A Global Algorithm for AC Optimal Power Flow Based on Successive Linear Conic Optimization

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***Abstract*— Newly, there has been significant research interest in the exact solution of the AC optimal power flow (AC-OPF) problem. A semidefinite relaxation solves many OPF problems globally. However, the real problem exists in which the semidefinite relaxation fails to yield the global solution. The appropriation of relaxation for AC-OPF depends on the success or unfulfillment of the SDP relaxation. This paper demonstrates a quadratic AC- OPF problem with a single negative eigenvalue in objective function subject to linear and conic constraints. The proposed solution method for AC-OPF model covers the classical AC economic dispatch problem that is known to be NP-hard. In this paper, by combining successive linear conic optimization (SLCO), convex relaxation and line search technique, we present a global algorithm for AC-OPF which can locate a globally optimal solution to the underlying AC-OPF within given tolerance of global optimum solution via solving linear conic optimization problems. The proposed algorithm is examined on modified IEEE 6-bus test system. The promising numerical results are described.**

***Index Terms*—AC optimal power flow, successive linear conic optimization, convex relaxation, line search technique.**

I. Introduction

THE AC optimal power (AC OPF) problem is to determine the most efficient, least-cost operation of a power system by dispatching the available electricity generation resources to supply the system load while satisfying the operational constraints of available generation resources. Typical objectives are the minimization of losses or generation costs. The AC-OPF problem is generally non-convex due to the non-linear power flow equations [1] and may have local solutions [2], [3], which made different solution techniques an ongoing research topic. Recently, there has been substantial attention in a Semidefinite Program (SDP) relaxation of the AC-OPF problem [4]. The AC-OPF problem is reformulated as a convex semidefinite program by rank minimization or rank relaxation techniques. In a sense that, if the convex relaxed problem satisfies a rank condition (in general rank one) the relaxation is exact, and then the global solution to the original AC-OPF problem can be determined in polynomial solution time. Not only the most AC-OPF solution methods do not guarantee to find a global solution in polynomial time, but also the rank condition is not satisfied for all practical AC-OPF problems [2], [5]-[7]. There is substantial interest in developing new algorithmic solution methodology of the SDP relaxation by using the successive linear programming and convex relations methods.

The main motivation of this work comes from a perspective of algorithm design. Mathematically speaking, AC-OPF belongs to the class of non-convex quadratic optimization problems that has been a major concern in the optimization community for long.

The first contribution of this work is to develop a new global algorithm for AC-OPF based on several simple optimization techniques such as successive linear optimization for conic constraints and line search. To the best of our knowledge, this is the first time in the literature showing that such a hard non-convex AC-OPF can be solved effectively by using simple optimization techniques. We show that the new algorithm enjoys a complexity bound $O([g/\sqrt{\varepsilon}]\log[g/\sqrt{\varepsilon}])$, that the g is the solution upper and lower bounds of the algorithm of the negative eigenvalues terms in the objective function. It should be pointed out that the complexity bound $O(\cdot)$ in most of the applications in literature are the best-case estimate due to the usage of the discretization scheme, while the complexity bound in this work, $O([g/\sqrt{\varepsilon}]\log[g/\sqrt{\varepsilon}])$ is the worst-case estimate.

The second contribution of this paper is to propose a new successive linear optimization for conic constraints approach that includes several attractive properties that are not shared by the classical successive linear optimization approach.

This paper is organized as follows. Section II introduces the AC-OPF problem. Section III gives the solution methodology. Section IV presents the example AC-OPF problem. Section V concludes the paper.

II. AC-OPF Problem Formulation

Let us present a formulation of the OPF problem in terms of rectangular voltage coordinates and active and reactive power generation. Consider an n-bus power system, where $N=\{1,2,...,n\}$ the set of all buses and G is the set of generator buses. Let $P_{Dk}+jQ_{Dk}$ represent the active and reactive load demand at each bus $k \in N$. Let $V_k = V_{dk} + jV_{qk}$ represent the

