

Strategic Equilibrium Bidding for Electricity Suppliers in A Day-Ahead Market Using Inverse Optimization*

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Abstract—We consider the problem of devising optimal bidding strategies for electricity suppliers in a day-ahead market where each supplier bids a linear non-decreasing function of its generating capacity for each of the 24 hours. The market operator schedules suppliers based on their bids to meet demand during each hour and determines hourly market clearing prices. Each supplier strives to submit bids that maximize her individual profit, conditional upon other suppliers bids. This process achieves a Nash equilibrium when no supplier is motivated to modify her bid. Solving the profit maximization problem requires information of rivals' bids which are typically not available. We develop an inverse optimization approach for estimating rivals' cost functions given historical market clearing prices and production levels, and use these functions to compute the Nash equilibrium bids. We propose sufficient conditions for the existence and uniqueness of the Nash equilibrium, and provide out-of-sample performance guarantees for the estimated cost parameters. Numerical experiments show that our approach achieves higher profit than the one proposed in [16], which relies instead on the assumption that other suppliers' bids are normally distributed.

I. INTRODUCTION

In the past several decades, the electricity industry has been experiencing a transformation from vertical integrated regulated monopolies to competitive supply and demand market participants with equal access to a regulated transmission and distribution network. Nevertheless, due to special features of the power industry, such as a limited number of producers, large capital investments which introduce barriers to entry, and congestion caused by occasionally binding transmission constraints, the electricity market is characterized by oligopolistic conditions [7]. Whereas under perfect competition, suppliers bid their marginal costs, a condition for social welfare and efficiency maximization, in an imperfect (perhaps real) oligopolistic energy market, suppliers can exploit market manipulation opportunities to increase their profits by bidding above their marginal cost, a behavior referred to as strategic bidding, whose investigation is of dual interest. First, to power suppliers, who wish to devise optimal bidding strategies allowing them to stand out

among competitors and realize profits that exceed those that a perfectly competitive market would warrant. Second, it is also of interest to market regulators, helping them to identify abuse of market power and develop policies to regulate it in the interest of economic efficiency and social welfare.

Electricity markets are generally auction based. The auction models can be classified as static or dynamic according to whether bidders are allowed to modify their bids sequentially. The most widely used type is the sealed single energy part bid auction resulting in a single Market Clearing Price (MCP) employed to credit all suppliers and charge all consumers. An exception is the England-Wales electricity market where the market clears with multiple bilateral transactions involving multi-part bids that include separate prices for start-up costs, shut-down costs, no-load operation, and energy. Multiple part bids, often called complex bids, are also used in the Spanish day-ahead market. Wholesale electricity markets in the U.S., (ISONE, NYISO, PJM, ERCOT, CAISO, MISO), Canada and most European markets involve single-part bids for energy and result in a single clearing price. In this paper we focus on the California Independent System Operator (CAISO) single-part bid protocol.

[7], [12] are good literature surveys on strategic bidding in electricity markets. There are mainly two types of methods for constructing optimal bidding strategies. One is concerned with solving a single supplier profit-maximization problem by estimating the bidding behavior of rivals, while the other models the interaction among all participants and relies on game theoretic methods to solve for Nash equilibrium strategies that, as a whole, render each participant unable to improve her individual profit by changing her individual bid. [8] proposes a recursive dynamic programming approach for determining the optimal bid price for each block of generation. In this approach, each supplier maximizes her individual profit while modeling the uncertainty about competitor bid prices by a probability distribution. In [16], two different bidding schemes are developed; one maximizes the hourly profit assuming all other producer bids are represented by a multivariate normal distribution whose parameters are estimated from historical data. If a supplier cannot be dispatched using this scheme, then an alternative bidding strategy, which guarantees production at the minimum stable output level, is used. [17] proposes a decomposition-based particle swarm optimization method to solve the expected profit maximization problem with MCP being the uncertain, exogenous variable. [14] presents a Q-Learning algorithm to find the optimal bidding strategies in Iran's power market with a pay-as-bid auction rule.

