Control of Wave Energy Converters Using A Simple Dynamic Model

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***Abstract*—This paper derives a control law within the context of optimal control theory for a heaving wave energy converter (WEC) and presents its implementation procedure. The proposed control assumes the availability of measurements of pressure distribution on the buoy surface, buoy position, and buoy velocity. This control has two main characteristics. First, this control is derived based on a simple dynamic model. The forces on the WEC are modeled as one total force, and hence there is no need to compute excitation or radiation forces. Second, this control can be applied to both linear and nonlinear WEC systems. The derived control law is optimal, yet its implementation requires estimation of some force derivatives which render the obtained control force sub-optimal. Numerical testing demonstrates in this paper that the proposed simple model control can achieve levels of harvested energy close to the maximum theoretical limit predicted by singular arc control in the case of linear WEC systems.**

***Index Terms*—Wave Energy Conversion, Optimal Control of Wave Energy Converters**

I. Introduction

Despite the enormous potential of ocean wave energy, the technology of wave energy converters (WECs) is not yet mature. The control system of a WEC has a significant impact on the amount of energy that can be harvested by WECs, and hence it influences the economics of wave energy conversion. Consider a heave only point absorber WEC. Over the years there have been significant advances in WEC control. Some control logics are simple to implement such as the damping control (resistive loading) in which the magnitude of the control force u is proportional to the buoy velocity $\dot{z}$ [1]. This control requires the only measurement of the buoy velocity, yet the harvested energy is relatively low. Another classical method of WEC control is the latching control [2], [3]. The latching control method involves computing the optimal latching delay; such computation requires the availability of a WEC dynamic model [4]. Reference [4] used the well known Cummins' equation [5] to model the WEC:

$$m_b\ddot{z}(t) = \overbrace{\int_{-\infty}^{\infty} h_f(\tau)\eta(t-\tau,z)d\tau}^{\text{excitation force } f_e} + f_s + u + f_d \underbrace{-\mu\ddot{z}(t) - \int_{-\infty}^{t} h_r(\tau)\dot{z}(t-\tau)d\tau}_{\text{radiation force } f_r} \quad (1)$$

where z is the heave displacement of the buoy from the sea surface, t is the time, m_b is the buoy mass, f_d is the viscous damping force, and f_s is the difference between the gravity and buoyancy force and it reflects the spring-like effect of the fluid. The pressure effect around the immersed surface of the floater is called the excitation force, f_e, where η is the wave surface elevation at buoy centroid and h_r is the impulse response function defining the radiation force in heave. The radiation force, f_r, is due to the radiated wave from the moving float, where h_r is the impulse response function defining the radiation force in heave, and μ is a frequency dependent added mass. The added mass at infinity frequency is μ_∞, and it is of particular importance since the radiation force can be separated as an inertia term that includes only μ_∞ in addition to the radiation damping forces based on the Kramers-Kronig relation.

There are challenges in using the Cummins' equation in control system design. Because the excitation force is modeled as a function of the wave elevation, η, the WEC controls problem is recognized as a non-causal problem, in the sense that computing the required control force in an optimal sense requires prediction in the future for the wave elevation or the excitation force [6], [7], [8]. This prediction adds complexity and cost to the energy conversion process. Also, the Cummins' equation is a linear model. Practically the WEC motion has nonlinear behavior, even for small motions, due to effects like the viscous friction or the actuation efficiency. This means a control computed based on a linear model may be suboptimal. The dynamic model in (1) also involves calculations for the excitation and radiation forces, which in turn require calculations for added masses at all frequencies in addition to the hydrodynamic coefficients.

Several other advanced concepts in WECs control have been developed that produce relatively higher levels of harvested energy. One of them is the model predictive control (MPC) [9], [10], [11], [12], and it needs a prediction in the future and a dynamic model to compute the control. The same can be said about the pseudospectral approach [13], [14] and the shape-based approach [15], [16]. Some other controls are causal (prediction is not needed) such as the optimal control presented in [17], [18], and the singular arc control presented in [19], [20]. Yet, the dynamic model in (1) is still used.

Unlike most existing control strategies, this study uses a simple dynamic model to derive a control law for WECs. Let the sum of all the forces on the right hand side of Eq. (1) (except the control u) be recognized as the total force F_T. The radiation force f_r in Eq. (1) can be approximated as the summation of an inertia term and a radiation damping force

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[21]. Hence the total force can be split into two parts:

$$F_T = F - \mu_\infty \ddot{z} \tag{2}$$

Instead of using the WEC model in Eq. (1), this paper uses a model that combines all the forces in one term as follows:

$$(m_b + \mu_\infty)\ddot{z} = F(z, \dot{z}, t) + u \tag{3}$$

where $F((z, \dot{z}, t)$ is a force term defined in Eq. (2). In section II, the control force $u(t)$ is derived within the context of optimal control theory using this WEC model. The resulting control law is valid whether $F((z, \dot{z}, t)$ is a linear or nonlinear function of the position z and the velocity $\dot{z}$. Section III details a process for applying this control law and the approximations that are needed for implementation. Numerical testing results and further discussion are detailed in Section IV.