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voltage phasors in rectangular coordinates at each bus $k \in N$. Superscripts "max" and "min" denote specified upper and lower limits. Buses without generators have the maximum and minimum generation set to zero (i.e., $P_{Gk,\max} = P_{Gk,\min} = Q_{Gk,\max} = Q_{Gk,\min} = 0 \quad \forall k \in N \setminus G$). Let $\mathbf{Y} = \mathbf{G} + j\mathbf{B}$, which denotes the network admittance matrix. The power flow equations of the power grid are given by the following the constraints:

$$P_{Gk} = V_{dk} \sum_{i=1}^{n} (G_{ik}V_{di} - B_{ik}V_{qi}) + V_{qk} \sum_{i=1}^{n} (B_{ik}V_{di} + G_{ik}V_{qi}) + P_{Dk} \quad \forall k \in N \qquad (1a)$$

$$Q_{Gk} = V_{qk} \sum_{i=1}^{n} (G_{ik}V_{di} - B_{ik}V_{qi}) - V_{dk} \sum_{i=1}^{n} (B_{ik}V_{di} + G_{ik}V_{qi}) + Q_{Dk} \quad \forall k \in N \qquad (1b)$$

The OPF problem considered in this paper is:

$$\min_{V_q, V_d} \quad \sum_{k \in G} c_{Gk} P_{Gk} \qquad (2a)$$

$$
\begin{aligned}
s.t. \quad & P_{Gk,\min} \le P_{Gk} \le P_{Gk,\max} && \forall k \in N \quad (2b) \\
& Q_{Gk,\min} \le Q_{Gk} \le Q_{Gk,\max} && \forall k \in N \quad (2c) \\
& | PL_{mn} | \le PL_{mn,\max} && \forall (m,n) \in L \quad (2d) \\
& (V_{k,\min})^2 \le V_{dk}^2 + V_{qk}^2 \le (V_{k,\max})^2 && \forall k \in N \quad (2e) \\
& V_{q1} = 0 && (2f)
\end{aligned}
$$

Where, $\overline{V}_k = V_{dk} + jV_{qk}$ is the complex voltage of the bus k expressed in Cartesian form. The c_{Gk} is the marginal cost of the generation unit $k \in G$. The constraint (2*f*) shows the phase angle zero of the slack bus in the AC-OPF model ($\overline{V}_1 = V_{d1} + j0 = V_{d1}\angle 0$). Let $\mathbf{e}_k$ denote the k^{th} standard basis vector. Define $Y_k = \mathbf{e}_k \mathbf{e}_k^T \mathbf{Y}$. Matrices employed in the bus power injection, voltage magnitude, and angle reference constraints are

$$\mathbf{Y}_k = \frac{1}{2} \begin{bmatrix} \mathrm{Re}(Y_k + Y_k^T) & \mathrm{Im}(Y_k^T - Y_k) \\ \mathrm{Im}(Y_k^T - Y_k) & \mathrm{Re}(Y_k + Y_k^T) \end{bmatrix} \qquad (2g)$$

$$\overline{\mathbf{Y}}_k = -\frac{1}{2} \begin{bmatrix} \mathrm{Im}(Y_k + Y_k^T) & \mathrm{Re}(Y_k - Y_k^T) \\ \mathrm{Re}(Y_k^T - Y_k) & \mathrm{Im}(Y_k + Y_k^T) \end{bmatrix} \qquad (2h)$$

$$\mathbf{Y}_{mn} = \frac{1}{2} \begin{bmatrix} \mathrm{Re}(Y_{mn} + Y_{mn}^T) & \mathrm{Im}(Y_{mn}^T - Y_{mn}) \\ \mathrm{Im}(Y_{mn}^T - Y_{mn}) & \mathrm{Re}(Y_{mn} + Y_{mn}^T) \end{bmatrix} \qquad (2i)$$

$$\mathbf{M}_k = \begin{bmatrix} \mathbf{e}_k \mathbf{e}_k^T & \mathbf{0} \\ \mathbf{0} & \mathbf{e}_k \mathbf{e}_k^T \end{bmatrix} \qquad (2j)$$

$$\mathbf{N}_k = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{e}_k \mathbf{e}_k^T \end{bmatrix} \qquad (2k)$$

$$\mathbf{M}_{mn} = \begin{bmatrix} (\mathbf{e}_m - \mathbf{e}_n)(\mathbf{e}_m - \mathbf{e}_n)^T & \mathbf{0} \\ \mathbf{0} & (\mathbf{e}_m - \mathbf{e}_n)(\mathbf{e}_m - \mathbf{e}_n)^T \end{bmatrix} \qquad (2l)$$

The transmission line-flow constraint is modeled in (2*d*). It should be noted that different line-flow limit formulations determine success or failure of the semidefinite relaxation [7].

