

PAPER • OPEN ACCESS

Numerical Modeling of Electron Transport in Solar Wind: Effects of Whistler Turbulence and Coulomb Collisions

To cite this article: Bofeng Tang *et al* 2018 *J. Phys.: Conf. Ser.* **1100** 012025

View the [article online](#) for updates and enhancements.



IOP | ebooks™

Bringing you innovative digital publishing with leading voices to create your essential collection of books in STEM research.

Start exploring the collection - download the first chapter of every title for free.

Numerical Modeling of Electron Transport in Solar Wind: Effects of Whistler Turbulence and Coulomb Collisions

Bofeng Tang¹, Gary P. Zank^{1,2}, Vladimir Kolobov³

¹ Department of Space Science, University of Alabama in Huntsville, Huntsville, AL 35899, USA

² Center for Space Plasma and Aeronomic Research (CSPAR), University of Alabama in Huntsville, Huntsville, AL 35805, USA

³ CFD Research Corporation, Huntsville, AL 35805, USA

E-mail: bt0008@uah.edu, garyp.zank@gmail.com, vik0001@uah.edu

Abstract. The electron distribution function (eVDF) in the solar wind deviates significantly from an equilibrium Maxwellian distribution, and is comprised of a Maxwellian core, a suprathermal halo, a field-aligned component strahl, and a higher energy superhalo. Charged particle Coulomb collisions are ineffective in relaxing such a velocity distribution beyond a few solar radii. Therefore wave-particle interactions need to be considered. A wave-particle interaction term was introduced into the kinetic equation that describes the interaction of electrons with whistler waves, as well as particle collision terms. The kinetic equation has the form of an advection-diffusion-like equation in which the advection and diffusion coefficients describe the scattering and drag of electrons in whistler turbulence. A reliable numerical method has been developed to solve a full form of the advection-diffusion-like kinetic equation. Preliminary applications of the numerical method to the solar wind electron problem are presented. Comparison and analysis of the electron VDFs in the presence of Coulomb collisions and resonant wave-particle interactions are made.

1. Introduction

The sun constantly emits a flux of electrically charged particles into space, mostly protons and electrons, known as the solar wind. This plasma affects the entire solar system, including the Earth's magnetic field and is therefore crucial to our understanding of space weather. In situ observations show that the solar wind electron distribution function (eVDF) deviates significantly from a Maxwellian distribution, which is the thermal equilibrium form. The solar wind electron VDF is comprised of a Maxwellian core, halo, and a field-aligned component called the strahl which is typically associated with the high-speed solar wind. The Maxwellian core is characterized by energies of $< \sim 10$ eV, the halo and strahl falls in the energy range of about $10^2 - 10^3$ eV, and the super-halo in the range $10^3 - 10^5$ eV. At 1 AU the Maxwellian component comprises about 95% of the total electron number density, the halo approximately 4%, and the strahl the remaining 1%.

The strahl is an anti-sunward magnetic field-aligned beam, although “counterstreaming” strahls, which are directed towards the sun, are also observed [1]. Observations show the strahl has a finite width. The strahl vanishes at electron energies above approximately 1 keV. At



higher energies, above 2 keV, the superhalo appears to be isotropic. The relative number density of strahl electrons decreases with increasing heliocentric distance, whereas the relative number density of halo electrons increases [2,3]. Although strahl electrons comprise a part of the electron number density, they are responsible for most of the heat flux transported away from the Sun due to their high energy [2]. Štverák et al. [2] combined measurements obtained from three spacecraft (*Helios*, *Cluster II* and *Ulysses*) for low ecliptic latitudes covering a heliocentric distance from 0.3 to 4 AU. They confirm that the halo and the strahl relative densities vary in an opposite sense (Figure 1).