II. Control Law For Maximum Energy

The simple dynamic model in Eq. (3) is used to derive the control force such that the harvested energy is maximized over a given time horizon T. The resulting control law will be referred to as Simple-Model Control (SMC) in the rest of this paper. Note that the force $F(z, \dot{z}, t)$ is assumed a function of time, buoy position, and buoy velocity. Although, the dependency of the total force is determined, the explicit format of the total force is assumed to be unknown. The dependence on time is intuitive since part of this force is due to the wave pressure on the buoy surface, and the wave pressure is time dependent. The buoy position determines the hydrostatic force, and hence the force F should be function of the position z. Also, the buoy velocity creates waves which affects the force on the buoy, and hence F is made also function of $\dot{z}$. Let the state vector $\vec{x} = [x_1, x_2, x_3]^T$ and $m = m_b + \mu_\infty$, the dynamic model in Eq. (3) can be written in the state space form:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{m}(F(x_1, x_2, x_3) + u) \\ \dot{x}_3 &= 1 \end{aligned} \tag{4}$$

where the x_1 and x_2 represent the position and velocity of the buoy respectively, x_3 represents the time t. The objective is to maximize the harvested energy over a time interval $[0\ T]$, defined as $E = -\int_0^T \{u(t)x_2(t)\}dt$. Assuming no limits on the control force, the optimal control problem is then defined as:

$$\begin{aligned} &Min : J((x(t), u(t)) = \int_0^T \{u(t)x_2(t)\}dt \\ &Subject\ to : Equations\ (4) \end{aligned} \tag{5}$$

The Hamiltonian [22] in this problem is defined as:

$$H(x_1, x_2, x_3, F, \lambda_1, \lambda_2, \lambda_3) = ux_2 + \lambda_1 x_2 + \frac{\lambda_2}{m}(F + u) + \lambda_3 \tag{6}$$

Where $\vec{\lambda} = [\lambda_1, \lambda_2, \lambda_3]^T$ are lagrange multipliers. The necessary conditions for optimality show that the optimal solution should satisfy the Euler-Lagrange equations:

$$H_\lambda = \dot{x}, \quad H_x = -\dot{\lambda}, \quad H_u = 0 \tag{7}$$

Evaluating the Hamiltonian partial derivatives in Eq. (7), we find that the optimal trajectory should satisfy the motion constraints in (4) in addition to:

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial x_1} = -\frac{\lambda_2}{m}\frac{\partial F}{\partial x_1} \tag{8}$$

$$\dot{\lambda}_2 = -\frac{\partial H}{\partial x_2} = -u - \lambda_1 - \frac{\lambda_2}{m}\frac{\partial F}{\partial x_2} \tag{9}$$

$$\dot{\lambda}_3 = -\frac{1}{m}\frac{\partial F(x_3)}{\partial x_3}\lambda_2 \tag{10}$$

$$\frac{\partial H}{\partial u} = x_2 + \frac{\lambda_2}{m} = 0 \tag{11}$$

It is possible to eliminate the co-states from the above equations and solve for the control force $u(t)$ as follows. Taking the derivative of Eq. (11), we get:

$$\dot{\lambda}_2 + m\dot{x}_2 = 0 \tag{12}$$

Substitute the second of Eqs. (4) and Eq. (12) into Eq. (9), we get:

$$-F - u = -u - \lambda_1 - \frac{\lambda_2}{m}\frac{\partial F}{\partial x_2} \tag{13}$$

Substituting Eq. (11) into Eq. (13) and rearranging we get:

$$F - \lambda_1 + x_2\frac{\partial F}{\partial x_2} = 0 \tag{14}$$

Evaluating the time derivative of the above equation we get:

$$\dot{F} - \dot{\lambda}_1 + \dot{x}_2\frac{\partial F}{\partial x_2} + x_2\frac{d}{dt}\frac{\partial F}{\partial x_2} = 0 \tag{15}$$

Substituting Eq. (8), Eq. (11) and the second of Eqs. (4) into the above equation we get:

$$m\dot{F} - mx_2\frac{\partial F}{\partial x_1} + (F + u)\frac{\partial F}{\partial x_2} + mx_2\frac{d}{dt}\frac{\partial F}{\partial x_2} = 0 \tag{16}$$

We can solve Eq. (16) for the controller to get:

$$u^* = m\frac{x_2\frac{\partial F}{\partial x_1} - \dot{F} - \frac{\partial F}{\partial x_2}F/m - x_2\frac{d}{dt}\frac{\partial F}{\partial x_2}}{\frac{\partial F}{\partial x_2}} \tag{17}$$

where u^* denotes the optimal control. To implement this control law in time domain, we need to compute F, $\dot{F}$, x_2, $\frac{\partial F}{\partial x_1}$ and $\frac{\partial F}{\partial x_2}$. These calculations are discussed in Section III. The equation of the optimal controller also indicates the SMC is adaptive to different format of the total force which means it is adaptive to different dynamics or different nonlinearities. This paper aims at introducing the concept of the simple model controller, and hence the constraints on the control force, position and velocity are not included in this paper. However, these constraints can be included in the optimization problem and also the Hamiltonian. Additionally, the control limitation can be implemented by using a Bang-Singular-Bang control law as described in [23].