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The game theory based approaches can be further classified into three subgroups based on the competition rules: Bertrand (price competition) [5], [6], Cournot (quantity competition) [4], and *Supply Function Equilibrium (SFE)* [15]. These methods analyze the economic equilibria of the electricity market by focusing on the mutual interactions among participants. In [6], a decentralized Nash equilibrium learning strategy is presented in a Bertrand competition framework to solve the economic dispatch problem. In [11], the authors study the problem of building bidding strategies in oligopolistic dynamic electricity double-sided auctions (both supply and demand bids are submitted) from a Nash-Cournot point of view. The SFE, as the most widely analyzed model in this category, assumes that participants set supply functions which link the bidding prices with their generation quantities. It has been recognized that the SFE model constitutes a good compromise between the Cournot and Bertrand models, and most accurately reflects the behavior of suppliers in real power markets [12]. For more SFE related work, see [3], [9], [10], [13].

In this paper we solve the SFE-based equilibrium strategies for suppliers in a California type electricity market. There have been two main criticisms regarding game-theoretic approaches. One is concerned with the unrealistic assumption that the payoff functions of all participants are publicly available. The other points to the multiple equilibria issue. We propose to use an inverse optimization [2] approach for the estimation of payoff functions. As such, our method could be easily applied without the need to know all profit functions. In addition, we propose a sufficient condition that is easy to satisfy in real systems and guarantees the uniqueness of the Nash equilibrium.

The rest of the paper is organized as follows. In Section II, we formulate the strategic bidding problem. Section III gives conditions for the existence and uniqueness of the Nash equilibrium and establishes out-of-sample performance guarantees for the estimated cost parameters. The algorithms, as well as numerical results are presented in Section IV. We conclude the paper in Section V.

II. PROBLEM FORMULATION

We consider a day-ahead California type electricity market which is composed of n energy suppliers, a group of customers, and a market operator instantiating a *Power Exchange (PX)*. For each of the 24 hours in the next day, every supplier submits a bid curve (or supply curve) that describes the relationship between energy price and production quantity. In real power markets, this is a piecewise constant curve [15] reflecting the constant bid price for each block of generation. Nonetheless, we assume the affine bid curve as commonly used in the SFE literature to facilitate our analysis. After receiving the bidding functions from all suppliers, the PX clears the market by balancing aggregate supply and demand, and derives the hourly MCP and supplier-specific dispatch schedule. This market clearing process is repeated for each of the 24 hours simultaneously and independently, since we assume no intertemporal coupling.

Suppliers determine the bidding coefficients in such a way as to maximize their own profits based on the historical information about past bids.

Since the auctions for different hours are performed separately and independently, we will only consider the bidding strategy for a specific hour and omit the time index in the following analysis. Assume that supplier i submits a linear non-decreasing bid function to PX,

$$B_i = \alpha_i + \beta_i P_i, \quad i = 1, \dots, n,$$

where B_i is the price per unit of power at production level P_i , and α_i, β_i are the bidding coefficients to be determined. With these linear bidding functions, the PX solves the following system to derive the MCP and dispatch plans:

$$\begin{aligned} R &= \alpha_i + \beta_i P_i, \quad i = 1, \dots, n, \\ Q &= \sum_{i=1}^n P_i, \\ P_i^{\min} &\leq P_i \leq P_i^{\max}, \quad i = 1, \dots, n, \end{aligned} \quad (1)$$

where R is the MCP; Q is the demand forecast known by all participants; and $P_i^{\min}, P_i^{\max}$ are the minimum and maximum generation levels of supplier i , respectively. Define $\alpha \triangleq (\alpha_1, \dots, \alpha_n)$, $\beta \triangleq (\beta_1, \dots, \beta_n)$. Ignoring the capacity constraints, the solution to (1) is,

$$\begin{aligned} R(\alpha, \beta) &= \frac{Q + \sum_{i=1}^n \alpha_i / \beta_i}{\sum_{i=1}^n 1 / \beta_i}, \\ P_i(\alpha, \beta) &= \frac{R - \alpha_i}{\beta_i}, \quad i = 1, \dots, n, \end{aligned} \quad (2)$$

where we write $R(\alpha, \beta), P_i(\alpha, \beta)$ to explicitly express the dependency of these quantities on the bids.