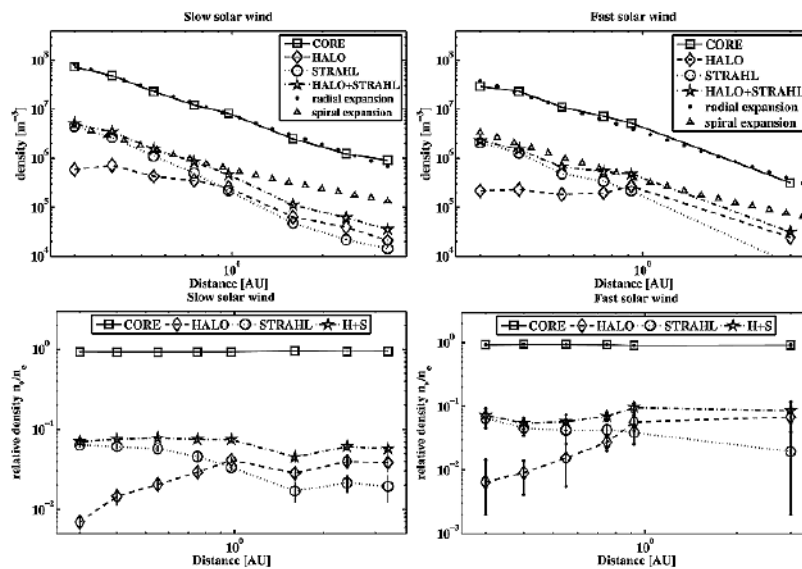


Figure 1. Radial evolution of the number densities profiles in the slow wind (top left) and fast wind (top right). Radial evolution of the relative number densities (i.e. the ratio of the density of individual eVDF components to the total electron density) of the eVDF components for the slow (bottom left) and fast (bottom right) solar wind. [2]

However the connection between the strahl and the halo electron population remains poorly understood. Both strahl and halo electrons fall in a similar energy regime and it is frequently speculated that the halo corresponds simply to strahl electrons that have been scattered gradually in pitch angle towards an isotropic distribution. There are two primary scattering mechanisms for electrons in the solar wind. Charged particle collisions are effective in the low corona, leading to the formation of an equilibrium Maxwellian distribution thus. Salem et al. [4] show that the electron temperature anisotropy actually depends mainly on Coulomb collisions. They also show that the role of Coulomb collisions in regulating the electron heat flux is not as negligible as has been suggested by other authors. For a large number of collisions, the more the distribution function is isotropized, smaller the heat flux. Horatius et al. [5,6] proposed a self-similar kinetic theory of thermal conductivity in a magnetized collisional plasma. Their kinetic equation included only Coulomb collisions, and the plasma parameters of the solar wind (electron number density, temperature and magnetic field) were approximated as power laws in heliocentric distance. They developed a model for the strahl and compared their results to *Wind* observations at 1 AU.

Beyond a few solar radii, intra-particle collisions are rare, and only wave-particle interactions can scatter electrons. The basic effect of wave-particle interactions on the electron VDF is pitch-angle diffusion in the reference frame of the waves. Vocks et al. [7,8] studied the formation of the halo and strahl in the electron VDF in the solar corona and solar wind due to

whistler turbulence wave-particle interactions at electron energies below 1 keV. Vocks et al. [9] subsequently extended their work to electron energies of more than 100 keV, thus including superhalo electrons. Their studies suggest that the quiet solar corona is capable of producing suprathermal electron VDFs by whistler turbulence wave-particle interactions. They asserted that such an electron population should appear in the solar wind. Kim et al. [10,11], and Yoon et al. [12] considered the interaction between electrons and spontaneously emitted electromagnetic fluctuations. They assumed that halo electrons interact with not only whistler turbulence but also Langmuir turbulence, and superhalo electrons interact with Langmuir turbulence. They show that the Langmuir fluctuation terms are orders of magnitude higher than the whistler related terms, and thus, whistler fluctuations and cyclotron resonance terms contribute virtually nothing to the overall halo electron VDF [12]. They hence proposed a new model that only Langmuir turbulence interacts with suprathermal electrons [13]. They treated the suprathermal tail found in the electron VDF as a whole, rather than dividing the suprathermal tail into three different parts. However they presumed that the wave-particle interaction is a very rapid temporal processes, and ignored all interactions associated with electric and magnetic fields. This assumption renders their equation incomplete. In examining their model [10–13], we found that the diffusion coefficients were too small to affect the electron VDF. Pierrard et al. [14] developed a kinetic model that includes both Coulomb collisions and wave-particle resonant interactions with whistler turbulence.

In this study, we first introduce a wave-particle interaction term into the kinetic equation that describes the interaction of electrons with whistler waves, as well particle collision terms. We developed a numerical method using adaptive mesh refinement to solve an advection-diffusion-like Fokker-Planck kinetic equation in 3D phase space. We apply the numerical method to analyze the effects of Coulomb collisions and resonant wave-particle interactions on the strahl found in the solar wind. By studying the temporal and radial evolution of the electron velocity distribution function, we confirm that for large electron energies and away from the corona Coulomb collisions are weak. We also show that resonant wave-particle interactions associated with whistler turbulence can diffuse the electron VDF significantly. We hence propose that whistler turbulence in the solar wind diffuses the electron VDF from an anisotropic strahl into an isotropic halo.

