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A novel two-stage optimization approach to machining process selection using error equivalence method

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A R T I C L E I N F O

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A B S T R A C T

The selection of a machining process involves the choice of machine tools, fixture elements, and fixture locator layout, as well as the allocation of tolerance in each operation. In practice, manufacturers frequently choose identical machine tools and fixture elements for each operation to reduce purchase cost. As such, fixture layout and tolerance allocation are critical in selecting or designing appropriate manufacturing processes. Conventional research deals with robust fixture layout design and simultaneous tolerance allocation for multiple types of error source separately. However, fixture layout design could also affect tolerance stackup caused by multiple error (not only the fixture error) sources. Therefore, considering the interaction between fixture layout and other types of error source is critical in the process selection to improve the process selection strategy. In this paper, a two-stage framework is proposed to optimize the process selection based on our previously developed error equivalence model, which transforms multiple errors into equivalent errors that occur on a fixture. In the first stage, a process is selected by determining the allowable tolerance for an aggregated base error given a fixture layout. In the second stage, a computer experiment model is established to search for the globally optimal fixture layout by exploring a large number of fixture layout alternatives. A real-world case study based on a two-operation machining process demonstrated the effectiveness of the proposed strategy in controlling manufacturing cost while ensuring product quality via proper fixture layout design.

1. Introduction

Machining process selection involves choosing machine tools, fixture elements, and fixture locator layout, as well as allocating tolerance for multiple tools in each manufacturing operation. Given that manufacturers frequently select identical machine tools and fixture elements to obtain increased discounts from vendors, the process selection can be simplified into two problems, namely, fixture layout and process tolerance design.

Fixture layout design involves the automated generation of a robust fixture layout that reduces the influence of fixture variation on product quality. The early approaches to fixture layout design are deterministic [1] because they do not consider fixture variations, such as those caused by worn or loose locating pins. Recent research on fixture layout design investigates the robustness of fixtures [2–4] with optimization aiming to minimize the sensitivity of the fixture layout. The optimal fixture layout design for multi-operation assembly processes was investigated by Kim et al. [5]. A fixture layout design for a machining process was developed by Huang and Shi [6] by using a 2D case study. Fixture layout optimization was also addressed by using metaheuristics, such as evolutionary techniques [7] and augmented ant colony algorithm [8]. These studies on fixture layout design mostly focused on kinematic analysis to reduce potential quality problems induced by fixture errors (e.g., clamping and locator inaccuracies).

Process tolerance design involves estimating tolerance stackup and allocating tolerances corresponding to multiple error sources to ensure quality or robustness at reasonable manufacturing cost. In this line of research, the focus is on the simultaneous optimization of assigning candidate processes to operations or parts, allocating process or product tolerances, and/or designing process parameters. Nagarwala et al. [9] solved the tolerance design problem in process selection by using a slope-based approach, which exhibits high computational efficiency.

Abbreviations: EFE, Equivalent fixture errors; NP, Non-deterministic Polynomial; FDM, Fused deposition modeling; CAD, Computer Aided Design; LH, Latin Hypercube; RMSE, Root-mean-square error; MSE, Mean square error

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Nomenclature

σ_γ	Standard deviation of cutter path orientation
σ_θ	Standard deviation of part orientation due to fixture error
$\mathbf{x}_i$	Error source i
$\mathbf{s}_i$	The ith candidate layout in the explored design space for LH sampling in computer experiments
$\mathbf{w}$	The fixture layout in the unexplored design space
$\mathbf{w}^*$	The globally optimal fixture layout
$\mathbf{K}_i$	Transformation matrix in EFE.
$\mathbf{\Gamma}_j$	Mapping matrix that reflects the impact of process errors on the jth quality feature (also called a sensitivity matrix)
$\mathbf{y}_j$	The jth quality feature deviation
$\mathbf{u}(k)$	The aggregated process error at the kth operation
$\boldsymbol{\varepsilon}(k)$	Noise term at the kth operation
$\mathbf{I}$	Identity diagonal matrix
$\Sigma_{\mathrm{u}(k)}$	Covariance matrix for the process error $\mathbf{u}(k)$
$\Sigma_{\mathrm{y}_j(k)}$	Covariance matrix for the deviation of feature j
$\mathbf{\Theta}$	A vector of the process errors for tolerance allocation
$\sigma\Theta$	The standard deviations of process errors $\mathbf{\Theta}$ (process tolerance)
$\mathbf{C}^T$	The coefficient matrix
$\mathbf{c}$	A row vector in matrix $\mathbf{C}$ with the highest dimension to compute overall cost
$\mathbf{b}_1$	The upper bound of the variation components of surfaces or dimensions
$\mathbf{b}_2$	The upper bound of tooling variations
$\mathbf{F}^c$	The reaction force between the workpiece and the fixture locator
$\mathbf{f}_i$	The positions of the fixture locators
$\mathrm{Y}(\mathbf{w})$	The response given the input $\mathbf{w}$
$Z(\cdot)$	Zero-mean Gaussian
$R(\cdot,\cdot)$	Correlation function between the responses
D	The number of the design variables
$\mathbf{Y}_j$	The quality features j

Qin et al. [10] proposed a unified point-by-point planning algorithm for machining fixture layout by considering practical degrees of freedom and determining the location of the locating pins. A tolerance design approach based on the Shapley value method was developed by considering the demands of manufacturing cost and product quality [11]. Other approaches developed for process selection considering tolerance design include functional group approach [12], exhaustive search methods [13,14], simulated annealing [15], genetic algorithms [16], and artificial intelligence techniques [17]. Optimization problems in previous studies target a certain manufacturing cost as their sole objective function. Wang and Liang [18] developed a dual-objective optimization approach to simultaneously assigning processes to operations, determining machining parameters, and designing product dimensional tolerances. Andolfatto et al. [19] proposed to allocate geometrical tolerances by solving a multi-objective optimization problem, aiming to minimize the cost and the nonconformity of the assembly plan. Comprehensive reviews of the tolerancing strategy for process selection were also conducted [20,21].

Most studies on process selection separately investigate the problems of fixture layout optimization and process tolerance design, and do not directly consider the influence of the fixture layout on the allocation of tolerance for multiple error sources. For example, tolerance allocation is optimized in a fixed fixture layout only. Such studies are only reasonable when multiple error sources are independent of each other.

This study provides additional insights into process selection by considering the interactions between the fixture layout and the tolerance allocation for non-fixture errors. The significance of such interactions is illustrated in Fig. 1, in which a 2D prismatic part is installed on a fixture and milled on the top surface. For simplicity of illustration, we assume that the machine tool error occurs rotating around the z direction (denoted by angle γ). In fixture layout 1 (Layout 1 in Fig. 1[a]), in which two locating pins are close to each other, the variation in the top surface is more sensitive to the fixture error. Thus, a tight tolerance ($\sigma_{\gamma 1}$) for the machine tool error is required to ensure product tolerance (i.e., thickness). By contrast, only a loose tolerance ($\sigma_{\gamma 2}$) is necessary for the machine tool error in fixture layout 2 (Layout2 in Fig. 1[b]) because the influence of the fixture error on the surface variation is small. Process selection can be improved by considering the mechanism by which the fixture layout affects the tolerance allocation for multiple error sources. Therefore, properly identifying the link between fixture errors and other types of error sources is important for tolerance stackup modeling.

Only limited research has been conducted on the joint optimization of fixture layout and tolerance design. For instance, Li [22] implemented a dual-objective optimization problem to obtain a robust fixture layout while minimizing the cost associated with the tolerances caused by fixture errors. A study on tolerance synthesis showed that fixture layout might have a significant influence on tolerance stackup because of the fixture, machine tool, and datum errors [6]. In the said study [6], a two-step optimization procedure was proposed, and a sensitivity analysis was conducted by using a 2D example; however, the fixture layout was not optimized.

Through a review of the literature, the following research gaps are identified:

- Studies on the relationship between the problems of fixture layout and process tolerance allocation are few. As mentioned, the fixture layout significantly affects the process tolerance allocation. However, prior research studied the two problems separately. For example, tolerance allocation was optimized in a fixed fixture layout only. Thus, the potential to improve the optimization process further is missing.
- A comprehensive understanding of how different types of process error interact to affect the process tolerance allocation problem is lacking. Prior research deals with the tolerance allocation for multiple types of process errors, such as machine tool, fixture, and datum errors, independently without considering their interaction effects on the process selection problem.