III. WEC Control System

In this study, it is assumed that we measure the buoy position x_1, its velocity x_2, and also the total pressure using multiple pressure sensors on the buoy surface, as detailed in reference [20]. The surface pressure can then be used to compute the total force F_T. Let $\tilde{x}_i$ be the measurement of x_i and $\tilde{F}_T$ be the measurement of F_T. The measurement $\tilde{F}_T$ is then added to the quantity $\mu_\infty \dot{\tilde{x}}_2$ to obtain $\tilde{F}$ as a pseudo measurement. These measurements will also be used to estimate the quantities $\dot{F}$, $\partial F/\partial x_1$ and $\partial F/\partial x_2$ as described below.

Since we need to estimate the derivatives of the force F with respect to time and the states, it is convenient to approximate the force F using a series expansion. This way, it is possible to compute approximate expressions for the derivatives once the coefficients of the polynomial are determined. Moreover, it is possible to compute approximate expressions for the control force and the harvested energy analytically. Toward that end, it is assumed that the following series expansion approximates the force F:

$$\begin{aligned}\bar{F} &= a_1 x_1 + a_2 x_2 + b_1 x_1^3 + b_2 sign(x_2) x_2^2 \\ &+ \sum_{n=1}^{N} (c_n \cos(\omega_n t) + d_n \sin(\omega_n t))\end{aligned} \tag{18}$$

The above series expansion is selected intuitively and in a general form. Higher order terms can be added to the polynomial terms if needed. While this series expansion is suitable for 1-DoF point absorbers, it is straightforward to write similar expansions for other types of WECs or extend it to account for multi-DoF WECs.

The coefficients in the Eq. (18) are estimated using a Kalman filter such that the square error between $\bar{F}$ and F is minimized. The frequencies $\omega_n, \forall n$, are assumed fixed. In the study presented in this paper the values of ω_n are assumed equally spaced in the range from 0.5 rad/sec to 3 rad/sec. This assumption enables the use of a linear Kalman filter. If desired, these frequencies could be appended to the Kalman filter state vector to be estimated; in such case, an extended Kalman filter would be needed for the resulting nonlinear system. If we use a sufficiently large number of frequencies, the assumption of fixed frequencies provides reasonable accuracy as demonstrated in Section IV where 45 fixed equally spaced frequencies are used.

The Kalman filter uses the measurements to update the estimates of the coefficients in $\bar{F}$ sequentially in time. The state vector of the Kalman filter is selected as:

$$\hat{\vec{x}} = [\hat{a}_1, \, \hat{a}_2, \, \hat{b}_1, \hat{b}_2, \, \hat{c}_1, \cdots, \, \hat{c}_N, \, \hat{d}_1, \cdots, \, \hat{d}_N]^T \tag{19}$$

The dynamic equation of the Kalman filter is:

$$\dot{\hat{\vec{x}}} = \vec{0} \tag{20}$$

where $\vec{0}$ is a vector which components are all zeros. The coefficiensts $\hat{\vec{x}}$ are assumed to be constant in the dynamic model; yet they can be updated based on measurements in the Kalman update step. Each measurement is simulated as a zero mean white noise added to its true signal. The Kalman filter output equation is:

$$\begin{aligned}\hat{y} = \hat{F} &= \hat{a}_1 \tilde{x}_1 + \hat{a}_2 \tilde{x}_2 + \hat{b}_1 \tilde{x}_1^3 + \hat{b}_2 sign(\hat{x}_2) \tilde{x}_2^2 \\ &+ \sum_{n=1}^{N} (\hat{c}_n \cos(\omega_n t) + \hat{d}_n \sin(\omega_n t))\end{aligned} \tag{21}$$

The process of implementing the Kalman filter in this paper is standard and is not presented; reference [24] presents the details on the process of linear Kalman filters. At each time step, the partial derivatives of F can be approximated by taking the derivatives of Eq. (18), and using the estimated states as follows:

$$\begin{aligned}\frac{\partial F}{\partial t} &= \sum (-\hat{c}_n \omega_n \sin(\omega_n t) + \hat{d}_n \omega_n \cos(\omega_n t)) \\ \frac{\partial F}{\partial \tilde{x}_1} &= \hat{a}_1 + 3\hat{b}_1 \tilde{x}_1^2 \\ \frac{\partial F}{\partial \tilde{x}_2} &= \hat{a}_2 + 2\hat{b}_2 sign(\tilde{x}_2) \tilde{x}_2 \, , \; \frac{d}{dt} \frac{\partial F}{\partial \tilde{x}_2} = 2\hat{b}_2 sign(\tilde{x}_2) \dot{\tilde{x}}_2\end{aligned} \tag{22}$$

These partial derivatives are substituted in Eq. (17) to compute the control force. As a result, the proposed control force only requires the current states which can be obtained from the state estimation by using Kalman Filter. The wave prediction is not required for the SMC controller.