Remark: The $P_i(\alpha, \beta)$ derived from (2) might be infeasible. If it is below $P_i^{\min}$, we set $P_i(\alpha, \beta)$ to zero and this supplier is not dispatched. On the other hand, if it is above $P_i^{\max}$, we set $P_i(\alpha, \beta) = P_i^{\max}$. Essentially, we want to project the $P_i(\alpha, \beta)$ from Eq. (2) onto the interval $[P_i^{\min}, P_i^{\max}]$. Instead of directly projecting $P_i(\alpha, \beta)$, we consider projecting the bidding intercepts α_i , which allows us to retain the closed form expression of (2) and facilitates the following analysis. (We do not project β_i here since it is considered to be a fixed constant; the reason will be explained in Section II-A.) Specifically, α_i is restricted to the interval $[R(\alpha, \beta) - \beta_i P_i^{\max}, R(\alpha, \beta) - \beta_i P_i^{\min}]$, where $R(\alpha, \beta)$ admits the form in (2). It is worth mentioning that the projected α_i is no longer the true bidding intercept. However, using this projected version gives us the same amount of information and greatly simplifies our further analysis. With a little abuse of notation, we use α_i to denote the projected bidding intercept.

A. Forward Problem

Supplier i determines the bidding curve (α_i, β_i) in such a way as to maximize its own profit, which is formulated as:

$$\begin{aligned} \max_{\alpha_i, \beta_i} \quad & \phi_i(\alpha, \beta) = R(\alpha, \beta)P_i(\alpha, \beta) - C_i(P_i(\alpha, \beta)) \\ \text{s.t.} \quad & R(\alpha, \beta) - \beta_i P_i^{\max} \leq \alpha_i \leq R(\alpha, \beta) - \beta_i P_i^{\min}, \quad (3) \\ & \beta_i \geq 0, \end{aligned}$$

where $C_i(P_i(\alpha, \beta))$ is the production cost at generation level $P_i(\alpha, \beta)$ and $\phi_i(\alpha, \beta)$ is the profit. If we assume a quadratic cost structure, i.e.,

$$C_i(P_i) = \theta_{i0} + \theta_{i1}P_i + \theta_{i2}P_i^2,$$

where $\theta_{i0}, \theta_{i1}, \theta_{i2}$ are the cost parameters for supplier i , then based on (2), the profit could be expressed as:

$$\begin{aligned} \phi_i(\alpha, \beta) &= R(\alpha, \beta)P_i(\alpha, \beta) - C_i(P_i(\alpha, \beta)) \\ &= \frac{Q + \sum_{i=1}^n \alpha_i / \beta_i}{\sum_{i=1}^n 1 / \beta_i} \frac{Q + \sum_{i=1}^n \alpha_i / \beta_i}{\sum_{i=1}^n 1 / \beta_i} - \alpha_i \\ &\quad - \theta_{i0} - \theta_{i1} \frac{Q + \sum_{i=1}^n \alpha_i / \beta_i}{\sum_{i=1}^n 1 / \beta_i} - \alpha_i \\ &\quad - \theta_{i2} \left(\frac{Q + \sum_{i=1}^n \alpha_i / \beta_i}{\sum_{i=1}^n 1 / \beta_i} - \alpha_i \right)^2. \end{aligned} \quad (4)$$

Remark: We assume that β_i is a fixed, known constant and only the intercept α_i can be manipulated. The capacity and technology of individual generating plants is public information that provides useful partial information about their cost functions. Nevertheless, their fuel and variable maintenance cost, and their exact heat rate (efficiency), reflected primarily in the parameter α_i , is proprietary and not known or easy to estimate. Assuming a known slope β_i at which the bid increases over the generator's capacity is reasonable. It reflects (i) the smoothing/regularization of the bid conforming to the monotonically increasing market rule requirement (marginal costs are physically not strictly monotonic), and (ii) the advantage of and desire for achieving unique price-directed marginal generator schedules.