In this paper, we consider the following quadratic AC-OPF problem with few negative eigenvalues and linear conic constraints defined as

$$\min \quad f(\mathbf{x}, \mathbf{X}) = \sum_{k \in \{G\}} c_k (\mathbf{x}^T \mathbf{Y}_k \mathbf{x}) \qquad (3a)$$

$$s.t. \quad (\mathbf{X}, \mathbf{x}) \in C$$

Where,

$$C \in \left\{ \begin{array}{ll} \mathrm{tr}(\mathbf{N}_1 \mathbf{X}) = 0 & (3b) \\ \mathbf{X} - \mathbf{x}^T \mathbf{x} \in \mathbf{S} & (3c) \\ P_{k\min} - P_{kD} \le \mathrm{tr}(\mathbf{Y}_k \mathbf{X}) \le P_{k\max} - P_{kD} & (3d) \\ Q_{k\min} - Q_{kD} \le \mathrm{tr}(\overline{\mathbf{Y}}_k \mathbf{X}) \le Q_{k\max} - Q_{kD} & (3e) \\ - PL_{mn,\max} \le \mathrm{tr}(\mathbf{Y}_{mn} \mathbf{X}) \le PL_{mn,\max} & (3f) \\ V_{k\min}^2 \le \mathrm{tr}(\mathbf{M}_k \mathbf{X}) \le V_{k\max}^2 & (3g) \\ \mathrm{tr}(\mathbf{N}_1 \mathbf{X}) = 0 & (3h) \\ \qquad \forall k \in N, \forall (m,n) \in L & \end{array} \right\}$$

Here, $\mathbf{S}$ denotes the cone of positive semidefinite matrices and $\mathbf{x}$ is the vector of the voltage components, and $\mathbf{X}$ is the rank-one matrix $\mathbf{X} = \mathbf{x}\mathbf{x}^T$.

$$\mathbf{x} = [V_{d1} \quad V_{d2} \quad \dots \quad V_{dn} \,|\, V_{q1} \quad V_{q2} \quad \dots \quad V_{qn}]^T \qquad (3i)$$

The active and reactive power injections at bus k are then given by $\mathbf{x}^T \mathbf{Y}_k \mathbf{x} = \mathrm{tr}(\mathbf{Y}_k \mathbf{X})$ and $\mathbf{x}^T \overline{\mathbf{Y}}_k \mathbf{x} = \mathrm{tr}(\overline{\mathbf{Y}}_k \mathbf{X})$, respectively, where tr indicates the matrix trace operator (i.e., sum of the diagonal elements). The square of the voltage magnitude at bus k and transmission line flow at line (m,n) are replaced by $\mathrm{tr}(\mathbf{M}_k \mathbf{X})$ and $\mathrm{tr}(\mathbf{Y}_{mn} \mathbf{X})$ respectively.

III. The Proposed Solution Methodology

The solution methodology is based on the recasting the objective function. By using the singular value decomposition (SVD) on the admittance matrix of the power grid, the positive and negative eigenvalues $\mathbf{Y}_k$ can be splitted out to two terms including the positive and negative quadratic functions. Indeed, the problem (3) can be recast to a particular difference convex quadratic problem as the following problem.

$$\min \quad f(\mathbf{x}, \mathbf{X}) = \sum_{k \in \{G\}} \mathrm{tr}(\mathbf{G}_k \mathbf{X}) - (\mathbf{c}_k^T \mathbf{x})^2 \qquad (4)$$

$$s.t. \quad (\mathbf{X}, \mathbf{x}) \in C$$

The matrixes $\mathbf{G}_k = c_k \mathbf{Y}_k$ and $\mathbf{c}_k$ are two semidefinite matrixes, where the first term in (4) indicates the positive eigenvalues of the matrix $\mathbf{Y}_k$ and the second term $\mathbf{c}_k$ specifies the negative eigenvalues of $\mathbf{Y}_k$.

