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RESEARCH ARTICLE

Lump solutions to a generalized Bogoyavlensky-Konopelchenko equation

Shou-Ting CHEN[1], **Wen-Xiu MA**[2,3,4]

1 School of Mathematics and Physical Science, Xuzhou Institute of Technology, Xuzhou 221008, China
2 Department of Mathematics and Statistics, University of South Florida, Tampa, FL 33620, USA
3 College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao 266590, China
4 International Institute for Symmetry Analysis and Mathematical Modelling, Department of Mathematical Sciences, North-West University, Mafikeng Campus, Mmabatho 2735, South Africa

Abstract A (2 + 1)-dimensional generalized Bogoyavlensky-Konopelchenko equation that possesses a Hirota bilinear form is considered. Starting with its Hirota bilinear form, a class of explicit lump solutions is computed through conducting symbolic computations with Maple, and a few plots of a specific presented lump solution are made to shed light on the characteristics of lumps. The result provides a new example of (2 + 1)-dimensional nonlinear partial differential equations which possess lump solutions.

Keywords Symbolic computation, lump solution, soliton theory
MSC 35Q51, 35Q53, 37K40

1 Introduction

The Cauchy problem is one of the fundamental problems in the theory of differential equations, and its aim is to determine a solution of a differential equation satisfying what are known as initial data. Laplace's method is developed for solving Cauchy problems for linear ordinary differential equations, and the Fourier transform method, for linear partial differential equations. In modern soliton theory, the isomonodromic transform method and the inverse scattering transform method have been created for handling Cauchy problems for nonlinear ordinary and partial differential equations, respectively [1,32].

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Corresponding author: Wen-Xiu MA, E-mail: mawx@cas.usf.edu

Only the simplest differential equations, normally constant-coefficient and linear, are solvable explicitly. It is definitely difficult to determine exact solutions to nonlinear differential equations. However, some recent studies have been made on a kind of interesting explicit solutions called lumps, originated from solving soliton equations [30,35]. Lumps are a kind of rational function solutions that are localized in all directions in space, and solitons are analytic solutions exponentially localized in all directions in space and time, historically found for nonlinear integrable equations. Taking long wave limits of N-soliton solutions can engender special lumps [34]. Positon and complexiton solutions also exist for nonlinear integrable equations, adding to the diversity of solitons [20,39]. More recent studies show that there exist interaction solutions [28] between two different kinds of solutions to (2 + 1)-dimensional integrable equations [27], and they can be used to describe various nonlinear phenomena in sciences.

It is known that the Hirota bilinear method provides a powerful technique to look for exact solutions in soliton theory [2,8]. Let a polynomial P determine a Hirota bilinear form

$$P(\mathrm{D}_x, \mathrm{D}_t) f \cdot f = 0,$$

where D_x and D_t are Hirota's bilinear derivatives, for a given partial differential equation with a dependent variable u. Through the Hirota bilinear scheme, soliton solutions can be usually determined as follows:

$$u = 2(\log f)_{xx}, \ f = \sum_{\mu=0,1} \exp\left(\sum_{i=1}^{N} \mu_i \xi_i + \sum_{i<j} \mu_i \mu_j a_{ij} \right),$$

where $\sum_{\mu=0,1}$ denotes the sum over all possibilities for $\mu_1, \mu_2, \ldots, \mu_N$ taking either 0 or 1, and the wave variables and the phase shifts are given by

$$\xi_i = k_i x - \omega_i t + \xi_{i,0}, \quad 1 \leqslant i \leqslant N,$$

and

$$\mathrm{e}^{a_{ij}} = -\frac{P(k_i - k_j, \omega_j - \omega_i)}{P(k_i + k_j, \omega_j + \omega_i)}, \quad 1 \leqslant i < j \leqslant N,$$

with k_i and ω_i satisfying the corresponding dispersion relation and $\xi_{i,0}$ being arbitrary translation shifts.

It is recognized that the KPI equation possesses lump solutions [22], among which are special lump solutions derived from N-soliton solutions [31]. Other integrable equations which possess lump solutions include the three-dimensional three-wave resonant interaction [11], the BKP equation [6,42], the Davey-Stewartson equation II [34], the Ishimori-I equation [10], and many others (see, e.g., [35,49]). It is very interesting to enlarge this category of nonlinear partial differential equations that possess lump solutions.