2. Description of the Model

In the case of non-relativistic charged particles, the general kinetic equation for the evolution of the velocity distribution function $f(\mathbf{r}, \mathbf{v}, t)$ is:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{q}{m} \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right) \cdot \frac{\partial f}{\partial \mathbf{v}} = \left(\frac{\delta f}{\delta t} \right)_{\text{coll}} + \left(\frac{\delta f}{\delta t} \right)_{\text{wp}}, \quad (1)$$

where q is charge of particle, $\mathbf{v}$ the particle velocity, $\mathbf{r}$ the position and m the mass of particle, $\mathbf{E}$ the electric field and $\mathbf{B}$ the magnetic field. The right side $(\delta f/\delta t)_{\text{coll}}$ denotes the collisional operator, and $(\delta f/\delta t)_{\text{wp}}$ refers to the wave-particle interaction term associated with various plasma waves.

Since the electron gyroperiod is much smaller than any other characteristic timescale, it is reasonable to assume gyrotropy of the electron VDF. The gyrophase averaged kinetic equation (1) for solar wind electron distribution function $f(r, v, \mu)$ has the form [15]:

$$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial r} + \left[\frac{v(1 - \mu^2)}{r} - \frac{eE_{\parallel}}{m} \frac{1 - \mu^2}{v} \right] \frac{\partial f}{\partial \mu} - \frac{eE_{\parallel}}{m} \mu \frac{\partial f}{\partial v} = \left(\frac{\delta f}{\delta t} \right)_{\text{coll}} + \left(\frac{\delta f}{\delta t} \right)_{\text{wp}}, \quad (2)$$

where (v, μ) is the magnitude of velocity and pitch-angle measured in the solar wind frame, r the heliocentric distance, e the electron charge and $E_{\parallel}$ the parallel electric field with respect to the

magnetic field. Note that we assume $\mu v \gg U$, where U is the bulk velocity of the background plasma flow

We assume the collisional operator used by Horaites et al. [5, 6]:

$$\left(\frac{\delta f}{\delta t}\right)_{\text{coll}} = \frac{4\pi n e^4 \Lambda}{m^2} \left\{ \frac{1}{v^3} \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial f}{\partial \mu} \right] + \frac{1}{v^2} \frac{\partial f}{\partial v} + \frac{v_{\text{th-core}}^2}{2v^2} \left(\frac{1}{v} \frac{\partial^2 f}{\partial v^2} - \frac{1}{v^2} \frac{\partial f}{\partial v} \right) \right\}, \quad (3)$$

where Λ is the Coulomb logarithm and $v_{\text{th-core}}^2$ the thermal velocity of the Maxwellian core electrons.

The wave-particle interaction term associated with whistler turbulence is given by Pierrard et al. [14]:

$$\left(\frac{\delta f}{\delta t}\right)_{\text{wp}} = \frac{\partial}{\partial \mu} \left(D_{\mu\mu} \frac{\partial f}{\partial \mu} + D_{\mu p} \frac{\partial f}{\partial p} \right) + \frac{1}{p^2} \frac{\partial}{\partial p} \left[p^2 \left(D_{p\mu} \frac{\partial f}{\partial \mu} + D_{pp} \frac{\partial f}{\partial p} \right) \right] \quad (4)$$

where p is the particle's momentum, $D_{\mu p} = D_{p\mu}$, and for nonrelativistic electrons:

$$\frac{D_{pp}}{\Omega_e (mc)^2} = \frac{\pi A}{3 a} \left(\frac{\beta |\mu|}{a} \right)^{\frac{s-1}{3}} (1 - \mu^2), \quad (4a)$$

$$\frac{D_{\mu p}}{\Omega_e (mc)} = -\frac{\pi A}{3 a} \left[\frac{\mu}{|\mu|} \left(\frac{\beta |\mu|}{a} \right)^{\frac{s-2}{3}} + \frac{\mu}{\beta} \left(\frac{\beta |\mu|}{a} \right)^{\frac{s-1}{3}} \right] (1 - \mu^2), \quad (4b)$$

$$\frac{D_{\mu\mu}}{\Omega_e} = \frac{\pi A}{3 a} \left[\left(\frac{\beta |\mu|}{a} \right)^{\frac{s-3}{3}} + \frac{2\mu}{|\mu|} \frac{\mu}{\beta} \left(\frac{\beta |\mu|}{a} \right)^{\frac{s-2}{3}} + \left(\frac{\mu}{\beta} \right)^2 \left(\frac{\beta |\mu|}{a} \right)^{\frac{s-1}{3}} \right] (1 - \mu^2). \quad (4c)$$