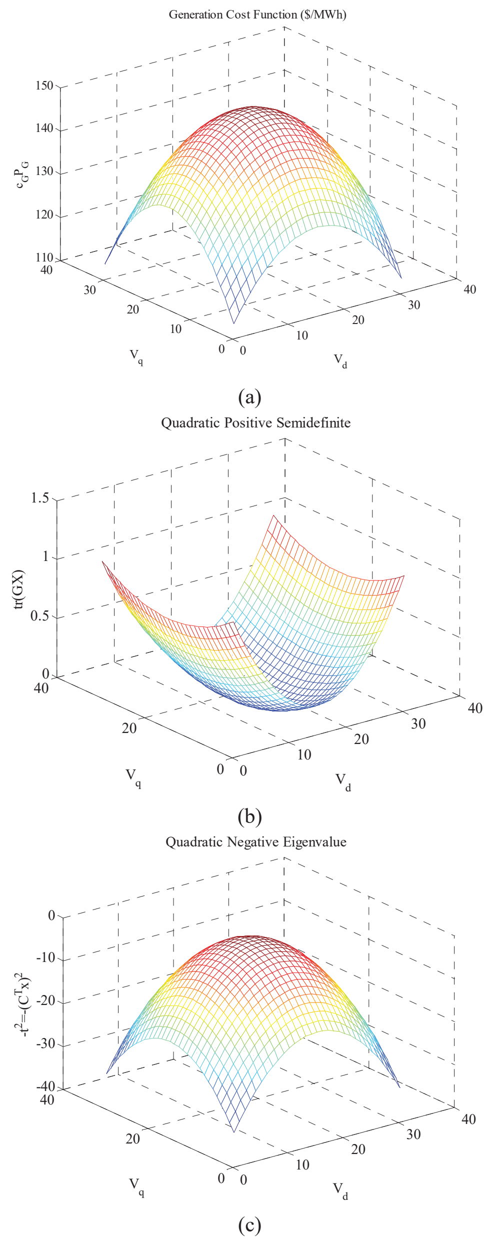

Fig. 1 Generation cost function (a) and its positive and negative quadratic terms (b) and (c)

The singular value decomposition of the $\mathbf{Y}_k$ is independed of the symmetric or asymmetric properties of the $\mathbf{Y}_k$, therefore the proposed methodology can be applied for any power grids even equipped with phase shifter transformers. Figure 1 shows the concave (non-convex) objective cost function of a simple generation unit in a two bus test system Fig. 1(a), and two decomposed positive and negative quadratic terms $\mathbf{x}^T\mathbf{G}_k\mathbf{x}$, and $(\mathbf{c}_k^T\mathbf{x})^2$ in Fig. 1(b), and (c).

To design the solution methodology of the proposed algorithm, we first observe that a globally optimal solution to a problem (4) can be located at a point $\mathbf{t}_k^* = \mathbf{c}_k^T\mathbf{x}^*$, where $\mathbf{x}^*$ is the $\mathbf{x}-$part optimal solution of the problem (4). Consequently, we can interpret problem (4) into the following lifted optimization problem (5),

$$\begin{aligned} \min_{\mathbf{X},\mathbf{x},\mathbf{t}} \quad & f(\mathbf{x},\mathbf{X}) = \sum_{k\in\{G\}} \mathrm{tr}(\mathbf{G}_k\mathbf{X}) - \mathbf{t}_k^2 \\ s.t. \quad & (\mathbf{X},\mathbf{x},\mathbf{t}) \in C_t \end{aligned} \tag{5}$$

where $\quad C_t = \{(\mathbf{X},\mathbf{x},\mathbf{t}) \,|\, (\mathbf{X},\mathbf{x}) \in C \cup \mathbf{t}_k \in [\mathbf{t}_{k,l},\mathbf{t}_{k,u}], \mathbf{t}_k = \mathbf{c}_k^T\mathbf{x}\}$ with the following lower bound and upper bound linear optimization problems:

$$\mathbf{t}_{k,l} = \mathbf{l}_k = \min_{(\mathbf{X},\mathbf{x})\in C} \mathbf{c}_k^T\mathbf{x} \qquad \mathbf{t}_{k,u} = \mathbf{u}_k = \max_{(\mathbf{X},\mathbf{x})\in C} \mathbf{c}_k^T\mathbf{x} \tag{6}$$