A. Initialization of The Kalman Filter States

The initial conditions of the state vector $\hat{\vec{x}}_0$ dictate the effectiveness of the Kalman filter in estimating the coefficients. In this problem, in particular, there are multiple local solutions that the Kalman filter can converge to that are not the true values of the coefficients. Hence, it is critical to have a good initial guess for the coefficients. In this study, the initial guess values are obtained via an optimization process that is here described. First, measurements are collected over some period, called the initialization period T_0. In the simulations conducted in Section IV, $T_0 = 100$ seconds and measurements are collected every $\Delta t = 0.05$ seconds. The optimization problem is to find the vector $\hat{\vec{x}}_0$ that minimizes the function:

$$J = \sum_{i=1}^{N_D} \| \bar{F} \left(\hat{\vec{x}}_0, \tilde{x}_1(i\Delta t), \tilde{x}_2(i\Delta t), i\Delta t \right) - \tilde{F}(i\Delta t) \|^2 \tag{23}$$

where N_D is the number of data points. A sequential quadratic programming algorithm was used to solve this optimization problem. It is noted here that this process works even when sea state changes over time. This is because data are collected continuously and used to update the estimate of the coefficients $\hat{x}_{arrow}$ via the Kalman update step. In the simulations conducted in this paper, data collected over 100 s are used for this update step. Hence the introduced initialization of Kalman Filter is adaptive to the changing environment using the collected data.

IV. Numerical Testing and Discussion

Numerical testing was conducted using the buoy shown in Figure 1 in a wave field that is modeled using linear wave theory. The dynamic model applied in the WEC plant in the

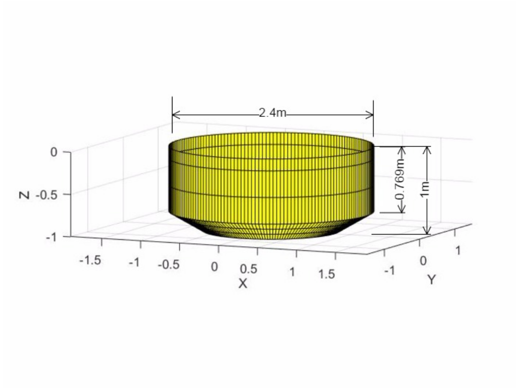

Fig. 1. Geometry of the buoy used in the numerical simulations in this paper.

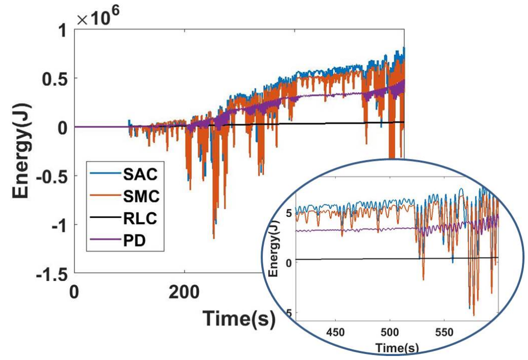

Fig. 2. Comparison between the SMC, the SAC, the RL and the PD in terms of harvested energy. The SMC performance is close to the ideal SAC.

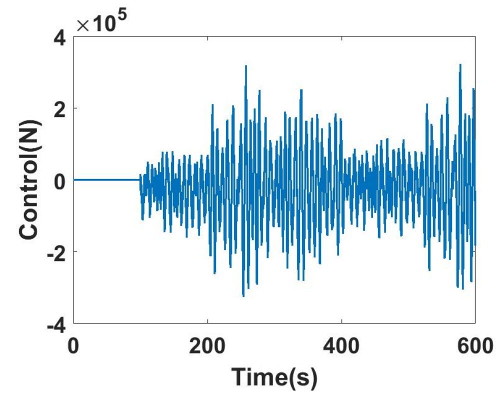

Fig. 3. Control force computed using SMC. No control limits are assumed in this study.