This paper is the first to provide insights into the relationship between the two problems based on an error equivalence model and present a method for jointly optimizing process tolerance and fixture layout design for process selection. Considering an error equivalence mechanism by which multiple types of error source result in identical

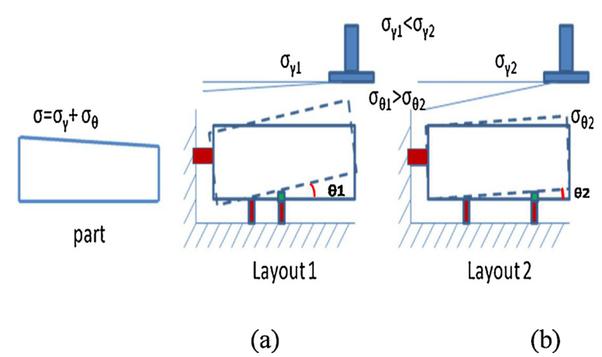

Fig. 1. Impact of fixture layout on tolerance allocation for a machine tool.

part variations and can be transformed into a fixture error [23,24], this study proposes using the fixture as a design parameter to control the tolerance stackup caused by multiple types of error source. In this manner, the tolerances for multiple error sources are aggregated into those for fixture errors to reduce the design space for optimization. A two-stage process selection problem for joint fixture layout design and tolerance allocation is formulated. In the first stage, the tolerance allocation is determined given a fixture layout. In the second stage, the optimal fixture layout is identified. A sequential computer experiment model is developed to solve this NP optimization problem without relying on exhaustive solution search. Unlike the traditional process tolerance allocation, which depends on a fixed fixture layout only, the proposed method explores the candidate fixture layouts in the design space thoroughly.

Following this introduction, Section 2 develops an error equivalence-based simultaneous tolerance synthesis and optimal fixture layout design. Section 3 demonstrates the proposed method by using a multi-operation machining process. A sensitivity analysis is also conducted to evaluate the robustness of the process tolerance design. Section 4 provides the conclusions of this study.

2. Error equivalence-based process selection method

Given the tolerance for certain product features, the process selection in this study involves determining the fixture layout for six locating pins (under the 3-2-1 locating scheme) and the manufacturing tolerances for the pins $\mathbf{x}_1$ (fixture error), datum surface $\mathbf{x}_2$ (datum error), and machine tool path $\mathbf{x}_3$ (machine tool error) by considering the interactions among these variation sources.

The two-stage process selection is developed to solve the problem, as shown in Fig. 2. First, a set of fixture layouts or *design variables* $\mathbf{s}_i$ are generated as the starting points in the space-filling method. The total quantity of the candidate layouts is denoted by n. In stage 1 selection, for a given fixture layout $\mathbf{s}_i$, the multiple types of error source are transformed into equivalent amounts of fixture errors Y_i [23], and the optimal tolerance allocation procedure for fixture error is implemented (Section 2.1). The tolerance that yields the lowest cost is allocated to the aggregated fixture errors at each manufacturing operation. Error equivalence modeling and tolerance allocation are implemented for all design variables $\mathbf{s}_i$. In stage 2 selection, the cost associated with the process tolerance, along with the corresponding design variables, is used to train a computer experiment model to determine the globally optimal fixture layout $\mathbf{w}^*$ (Section 2.2). This method allows for the investigation of how different fixture layouts (generated in stage 2 selection) impact the performance of the tolerance allocation (determined in stage 1 selection) by deriving a metamodel or surrogate model (Kriging model) based on the results of the first-stage (input) versus the second-stage (output) designs. Stage 1 selection plays the role of a simulation model (although it is not the traditional simulation).

2.1. Modeling and tolerance allocation of the equivalent error source given a fixture layout

This section discusses the tolerance allocation strategy, which considers the interactions between multiple types of error source (i.e., error equivalence). An optimization algorithm is used to generate the responses to the corresponding design variables $\mathbf{s}_i$.

(1) Error equivalence-based process modeling

Two types of error source are considered equivalent if they generate similar dimensional deviation. Thus, equivalent error sources at each manufacturing operation can be aggregated when feature deviations are predicted. Specifically, multiple types of error $\mathbf{x}_i$ can be transformed into an equivalent amount of the same type of error source, called base error, through a linear transformation: $x_i^* = \mathbf{K}_i\mathbf{x}_i$, $i = 1, 2,\ldots, m$ (the transformation matrix $\mathbf{K}_i$ is shown in Appendix A). For a machining process, the machine tool and datum errors can be transformed into a fixture error. In this study, the fixture error is also named the "base error" because it provides a reference to evaluate other types of error source and can be conveniently monitored or controlled. Given that the error from the fixture is convenient to control and monitor, the fixture error is selected as the base error in this study, and the linear transformation matrix $\mathbf{K}_i$ is used to transform different types of errors into equivalent fixture errors (EFE). Thus, the dimensional deviation of the product can be predicted by [23]

$$\mathbf{y}_j(k) = \mathbf{\Gamma}_j\mathbf{u}(k) + \varepsilon(k), \tag{1}$$

where $\mathbf{\Gamma}_j$ is the sensitivity matrix, which aggregates the coefficients of the quality prediction model given certain process errors and is determined based on the process and product design. $\mathbf{y}_j(k)$ denotes the jth quality feature deviation caused by the base error $\mathrm{u}(k) = \sum_i \mathrm{x}_i^*(k)$ and process noise $\varepsilon(k)$ at the kth operation; it is also called a sensitivity matrix because it reflects how the jth quality feature change is sensitive to any unit change in fixture errors (e.g., worn locator). A high sensitivity indicates that the process quality is prone to be affected by small process variations. Notably, in the traditional prediction model of process errors, the right side of Eq. (1) contains not only an aggregated error equivalence vector in each operation but also a vector that consists of different individual error sources, for example, $\mathbf{y}_j(k) = \mathbf{\Gamma}_j[\mathbf{x}_1(k)|\mathbf{x}_2(k)|\mathbf{x}_3(k)]^T + \varepsilon(k)$, where $[\mathbf{x}_1(k)|\mathbf{x}_2(k)|\mathbf{x}_3(k)]$ represent the machine tool, datum, and fixture errors, respectively. Through the aggregation of the errors from different sources, we can focus on the machining process with the base error only, thereby significantly reducing the design variables in the process selection. Multivariate normal distributions can be assumed to characterize the base error $\mathbf{u}(k)$ and noise $\varepsilon(k)$.

The model dimensions may also be reduced by studying the linear dependency of the columns in matrix $\mathbf{\Gamma}_j$ through, for example, diagnosability analysis [25]. The error equivalence method is adopted for the following reasons: (1) the machining process generally has multiple types of error, and (2) the machining process modeling directly captures the kinematic relationship between multiple types of error. The method is useful for process control, such as the equivalent error compensation (Wang et al. [24]).

For feature j, the variance-covariance matrix is estimated by using Eq. (1) as follows:

$$(\Sigma_{\mathrm{y}_j(k)}) = \Gamma_j(\Sigma_{\mathrm{u}(k)})\Gamma_j^T + \sigma_\varepsilon^2\mathrm{I} \tag{2}$$

where $\Sigma_{\mathrm{u}(k)}$ and $\Sigma_{\mathrm{y}_j(k)}$ are the variance-covariance matrices of the process errors and deviations of feature j, respectively. Since $\mathrm{diag}(\mathbf{\Sigma}_{\mathbf{y}_j(k)})$ reflects the tolerance, the tolerance stackup can be found from the diagonal term $\mathrm{diag}(\mathbf{\Sigma}_{\mathbf{y}_j(k)})$.