Since $-\mathbf{t}_k^2$ is the only quadratic term in the lifted problem then, the problems (1) and (5) are equivalent. We can use a linearized objective function to approximate the original quadratic function for a given $\mathbf{t}_k \in [\mathbf{t}_{k,l},\mathbf{t}_{k,u}]$. This result is given in the following linear approximation.

$$\begin{aligned} \min_{\mathbf{X},\mathbf{x},\mathbf{t}} \quad & f(\mathbf{x},\mathbf{X}) = \sum_{k\in\{G\}} \mathrm{tr}(\mathbf{G}_k\mathbf{X}) - 2\mathbf{t}_k\boldsymbol{\tau}_k \\ s.t. \quad & \boldsymbol{\tau}_k = \mathbf{c}_k^T\mathbf{x} \\ & (\mathbf{X},\mathbf{x}) \in C, \boldsymbol{\tau}_k \in [\mathbf{t}_{k,l},\mathbf{t}_{k,u}]. \end{aligned} \tag{7}$$

The linearized negative quadratic term of the generation cost function, with its upper and lower bound in Fig. 1 is shown in Fig. 2.

According to the solution to the problem (7), a successive linear optimization for conic constraints (SLCO) is developed to the problem (5) that updates $(\mathbf{X},\mathbf{x})$ and $\mathbf{t}_k$, alternatively. It should be emphasized that just only the negative quadratic term in the objective function is replaced by a linearized term in the proposed SLCO.

Our proposed method is very different from the classical SLO where all the quadratic terms are approximated by linearized functions [8].

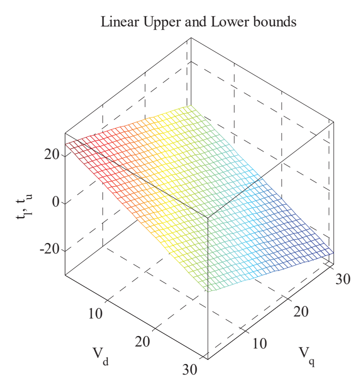

Fig.2 linear approximation of negative quadratic term, with its upper and lower bounds.

As we discuss in next section, such a change gives rise to several appealing properties liked by the sequence generated from the new SLCO. One of the main contributions of this paper is that the sequence generated by the new SLCO converges in a monotonic manner to a so-called approximate local optimal solution of problem (2), while the classical SLO can only provide a sequence converging to a stationary point the underlying problem [8].

To establish the global convergence of the new algorithm, we further rewrite problem (5) as the following global optimization problem with a single variable

$$\min_{\mathbf{t}_k \in [\mathbf{t}_{k,l}, \mathbf{t}_{k,u}]} \quad \sum_{k\in\{G\}} g(\mathbf{t}_k) = \sum_{k\in\{G\}} f(\mathbf{x}(\mathbf{t}_k), \mathbf{X}(\mathbf{t}_k)) \quad (8)$$

Where $(\mathbf{x}(\mathbf{t}_k), \mathbf{X}(\mathbf{t}_k))$ is an optimal solution to the problem (7). The properties of the problem (8) show that the generated $\mathbf{t}_k$ −sequence converges in a monotonic manner to a so-called semi-local minimum of $\sum_{k\in\{G\}} g(\mathbf{t}_k)$, a notion to be introduced later. Such a property allows us to run the new SLCO algorithm from $\mathbf{t}_{k,l}$ and $\mathbf{t}_{k,u}$ separately. Let $\mathbf{t}^*_{k,l}$ and $\mathbf{t}^*_{k,u}$ indicate the resultant accumulation points of the two sequences from these two runs, it is shown that there exists at least one global optimal solution $\mathbf{t}^*_k$ to problem (8) in the interval $[\mathbf{t}^*_{k,l}, \mathbf{t}^*_{k,u}]$. This denotes that we can use these two runs of the new SLCO algorithm to diminish the search space of $\mathbf{t}^*_k$ significantly.

To cope with the scenario $\mathbf{t}^*_{k,l} < \mathbf{t}^*_{k,u}$, a new line search procedure based on convex relaxation for the problem (2) that has a complexity bound $O(\log(\mathbf{t}_u - \mathbf{t}_l / \sqrt{\varepsilon}))$. One of the big advantages of the linear approximation in (7) is the polynomial linear search procedure of the SLO technique based on convex relaxation that can further cut the interval $[\mathbf{t}^*_{k,l}, \mathbf{t}^*_{k,u}]$ without missing a potential global optimal solution. By combining the new SLCO and the new line search technique, a new global algorithm is developed for AC-OPF, establish its convergence and estimate its complexity. This technique and required algorithm are presented in the subsection A and B.