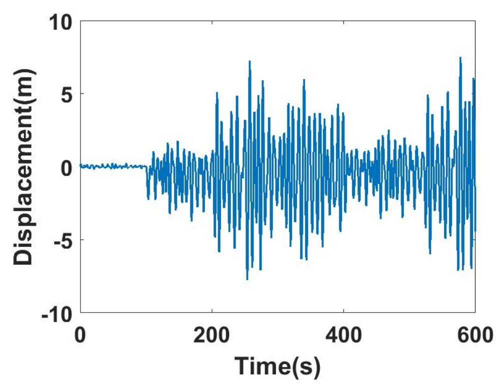

Fig. 4. Displacement of the buoy when using SMC. No displacement constraints are assumed in this study.

simulation is the Cummins' equation (Eq. (1)), although the controller is derived based on the simple model by defining a total force. A Bretschneider wave is realized using 200 frequencies equally spaced in the range $0 - 4$ rad/s. The significant wave height is 0.3 m, and the peak period is 7 s. The hydrodynamic and hydrostatic forces on the buoy are simulated using force coefficients that are computed using NEMOH [25]. These forces, in addition to the viscous damping force, are considered as data that simulates the force measurements $\tilde{F}_T$. These forces are also used to propagate the buoy motion and generate simulated measurements for the buoy position and velocity. The derivative of the measured velocity is computed at each time step and is used to compute the force $\tilde{F}$. The numerical parameters used to generate the data are as follows: the mass of the buoy is 4.637×10^3 kg, the stiffness of hydrostatic force is 4.437×10^4 N/m, and the viscous damping coefficient is 6.1525 Nm/s.

The effectiveness of the proposed control system is assessed by comparing the harvested energy obtained using the proposed SMC to the optimal harvested energy as computed by the singular arc control (SAC) [19]. The SAC is computed assuming perfect measurements (noise free measurements) so that we can use the harvested energy from SAC as a reference. Figure 2 shows the harvested energy over 10 minutes. In the first 100 seconds, no control was applied; rather only measurements were collected and used to initialize the Kalman filter. Three different controls are presented in Figure 2. The SAC is the maximum energy curve computed using the singular arc control. The SMC line is the energy harvested using SMC. The RL line is the energy harvested using the resistive loading control: $u = -R_m x_2$. Finally, the PD line is the energy extracted using the Proportional Derivative control $u = -K_p x_1 - K_d x_2$. The feedback gain of the RL and PD controllers are optimized in terms of energy extraction.

As can be seen in Figure 2, the harvested energy using the SMC is very close to the optimal one, and it is significantly larger than the RL harvested energy. The control force produced using the SMC is shown in Figure 3, and the displacement of the buoy over time is shown in Figure 4. To emphasize the accuracy of the assumed force series expansion in Eq. (18) when the actual forces on the buoy are linear, Figure 5 shows both the true and the approximate forces on the buoy surface.

The proposed SMC is valid for both linear and nonlinear systems. It is still valid even when the hydrodynamic and/or the hydrostatic forces are nonlinear functions of the states. In fact, in the case of a nonlinear system, the SMC control law in Eq. (17) is locally optimal. To demonstrate this advantage numerically, a nonlinear stiffness term, $K_2 x_1^3$ N, is added to the force acting on the buoy, where $K_2 = 10000$ N/m^3. Some references point out that when the shape of the buoy is complex (not cylinder), it is possible to have a cubic nonlinear hydrostatic force in the dynamics [26] [27]. Hence

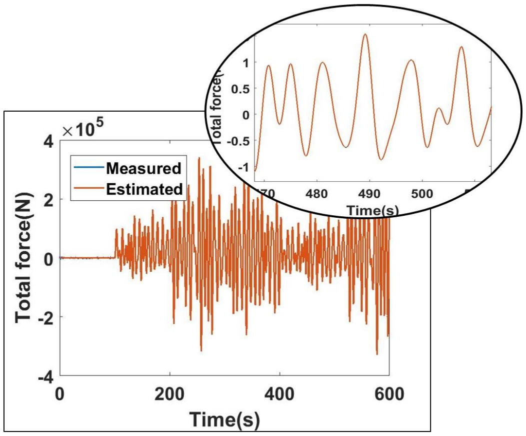

Fig. 5. The series expansion for the force F on the buoy is a good approximation for the true force for the linear force test case

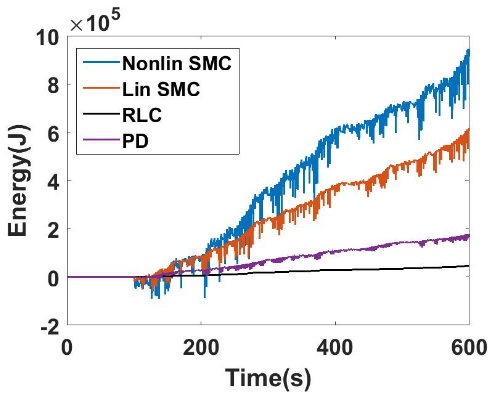

Fig. 6. The harvested energy using SMC when hydrodynamic/hydrostatic forces are nonlinear. The Nonlin SMC can harvest more energy than the Lin SMC designed to harvest energy based on a linear WEC model