This paper aims to add an equation to that category of nonlinear equations by exploiting lump solutions to a (2+1)-dimensional generalized Bogoyavlensky-Konopelchenko equation, via Maple symbolic computations starting with its

Hirota bilinear form. Explicit formulas of the parameters involved in the obtained solutions will be given, and three-dimensional plots, contour plots, and plots of t-, x-, and y-curves of a specific example of the solutions will be made via Maple plot tools. A few concluding remarks will be presented in the last section.

2 A study on lump solutions

We consider a $(2+1)$-dimensional generalized Bogoyavlensky-Konopelchenko (gBK) equation

$$\begin{aligned} P_{\mathrm{gBK}}(u,v) &:= u_t + \alpha(6uu_x + u_{xxx}) + \beta(u_{xxy} + 3uu_y + 3u_x v_y) \\ &\quad + \gamma_1 u_x + \gamma_2 u_y + \gamma_3 v_{yy} \\ &= 0, \end{aligned} \tag{2.1}$$

where $v_x = u$, and $\alpha, \beta, \gamma_1, \gamma_2$, and γ_3 are constant coefficients. This is equivalent to the following equation:

$$\begin{aligned} v_{tx} + \alpha(6v_x v_{xx} + v_{xxxx}) + \beta(v_{xxxy} + 3v_x v_{xy} + 3v_{xx} v_y) \\ + \gamma_1 v_{xx} + \gamma_2 v_{xy} + \gamma_3 v_{yy} = 0, \end{aligned} \tag{2.2}$$

which is a generalization of the $(2+1)$-dimensional gBK equation (see, e.g., [33,37]):

$$v_{tx} + \alpha(6v_x v_{xx} + v_{xxxx}) + \beta(v_{xxxy} + 3v_x v_{xy} + 3v_{xx} v_y) = 0.$$

A direct computation tells that this gBK equation (2.1) can be written as a Hirota bilinear form

$$\begin{aligned} B_{\mathrm{gBK}}(f) &:= (\mathrm{D}_t\mathrm{D}_x + \alpha \mathrm{D}_x^4 + \beta \mathrm{D}_x^3 \mathrm{D}_y + \gamma_1 \mathrm{D}_x^2 + \gamma_2 \mathrm{D}_x \mathrm{D}_y + \gamma_3 \mathrm{D}_y^2) f \cdot f \\ &= 2[f_{tx} f - f_t f_x + \alpha(f_{xxxx} f - 4f_{xxx} f_x + 3f_{xx}^2) \\ &\quad + \beta(f_{xxxy} f - f_{xxx} f_y - 3f_{xxy} f_x + 3f_{xx} f_{xy}) \\ &\quad + \gamma_1(f_{xx} f - f_x^2) + \gamma_2(f_{xy} f - f_x f_y) + \gamma_3(f_{yy} f - f_y^2)] \\ &= 0, \end{aligned} \tag{2.3}$$

under the transformations

$$u = 2(\log f)_{xx} = \frac{2(f_{xx} f - f_x^2)}{f^2}, \quad v = 2(\log f)_x = \frac{2f_x}{f}. \tag{2.4}$$

Such logarithmic transformations play a prominent role in Bell polynomial theories for soliton equations and their generalized counterparts (see, e.g., [5,21]). Actually, we have

$$P_{\mathrm{gBK}}(u,v) = \Big(\frac{B_{\mathrm{gBK}}(f)}{f^2}\Big)_x,$$

and thus, when f solves the bilinear gBK equation (2.3), $u = 2(\log f)_{xx}$ and $v = 2(\log f)_x$ will solve the $(2+1)$-dimensional gBK equation (2.1).