A. A New Line Search Algorithm

A new line search procedure to find the global optimum solution is introduced in this part. To start, we consider a restricted version of the lifted problem (2) where the variable $\mathbf{t}_k$ is in a sub interval $[\mathbf{l}_k, \mathbf{u}_k]$. Let $\mathbf{s}_k = \mathbf{t}_k^2$ and $\mathbf{t}_k^2 \le (\mathbf{l}_k + \mathbf{u}_k)\mathbf{t}_k - \mathbf{l}_k \mathbf{u}_k$, the following convex relation could be derived,

$$\begin{aligned}
\min_{\mathbf{X},\mathbf{x},\mathbf{t},\mathbf{s}} \quad & f(\mathbf{x},\mathbf{X}) = \sum_{k\in\{G\}} \mathrm{tr}(\mathbf{G}_k \mathbf{X}) - \mathbf{s}_k \qquad (9) \\
s.t. \quad & \mathbf{t}_k = \mathbf{c}_k^T \mathbf{x} \\
& \mathbf{t}_k^2 \le \mathbf{s}_k,\ \mathbf{s}_k \le (\mathbf{l}_k + \mathbf{u}_k)\mathbf{t}_k - \mathbf{l}_k \mathbf{u}_k \\
& (\mathbf{X},\mathbf{x}) \in C,\ \mathbf{t}_k \in [\mathbf{l}_k, \mathbf{u}_k].
\end{aligned}$$

B. A new SLCO Method and Solution Algorithm

This section, a new successive linear optimization approach for conic constraints (SLCO) is proposed for the problem (8). The input of this procedure is the parameter $\mathbf{t}_k^{(0)}$ and stopping criterion $\varepsilon > 0$, and the output of this procedure at each iteration is the value of $(\mathbf{x}^{(n)}, \mathbf{X}^{(n)}, \mathbf{t}_k^{(n)})$. The procedure catches the following steps:

Step 01) set $n = 0$;

Step 02) Solve the problem (6) to find the upper and lower bounds: $\mathbf{l}_k$ and $\mathbf{u}_k$.

Step 03) Solve the problem (9), with $\mathbf{t}_k = \mathbf{t}_k^{(n)}$ for optimal solution results $(\mathbf{x}(\mathbf{t}_k^{(n)}), \mathbf{X}(\mathbf{t}_k^{(n)}))$. Set $\mathbf{x}^{(n+1)} = \mathbf{x}(\mathbf{t}_k^{(n)})$, $\mathbf{X}^{(n+1)} = \mathbf{X}(\mathbf{t}_k^{(n)})$ and $\mathbf{t}_k^{(n+1)} = \mathbf{c}_k^T \mathbf{x}^{(n+1)}$

Step 04) if $|\mathbf{t}_k^{(n+1)} - \mathbf{t}_k^{(n)}| > \sqrt{\varepsilon}$, then set $n = n+1$ and go back to step 02, otherwise stop, and output $(\mathbf{x}^{(n)}, \mathbf{X}^{(n)}, \mathbf{t}_k^{(n)})$ would be the final solution result.

IV. Simulation Results

The three-bus test system depicted in Fig. 3, is applied to show the efficiency of the proposed algorithm. The based value of the complex power is 100 MVA. For the sake of simplicity, we supposed that the generation unit at bus 3 is a synchronous condenser and generates the reactive power at bus 3. The active and reactive power outputs of generators 1, 2 and reactive power generation limits of the bus 3 have large with infinite limits. The generator real power cost coefficients for generators 1 and 2 are $c_1 = \$5/\mathrm{MWh}$ and $c_2 = \$1.2/\mathrm{MWh}$. The line and branch data are given in Table I. The voltage magnitudes at all buses are restricted to the range 0.9 (pu) to 1.1 (pu). First consider a line-flow limit of 60 MVA enforced on both ends of the line between bus 2 and bus 3. The other two lines 1-2 and 1-3 have no flow limits.