the cubic nonlinear term is included in the model to examine the performance of the proposed SMC in such cases. Figure 6 shows the harvested energy, where two versions of the SMC are plotted. The Nonlin SMC is the SMC when all coefficients in the series expansion (Eq. (18)) are included. The Lin SMC is the SMC when forcing $b_i = 0, \forall i$; i.e. when forcing the series expansion to be a linear function of the states. Clearly, the SMC (Nonlin SMC in Figure 6) is able to capture more energy than a control that is designed based on a linear WEC model when they are applied to a nonlinear model. To further highlight this advantage, note that the SAC is designed based on a linear WEC model and if we run it in a simulation that has nonlinear forces, it will have a poor performance as shown in Figure 7, for different values of the nonlinear coefficient K_2. The figure also indicates the advantage of the SMC that it can handle different nonlinearities without the need to change the control law. This is unlike the SAC which is designed to work only on a linear dynamic model.

The control law presented in Section II is optimal for the system described by Eq. (3). The optimality of this control law is valid even when the WEC force model is nonlinear. However, the implementation of this control law requires the estimation of the partial derivatives in Eq. (17), which involved

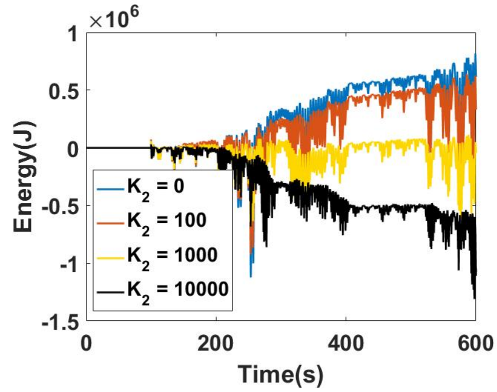

Fig. 7. The SAC is designed based on a linear WEC model. Its performance degrades when tested in a nonlinear environment; $K_2 = 0$ corresponds to a linear WEC model. As K_2 value increases, the significance of the nonlinearity increases.

approximating the force using a series expansion. Hence, the resulting control force is sub-optimal. A significant advantage of this control is the ability to generate a sub-optimal solution despite using a simple dynamic model. It is not needed to compute radiation or excitation forces.

Estimating the coefficients in the force series expansion can benefit significantly from some basic knowledge of the buoy properties and/or the incoming wave. For example, the coefficients a_1 and b_1 represent stiffness terms, which means they represent hydrostatic forces. These coefficients then can be removed from the Kalman filter state vector and can be determined a priori from the geometry and mass properties of the buoy, or from the measured data during the initialization phase. Similarly, if we know that for a given buoy and wave conditions the forces are strictly linear, then we can set $b_i = 0, \forall i$, as was the case in Figure 2. Higher order terms can be added to the force series expansion if needed. Fixing some of the coefficients, if they can be determined, and selecting the ranges for the other coefficients is a precursor that may require some trial and error.

To demonstrate the effectiveness of the proposed SMC, simulations were conducted on a range of sea states (Bretschneider wave with a significant height of 0.3 m for a range of wave periods) and the average power harvested is plotted in Figure 8. Clearly, the SMC is very close to the SAC most of the time. Note that the SAC solution is a steady state solution, and hence its performance is not optimal in periods of transient behavior. The SMC is not restricted to steady-state situations and hence might have better performance when motion is in the transient period. The transient response may occur due to the arbitrary initial conditions. Another reason that leads to transient periods is the use of stochastic estimators. The use of stochastic estimation results in estimates that are not error free; when these estimates are used to compute the control the errors propagate to the control. This effect creates transient behavior in the response of the WEC. To further validate the performance of the SMC, the controller is simulated with a more realistic dynamic model that models the cases when the buoy is fully in the air or fully in the water. When the buoy is totally in the air, we assume no hydrodynamic or hydrostatic

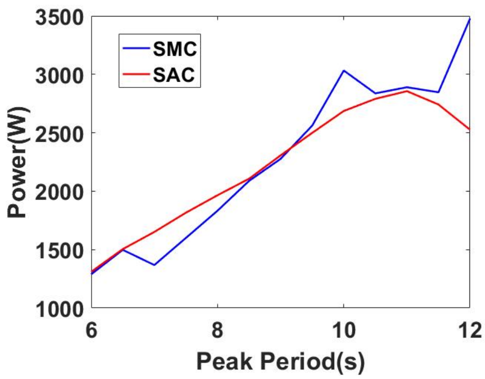

Fig. 8. The average harvested power using both the SMC and the SAC for a range of peak periods of Bretschneider waves