(2) Error equivalence-based simultaneous optimal tolerance allocation

The tolerance allocation aims to assign tolerances to process errors to meet certain specification at the lowest manufacturing cost. Here, $\mathbf{\Theta}$ represents a vector of process errors $(\mathbf{u}(1)^T,\ldots, \mathbf{u}(k)^T)^T$, and their standard deviations are denoted by $\boldsymbol{\sigma}_\Theta = (\boldsymbol{\sigma}_{\mathbf{u}(1)},\ldots, \boldsymbol{\sigma}_{\mathbf{u}(k)})^T$. Given that a

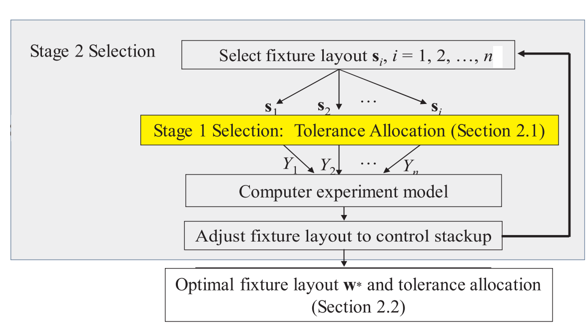

Fig. 2. Two-stage process selection using the error equivalence concept.

large process tolerance can reduce the manufacturing cost, $\max\boldsymbol{\sigma}_\Theta$ can be used as the objective [6]. The variance of the surfaces to be machined at each operation can be represented by a set of linear combinations of σ_Θ^2, denoted by $C^T\sigma_\Theta^2$. It can be derived by extracting the diagonal terms in the variance-covariance matrix $\boldsymbol{\Sigma}_{\mathbf{y}_j}$ [6], which is estimated by Eq. (2). The objective function, that is, the cost of achieving the specified process tolerance, can be used to maximize the process tolerance estimated as a linear combination of $\boldsymbol{\sigma}_\Theta$, denoted by $c^T\sigma_\Theta^2$. Here, $\mathbf{c}$ is the row vector with the maximum dimension in matrix $\mathbf{C}$, that is, $\mathbf{c} = \max[\mathrm{Dim}(\mathbf{c}_i)]$, where $i = 1, 2, 3,\ldots, m$ (where m is the number of rows in $\mathbf{C}$) and Dim() stands for dimensionality [6]. Therefore, the error equivalence-based simultaneous tolerance allocation is formulated as follows:

$$
\begin{aligned}
s.\,t \quad & \mathrm{Max}\,\boldsymbol{c}^T\boldsymbol{\sigma}_\Theta^2,\\
& \boldsymbol{C}^T\boldsymbol{\sigma}_\Theta^2 \le \boldsymbol{b}_1, \text{constraints from the specification},\\
& \mathbf{0} < \boldsymbol{\sigma}_\Theta \le \boldsymbol{b}_2, \text{the tooling constraints},\\
& \mathbf{F}^c > 0, \text{ the load equilibrium condition},
\end{aligned} \tag{3}
$$

where $\mathbf{b}_1$ denotes the upper bound of the variation components of the surfaces or dimensions. The values of $\mathbf{b}_2$ constrain the tool variations. $\mathbf{F}^c$ is the reaction force from the workpiece against the locator at the contact point between the workpiece and the fixture locator, and it is related to the clamping forces and the workpiece shape at that contact point [2]. The equilibrium condition ensures that the workpiece contacts with the locating pins [2]. The tolerances can be first assigned to the aggregated base errors and then further distributed to other types of errors by using the error equivalence relationship. The rationale of the formulation in Eq. (3) can be explained as follows: the maximization of process tolerance is equivalent to the minimization of manufacturing cost due to the process tolerance design. A tight tolerance designed for process errors can result in a significantly high investment cost for manufacturing processes (e.g., an expensive high-precision machine tool may be selected to achieve the target quality feature). However, if the final manufactured quality is not sensitive to certain process errors in certain directions, then the assignment of a tight tolerance is a waste of resources. Manufacturers always prefer to loosen the process tolerance as much as possible to reduce the potential cost while still ensuring that the output quality remains within the product tolerance.

The parameters $\mathbf{c}$, $\mathbf{C}$, $\mathbf{b}_1$, and $\mathbf{b}_2$ are assumed to be known in the objectives and constraints. The simultaneous change of these parameters is not within the original scope of this study. However, we can envision a solution by treating these parameters as decision variables in addition to the fixture layout in stage 1 selection and then conduct computer experiments to explore different fixture layouts and parameters ($\mathbf{c}$, $\mathbf{C}$, $\mathbf{b}_1$, and $\mathbf{b}_2$) to find the optimal fixture layout.

2.2. Error equivalence-based optimal fixture layout design

This section discusses the influence of the fixture layout on the optimal tolerance allocation (discussed in Section 2.1) for multiple types of error source and the identification of the globally optimal layout to minimize the manufacturing cost associated with process tolerance. We let the process tolerance design variables be the positions of the fixture locators (i.e., $\mathbf{f}_i$). The two-stage algorithm improves the fixture layout design by exploring the values of the parameters mentioned in Section 2.1 given different candidate fixture layouts and identifying the optimal fixture layout that leads to the minimal tolerance cost (response). Computer experiments are conducted to examine the response surface of max $\mathbf{c}^T\sigma_\Theta^2$ for different fixture layout alternatives. The computer experiment is chosen because tolerance synthesis involves extensive symbolic computation when examining all possible fixture layouts and an exhaustive search method is usually infeasible. In addition, the lack of random errors in the computational tolerance synthesis renders the computer experiment method preferable to the traditional regression analysis. The computer experiment method can approximate the optimal global solution by preventing the solution from being trapped in the local optima. Nevertheless, this method is not the only approach to solving the given two-stage decision problem. Various metaheuristic methods, such as evolutionary algorithms, may also be applicable. The computer experiment method is chosen for this particular problem because it can explicitly reveal the impact of the equivalent fixture on the cost of process tolerance allocation (response), thus providing in-depth insights into the proposed equivalent fixture concept. This method also allows for a sequential search strategy by refining more design points than other methods can to ensure the efficient exploration of the search space where the root-mean-square error (RMSE) of the response is large.

Remark 1: In addition to the tolerance cost, the fixture layout has one more objective to satisfy, namely, the sensitivity index, which is the ratio between the sum of the squares of the product deviations and that of the aggregated fixture errors, that is, $\frac{(\Gamma\mathbf{u})^T(\Gamma\mathbf{u})}{\mathbf{u}^T\mathbf{u}}$. A similar dual-objective optimization problem was solved by Li (2006) for fixture layout and fixture tolerance design. The solution is beyond the scope of this study and is not discussed in detail.

The optimal fixture layout can be obtained through the computer experiment model or Kriging model [26–28], which generally depicts the relationship between the fixture layout variables and max $\mathbf{c}^T\sigma_\Theta^2$. The model consists of a trend $\mathbf{g}^T(\mathbf{w})\boldsymbol{\beta}$ and a stochastic process $Z(\mathbf{w})$ and is expressed as follows:

$$
Y(\mathbf{w}) = \mathbf{g}^T(\mathbf{w})\boldsymbol{\beta} + Z(\mathbf{w}), \tag{4}
$$

where $Y(\mathbf{w})$ is the response (which, represents max $\mathbf{c}^T\sigma_\Theta^2$ in this study) given the input $\mathbf{w} = (w_1,\ldots, w_d)$, where d is the quantity of the design variables. The input $\mathbf{w}$ is for the fixture layout in the unexplored design space, which contains the coordinates of all the fixture locators as $\mathbf{s}_i$. The responses in layouts $\mathbf{s}_i$ are all tested or explored by the following computer experiments, whereas the response in layout $\mathbf{w}$ should be predicted by the established computer experiment model. The stochastic term $Z(\cdot)$ is a zero-mean Gaussian variable with a covariance $Cov(\mathbf{w}_1, \mathbf{w}_2) = \sigma^2 R(\mathbf{w}_1, \mathbf{w}_2)$, where $R(\mathbf{w}_1, \mathbf{w}_2)$ is a correlation function between the responses in the two inputs $\mathbf{w}_1$ and $\mathbf{w}_2$, and σ^2 is the standard error, which can be estimated using Eq. (B3) in Appendix B. Appendix B also presents the details of the computer experiment model.

Before the optimal fixture layout can be obtained, we need to first estimate the structure of $\mathbf{g}^T(\mathbf{w})\boldsymbol{\beta}$ and the stochastic process $Z(\mathbf{w})$. From [27], highly elaborated polynomial functions exhibit few advantages in modeling. Thus, a constant β was first selected for $\mathbf{g}^T(\mathbf{w})\boldsymbol{\beta}$. In addition, a power exponential correlation function, which is well known in the literature on computer experiments, is chosen. The correlation function is determined by the multiplication of correlations as $R(\mathbf{w}_1, \mathbf{w}_2) = \prod_{j=1}^{d}\exp(-\theta_j|w_{1j} - w_{2j}|^{p})$, where $0 \le p_j \le 2$ and $\theta_j \ge 0$. Here, $p_j = 2$ is selected because the correlation therewith leads to smooth stochastic processes [28]. For instance, a two-operation machining process has a total of $d = 12$ design variables. In summary, the unknown model parameters include the constant β, the stochastic process variance σ^2, and $\boldsymbol{\theta} = (\theta_1,\ldots, \theta_{12})$.