The following is a list of notations used in the tables:

- n_{qc} , n_{lc}, n and n_r stand for the number of quadratic, linear constraints, the variables, and the negative eigenvalue, respectively;
- “iter” indicates the average number of iterations of SLCO;
- "focal" symbolizes the average optimal value obtained by SLCO;

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- $t_{\min}$, $t_{\max}$ and t_{ave} stand for the minimum, maximum and average CPU time of SLCO in seconds, respectively;

We present computational results of the SLCO algorithm and SDP model for the AC-OPF problem. The algorithm is coded in Matlab R2015b and run on a PC (3.33GHz, 8GB RAM). All the convex quadratic subproblems in SLCO are solved by the QP solver in CPLEX 12.3 with Matlab interface. The SLCO algorithm yields a physically meaningful result, as evidenced by the final solution result of SDP methods in MATPOWER 5.1 [9] that matches the solution of the SDP formulation. The SDP model is adopted form reference [7]. The detail solution results and its comparisons are shown in Tables II and TABLE IV. The optimal objective values for both the proposed SLCO algorithm and SDP formulation in [7] are \$5871.45 and \$5707.07 per hour, respectively.

From Table IV, one can see that for three bus test system problem, both SLCO and SDP can find the global optimal solution. However, SLCO usually takes much longer time than the SDP. The maximum solution CPU time is 40.56 (s) and compare to the solution time of SDP; it is increased 10%. We also observed that the CPU time of SDP increases very fast as the size of the test problem grows, in compare to the solution time of SLCO.

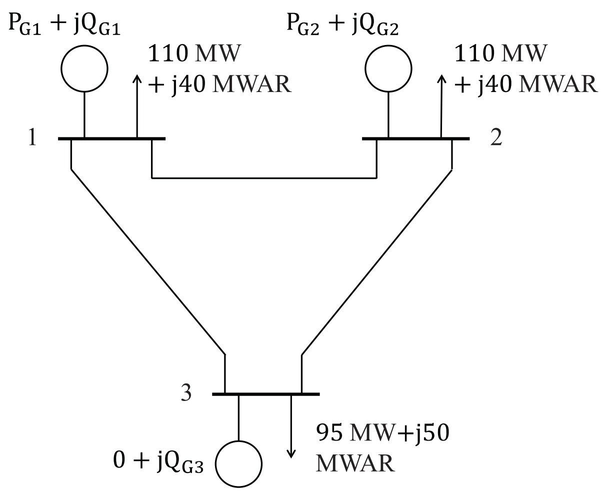

Fig. 3 Three bus test system

TABLE I
Three-bus Test System Raw Data

From bus	To bus	R(pu)	X(pu)	B(pu)	PLmax(MW)
1	3	0.065	0.62	0.45	inf
3	2	0.025	0.75	0.7	60
1	2	0.042	0.9	0.3	inf

TABLE II
Final Numerical Results: Comparison of SLCO and [7]

	Bus 1		Bus 2		Bus 3	
	SLCO	SDP [7]	SLCO	SDP [7]	SLCO	SDP [7]
|V| (pu)	1.072	1.069	1.019	1.028	1.008	1.001
Angle (Degrees)	0	0	9.92	9.916	-13.57	-13.561
P_G (MW)	131.1	131.9	185.85	185.93	0	0
Q_G (MVAR)	20.32	17.02	-8.5	-3.5	0.15	0.06

TABLE III
Final Numerical Results of Line Flow:
Comparison of SLCO and [7]

		From MVA		To MVA	
From	To	SLCO	SDP [7]	SLCO	SDP [7]
1	3	44.8	43.9	50.1	47.47
3	2	60	60	60	60
1	2	23.52	22.72	30.92	28.69

TABLE IV
Average Numerical Results of SLCO

n_{qc}	n_{lc}	n	fval	t_{min}	t_{max}	t_{ave}
14	13	19	5871.45	5.33	40.56	10.35

V. Conclusion

In this paper, we considered the AC-OPF problem that arises from various disciplines and known to be NP-hard. By combining the classical linear approximation, modern convex relaxation, and new line search technique, we developed a global algorithm to find the global optimal solution of the AC-OPF problem. We established the global convergence of the proposed algorithm and estimated its complexity. Preliminary experiments illustrated that the new SLCO algorithm can effectively find the global optimal solution for test case problem described in this paper.

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