Here $\beta = v/c$, $a = \omega_{\text{pe}}^2 / \Omega_e^2$, $A = 0.1$, $s = 3/2 - 5/3$ and the electron plasma and gyrofrequencies are given by

$$\omega_{\text{pe}}^2 = \frac{4\pi n e^2}{m}, \quad \Omega_e = \frac{eB}{mc}.$$

The background plasma parameters are assumed have the form of power laws, as used by Horaites et al. [5, 6]:

$$B = B_0 \left(\frac{r}{R} \right)^{-2}; \quad n = n_0 \left(\frac{r}{R} \right)^{-1.8};$$

$$T = T_0 \left(\frac{r}{R} \right)^{-0.4}; \quad E_{\parallel} = 2.2 \frac{T_0}{eR} \left(\frac{r}{R} \right)^{-1.4}.$$

where B_0 , n_0 and T_0 are the magnetic field, electron number density and electron temperature at 1 AU, r is the helocentric distance and R is 1 AU.

3. The Fokker-Planck equation for solar wind electrons

We developed a numerical method using adaptive mesh refinement [16–20] in a 3D Cartesian coordinates system to solve an equation that has the form of:

$$\frac{\partial f}{\partial t} + \nabla \cdot (\mathbf{V} f) = \nabla \cdot (\mathbb{D} \cdot \nabla f) + \beta f + r. \quad (5)$$

where $f(x_1, x_2, x_3)$ is a general function, $\mathbf{V} = (V_1, V_2, V_3)$ is the phase space advection velocity vector and the phase space diffusion tensor $\mathbb{D}$ here has only the simplest diagonal form in which there are no cross terms. A reduced form of diffusion coefficient is sufficient for our purposes.

$$\begin{pmatrix} \mathbb{D}_{11} & 0 & 0 \\ 0 & \mathbb{D}_{22} & 0 \\ 0 & 0 & \mathbb{D}_{33} \end{pmatrix}.$$

On combining equation (2) and equation (4), changing the variable from p to v and ignoring the two cross terms ($D_{p\mu}$, $D_{\mu p}$), we then obtain the kinetic equation with wave-particle interaction terms associated with whistler turbulence for solar wind electrons:

$$\begin{aligned} \frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial r} + \left[\frac{v(1-\mu^2)}{r} - \frac{eE_{\parallel}}{m} \frac{1-\mu^2}{v} \right] \frac{\partial f}{\partial \mu} - \frac{eE_{\parallel}}{m} \mu \frac{\partial f}{\partial v} \\ = \frac{\partial}{\partial \mu} \left(D_{\mu\mu} \frac{\partial f}{\partial \mu} \right) + \frac{1}{v^2} \frac{\partial}{\partial v} \left(\frac{v^2 D_{pp}}{m^2} \frac{\partial f}{\partial v} \right). \end{aligned} \quad (6)$$

On setting $f = Y/(v^2 r^2)$ allows the kinetic equation above to be rewritten as:

$$\begin{aligned} \frac{\partial Y}{\partial t} + \frac{\partial}{\partial r} (\mu v Y) + \frac{\partial}{\partial \mu} \left\{ \left[\frac{v(1-\mu^2)}{r} - \frac{eE_{\parallel}}{m} \frac{1-\mu^2}{v} \right] Y \right\} \\ + \frac{\partial}{\partial v} \left\{ \left[-\frac{eE_{\parallel}}{m} \mu + \frac{\pi c^2 \Omega_e^3}{15 v \omega_{pe}^2} \left(\frac{v \Omega_e^2}{c \omega_{pe}^2} \right)^{1/6} |\mu|^{1/6} (1-\mu^2) \right] Y \right\} \\ = \frac{\partial}{\partial v} \left[\frac{\pi c^2 \Omega_e^3}{30 \omega_{pe}^2} \left(\frac{v \Omega_e^2}{c \omega_{pe}^2} \right)^{1/6} |\mu|^{1/6} (1-\mu^2) \frac{\partial Y}{\partial v} \right] \\ + \frac{\partial}{\partial \mu} \left\{ \frac{\pi \Omega_e^3}{30 \omega_{pe}^2} \left[\left(\frac{v \Omega_e^2}{c \omega_{pe}^2} \right)^{-\frac{1}{2}} |\mu|^{-\frac{1}{2}} + \frac{c}{v} \left(\frac{v \Omega_e^2}{c \omega_{pe}^2} \right)^{-\frac{1}{6}} \frac{2\mu^2}{|\mu|^{7/6}} \right. \right. \\ \left. \left. + \frac{c^2}{v^2} \left(\frac{v \Omega_e^2}{c \omega_{pe}^2} \right)^{\frac{1}{6}} |\mu|^{\frac{1}{6}} \mu^2 \right] (1-\mu^2) \frac{\partial Y}{\partial \mu} \right\}. \end{aligned} \quad (7)$$