The design of experiments should be representative, that is, covering the entire design space, and can provide precise responses at the inputs of interest (fixture layout that can effectively reduce tolerance stackup) to construct an appropriate computer experiment model. A sequential procedure [29,30] is adopted to search such a design space. The procedure is explained as follows:

2.2.1. Initial design

The initial designs are obtained by using a Latin hypercube (LH) sampling method to ensure that the initial design spreads over the entire design space. Introduced by McKay et al. [31], LH sampling is one of the most prevalently used space-filling methods. It is the generalization of a Latin square to an arbitrary number of dimensions. In this method, each sample is the only one in each axis-aligned hyperplane that contains it. The procedure of LH includes the following steps: (1) the range of each decision variable is divided into multiple equal intervals, and (2) candidate fixture layouts (samples) that can meet the requirement of the LH are selected. For each component of the input site w_j, a uniform distribution can be assumed. In practice, the initial design can sometimes be obtained by experienced practitioners, whose expertise may further reduce the effort in searching the design space.

The guidelines for choosing the initial layout are to (1) utilize prior engineering knowledge to refine the selection of initial solutions and/or (2) generate the initial solutions that can representatively fill the design space by using the LH sampling method if prior knowledge is lacking.

2.2.2. Design and model refinement

The initial design can be used as input to the tolerance allocation strategy to obtain the optimal tolerance setup with the lowest tolerance cost in that fixture layout. For a high-dimensional case, the initial model may be unable to predict the true tolerance responses well at the unexplored input sites and must be improved. Given the predictor $\hat{y}(\mathrm{w})$ expressed in Eq. (B4) in Appendix B, the predictor RMSE $RMSE(\hat{y}(\mathrm{w}))$, which is the square root of Eq. (B5), is first calculated at several tested input sites. If the RMSE is large, then these sites should be included in the design space. The untested sites can be selected using the LH method following a maximin criterion. The maximin criterion ensures that no testing inputs are very close, such that all the experiment points to be tested can representatively cover the design space. Notably, Gupta et al. [32] developed a zoom-in criterion to refine the computer experiment model by employing a contour plot to estimate the distribution of the MSE over the entire design space. More design points will be added to the areas on the contour plot with considerably large MSEs. In this study, the RMSE is estimated based on a set of LH design sites at each iteration, without using a contour plot, for the following reasons: (1) the contour plot cannot efficiently cover the design space with high dimensions, and (2) the maximin-LH design-based test point selection can efficiently cover the entire design space. The iterative model refinement steps are summarized as follows:

Step 1: Estimate the parameters of the computer experiment model. At the ith iteration, the model is constructed based on the n_i designs $\mathbf{S} = \{\mathbf{s}_1, \ldots, \mathbf{s}_{n_i}\}$ with response data $\mathbf{y_s} = \{y_1, \ldots, y_{n_i}\}$, that is, $\hat{y}(\mathbf{w}) = \hat{\beta} + \mathbf{r}^T(\mathbf{w})_{n_i \times n_i}\ \mathrm{R}^{-1}_{n_i \times n_i}\ (\mathbf{y_s} - e_{n_i}\ \hat{\beta})$, where e_{n_i} is the all-in-one vector of length n_i, $i = 0, 1, 2, \ldots$

Step 2: Improve the modeling with more inputs. The RMSEs at the test points generated using the maximin–LH design are calculated. The points that yield large RMSEs are added to the experimental design $\mathbf{S}$.

Step 3: Set $i \longleftarrow i + 1$, and return to steps (1) and (2).

Step 4: Stop the refinement when the maximum of $\hat{y}$ does not exhibit significant changes between consecutive iterations and the RMSE is sufficiently small.

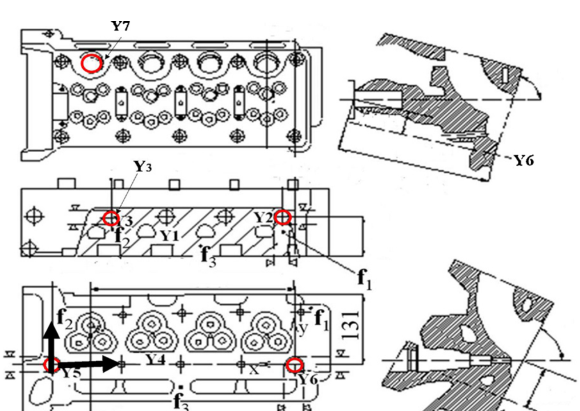

Fig. 3. Workpiece and locating scheme [23].

3. Case study

This section demonstrates the proposed error equivalence-based process selection by using a two-operation machining process. The two-operation example can be conveniently generalized to a multi-operation case. The generalization is enabled by the multi-operation modeling of variation propagation, which was developed in our prior study [23]. Given that the EFE can estimate the datum error, which can be transmitted from one operation to another, we can estimate the influence of the EFE in the upstream operations on the variation generated in the downstream operations.

The examples in our prior publications [23,33] are adopted in this case study. As shown in Fig. 3, the part to be machined is an engine head that includes seven quality features $\mathbf{Y}_1$–$\mathbf{Y}_7$. Two planar surfaces are represented by $\mathbf{Y}_1$ and $\mathbf{Y}_4$, and cylindrical holes are denoted by $\mathbf{Y}_2$, $\mathbf{Y}_3$, $\mathbf{Y}_5$, $\mathbf{Y}_6$, and $\mathbf{Y}_7$. A global coordinate system is established to represent the dimensions. The origin of the system is set as the center of hole $\mathbf{Y}_6$. Thus, the quality feature $\mathbf{Y}_1$ can be denoted by $\mathbf{Y}_1 = (0, 1, 0, 0, 131, 0)^T$, following a vectorial surface model [35]. This model represents any feature with a vector consisting of the orientation, position, and size of the feature under a specified coordinate system. The value for nominal feature $\mathbf{Y}_1$ can be determined in Fig. 3 as follows: the origin of the coordinate system is on the center axis of the cylindrical hole $\mathbf{Y}_5$, and (0 1 0) represents the normal vector for top surface $\mathbf{Y}_1$, which is offset 131 mm above the horizontal axis (aligned with the Y5–Y6 direction).

Fig. 4 shows that the part with the seven features undergoes the following two operations: (1) milling of plane $\mathbf{Y}_1$ and drilling of two holes $\mathbf{Y}_2$ and $\mathbf{Y}_3$ using $\mathbf{Y}_4$, $\mathbf{Y}_5$, and $\mathbf{Y}_6$ as datum surfaces and (2) drilling of hole $\mathbf{Y}_7$ using plane $\mathbf{Y}_1$ and the two holes from Operation 1 as datum surfaces. The locating positions on the datum surfaces in each operation are denoted by $\mathbf{f}_1$–$\mathbf{f}_6$ (Fig. 3.) For example, the coordinates of fixture locator 1 in Operation 1 is $\mathbf{f}(1)_1 = (f(1)_{1x}, f(1)_{1y}$ and $f(1)_{1z})^T = (-7, 109, 0)^T$, where $f(k)_j$, k denotes Operation k, and the subscript j denotes the jth locator. The machine tool, datum, and fixture errors are denoted by $\mathbf{x}_1(k)$, $\mathbf{x}_2(k)$, and $\mathbf{x}_3(k)$, respectively. The fixture error is chosen to be the base error to which other error sources are equivalently transformed. For Operation 1, the fixture error is characterized by the fixture locator deviations, $(\Delta f(1)_{1z}, \Delta f(1)_{2z}, \Delta f(1)_{3z}, \Delta f(1)_{4y}, \Delta f(1)_{5y}, \Delta f(1)_{6x})^T$. The procedure consists of two steps, which are summarized as follows:

3.1. Tolerance synthesis in a specific fixture locator layout

Tolerance stackup modeling is first conducted by considering the fixture and machine tool errors in Operation 1. The machined surface errors generated a datum error in the next operation. The EFE caused by the machine tool and datum errors in Operation k are denoted by $\mathrm{x}_1^*(k)$ and $\mathrm{x}_2^*(k)$, respectively. Accordingly, $\mathbf{u}(2) = \mathrm{x}_1^*(2) + \mathrm{x}_2^*(2) + \mathbf{x}_3(2)$, where $\mathrm{x}_2^*(2)$ is generated by $\mathbf{u}(1)$ (Eq. (A4)). The final product feature deviation $\mathbf{y}$ is expressed as follows:

$$\mathrm{y} = \Gamma \begin{bmatrix} \mathrm{u}(1) \\ \mathrm{u}(2) \end{bmatrix} + \varepsilon, \tag{5}$$

where $\Gamma = \begin{bmatrix} \Gamma_1 & 0_{6\times 6} \\ 0_{6\times 6} & \Gamma_7 \end{bmatrix}_{12\times 12}$ and $y = [y_1^T y_7^T]^T$. The matrices $\mathbf{\Gamma}_1$ and $\mathbf{\Gamma}_7$ are