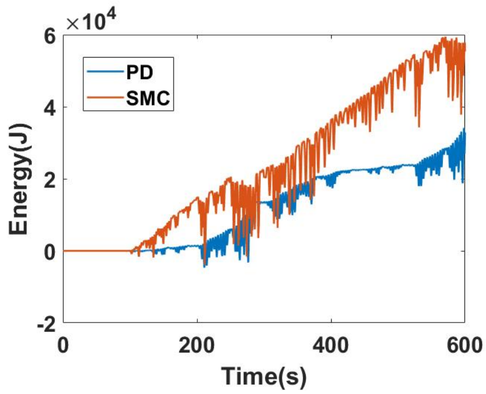

Fig. 9. The harvested energy using SMC with the realistic model. The SMC can harvest more energy than the PD

forces on the body and we turn off the control. When it is fully under water, hydrostatic force becomes constant. The performance of the SMC is shown in Fig. 9 which is simulated with a wave that has a significant wave height of 0.08 m and a peak period of 7 s. A small significant wave height is elected to help keep the buoy in the water for longer times. The SMC controller extracts better energy than the PD controller in this case.

The SMC assumes the availability of the pressure, position, and velocity, measurements of the buoy. Measurements of the buoy position are quite common in many WEC control algorithms. It is not unusual also to assume the availability of the velocity measurement in the WEC literature. The pressure measurement on the buoy surface was also proposed in reference [20], and was actually implemented recently in the scaled experimental buoy developed by Sandia National Labs. Finally, this paper is a proof of concept for the SMC. Improvements can be made to the method presented in Section III in estimating the force and its derivatives. For example, system identification methods such as those presented in references [28], [29], [30] can be investigated to improve the structure of the force model in Eq. (18), or the coefficients of that model. The Kalman filter estimation vector can be extended to include the buoy position and velocity to improve the estimation accuracy. These will be investigated in future work.

V. Conclusion

This paper demonstrated that it is possible to design a control system utilizing measurements of the pressure distribution on the buoy surface, the buoy's position, and its velocity. The main advantage of the proposed control method is that it uses a simple dynamic model that does not require calculations of radiation or excitation forces. The proposed SMC is valid for linear and nonlinear WECs. The SMC, despite developed using optimal control theory, is sub-optimal due to the assumption of a series expansion for the force, in the estimation step. Numerical simulations demonstrated the SMC can generate near-optimal harvested energy and can work in nonlinear force environments. Future research will investigate the situations of changing sea states and how the proposed SMC adapts to these changes.

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References

[1] J. Falnes, "A review of wave-energy extraction," *Marine structures*, vol. 20, no. 4, pp. 185–201, 2007.

[2] A. Babarit, G. Duclos, and A. H. Clment, "Comparison of latching control strategies for a heaving wave energy device in random sea," *Applied Ocean Research*, vol. 26, pp. 227–238, 2004.

[3] M. Durand, A. Babarit, B. Pettinotti, O. Quillard, J. Toularastel, and A. Clément, "Experimental validation of the performances of the searev wave energy converter with real time latching control," *EWTEC, Porto*, 2007.

[4] A. Babarit and A. Clment, "Optimal latching control of a wave energy device in regular and irregular waves," *Applied Ocean Research*, vol. 28, no. 2, pp. 77 – 91, 2006. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0141118706000423

[5] W. Cummins, D. T. M. B. W. D. C., and D. W. T. M. Basin, *The Impulse Response Function and Ship Motions*, ser. Report (David W. Taylor Model Basin). Navy Department, David Taylor Model Basin, 1962. [Online]. Available: https://books.google.com/books?id=GLLANwAACAAJ

[6] G. Li, G. Weiss, M. Mueller, S. Townley, and M. R. Belmont, "Wave energy converter control by wave prediction and dynamic programming," *Renewable Energy*, vol. 48, no. 0, pp. 392 – 403, 2012. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0960148112003163

[7] B. A. Ling, "Real-time estimation and prediction of wave excitation forces for wave energy control applications," MS, Oregon State University, June 2015.

[8] F. Fusco and J. V. Ringwood, "Short-term wave forecasting for real-time control of wave energy converters," *IEEE Transactions on Sustainable Energy*, vol. 1, no. 2, pp. 99–106, July 2010.

[9] G. Li and M. R. Belmont, "Model predictive control of sea wave energy converters part i: A convex approach for the case of a single device," *Renewable Energy*, vol. 69, no. 0, pp. 453 – 463, 2014. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0960148114002456

[10] J. Hals, J. Falnes, and T. Moan, "Constrained optimal control of a heaving buoy wave-energy converter," *Journal of Offshore Mechanics and Arctic Engineering*, vol. 133, no. 1, pp. 1–15, 2011.

[11] G. Li, "Nonlinear model predictive control of a wave energy converter based on differential flatness parameterisation," *International Journal of Control*, vol. 89, no. 0, pp. 1–10, 0. [Online]. Available: http://dx.doi.org/10.1080/00207179.2015.1088173

[12] J. Cretel, G. Lightbody, G. Thomas, and A. Lewis, "Maximisation of energy capture by a wave-energy point absorber using model predictive control," in *IFAC Proceedings*, vol. 44.1, 2011, pp. 3714–3721.