On combining equation (2) and equation (3), we also obtain the kinetic equation for solar wind electrons with the diffusion term associated with Coulomb collisions:

$$\begin{aligned} \frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial r} + \left[\frac{v(1-\mu^2)}{r} - \frac{eE_{\parallel}}{m} \frac{1-\mu^2}{v} \right] \frac{\partial f}{\partial \mu} - \frac{eE_{\parallel}}{m} \mu \frac{\partial f}{\partial v} \\ = \frac{4\pi n e^4 \Lambda}{m^2} \left\{ \frac{1}{v^3} \frac{\partial}{\partial \mu} \left[(1-\mu^2) \frac{\partial f}{\partial \mu} \right] + \frac{1}{v^2} \frac{\partial f}{\partial v} + \frac{v_{th}^2}{2v^2} \left(\frac{1}{v} \frac{\partial^2 f}{\partial v^2} - \frac{1}{v^2} \frac{\partial f}{\partial v} \right) \right\}. \end{aligned} \quad (8)$$

Similarly, let $f = Y/(v^2 r^2)$, allowing the above equation to be rewritten as:

$$\begin{aligned} \frac{\partial Y}{\partial t} + \frac{\partial}{\partial r}(\mu v Y) + \frac{\partial}{\partial \mu} \left\{ \left[\frac{v(1-\mu^2)}{r} - \frac{eE_{\parallel}}{m} \frac{1-\mu^2}{v} \right] Y \right\} \\ + \frac{\partial}{\partial v} \left[\left(-\frac{eE_{\parallel}}{m} - \frac{4\pi n e^4 \Lambda}{m^2} \frac{1}{v^2} + \frac{4\pi n e^4 \Lambda}{m^2} \frac{v_{\text{th-core}}^2}{v^4} \right) Y \right] \\ = \frac{\partial}{\partial v} \left(\frac{1}{v^3} \frac{4\pi n e^4 \Lambda v_{\text{th-core}}^2}{2m^2} \frac{\partial Y}{\partial v} \right) + \frac{\partial}{\partial \mu} \left(\frac{4\pi n e^4 \Lambda}{m^2} \frac{1-\mu^2}{v^3} \frac{\partial Y}{\partial \mu} \right). \end{aligned} \quad (9)$$

Here we have rewritten equation (6) and equation (8) for an electron VDF $f(r, v, \mu)$ in the form of an advection-diffusion-like equation for $Y(r, v, \mu)$ in the 3D phase space (r, v, μ) , i.e. equations (7) and (9) which have the same form as equation (5). The phase space advection velocities and phase space diffusion coefficients associated with whistler turbulence and Coulomb collision separately are as follows.

For the advection-diffusion-like kinetic equation with whistler turbulence diffusion terms we have:

$$\begin{aligned} V_r &= \mu v; \\ V_v &= -\frac{eE_{\parallel}}{m} \mu + \frac{\pi c^2 \Omega_e^3}{15 v \omega_{pe}^2} \left(\frac{v}{c} \frac{\Omega_e^2}{\omega_{pe}^2} \right)^{1/6} |\mu|^{1/6} (1-\mu^2); \\ V_{\mu} &= \frac{v(1-\mu^2)}{r} - \frac{eE_{\parallel}}{m} \frac{1-\mu^2}{v}; \\ \mathbb{D}_{rr} &= 0; \\ \mathbb{D}_{vv} &= \frac{\pi c^2 \Omega_e^3}{30 \omega_{pe}^2} \left(\frac{v}{c} \frac{\Omega_e^2}{\omega_{pe}^2} \right)^{1/6} |\mu|^{1/6} (1-\mu^2); \\ \mathbb{D}_{\mu\mu} &= \frac{\pi \Omega_e^3}{30 \omega_{pe}^2} \left[\left(\frac{v}{c} \frac{\Omega_e^2}{\omega_{pe}^2} \right)^{-\frac{1}{2}} |\mu|^{-\frac{1}{2}} + \frac{c}{v} \left(\frac{v}{c} \frac{\Omega_e^2}{\omega_{pe}^2} \right)^{-\frac{1}{6}} \frac{2\mu^2}{|\mu|^{7/6}} + \frac{c^2}{v^2} \left(\frac{v}{c} \frac{\Omega_e^2}{\omega_{pe}^2} \right)^{\frac{1}{6}} |\mu|^{\frac{1}{6}} \mu^2 \right] (1-\mu^2). \end{aligned}$$