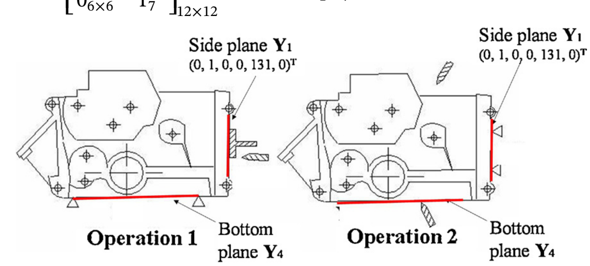

Fig. 4. Machining and fixturing schemes for two operations (Side view) [23].

expressed as follows:

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 & -0.0025 & 0.0025 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0042 & -0.0042 & 0.0083 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.3725 & 0.3725 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ -1.0748 & -0.1074 & 0.1833 & 0 & 0 & 0 \end{pmatrix}$$

and

$$\Gamma_7 = \begin{pmatrix} -0.0014 & 0.0014 & 0 & -0.0030 & 0.0030 & 0 \\ -0.0090 & -0.0090 & 0.0180 & 0 & 0 & 0 \\ 0.0043 & 0.0043 & -0.0086 & 0 & 0 & 0 \\ 0.2610 & -0.2610 & 0 & 0.2708 & -0.2708 & -1 \\ 0.7150 & -0.4900 & -1.2250 & 0 & 0 & 0 \\ -0.7831 & -0.7831 & 1.5662 & 0.1025 & -1.1025 & 0 \end{pmatrix}.$$

The variance-covariance matrix $\boldsymbol{\Sigma}_{\mathrm{y}}$ can be obtained by using Eq. (2). The covariance structure is influenced by the final product tolerance. The deviation of feature j consists of the orientation deviations α_j, β_j, and γ_j, as well as position deviations x_j, y_j, and z_j in three orthogonal directions. The quality of feature j can be represented by $\mathrm{y}_j\,(k) = (\alpha_j, \beta_j, \gamma_j, x_j, y_j, z_j)^T$. The diagonal terms in $\boldsymbol{\Sigma}_{\mathrm{y}}$ characterize the variances of features $\mathbf{Y}_1$ and $\mathbf{Y}_7$ as

$$\sigma^2_{y_1} = \begin{pmatrix} 6.25\times10^{-6}\sigma^2_{f(1)_{4y}} + 6.25\times10^{-6}\sigma^2_{f(1)_{5y}} + \sigma^2_{\varepsilon} \\ \sigma^2_{\varepsilon} \\ 1.74\times10^{-5}\sigma^2_{f(1)_{1z}} + 1.74\times10^{-5}\sigma^2_{f(1)_{2z}} + 6.94\times10^{-5}\sigma^2_{f(1)_{3z}} \\ + \sigma^2_{\varepsilon} \\ \sigma^2_{f(1)_{6x}} + 0.17\sigma^2_{f(1)_{4y}} + 0.17\sigma^2_{f(1)_{5y}} + \sigma^2_{\varepsilon} \\ \sigma^2_{f_{14y}} + \sigma^2_{\varepsilon} \\ 1.16\sigma^2_{f(1)_{1z}} + 0.01\sigma^2_{f(1)_{2z}} + 0.03\sigma^2_{f(1)_{3z}} + \sigma^2_{\varepsilon} \end{pmatrix}$$

and

$$\sigma^2_{y_7} = \begin{pmatrix} 1.16\times10^{-6}\sigma^2_{f(1)_{4y}} + 1.16\times10^{-6}\sigma^2_{f(1)_{5y}} + 2.05\times10^{-6}\sigma^2_{f(2)_{1y}} \\ + 2.05\times10^{-6}\sigma^2_{f(2)_{2y}} \\ +3.27\times10^{-6}\sigma^2_{f(1)_{1z}} + 4.73\times10^{-6}\sigma^2_{f(1)_{2z}} + 9\times10^{-6}\sigma^2_{f(2)_{4z}} \\ + 9\times10^{-6}\sigma^2_{f(2)_{5z}} + \sigma^2_{\varepsilon} \\ 8.10\times10^{-5}\sigma^2_{f(2)_{1y}} + 8.10\times10^{-5}\sigma^2_{f(2)_{2y}} + 3.24\times10^{-4}\sigma^2_{f(2)_{3y}} \\ + 3.66\times10^{-5}\sigma^2_{f(1)_{1z}} \\ +1.40\times10^{-5}\sigma^2_{f(1)_{2z}} + 5.63\times10^{-5}\sigma^2_{f(1)_{3z}} + \sigma^2_{\varepsilon} \\ 1.85\times10^{-5}\sigma^2_{f(2)_{1y}} + 1.85\times10^{-5}\sigma^2_{f(2)_{2y}} + 7.40\times10^{-5}\sigma^2_{f(2)_{3y}} \\ + 8.35\times10^{-6}\sigma^2_{f(1)_{1z}} \\ +3.21\times10^{-6}\sigma^2_{f(1)_{2z}} + 1.28\times10^{-5}\sigma^2_{f(1)_{3z}} + \sigma^2_{\varepsilon} \\ \sigma^2_{f(1)_{6x}} + \sigma^2_{f(2)_{6x}} + 0.020\sigma^2_{f(1)_{4y}} + 0.020\sigma^2_{f(1)_{5y}} + 0.068\sigma^2_{f(2)_{1y}} \\ + 0.070\sigma^2_{f(2)_{2y}} \\ +0.0044\sigma^2_{f(1)_{1z}} + 0.073\sigma^2_{f(2)_{4z}} + 0.073\sigma^2_{f(2)_{5z}} + \sigma^2_{\varepsilon} \\ 0.0156\sigma^2_{f(1)_{4y}} + 0.766\sigma^2_{f(1)_{5y}} + 0.512\sigma^2_{f(2)_{1y}} + 0.240\sigma^2_{f(2)_{2y}} \\ + 1.500\sigma^2_{f(2)_{3y}} \\ +0.033\sigma^2_{f(1)_{1z}} + \sigma^2_{\varepsilon} \\ 0.613\sigma^2_{f(2)_{1y}} + 0.613\sigma^2_{f(2)_{2y}} + 2.453\sigma^2_{f(2)_{3y}} + 0.088\sigma^2_{f(1)_{1z}} \\ + 0.394\sigma^2_{f(1)_{2z}} \\ +0.220\sigma^2_{f(1)_{3z}} + 0.011\sigma^2_{f(2)_{4z}} + 1.216\sigma^2_{f(2)_{5z}} + \sigma^2_{\varepsilon} \end{pmatrix}. \tag{6}$$

The hole feature $\mathbf{Y}_7$ is a key in the future assembly operations. The tolerances in the x and y positions of $\mathbf{Y}_7$ are set as the final product tolerances, which are related to $\sigma^2_{x_7}$ and $\sigma^2_{y_7}$, respectively. The objective is to maximize the following expression:

$$0.5(\sigma^2_{x_7} + \sigma^2_{y_7}), \tag{7.1}$$

where equal importance of the tolerances in two directions is assumed. Eq. (7.1) contains the variation components of the dimensions with the maximum number of $\boldsymbol{\sigma}_{\Theta}$ (Eq. [3]). This objective function is subject to the following constraints:

$$\begin{aligned} &\sigma^2_{\alpha_1} \le \mathrm{b}_{\alpha_1},\ \sigma^2_{\gamma_1} \le \mathrm{b}_{\gamma_1},\ \sigma^2_{y_1} \le \mathrm{b}_{y_1} \text{ for } \mathrm{Y}_1, \text{and} \\ &\sigma^2_{\alpha_7} \le \mathrm{b}_{\alpha_7},\ \sigma^2_{\beta_7} \le \mathrm{b}_{\beta_7},\ \sigma^2_{\gamma_7} \le \mathrm{b}_{\gamma_7},\ \sigma^2_{x_7} \le \mathrm{b}_{x_7}, \\ &\sigma^2_{y_7} \le \mathrm{b}_{y_7},\ \sigma^2_{z_7} \le \mathrm{b}_{z_7} \text{ for } \mathrm{Y}_7, \end{aligned} \tag{7.2}$$

where b_{β_1}, b_{x_1}, and b_{z_1} need not be considered because the plane is free to move in the x and z directions, and the orientation of plane $\mathbf{Y}_1$ can be set free in the y direction.