[13] G. Bacelli, "Optimal control of wave energy converters," PhD, National University of Ireland, Maynooth, Maynooth, Ireland, 2014.

[14] D. R. Herber and J. T. Allison, "Wave energy extraction maximization in irregular ocean waves using pseudospectral methods," in *ASME 2013 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*. American Society of Mechanical Engineers, 2013, pp. V03AT03A018–V03AT03A018.

[15] O. Abdelkhalik, R. Robinett, S. Zou, G. Bacelli, R. Coe, D. Bull, D. Wilson, and U. Korde, "On the control design of wave energy converters with wave prediction," *Journal of Ocean Engineering and Marine Energy*, vol. 2, no. 4, pp. 473–483, 2016. [Online]. Available: http://dx.doi.org/10.1007/s40722-016-0048-4

[16] O. Abdelkhalik, R. Robinett, G. Bacelli, R. Coe, D. Bull, D. Wilson, and U. Korde, "Control optimization of wave energy converters using a shape-based approach," in *ASME Power & Energy 2015*. San Diego, CA: ASME, June 2015.

[17] J. Scruggs, S. Lattanzio, A. Taflanidis, and I. Cassidy, "Optimal causal control of a wave energy converter in a random sea," *Applied Ocean Research*, vol. 42, pp. 1 – 15, 2013. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0141118713000205

[18] R. Nie, J. Scruggs, A. Chertok, D. Clabby, M. Previsic, and A. Karthikeyan, "Optimal causal control of wave energy converters in stochastic waves accommodating nonlinear dynamic and loss models," *International Journal of Marine Energy*, vol. 15, pp. 41 – 55, 2016, selected Papers from the European Wave and Tidal Energy Conference 2015, Nante, France. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S2214166916300169

[19] S. Zou, O. Abdelkhalik, R. Robinett, G. Bacelli, and D. Wilson, "Optimal control of wave energy converters," *Renewable Energy*, pp. –, 2016. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0960148116310059

[20] O. Abdelkhalik, S. Zou, R. Robinett, G. Bacelli, and D. Wilson, "Estimation of excitation forces for wave energy converters control using pressure measurements," *International Journal of Control, Taylor & Francis*, 2016.

[21] Z. Yu and J. Falnes, "State-space modelling of a vertical cylinder in heave," *Applied Ocean Research*, vol. 17, no. 5, pp. 265–275, Oct. 1995. [Online]. Available: http://www.sciencedirect.com/science/article/pii/0141118796000028

[22] J. A. E. Bryson and Y.-C. Ho, *Applied Optimal Control optimization,estimation and control*. CRC Press,, 1975.

[23] S. Zou, O. Abdelkhalik, R. Robinett, G. Bacelli, and D. Wilson, "Optimal control of wave energy converters," *Renewable Energy*, vol. 103, pp. 217–225, 2017.

[24] J. L. Crassidis and J. L. Junkins, *Optimal Estimation of Dynamic Systems*. CHAPMAN&HALL/CRC, 2004.

[25] A. Babarit and G. Delhommeau, "Theoretical and numerical aspects of the open source BEM solver NEMOH," in *11th European Wave and Tidal Energy Conference (EWTEC2015)*, ser. Proceedings of the 11th European Wave and Tidal Energy Conference, Nantes, France, Sep. 2015. [Online]. Available: https://hal.archives-ouvertes.fr/hal-01198800

[26] J. M. Kluger, "Synergistic design of a combined floating wind turbine-wave energy converter," Ph.D. dissertation, Massachusetts Institute of Technology, 2017.

[27] R. G. Coe and D. L. Bull, "Sensitivity of a wave energy converter dynamics model to nonlinear hydrostatic models," in *ASME 2015 34th International Conference on Ocean, Offshore and Arctic Engineering*. American Society of Mechanical Engineers, 2015, pp. V009T09A030–V009T09A030.

[28] S. Giorgi, J. Davidson, and J. V. Ringwood, "Identification of wave energy device models from numerical wave tank datapart 2: Data-based model determination," *IEEE Transactions on Sustainable Energy*, vol. 7, no. 3, pp. 1020–1027, 2016.

[29] G. Bacelli, R. G. Coe, D. Patterson, and D. Wilson, "System identification of a heaving point absorber: Design of experiment and device modeling," *Energies*, vol. 10, no. 4, p. 472, 2017.

[30] B. Bosma, T. Brekken, P. Lomonaco, A. McKee, B. Paasch, and B. Batten, "Physical model testing and system identification of a cylindrical owc device," in *Proceedings of the 12th European Wave and Tidal Energy Conference*, 2017.

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