For the advection-diffusion-like kinetic equation with Coulomb collision diffusion terms we have:

$$\begin{aligned} V_r &= \mu v; \\ V_v &= -\frac{eE_{\parallel}}{m} - \frac{4\pi n e^4 \Lambda}{m^2} \frac{1}{v^2} + \frac{4\pi n e^4 \Lambda}{m^2} \frac{v_{\text{th-core}}^2}{v^4}; \\ V_{\mu} &= \frac{v(1-\mu^2)}{r} - \frac{eE_{\parallel}}{m} \frac{1-\mu^2}{v}; \\ \mathbb{D}_{rr} &= 0; \\ \mathbb{D}_{vv} &= \frac{4\pi n e^4 \Lambda}{m^2} \frac{v_{\text{th-core}}^2}{2v^3}; \\ \mathbb{D}_{\mu\mu} &= \frac{4\pi n e^4 \Lambda}{m^2} \frac{1-\mu^2}{v^3}. \end{aligned}$$

4. Numerical solutions

Equations (7) and (9) can be solved numerically by the method we developed for appropriate boundary and initial conditions. The solutions yield the temporal and radial evolution of the function Y (hence the electron VDF $f = Y/(v^2 r^2)$). At the left boundary $r = 0$, a narrow beam of particles is injected into interplanetary space. The beam is comprised of a δ -function in v and a δ -function in μ . For simplicity we use normal distributions to substitute for the two δ -functions. In this case, the boundary condition at $r = 0$ has the form:

$$f(r = 0, v, \mu) = \frac{n_0}{2\pi v^2} \frac{1}{\sqrt{4\pi^2 \sigma_\mu^2 \sigma_v^2}} \exp \left[-\frac{(\mu - \mu_0)^2}{2\sigma_\mu^2} \right] \exp \left[-\frac{(v - v_0)^2}{2\sigma_v^2} \right] \quad (10)$$

where n_0 is electron number density at 1 AU, and the following parameters are used:

$$\begin{aligned} \sigma_\mu^2 &= 0.02; & \sigma_v^2 &= 0.2; \\ \mu_0 &= 0.95; & v_0 &= 3. \end{aligned}$$

The initial distribution is plotted in Figure (2), shown in normalized phase space coordinates $(v_\parallel, v_\perp)$, $v_\parallel$ is the velocity component parallel to the mean magnetic field and $v_\perp$ the perpendicular component.

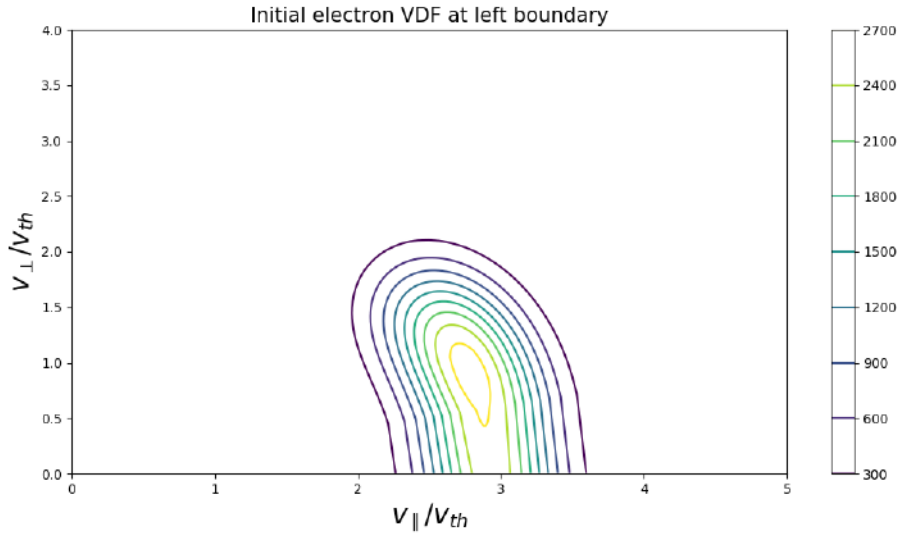


Figure 2. The left boundary condition for the injected electron VDF, plotted in $(v_\parallel, v_\perp)$, $v_\parallel$ phase space.