Since all suppliers choose their bids by solving the profit maximization problem (3), it is reasonable to construct a SFE model describing the game among all generators where the actions are the bids and the payoffs are the profits. We say that α is a Nash equilibrium if no single supplier can increase her profit by unilaterally changing her own bid.

Note that for each supplier i , $\phi_i(\alpha, \beta)$ is determined by the actions of all players and its own cost parameter $\theta_i \triangleq (\theta_{i0}, \theta_{i1}, \theta_{i2})$ (cf. Eq. (4)), and thus, henceforth, we write $\phi_i(\alpha, \theta_i)$ to emphasize this dependency (β is removed since it is fixed and known). In order to solve her own profit maximization problem, supplier i must have knowledge of how her rivals behave. Historical data could be utilized to infer the behavior of competitors. Suppose that we have obtained the MCP, as well as the corresponding dispatch plans, for the same time period in the past N days, denoted by $(R^j, P_1^j, \dots, P_n^j)$, $j = 1, \dots, N$. Then the bids $\alpha^j =$

$(\alpha_1^j, \dots, \alpha_n^j)$, $j = 1, \dots, N$, could be computed via the market clearing condition in (1) (β_i^j 's are assumed to be known). Given these past bidding histories, we propose to first estimate rivals' profit functions (or cost parameters) and then compute the current equilibrium bids using these estimated profit functions.

B. Inverse Problem

The inverse problem deals with the task of estimating competitors' cost parameters. The main theoretical foundation is attributed to [2], where the authors estimate the utility functions of players in a game from their observed actions by combining ideas from inverse optimization with the theory of variational inequalities. In our case, we are given the past bids $\alpha^j = (\alpha_1^j, \dots, \alpha_n^j)$, $j = 1, \dots, N$, and are interested in obtaining estimates of θ_i , $i = 1, \dots, n$. We assume that each of these bids lies in the interval $[0, \bar{\alpha}]$, where $\bar{\alpha}$ is an upper bound known a priori. According to Theorem 3 in [2], which is derived through duality, the solution to the following optimization problem gives us the estimated cost parameters.

$$\begin{aligned} \min_{\theta_1, \dots, \theta_n} \quad & \|\epsilon\|_\infty \\ \text{s.t.} \quad & y_i^j \geq 0, \quad i = 1, \dots, n; \quad j = 1, \dots, N, \\ & y_i^j \geq \nabla_i \phi_i(\alpha^j, \theta_i), \quad i = 1, \dots, n; \quad j = 1, \dots, N, \\ & \sum_{i=1}^n (\bar{\alpha} y_i^j - \alpha_i^j \nabla_i \phi_i(\alpha^j, \theta_i)) \leq \epsilon_j, \quad j = 1, \dots, N, \end{aligned} \quad (5)$$

where $\|\cdot\|_\infty$ is the infinity norm operator defined as the maximum of the absolute elements of the argument; and $\nabla_i \phi_i(\alpha^j, \theta_i)$ is the partial derivative of ϕ_i w.r.t. α_i evaluated at (α^j, θ_i) , which is given by:

$$\begin{aligned} \nabla_i \phi_i(\alpha^j, \theta_i) &= \frac{1/\beta_i^j}{\sum_{i=1}^n 1/\beta_i^j} \cdot \frac{Q^j + \sum_{i=1}^n \alpha_i^j / \beta_i^j}{\sum_{i=1}^n 1/\beta_i^j} - \alpha_i^j \\ &\quad + \frac{Q^j + \sum_{i=1}^n \alpha_i^j / \beta_i^j}{\sum_{i=1}^n 1/\beta_i^j} \cdot \frac{1/\beta_i^j}{\sum_{i=1}^n 1/\beta_i^j} - 1 \\ &\quad - \theta_{i1} \cdot \frac{1/\beta_i^j}{\sum_{i=1}^n 1/\beta_i^j} - 1 \\ &\quad - 2\theta_{i2} \cdot \frac{Q^j + \sum_{i=1}^n \alpha_i^j / \beta_i^j}{\sum_{i=1}^n 1/\beta_i^j} \cdot \frac{1/\beta_i^j}{\sum_{i=1}^n 1/\beta_i^j} - 1, \end{aligned} \quad (6)$$

where Q^j is the demand level for the j th day and β_i^j is the known bidding slope for supplier i on the j th day. Note that (5) is a *Linear Programming (LP)* problem and can be solved to optimality very efficiently. We note that this inverse optimization technique still applies even when more constraints (the network flow constraints) are imposed in the forward problem setup, or when the bid function is changed, as long as R and P_i have closed form expressions w.r.t. the bidding coefficients. This can be achieved by the projection trick used to incorporate the production capacity

constraints. In addition to estimating competing generators' cost functions, our method is particularly useful in estimating the underlying cost functions of market participants who bid synthetic or virtual generators corresponding to contracts with either physical generation owners or a portfolio of demand-response-capable consumers.

Remark: Our method appears to assume away the fact that network connected markets result in location dependent clearing price differentials driven by (i) small effects of location-specific line loss contributions, but also, (ii) significant contributions during network congestion events. It can capture and address the significant congestion caused differentials by detecting market-splitting occurrences that result in "price islands" with essentially homogeneous prices within each island. Although this limits the number of relevant observations when price islanding occurs, it utilizes the unusually high or low price events associated with congestion.

III. THEORETICAL GUARANTEES

Theorem III.1 provides sufficient conditions for the existence and uniqueness of the Nash equilibrium, which coincides with the 1954 Arrow-Debreu Theorem [1]. It essentially requires the convexity of all cost functions, which is a reasonable and practically meaningful assumption. We omit the proof due to space limitations.

Theorem III.1. *Suppose that for every supplier i , its cost function $C_i(P_i(\alpha, \beta))$ is convex in $P_i(\alpha, \beta)$, i.e., $\theta_{i2} \geq 0$, $\forall i$. Then, there exists a unique Nash equilibrium $\alpha^* = (\alpha_1^*, \dots, \alpha_n^*)$ for the SFE model we defined.*

The quality of the computed equilibrium strategies depends heavily on the reliability of the estimated cost parameters. Intuitively, good estimators should be able to explain well new, future equilibria. The following theorem, which is a restatement of Theorem 6 in [2], ensures the quality of the estimated cost functions under some mild conditions.

Theorem III.2. *Suppose that α^j , $j = 1, \dots, N$ are i.i.d. realizations of a random variable $\tilde{\alpha}$, and $\tilde{\alpha} \in \{\alpha : 0 \leq \alpha_i \leq \bar{\alpha}, \forall i\}$ almost surely. Then, for any $0 < \delta < 1$, with probability at least $1 - \eta$ w.r.t. the sampling,*

$$\mathbb{P}(\tilde{\alpha} \text{ is a } z\text{-approximate equilibrium for the game} \\ \text{with payoffs defined through } \hat{\theta}_1, \dots, \hat{\theta}_n) \geq 1 - \delta, \quad (7)$$

where $\eta = \sum_{i=0}^{2n} \binom{N}{i} \delta^i (1 - \delta)^{N-i}$; z is the optimal value of problem (5); and $\hat{\theta}_1, \dots, \hat{\theta}_n$ are the optimal solutions to (5).