To illustrate, we assume that 0.1 rad^2 can be assigned to b^{α_1}, b^{γ_1}, b^{α_7}, b^{β_7}, and b^{γ_7}, and 5 mm^2 can be assigned to b^{y_1}, b^{x_7}, b^{y_7}, and b^{z_7}. A value of 1.732 mm is set for all the elements of $\mathbf{b}_2$ in Eq. (3). The allowable standard deviations (or tolerances) for the EFE are $\boldsymbol{\sigma}_{\Theta}$ = (0.01, 0.01, 0.01, 0.01, 1.415, 1.732, 1.732, 0.01, 1.135, 0.01, 0.01, 1.327) and $\mathrm{c}^{\mathrm{T}}\sigma^2_{\Theta} = 4.99\,\mathrm{mm}^2$.

When additional information about the process tolerance design is available, such as the weights between the process tolerances of the fixture and machine tool errors considering manufacturing cost, the tolerances can be further distributed to the EFEs induced by different types of error source. We can allocate 80% of the tolerance for EFE to the machine tool error to reduce equipment cost. Thus, in Operation 1 (when no datum error is present), 80% of $\boldsymbol{\sigma}_{\mathbf{u}(1)}$ is allocated to $\sigma_{\mathrm{x}_1^*(1)}$, that is, $\sigma_{\mathrm{x}_1^*(1)} = 0.8\boldsymbol{\sigma}_{\mathbf{u}(1)}$, where $\sigma_{\mathrm{x}_1^*(1)}$ denotes the standard deviation forf the fixture error and the EFE caused by the machine tool error in Operation 1. The variance-covariance matrix of the machine tool error at the first operation is expressed as follows:

$$\Sigma_{\mathrm{x}_1(1)} = \mathbf{K}(1)_2^{-1}\Sigma_{\mathrm{x}_1^*(1)}(\mathbf{K}(1)_2^{-1})^T \tag{8}$$

where $\Sigma_{\mathrm{x}_1^*(1)} = \mathrm{diag}(0.8^2\ \sigma^2_{\mathrm{u}(1)})$. The matrices $\mathbf{K}_1$ and $\mathbf{K}_2$ are given in Appendix A. Solving Eq. (8) and extracting the terms in $\mathrm{diag}(\Sigma_{\mathrm{x}_1(1)})$ yield $\sigma^{\mathrm{x}_1(1)}$ = (1.386 mm, 0.008 mm, 0.009 mm, 2.733×10^{-5} rad, 8.165×10^{-5} rad, 0.003 rad), where the first three values represent the translational error of the machine tool, and the last three values correspond to the rotational error (around x, y, and z directions). On the premise that the cutting path of the tool head varies considerably when machining multiple features, the tightest tolerance can be equally set for the machine tool error in all directions, that is, $\sigma^{\mathrm{x}_1(1)}$ = (8 μm, 8 μm, 8 μm, 2.733×10^{-5} rad, 2.733×10^{-5} rad, 2.733×10^{-5} rad).

The datum error in Operation 2 is introduced from Operation 1. The variance in EFE caused by the datum error is diag $(\mathbf{K}\boldsymbol{\Sigma}_{\mathbf{u}(1)}\mathbf{K}^T) = \mathrm{diag}(\Sigma_{\mathrm{x}_2^*(2)}) = \sigma^2_{\mathrm{x}_2^*(2)}$ = (0.069 mm^2, 1.129 mm^2, 0.599 mm^2, 0.003 mm^2, 0.009 mm^2, 1.793 mm^2) in Eq. (A4). Tolerance allocation for the datum error can be expressed as follows:

$$\mathrm{diag}\mathrm{K}(2)_1\Sigma_{\mathrm{x}_2(2)}(\mathrm{K}(2)_1)^T = \mathrm{diag}(\Sigma_{\mathrm{x}_2^*(2)}) \tag{9}$$

The following issue is encountered in solving the tolerance allocation problem: given that $\mathbf{K}_1$ is a 6 × 18 matrix (Eq. [A3]), $\mathrm{diag}(\Sigma_{\mathrm{x}_2(2)})$ cannot be obtained by solving Eq. (9). In consideration of the characteristics of $\mathbf{K}_1$, the information about the tolerance for the second datum surface and the tertiary datum surface should be specified. Let $\mathbf{x}_2 = (\mathbf{v}_{\mathrm{I}}, \mathbf{p}_{\mathrm{I}}, \mathbf{v}_{\mathrm{II}}, \mathbf{p}_{\mathrm{II}}, \mathbf{v}_{\mathrm{III}}, \mathbf{p}_{\mathrm{III}})$, where $\mathbf{v}$ and $\mathbf{p}$ represent the rotational and translational errors of the datum surfaces, respectively, in three directions. The indices I, II, and III, represent the three datum surfaces, respectively. For example, $v(2)_{\mathrm{I}x}$ represents the rotational variation of the primary datum in the x direction in Operation 2. We assign 1×10^{-9} rad^2 to $\sigma^2_{v(2)_{IIy}}$, 1×10^{-7} rad^2 to $\sigma^2_{v(2)_{IIIy}}$, and 1×10^{-6} mm^2 to $\sigma^2_{p(2)_{IIIx}}$. By solving Eq. (9), we obtain $\sigma^{\mathrm{x}_2(2)} = (\sigma^{v(2)_{Ix}}, \sigma^{v(2)_{Iz}}, \sigma^{p(2)_{Iy}}, \sigma^{v(2)_{IIx}},$

$\sigma^{v(2)IIy}$, $\sigma^{p(2)IIz}$, $\sigma^{v(2)IIIy}$, $\sigma^{v(2)IIIz}$, $\sigma^{p(2)IIIx}$) = (0.004 rad, 1.400 × 10^{-4} rad, 7.5 μm, 2.473 × 10^{-5} rad, 3.162 × 10^{-5} rad, 3.2 μm, 3.162 × 10^{-4} rad, 0.022 rad, 1 μm).

We utilize $\sigma_{x_1^*(2)} = 0.8(\sigma^{u(2)} - \sigma_{x_2^*(2)})$ = (1.191 mm, 0.0002 mm, 0.480 mm, 0.041 mm, 0.0004 mm, 0.437 mm) to distribute the tolerances for the fixture and machine tool errors in Operation 2. We can estimate the tolerance for the machine tool error in Operation 2 as follows:

$$\Sigma_{x_1(2)} = \mathbf{K}(2)_2^{-1}\Sigma_{x_1^*(2)}(\mathbf{K}(2)_2^{-1})^T \tag{10}$$

Thus, $\sigma^2_{x_1(2)}$ = (0.462 mm^2, 0.636 mm^2, 4.021 mm^2, 0.0002 rad^2, 1.894 × 10^{-8} rad^2, 0.000016 rad^2). Assigning equal tolerances for the translational and rotational errors, we obtain $\sigma^{x^1(2)}$ = (0.679 mm, 0.679 mm, 0.679 mm, 0.00014 rad, 0.00014 rad, 0.00014 rad).

4. Optimal fixture layout design within the allowable design range

Given that all the error sources are transformed into fixture errors, the input site **w** reflects the layout of the fixture locating pins and can be estimated under the 3–2–1 locating scheme. In this case study, only locators 1, 2, and 3 are movable over the allowable design space because locators 4–6 are fixed with cylindrical holes. In addition, each of locators 1–3 can be freely placed in the primary datum plane. Therefore, a total of 12 design variables are involved in the two-operation machining process, that is, $\mathbf{\Omega}$ = ($f(1)_{1x}$, $f(1)_{1y}$, $f(1)_{2x}$, $f(1)_{2y}$, $f(1)_{3x}$, $f(1)_{3y}$, $f(2)_{1x}$, $f(2)_{1z}$, $f(2)_{2x}$, $f(2)_{2z}$, $f(2)_{3x}$, $f(2)_{3z}$). The design space for each design variable is summarized in Table 1 and illustrated in Fig. 5. The input sites **w** for $\mathbf{\Omega}$ are normalized to $[0,1]^d$, where $d = 12$, $0 \le w_i \le 1$, $i = 1, 2,. . ., 12$.