Bearing in mind that the gBK equation (2.1) has a Hirota bilinear form, we search for a class of quadratic function solutions to the $(2+1)$-dimensional bilinear gBK equation (2.3), defined by

$$f = \xi_1^2 + \xi_2^2 + a_9, \tag{2.5}$$

where

$$\xi_1 = a_1 x + a_2 y + a_3 t + a_4, \quad \xi_2 = a_5 x + a_6 y + a_7 t + a_8, \tag{2.6}$$

a_i, $1 \leqslant i \leqslant 9$, being constant parameters to be determined. Inserting such a function f into the gBK equation (2.1) yields a system of algebraic equations on the parameters and the constant coefficients. Then, direct symbolic computations with Maple show that the resulting system of algebraic equations has a class of explicit solutions:

$$\begin{cases} a_3 = -a_1\gamma_1 - a_2\gamma_2 - \dfrac{a_1(a_2^2 - a_6^2) + 2a_2a_5a_6}{a_1^2 + a_5^2}\gamma_3, \\ a_7 = -a_5\gamma_1 - a_6\gamma_2 - \dfrac{2a_1a_2a_6 - a_5(a_2^2 - a_6^2)}{a_1^2 + a_5^2}\gamma_3, \\ a_9 = -\dfrac{3(a_1^2 + a_5^2)^2[\alpha(a_1^2 + a_5^2) + \beta(a_1a_2 + a_5a_6)]}{(a_1a_6 - a_2a_5)^2\gamma_3}, \end{cases} \tag{2.7}$$

and the other parameters could be arbitrary provided that the solutions of u and v presented by (2.4) will make sense. The constant coefficient γ_3 in the solutions by (2.7) should not be zero, in order to produce lump solutions, but it could be either positive or negative, which is different from the case in the KPI equation [22].

Now, the transformations in (2.4) generate a large class of lump solutions to the $(2+1)$-dimensional gBK equation (2.1), determined by

$$\begin{cases} u = \dfrac{2(f_{xx}f - f_x^2)}{f^2} = \dfrac{4(a_1^2 + a_5^2)}{f} - \dfrac{8(a_1\xi_1 + a_5\xi_2)^2}{f^2}, \\ v = \dfrac{2f_x}{f} = \dfrac{4(a_1\xi_1 + a_5\xi_2)}{f}. \end{cases} \tag{2.8}$$

It is known that the requirement

$$a_1a_6 - a_2a_5 \neq 0 \tag{2.9}$$

is a necessary and sufficient condition for a solution f, defined by (2.5) and (2.6), to yield a lump solution in (2+1)-dimensions through (2.8). The condition (2.9) also guarantees $a_1^2 + a_5^2 \neq 0$. Once we require the condition (2.9), we can solve

$$f_x(x(t), y(t), t)) = 0, \quad f_y(x(t), y(t), t) = 0, \tag{2.10}$$

to get all critical points of f:

$$\begin{cases} x = x(t) = \dfrac{(a_2a_7 - a_3a_6)t + (a_2a_8 - a_4a_6)}{a_1a_6 - a_2a_5}, \\ y = y(t) = -\dfrac{(a_1a_7 - a_3a_5)t + (a_1a_8 - a_4a_5)}{a_1a_6 - a_2a_5}, \end{cases} \tag{2.11}$$

where t is a time parameter arbitrarily fixed. Since the sum of two squares, i.e., the function $f - a_9$, vanishes at this set of critical points, we see that $f > 0$ if and only if $a_9 > 0$. This implies that u and v defined by (2.8) are analytical in $\mathbb{R}^3$, if and only if $a_9 > 0$. Further according to (2.7), u and v by (2.8) are analytical, if and only if

$$[\alpha(a_1^2 + a_5^2) + \beta(a_1a_2 + a_5a_6)]\gamma_3 < 0. \tag{2.12}$$

For any given time t, the point $(x(t), y(t))$ defined by (2.11) is also a critical point of the function $u = 2(\log f)_{xx}$, and thus, by the second derivative test, the lump solution u has a peak at this point $(x(t), y(t))$, because we have

$$\begin{aligned} u_{xx} &= -\frac{24(a_1^2 + a_5^2)^2}{a_9^2} < 0, \\ u_{xx}u_{yy} - u_{xy}^2 &= \frac{192(a_1^2 + a_5^2)^2(a_1a_6 - a_2a_5)^2}{a_9^4} > 0, \end{aligned} \tag{2.13}$$

at the critical point $(x(t), y(t))$ of f. The peak of u has a value of

$$u_{\max} = \frac{4(a_1^2 + a_5^2)}{a_9}. \tag{2.14}$$

At the critical point $(x(t), y(t))$ of f, we also have

$$v = 0, \quad v_x = \frac{4(a_1^2 + a_5^2)}{a_9} > 0, \quad v_y = \frac{4(a_1a_2 + a_5a_6)}{a_9}, \tag{2.15}$$

and thus, $(x(t), y(t))$ is definitely not a critical point of v, and the x-curve of v is increasing but the y-curve of v could be either increasing or decreasing at $(x(t), y(t))$, which depends on the sign of $a_1a_2 + a_5a_6$.