Remarks: For the definition of z -approximate equilibrium, see [2, Section 2.2]. Roughly speaking, it describes the situation where each generator does not necessarily play its best action given what others are doing, but plays a strategy which is no worse than z from the best response.

There are two probability measures in the statement of Theorem III.2. One is related to the new data $\tilde{\alpha}$, while the

other is related to the samples $\alpha^1, \dots, \alpha^N$. The probability in (7) is taken w.r.t. the new data $\tilde{\alpha}$. For a fixed set of samples, (7) holds with probability at least $1 - \eta$ w.r.t. the measure of samples. Theorem III.2 essentially says that given typical samples, the probability that the estimated cost functions explain well new, future equilibria is bounded below. It guarantees the accuracy of the estimated cost parameters under mild conditions.

IV. NUMERICAL IMPLEMENTATIONS

In this section we use synthetic input data to test the validity of our approach and compare it with the one used in [16] (referred to as Wen & David's method). It has been observed that multiple optimal solutions to (5) often exist, which implies that potentially many cost functions could generate the same observed equilibria. We design two algorithms to find practically meaningful cost parameters from the set of all optimal solutions, based on the principle that only estimators close to true costs have good out-of-sample performance. One is termed random search, which searches randomly in the set of optimal solutions until the one that performs well on a validation dataset is found, see Section IV-A. The other is called box search, which narrows down the search region and is thus more efficient compared to random search, see Section IV-B.

Wen & David's method solves the profit maximization problem (3) under the assumption that the bidding coefficients of any other generator follow a bivariate normal distribution whose mean and covariance could be inferred from historical bids. We will see from the experimental results that the performance of their approach relies heavily on the normal distribution hypothesis. There is no evidence that generator's bids behave like a normal random variable. In addition, the hypothesis ignores the potential correlation among generators' actions.

A. Random Search

We first describe the experimental setup. Suppose there are 2 suppliers, whose bidding slopes β_i , $i = 1, 2$ are uniformly distributed in $[0.01, 0.1]$ and $[0.05, 0.2]$, respectively. We obtain $N = 300$ past bids α^j , $j = 1, \dots, 300$, where α^j is generated as the equilibrium bid to the game with demand $Q^j = 40\text{MW}$, $\bar{\alpha} = 20$, and β_i 's coming from the uniform distributions specified above. Several sets of true cost parameters are tested, see Table I. We note that in all tables, within each cell the first number is for supplier 1, and the second is for supplier 2. In all scenarios it is ensured that the cost functions are convex and non-decreasing in P_i . $\bar{\alpha}$ is changed to 100 for Scenario 4 and 50 for Scenario 5.

Our first step is to estimate the cost functions based on past bidding data by solving problem (5). Note that θ_{i0} only serves as a constant term in the profit function (4) and thus does not play a role in the determination of α . We therefore do not need to estimate θ_{i0} , $i = 1, \dots, n$. It is worth mentioning that (5) might give multiple optimal solutions. Our goal is to recover the true cost parameters from this set of optima. Although there might be multiple

TABLE I
TRUE COST PARAMETERS FOR SUPPLIERS.

	θ_{i0}	θ_{i1}	θ_{i2}
Scenario 1 (S1)	(10, 10)	(13, 15)	(0.01, 0.05)
Scenario 2 (S2)	(5, 10)	(3, 5)	(0.1, 0.2)
Scenario 3 (S3)	(3, 5)	(0.1, 0.2)	(0.05, 0.05)
Scenario 4 (S4)	(20, 10)	(20, 50)	(1, 2)
Scenario 5 (S5)	(30, 50)	(10, 5)	(0.5, 1.5)

cost functions that can explain the observed equilibria well, only true costs have good out-of-sample performance. The following Algorithm 1 is thus proposed to identify the true cost functions, based on which current equilibrium bids could be easily computed. Given the cost estimators and the bidding slopes, the equilibrium strategies are obtained through an iterative process in which each player plays her best response given others' actions.