Three locators have identical design ranges, which may lead to the colocation of the locating pins. Such a problem can be prevented by inspecting a locating condition, which requires the Jacobian matrix given a fixture layout that is in full rank [2]. In addition, the reaction force from fixture $\mathbf{F}^c$ should be nonnegative (i.e., in this study, $\mathbf{F}^c > 0.5$ kN) at each locating position so that the locators maintain close contact with the part. We apply the clamping load as follows: for Operation 1, $\mathbf{F}^A$ = (−52, −28, −25)kN, $\mathbf{T}^A$ = (−10,136, 18,300, −4489)Nm; for Operation 2, $\mathbf{F}^A$ = (−45, 294, 158)kN, $\mathbf{T}^A$ = (−149, 302, −51)Nm.

The number of design sites for the initial design should be determined. The number of input points n_0 for the initial design should be carefully selected to balance the computational complexity and fidelity of the computer experiment model. Bernardo et al. [29] and Gupta et al. [32] suggested as a rule of thumb that the number of design points should be thrice the parameter numbers in the computer experiment model. In this study, 14 parameters should be determined, including 12 design variables, 1 constant, and 1 process variance. Thus, n_0 should be between 12 and 42.

In this study, knowledge on the design space is lacking; therefore, LH sampling is employed. A total of sixteen points are selected. As a result, a 16 × 12 LH design is generated. The unknown parameters in the computer experiment model can be estimated by using the maximum likelihood estimation criterion, that is, by optimizing the objective function Eq. (B1) in Appendix B. The Torczon pattern search approach [34] is selected in solving the optimization problem because of its capacity to converge at stationary points. The method can be conveniently generalized to optimization with constraints. The initial design yielded 52 maximin–LH design sites, which are selected to refine the model. The additional design sites are those whose RMSEs are greater than 85% of the largest RMSE of all the tested sites. The largest value of RMSE of the tested sites is approximately 0.224. The RMSE test results lead to seven other points (by checking if RMSE > 0.19 (0.224 × 85%)) that should be added to the design.

The refinement steps should be terminated when the maximum responses do not significantly change between consecutive iterations

Table 1
Ranges of design variables under a global coordinate system (unit: mm).

Stage 1	$f(1)_{1x}$	$f(1)_{1y}$	$f(1)_{1z}$	$f(1)_{2y}$	$f(1)_{3x}$	$f(1)_{3y}$
Range	0–400	−10–130	0–400	−10–130	0–400	−10–130
Stage 2	$f(2)_{1x}$	$f(2)_{1z}$	$f(2)_{1x}$	$f(2)_{2z}$	$f(2)_{3x}$	$f(2)_{3z}$
Range	0–360	0–80	0–360	0–80	0–360	0–804

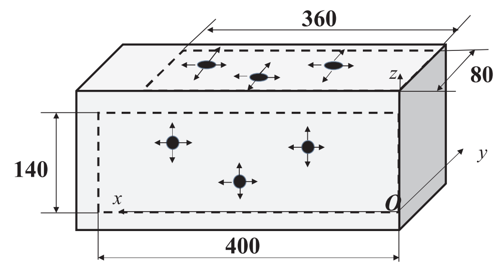

Fig. 5. Allowable ranges of the design variables.

and the RMSE in the explored design space is not excessively large. For this problem, the increase at the last iteration in $\hat{y}_{\max}$ is less than 1% of the increase in $\hat{y}_{\max}$ at the previous iteration. In addition, the RMSE is less than 85% of the largest RMSE of the total tested sites. Thus, both conditions are satisfied. The coefficients of the final refined model are 4.6957 and (0.1, 0.85, 0.1, 0.1, 0.725, 0.1, 1.6, 0.6, 0.1, 0.1, 0.6, 0.6). Through a simplex search, the optimal solution $\mathbf{w}^*$ in the computer experiment model is $\mathbf{w}^*$ = (0.438, 0.469, 0.188, 0.102, 0.734, 0.344, 0, 0.422, 0.781, 0.734, 0.453, 0.547). The layout of the fixture locating pins are $\mathbf{\Omega}^*$ = (175, 55.625, 75, 4.219, 293.750, 38.125, 0, 33.750, 281.250, 58.750, 163.125, 43.750) mm, with $\hat{y}(w^*)$ = 5.029 mm and RMSE($\hat{y}(w^*)$) = 0.087 mm. The reaction forces caused by the six locators are $\mathbf{F}^c$ = (18.263, 21.342, 12.754, 22.003, 5.611, 25.335) kN for Operation 1 and $\mathbf{F}^c$ = (0.567, 0.683, 0.825, 0.677, 0.753, 0.663) kN for Operation 2. The optimal tolerance allocation can then be obtained as described in Section 2.

Remark 2: A Discussion on the validation strategies

The proposed method can be validated in two ways.

1 The theory of error equivalence has been validated in our prior studies [23,24], which showed that the EFE could successfully predict the quality problems induced by machine tool and datum errors. Thus, we can rely on the equivalence transformation to derive an equivalent fixture for the tolerance allocation for the fixture, machine tool, and datum errors.
2 We adopt a sequential search strategy to refine the fixture layouts based on the output response from the second-stage design. This strategy examines the RMSEs of the response and will add more design points to those areas with large RMSEs.
3 The proposed approach adopts the error equivalence concept to reduce the design space in the two-stage process selection. If the error equivalence concept is not employed, then the number of design variables (or process errors) for optimization (3) are [6 (fixture errors) + 6 (machine tool errors) + 18 (datum errors)] × 2 = 60. A large number of design variables increase the computational load and lead to nonunique solutions. After introducing the error equivalence, the design variables are reduced to 12. The major advantage of this method is the reduction of the computational load involved in the simultaneous optimization of the fixture layout and tolerance allocation. Particularly, 68 points, including 16 initial and 52 refinement test points, are involved in the computer experiment. Moreover, tolerance allocation optimization (Eq. [3]) should be performed at every test point. As such, the error equivalence concept can remarkably reduce the unknown design variables by 68 × (60 − 12) = 3264 during optimization.

4 The proposed method is compared with the traditional tolerance allocation method. The proposed two-stage the process selection minimizes the process tolerance cost by exploring all candidate fixture layouts; thus, it outperforms the traditional tolerance allocation method, which relies on an empirically selected fixture layout only.

Remark 3: Generalization of the EFE approach in additive manufacturing

The concept of error equivalence can be extended to process tolerance design in additive manufacturing processes, such as fused deposition modeling, where the positional error of the extruder $\Delta\mathbf{t}$, the thermal shrinkage error $\Delta\mathbf{s}$, and the geometric error of the CAD design of a part $\Delta\mathbf{r}$ can generate equivalent shape variations of a final product. Given that the CAD design can be conveniently adjusted, $\Delta\mathbf{t}$ and $\Delta\mathbf{s}$ can be transformed to $\Delta\mathbf{r}$ via identity mapping. The process tolerance design can be simplified to tolerance allocation initially for an equivalent amount of $\Delta\mathbf{r}$ and then the further allocation for other process errors.

5. Conclusions

This study provides further insights into the machining process selection by investigating the influence of fixture layout on the tolerance allocation for multiple types of error source, particularly non-fixture error sources. The interaction between the problems of fixture layout and process tolerance design has rarely been investigated in previous studies on process selection to obtain engineering insights.

The developed method incorporates an error equivalence mechanism by which multiple error sources are transformed into the fixture error. Thus, process selection is simplified as an optimization problem that considers only the fixture layout. The simplified process selection consists of the following subproblems: (1) determining the optimal tolerance allocation for the given design specifications and fixture locating pin layout and (2) deriving the optimal fixture layout by examining all possible combinations of the process design variables. The computer experiment method establishes a surrogate model for the tolerance stackup prediction and optimization of the fixture locating pin layout. Specifically, a space-filling method (i.e., LH sampling with the maximin criterion) first generates random design points and estimates the optimal tolerance stackup at each design point (i.e., fixture locator layout). The computer experiment model is derived and refined by sequentially exploring the design space with high uncertainty. When additional process information is available, the tolerance assigned to fixture errors can be further allocated to other error sources. The approach is demonstrated on a two-operation machining process. All the errors are transformed into EFE. As such, the error equivalence mechanism can also significantly reduce the design space. The computational load of the tolerance optimization problem and computer experiments can be reduced. The EFE-based process selection method can be potentially extended to other manufacturing applications such as additive manufacturing.