We obtained the numerical solution of electron VDF after a steady state was reached. Figure (3) shows the VDF at 1 AU after being subjected to whistler turbulence and Coulomb collisions. Figure (3) illustrates that whistler turbulence has a much stronger diffusive effect on the narrow beam of particles than Coulomb collisions. Figure 3a and 3b are the electron VDFs at 0.3 and 1 AU assuming only Coulomb collisions from Horaites' formulation [5, 6]. Figure 3c and 3d are the electron VDFs at 0.3 and 1 AU assuming only whistler turbulence from Pierrard's formulation [14].

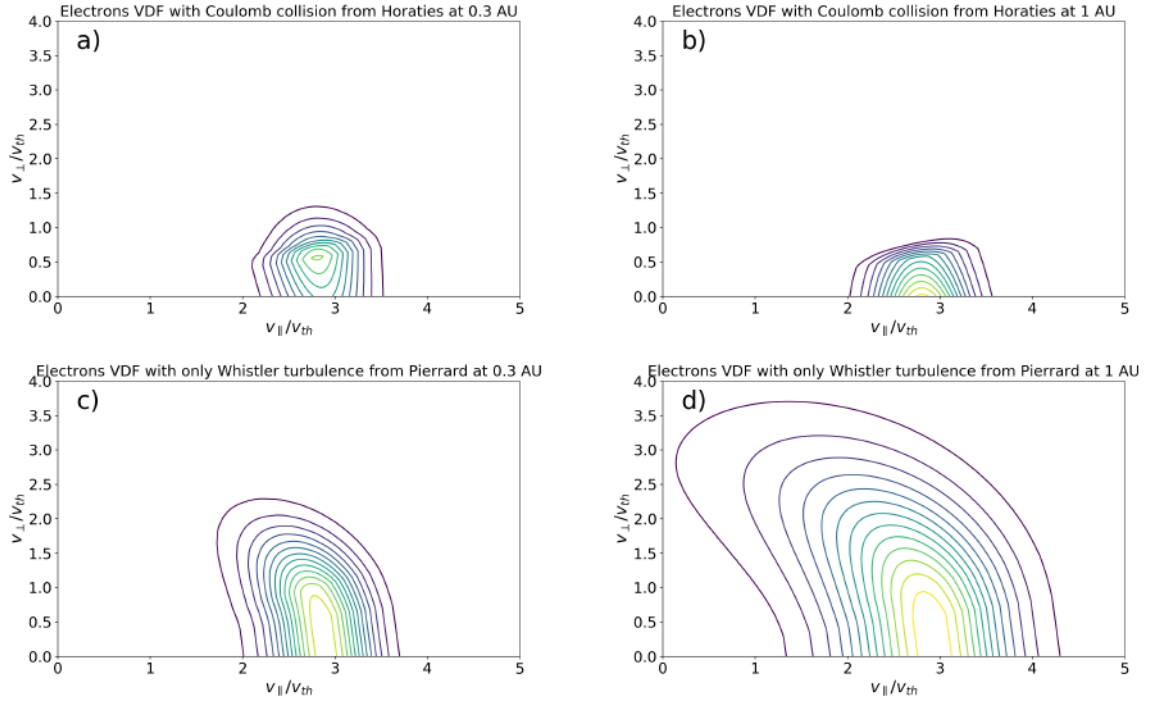


Figure 3. Comparison of the electron VDFs. The left column shows results at 0.3 AU and the right column at 1 AU. The top row shows the evolution of the electron VDF when subjected to Coulomb collisions only. The bottom row correspond to whistler turbulence only.

5. Summary and Conclusion

We have investigated the influence of whistler turbulence and Coulomb collisions separately on the temporal and radial evolution of the electron distribution function by numerically solving the Fokker-Planck kinetic equations. From the viewpoint of the method of characteristics, it is not difficult to identify that these phase space advection velocities are actually the equations for the characteristics. Therefore, in the 3D phase space (r, v, μ) , the function Y (hence the electron VDF or electrons) moves along these characteristics. Electrons will move to regions in phase space where $\mu \approx 1$ and move in the r direction. In the solar wind, electrons therefore focus and align with the magnetic field and the beam is narrowed. This effect is illustrated in Figures (2) and (3a,b). The left boundary condition distribution is narrowed and electrons are therefore focused and aligned with the magnetic field. However, there are diffusion terms on the right side of both equations (7) and (9). Besides moving along the characteristics, electrons will also experience diffusion. From the equations, diffusion in phase space is with respect to v and pitch-angle μ . Because of diffusion associated with whistler turbulence and Coulomb collisions, the injected narrow beam of electrons broadens in velocity space (v, μ) . Pitch-angle diffusion causes the beam to expand (or diffuse) in the μ direction, which will increase the width of beam. This effect can explain the observation that the strahl has a finite width.