Algorithm 1 Random Search

- 1: **Input:** Demand Q , the number of past bids N , the number of producers n , the percentage of training samples p , tolerance level τ , upper bound for bidding intercepts $\bar{\alpha}$, bidding slopes β^j , $j = 1, \dots, N$, as well as the mean value of the β distribution: $\bar{\beta} = (\bar{\beta}_1, \dots, \bar{\beta}_n)$, past bids α^j , $j = 1, \dots, N$.
 - 2: **Initialize:** $c = 1$
 - 3: **while** $c \geq \tau$ **do**
 - 4: Randomly choose $M = \lfloor Np \rfloor$ samples from all past bids to constitute the training dataset, and use the rest as the validation set.
 - 5: Obtain $\hat{\theta}_i$, $i = 1, \dots, n$, by solving problem (5) using the training dataset.
 - 6: Compute equilibrium strategies $\hat{\alpha}_{\text{val}}^j$, $j = 1, \dots, N - M$, on the validation dataset using $\hat{\theta}_i$, $i = 1, \dots, n$, and the bidding slopes on the validation set β_{val}^j , $j = 1, \dots, N - M$.
 - 7: Evaluate the average discrepancy between computed and true bids on the validation dataset as,

$$c = \frac{1}{N - M} \sum_j \|\alpha_{\text{val}}^j - \hat{\alpha}_{\text{val}}^j\|_1 \quad (8)$$

where α_{val}^j is the j -th true bid on the validation set, and $\|\cdot\|_1$ is the ℓ_1 norm operator defined as the sum of the absolute elements of the argument.
 - 8: **end while**
 - 9: Compute current equilibrium bids using $\hat{\theta}_i$, $i = 1, \dots, n$, and $\bar{\beta}$.
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One may argue that it could take infinite long time to find the estimators $\hat{\theta}_i$, $i = 1, \dots, n$ with satisfactory performance on the validation data. Instead of using the while loop, we could search the optimal solution set of (5) for a fixed number of times. In Section IV-B we present a different search algorithm to improve efficiency. Table II shows the comparison with Wen & David's method. The

loop in Algorithm 1 is executed for 100 times. The profit is computed for the current equilibrium bids using true cost functions and mean bidding slopes $\bar{\beta}$.

B. Box Search

To resolve the issue of a possibly endless loop with random search, we propose the box search method. The idea is to partition the feasible region and search under the guidance of the out-of-sample performance. Specifically, starting with some lower and upper bounds on the cost parameters, we divide the box-shaped region formed by these bounds into several subregions and solve problem (5) within these smaller sets. Compute the out-of-sample performance of the optimal solution from each subregion and proceed with the set that performs the best. By repeating this process, we expect to finally obtain some accurate estimators. See Algorithm 2.

Algorithm 2 Box Search

- 1: **Input:** Demand Q , the number of past bids N , the number of producers n , the percentage of training samples p , tolerance level τ , upper bound for bidding intercepts $\bar{\alpha}$, bidding slopes β^j , $j = 1, \dots, N$, as well as the mean value of the β distribution: $\bar{\beta} = (\bar{\beta}_1, \dots, \bar{\beta}_n)$, past bids α^j , $j = 1, \dots, N$.
 - 2: Randomly choose $M = \lfloor Np \rfloor$ samples from all past bids to constitute the training dataset, and use the rest as the validation set.
 - 3: **Initialize:** Set the lower and upper bounds for the $2n$ cost parameters, denoted as $\text{lb}_{2n \times 1}$ and $\text{ub}_{2n \times 1}$, respectively.
 - 4: **inter** = $\text{ub} - \text{lb}$.
 - 5: **while** $\|\text{inter}\|_2 \geq \tau$ **do**
 - 6: Divide the box-shaped region into 2^{2n} subregions by cutting the interval for each variable in half.
 - 7: Solve problem (5) within each subset using training samples.
 - 8: Compute the out-of-sample performance on the validation data of the optimal solutions from all subproblems as in Eq. (8), denoted as c_k , $k = 1, \dots, 2^{2n}$.
 - 9: Form a probability vector $\mathbf{p}$ based on c_k as in (9). p_k denotes the probability of proceeding within subregion k :