The proposed method is applicable to the process selection for multi-stage machining, where different types of process error coexist, and their equivalence relationships can be established. The equivalence model has been developed and validated in our prior studies [23,24]. The method considers the interaction between fixture layout and process tolerance allocation, thereby allowing for the joint optimization of both problems. Unlike the traditional tolerance allocation method, which relies on an empirically selected fixture layout, the joint optimization in the proposed method can help identify the fixture layout that achieves the lowest process tolerance cost. In addition, the error equivalence model significantly reduces the number of design variables in the two-stage optimization of the process selection, as well as the computational complexity and search space. The potential limitations of the proposed method include the following: (1) equivalent mapping must be established and integrated for the process errors under consideration; (2) the method cannot further improve the clamping and locating accuracies compared with the traditional fixture layout design methods. However, the proposed method does not aim to replace the traditional fixture layout design for improving fixturing. Instead, it provides an alternative to fixture layout design for tolerance cost reduction; and (3) allocation of equivalent fixture errors to other types of process errors may not be unique. A certain degree of process knowledge is required to guide the tolerance allocation. For example, the angular machine tool errors are more difficult to control compared with positional errors. Therefore, more tolerance should be allocated to the angular errors than the positional errors.

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Appendix A. Review of EFE and derivation of EFE due to datum errors

In our prior work, Wang et al. [23], the derivation of EFE are presented as follows,

$$\mathbf{x}_2^* = \mathbf{K}_1(\mathbf{v}_{\text{I}} \quad \mathbf{p}_{\text{I}} \quad \mathbf{v}_{\text{II}} \quad \mathbf{p}_{\text{II}} \quad \mathbf{v}_{\text{III}} \quad \mathbf{p}_{\text{III}})^T, \tag{A1}$$

and

$$\mathrm{x}_1^* = \mathrm{K}_2\mathrm{x}_1, \tag{A2}$$

where

$$\mathbf{K}_1 = \begin{pmatrix} \mathbf{G}_1 & & \\ & \mathbf{G}_2 & \\ & & \mathbf{G}_3 \end{pmatrix}. \tag{A3}$$

For the representation of matrices $\mathbf{K}_1$ and $\mathbf{K}_2$ in multi-operation machining processes, please refer to Wang et al. (2005). For instance, for Operation 1 in the case study, the matrices are:

$$\mathbf{K}(1)_2 = \begin{pmatrix} 0 & 0 & -1 & -f(1)_{1y} & f(1)_{1x} & 0 \\ 0 & 0 & -1 & -f(1)_{2y} & f(1)_{2x} & 0 \\ 0 & 0 & -1 & -f(1)_{3y} & f(1)_{3x} & 0 \\ 0 & -1 & 0 & f(1)_{4z} & 0 & -f(1)_{4x} \\ 0 & -1 & 0 & f(1)_{5z} & 0 & -f(1)_{5x} \\ -1 & 0 & 0 & 0 & -f(1)_{6z} & f(1)_{6y} \end{pmatrix} \text{for operation 1,}$$

and for Operation 2, we have:

$$\mathbf{K}(2)_2 = \begin{pmatrix} 0 & -1 & 0 & f(2)_{1z} & 0 & -f(2)_{1x} \\ 0 & -1 & 0 & f(2)_{2z} & 0 & -f(2)_{2x} \\ 0 & -1 & 0 & f(2)_{3z} & 0 & -f(2)_{3x} \\ 0 & 0 & -1 & f(2)_{4y} & -f(2)_{4x} & 0 \\ 0 & 0 & -1 & f(2)_{5y} & -f(2)_{5x} & 0 \\ -1 & 0 & 0 & 0 & -f(1)_{6z} & f(1)_{6y} \end{pmatrix}$$

and

$$\mathbf{G}(2)_1 = -\begin{pmatrix} f(2)_{1x} & 0 & f(2)_{1z} & 0 & 1 & 0 \\ f(2)_{2x} & 0 & f(2)_{2z} & 0 & 1 & 0 \\ f(2)_{3x} & 0 & f(2)_{3z} & 0 & 1 & 0 \end{pmatrix},$$

$$\mathbf{G}(2)_2 = -\begin{pmatrix} f(2)_{4x} & f(2)_{4y} & 0 & 0 & 0 & 1 \\ f(2)_{5x} & f(2)_{5y} & 0 & 0 & 0 & 1 \end{pmatrix},$$

and

$$\mathbf{G}(2)_3 = -(0 \quad f(2)_{6y} \quad f(2)_{6z} \quad 1 \quad 0 \quad 0)$$

To calculate $x_2^*(2)$, we can apply the feature deviation from Operation 1 under the nominal location of six locators in Operation 2. The relation between $x_2^*(2)$ and $\mathbf{u}(1)$ after linearization can be derived as

$$\mathbf{x}_2^*(2) = \mathbf{K}\mathbf{u}(1), \tag{A4}$$

where $\mathbf{K}$ is the coefficient matrix. Then the EFE due to datum errors can be added to Operation 2 in the stackup model. The EFE due to datum errors calculated thus obtained are:

$$x_2^*(2) = \begin{pmatrix} -0.952\Delta f(1)_{4y} - 0.048\Delta f(1)_{5y} + 0.255\Delta f(1)_{1z} + 0.255\Delta f(1)_{2z} - 0.510\Delta f(1)_{3z} \\ -0.202\Delta f(1)_{4y} - 0.798\Delta f(1)_{5y} + 0.255\Delta f(1)_{1z} + 0.255\Delta f(1)_{2z} - 0.510\Delta f(1)_{3z} \\ -0.577\Delta f(1)_{4y} - 0.423\Delta f(1)_{5y} + 0.047\Delta f(1)_{1z} + 0.047\Delta f(1)_{2z} - 0.094\Delta f(1)_{3z} \\ -1.075\Delta f(1)_{1z} - 0.109\Delta f(1)_{2z} + 0.183\Delta f(1)_{3z} \\ -1.075\Delta f(1)_{1z} - 0.109\Delta f(1)_{2z} + 0.183\Delta f(1)_{3z} \\ -\Delta f(1)_{6x} - 0.328\Delta f(1)_{4y} + 0.328\Delta f(1)_{5y} \end{pmatrix}.$$

Appendix B. Estimation based on the computer experiment model

Given an experimental design $\mathbf{S} = \{s_1, \ldots, \mathbf{s}_n\}$ with corresponding responses $y_s = \{y_1, \ldots, y_n\}$, the undetermined parameters in the correlation function can be estimated, using the maximum likelihood estimation (MLE) criteria to minimize

$$\frac{1}{2}(n \ln \hat{\sigma}^2 + \ln \det \mathbf{R}),$$

where R is correlation coefficient matrix, and

$$\mathbf{R} = \begin{pmatrix} R(s_1, s_1) & R(s_1, s_2) & \cdots & R(s_1, s_n) \\ R(s_2, s_1) & R(s_2, s_2) & \cdots & R(s_2, s_n) \\ \vdots & \vdots & \ddots & \vdots \\ R(s_n, s_1) & R(s_n, s_2) & \cdots & R(s_n, s_n) \end{pmatrix}.$$

By applying generalized least square estimation, we estimate parameters $\boldsymbol{\beta}$ and σ^2 as

$$\hat{\boldsymbol{\beta}} = (\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H})\mathbf{H}^T\mathbf{R}^{-1}\mathbf{y}_s$$

and

$$\hat{\sigma}^2 = \frac{1}{n}(\mathbf{y}_s - \mathbf{H}\hat{\boldsymbol{\beta}})^T\mathbf{R}^{-1}(\mathbf{y}_s - \mathbf{H}\hat{\boldsymbol{\beta}}),$$

where $H= [f(s_1),\ldots,f(s_n)]^T$. The estimation can be obtained by

$$\hat{y}(w) = \mathbf{g}^T(\boldsymbol{w})\hat{\boldsymbol{\beta}} + \mathbf{r}^T(\mathbf{w})\mathbf{R}^{-1}(\mathbf{y}_s - \mathbf{H}\hat{\boldsymbol{\beta}}),$$

where $\mathbf{r}^T = [\boldsymbol{R}(\mathbf{s}_1,\mathbf{w}),\ldots,\boldsymbol{R}(\mathbf{s}_n,\mathbf{w})]^T$ is a column matrix showing the correlation between the stochastic processes at given input sites and those at untried input sites. Williams et al. [30] estimated the MSE as

$$\mathrm{MSE}(\hat{y}(\mathrm{w})) = \hat{\sigma}^2\{1 - [\mathbf{g}^T(\mathbf{w}) \quad \mathbf{r}^T(\mathbf{w})]\begin{bmatrix} 0 & \mathbf{H}^T \\ \mathbf{H} & \mathbf{R} \end{bmatrix}^{-1}\begin{bmatrix} \mathbf{g}^T(\mathbf{w}) \\ \mathbf{r}^T(\mathbf{w}) \end{bmatrix}.$$

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