All the solutions computed this way provide a valuable supplement to the solution theories available on soliton solutions and dromion-type solutions, developed through powerful existing techniques such as the Hirota perturbation approach and symmetry constraints including symmetry reductions (see, e.g., [3,4,13–15,52]).

Let us take

$$\alpha = 1, \quad \beta = 1, \quad \gamma_1 = 1, \quad \gamma_2 = -1, \quad \gamma_3 = 1, \tag{2.16}$$

and then we arrive at a special gBK equation

$$u_t + 6uu_x + u_{xxx} + u_{xxy} + 3uu_y + 3u_xv_y + u_x - u_y + v_{yy} = 0, \tag{2.17}$$

where $u = v_x$. Now, further fixing

$$a_1 = 1, \quad a_2 = -1, \quad a_4 = -1, \quad a_5 = -1, \quad a_6 = 5, \quad a_8 = 1, \tag{2.18}$$

which ensures the conditions (2.9) and (2.12), and so, the positiveness of the generating function f, we can obtain a specific lump solution to the special gBK equation (2.17) as follows:

$$u = \frac{8}{f} - \frac{32(x-3y-13t-1)^2}{f^2}, \quad v = \frac{8(x-3y-9t-1)}{f}, \tag{2.19}$$

where

$$f = (x-y+5t-1)^2 + (-x+5y+23t+1)^2 + 3. \tag{2.20}$$

Three three-dimensional plots and contour plots, and t-, x-, and y-curves of this lump solution are made via Maple plot tools, to shed light on the characteristics of lump solutions, in Figures 1 and 2.

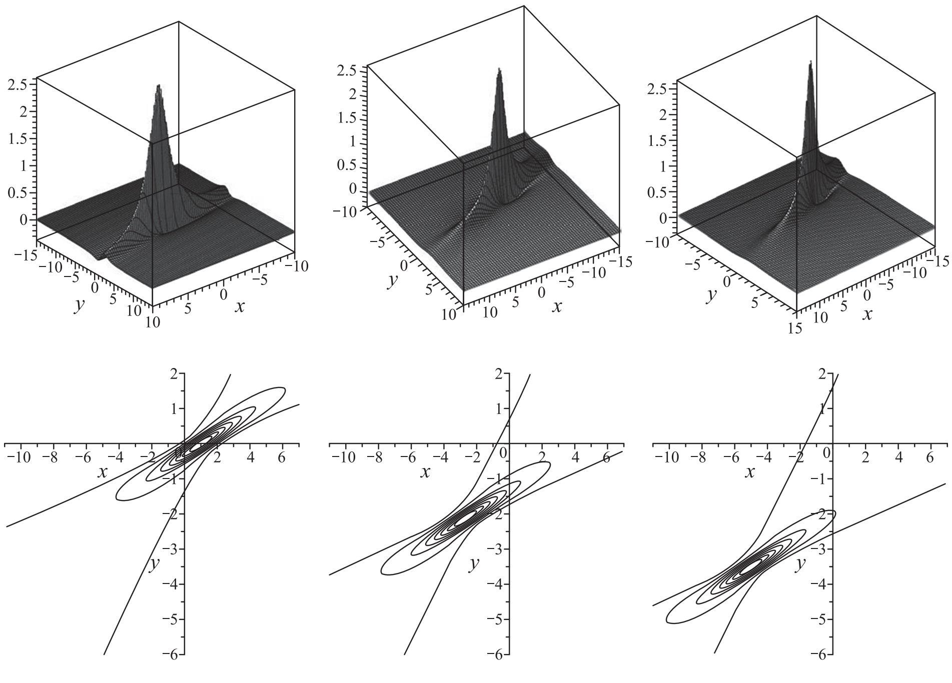

Fig. 1 Profiles of u when $t = 0$, 0.3, 0.5: 3d plots (top) and contour plots (bottom)