$$p_k = \frac{e^{-c_k/T}}{\sum_i e^{-c_i/T}}, \quad k = 1, \dots, 2^{2n}, \quad (9)$$

where T is a temperature variable inversely proportional to the square of the iteration number.
 - 10: Choose a subregion based on $\mathbf{p}$ and update lb , ub and inter .
 - 11: **end while**
 - 12: $\hat{\theta} = (\text{lb} + \text{ub})/2$.
 - 13: Compute current equilibrium bids using $\hat{\theta}$ and $\bar{\beta}$.
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The reason for the randomized selection in Step 10 lies in that simply choosing the subregion which gives the best out-of-sample performance might miss the subset that contains

the true cost parameters, especially when there exist multiple optimal solutions in the subregion. On the other hand, the randomized selection will become more deterministic (controlled by the temperature variable T) as the subregion becomes smaller.

TABLE II

COMPARISON BETWEEN RANDOM SEARCH / BOX SEARCH AND WEN & DAVID'S METHOD.

		Equilibrium bids	Profit
S1	Random Search	(15.85, 15.80)	(112.48, 21.69)
	Box Search	(15.51, 15.37)	(100.69, 17.47)
	Wen & David's method	(15.20, 15.32)	(93.81, 14.86)
S2	Random Search	(11.16, 10.80)	(180.54, 67.22)
	Box Search	(10.67, 10.26)	(166.35, 60.54)
	Wen & David's method	(9.74, 9.62)	(144.70, 50.45)
S3	Random Search	(3.54, 1.28)	(53.58, 72.90)
	Box Search	(3.65, 1.18)	(50.82, 76.02)
	Wen & David's method	(2.83, 0.82)	(47.81, 56.88)
S4	Random Search	(96.35, 96.85)	(1450.10, 275.50)
	Box Search	(87.03, 87.12)	(1139.60, 177.30)
	Wen & David's method	(84.12, 84.87)	(1080.40, 161.81)
S5	Random Search	(47.05, 47.22)	(697.03, 301.34)
	Box Search	(40.46, 40.46)	(502.78, 228.05)
	Wen & David's method	(39.77, 39.97)	(485.02, 221.49)

From Table II we see that both random search and box search achieve higher profits than Wen & David's method, which is reasonable since the normal distribution assumption embedded in their method becomes invalid. In addition, in almost all scenarios random search performs better than box search; the reason for this lies in that there is some chance of proceeding with the wrong subregion due to the probabilistic subset selection embedded in box search. By contrast, random search always deals with the whole feasible region. Moreover, when we fix the number of iterations in Algorithm 1, which in our case is 100, it is approximately 4 times faster than Algorithm 2. The running time for random search was about 10.82 seconds on a Dell Precision T7810 workstation with a 2.2 GHz dual Intel Xeon processor, compared with 42.72 seconds for box search. To summarize, random search finds better cost estimates and generates more profitable bids in a more efficient way.

V. CONCLUSIONS

We propose an inverse optimization method to estimate the cost functions of suppliers in the electricity market based on historical bidding data. Then, the problem of computing the optimal bidding strategies can be casted as an equilibrium computation problem given the estimated payoff functions. We present a sufficient condition for the existence and uniqueness of the Nash equilibrium, and provide out-of-sample performance guarantees for the estimated cost functions. Numerical experiments have shown the superiority of our approach compared to Wen & David's method which is designed on the basis of normally distributed bids.

Regarding future research directions, assuming that the specific form of the cost function is unknown, the parametric formulation (5) is not applicable any more. It would be of interest to develop non-parametric approaches that do not

require to assume a specific parametric form for the cost functions.

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