. How much can holding and/or limiting boarding improve transit performance? Transp. Res. Part B 46 (9), 1202–1217. doi:10.1016/j.trb.2012.04.005.

Delgado, F., Muñoz, J.C., Giesen, R., Cipriano, A., 2009. Real-time control of buses in a transit corridor based on vehicle holding and boarding limits. Transp. Res. Rec. (2090) 59–67.

Dessouky, M., Hall, R., Nowroozi, A., Mourikas, K., 1999. Bus dispatching at timed transfer transit stations using bus tracking technology. Transp. Res. Part C 7 (4), 187–208. doi:10.1016/S0968-090X(99)00019-4.

Eberlein, X.J., Wilson, N.H.M., Bernstein, D., 2001. The holding problem with real-time information available. Transp. Sci. 35 (1), 1–18.
Estrada, M., Mensión, J., Aymamí, J.M., Torres, L., 2016. Bus control strategies in corridors with signalized intersections. Transp. Res. Part C 71, 500–520. doi:10.1016/j.trc.2016.08.013.
Fu, L., Liu, Q., Calamai, P., 2003. Real-time optimization model for dynamic scheduling of transit operations. Transp. Res. Rec. (1857) 48–55.
Hickman, M.D., 2001. An analytic stochastic model for the transit vehicle holding problem. Transp. Sci. 35 (3), 215–237.
Jones, J., 2018. Now you can ride a driverless shuttle in Las Vegas, for free. http://www.latimes.com/travel/deals/la-tr-las-vegas-first-driverless-shuttle-to-loop-the-city-20171228-story.html.
Kay, M., Clark, M., Duffy, C., Laube, M., Lian, F., Plosky, E., 2011. Bus Lifecycle Cost Model for Federal Land Management Agencies: User's Guide. John A. Volpe National Transportation Systems Center.
Ling, K., Shalaby, A., 2004. Automated transit headway control via adaptive signal priority. J. Adv. Transp. 38 (1), 45–67.
Liu, H., Skabardonis, A., Zhang, W., 2003. A dynamic model for adaptive bus signal priority. 82nd Transportation Research Board Annual Meeting, Washington, DC. Preprint CD-ROM.
Liu, Z., Yan, Y., Qu, X., Zhang, Y., 2013. Bus stop-skipping scheme with random travel time. Transp. Res. Part C 35, 46–56. doi:10.1016/j.trc.2013.06.004.
Neff, J., Dickens, M., 2015. 2013 Public Transportation Association Fact Book. Technical Report. American Public Transportation Association, Washington, DC.
Newell, G.F., 1974. Control of pairing of vehicles on a public transportation route, two vehicles, one control point. Transp. Sci. 8 (3), 248–264.
Newell, G.F., Potts, R.B., 1964. Maintaining a bus schedule. In: Proceedings of the 2nd Australian Road Research Board Conference, 2, pp. 388–393.
Osuna, E.E., Newell, G.F., 1972. Control strategies for an idealized public transportation system. Transp. Sci. 6 (1), 52–72.
Pilachowski, J.M., 2009. An Approach to Reducing Bus Bunching. University of California, Berkeley Ph.D. thesis.
Powell, W.B., 2011. Approximate Dynamic Programming: Solving the curses of dimensionality, 703, 2nd ed. John Wiley & Sons, New York.
Sun, A., Hickman, M., 2005. The real–time stop–skipping problem. J. Intell. Transp. Syst. 9 (2), 91–109. doi:10.1080/15472450590934642.
Tao, L., 2017. Shenzhen bus operator joins Baidu, Google in autonomous driving race with public trial. http://www.scmp.com/tech/enterprises/article/2123880/shenzhen-bus-operator-joins-baidu-google-autonomous-driving-race.
Transportation Research Board, 2000. Highway Capacity Manual, 3rd ed. National Research Council, Washington DC.
Xuan, Y., Argote, J., Daganzo, C.F., 2011. Dynamic bus holding strategies for schedule reliability: optimal linear control and performance analysis. Transp. Res. Part B 45 (10), 1831–1845. doi:10.1016/j.trb.2011.07.009.