3 Concluding remarks

We have studied a (2 + 1)-dimensional generalized Bogoyavlensky-Konopelchenko (gBK) equation to exploit lump solutions, through symbolic computations with Maple. The result, enriching the theory of solitons, provides a new example of (2 + 1)-dimensional nonlinear integrable equations that possess lump solutions. Three-dimensional plots, contour plots, and t-, x-, and y-curves of a specially chosen solution were made by using plot tools in Maple.

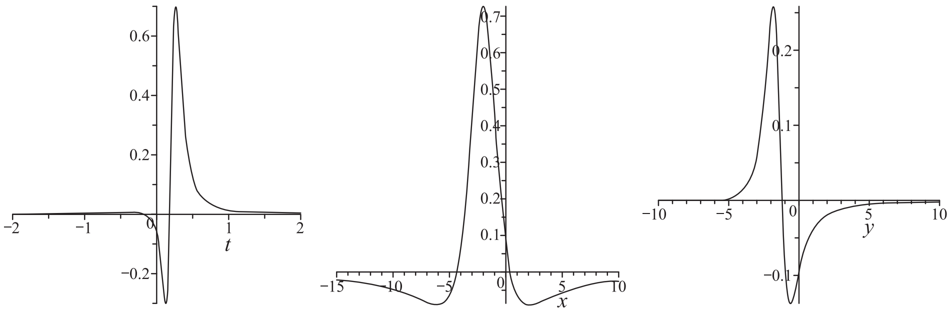

Fig. 2 Curves of u at $(x, y) = (1, -1)$, $(t, y) = (0, -1)$, and $(t, x) = (0, -5)$

On one hand, recent studies tell that many other nonlinear equations possess lump solutions, which include $(2+1)$-dimensional generalized KP, BKP, KP-Boussinesq, and Sawada-Kotera equations [18,19,26,29,47]. Abundant lump solutions provide valuable supplements to exact solutions generated from different kinds of combinations (see, e.g., [16,25,38,40,53]), and generate the corresponding Lie-Bäcklund symmetries, which might be helpful in determining conservation laws by symmetries and adjoint symmetries [9,24]. On the other hand, some more recent studies show that there exist interaction solutions between lumps and other kinds of exact solutions to nonlinear integrable equation in $(2+1)$-dimensions. They include lump-kink interaction solutions (see, e.g., [12,36,48,51]) and lump-soliton interaction solutions (see, e.g., [27,44–46]). In the $(3+1)$-dimensional case, lump-type solutions, which are rationally localized in almost all directions in space, were computed for the integrable Jimbo-Miwa equations. Various such solutions were worked out for the $(3+1)$-dimensional Jimbo-Miwa equation (see, e.g., [23,43,50]) and the $(3+1)$-dimensional Jimbo-Miwa like equation [7]. There are also Rossby wave solutions to generalized Boussinesq and Benjamin-Ono equations (see, e.g., [17,41]).

We point out that for the $(2+1)$-dimensional gBK equation (2.1), we can also find a set of traveling wave solutions with an arbitrary function:

$$u = 2(\log g(\xi))_{xx}, \quad v = 2(\log g(\xi))_x,$$
$$\xi = x - \frac{\alpha}{\beta}\, y - \frac{\alpha^2\gamma_3 - \alpha\beta\gamma_2 + \beta^2\gamma_1}{\beta^2}\, t + c,$$

where g is an arbitrary function and c is an arbitrary constant. Therefore, the gBK equation (2.1) can have various lump-type solutions as well. However, we failed to find any interaction solutions between lump or lump-type solutions and kink or soliton solutions for the $(2+1)$-dimensional gBK equation (2.1). We guess that the existence of such interaction solutions might strongly reflect complete integrability of the partial differential equations under consideration.

It is, of course, interesting to look for lump solutions and interaction solutions to partial differential equations in whatever dimensions. Conversely, the other interesting problem is to characterize partial differential equations, both linear and nonlinear, which could possess lump solutions and interaction solutions.

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