From Figures (2), (3c) and (3d), we see that the diffusion in $v_{\parallel}$ is a little larger than $v_{\perp}$. For the whistler turbulence, the isotropization is due to $\mathbb{D}_{\mu\mu}$ term, but there also appears to be quite significant energization due to $\mathbb{D}_{vv}$. It is clear that whistler turbulence is far more important than Coulomb collisions. Comparing with the initial beam, the beam is broadened along $v_{\parallel}$. While the beam is broadened along $v_{\perp}$ too, such broadening is more slowly. Therefore, the electron VDF retains some anisotropic with a beam along the parallel direction. This tendency is a consequence of the focusing term.

Although both Coulomb collisions and whistler turbulence can cause diffusion of electrons, their efficiency or ability to diffuse is different. When the heliocentric distance is large or the energies of the electrons are high enough, Coulomb collisions are a much weaker effect and wave-particle interactions becomes important. Figure (3b) is the electron VDF at 1 AU after electrons have been subjected to Coulomb collisions only. Figure (3d) is the electron VDF at 1 AU after electrons have been subjected to whistler turbulence only. Diffusion associated with whistler turbulence is indeed much stronger than that associated with Coulomb collisions.

Our present results confirm that whistler turbulence is much much efficient than Coulomb collisions in scattering particles in phase space in the supersonic solar wind.

References

- [1] Anderson B, Skoug R, Steinberg J and McComas D 2012 *Journal of Geophysical Research: Space Physics* **117**
- [2] Štverák Š, Maksimovic M, Trávníček P M, Marsch E, Fazakerley A N and Scime E E 2009 *Journal of Geophysical Research: Space Physics* **114**
- [3] Tao J, Wang L, Zong Q, Li G, Salem C S, Wimmer-Schweingruber R F, He J, Tu C, Bale S D, Wang L *et al.* 2016 Solar wind 0.1-1.5 keV electrons at quiet times *AIP Conference Proceedings* vol 1720 (AIP Publishing) p 070006
- [4] Salem C, Hubert D, Lacombe C, Bale S, Mangeney A, Larson D and Lin R 2003 *The Astrophysical Journal* **585** 1147
- [5] Horaites K, Boldyrev S, Krasheninnikov S, Salem C, Bale S and Pulupa M 2015 *Physical review letters* **114** 245003
- [6] Horaites K, Boldyrev S, Wilson III L B, Viñas A F and Merka J 2017 *Monthly Notices of the Royal Astronomical Society* **474** 115–127
- [7] Vocks C and Mann G 2003 *The Astrophysical Journal* **593** 1134
- [8] Vocks C, Salem C, Lin R and Mann G 2005 *The Astrophysical Journal* **627** 540
- [9] Vocks C, Mann G and Rausche G 2008 *Astronomy & Astrophysics* **480** 527–536
- [10] Kim S, Yoon P H, Choe G and Wang L 2015 *The Astrophysical Journal* **806** 32
- [11] Kim S, Yoon P H and Choe G 2016 *Physics of Plasmas* **23** 022111
- [12] Yoon P H, Kim S, Choe G *et al.* 2016 *The Astrophysical Journal* **826** 204
- [13] Kim S, Yoon P H, Choe G *et al.* 2016 *The Astrophysical Journal* **828** 60
- [14] Pierrard V, Lazar M and Schlickeiser R 2011 *Solar Physics* **269** 421–438
- [15] Zank G P 2014 *Transport Processes in Space Physics and Astrophysics (Lecture Notes in Physics, Berlin Springer Verlag* vol 877) (Springer-Verlag)
- [16] Kolobov V I 2003 *Computational materials science* **28** 302–320
- [17] Kolobov V and Arslanbekov R 2012 *Journal of Computational Physics* **231** 839–869
- [18] Arslanbekov R R, Kolobov V I and Frolova A A 2013 *Physical Review E* **88** 063301
- [19] Kolobov V I 2013 *Physics of Plasmas* **20** 101610
- [20] Zabelok S, Arslanbekov R and Kolobov V 2015 *Journal of Computational Physics* **303** 455–469