Towards a Form-Finding Process for Damage-Tolerant Tensegrity Structures

O. Aloui[1] and L. Rhode-Barbarigos[2]

[1]University of Miami, Dept. of Civil, Architectural & Environmental Engineering, 1251 Memorial Dr., McArthur Engineering Bldg, Room 307, Coral Gables, FL 33146; PH: (305) 284-3490; email: omar.aloui@miami.edu

[2]University of Miami, Dept. of Civil, Architectural & Environmental Engineering, 1251 Memorial Dr., McArthur Engineering Bldg, Room 307, Coral Gables, FL 33146; PH: (305) 284-3490; email: landolfrb@miami.edu

ABSTRACT

Tensegrity structures have been applied in various fields in science and engineering. In civil engineering, tensegrity structures have been used to construct lightweight yet strong structures, while in aerospace applications, it has been shown that tensegrity systems have great potential for active applications such as planetary exploration. Therefore, active tensegrity systems have been under investigation for small, light-weight and low-cost missions. However, the robustness or damage tolerance of a tensegrity structure is a function of its inherent self-equilibrated pre-stress states. Although research into tensegrity systems has resulted in reliable techniques for their form-finding and analysis, no technique has been found previously that integrates damage tolerance as an input or output parameter. This paper reflects an effort to integrate damage tolerance in the form-finding process of tensegrity structures through a bio-inspired approach that combines topology definition and form finding.

INTRODUCTION

The term “tensegrity” was invented by Buckminster Fuller in 1962 to describe Kenneth Snelson’s art work (Motro 2003). Despite its connection with art, the concept has been employed by scientists and engineers for applications spanning from civil (Rhode-Barbarigos et al. 2012) and aerospace engineering (Sabelhaus et al. 2015) to biology (Ingber et al. 2014) and mathematics (de Guzmán and Orden 2006). Tensegrity has also been proposed as a model for urban design (Châtelet 2007). Moreover, many definitions have been proposed. In this paper, the mathematical definition is followed where tensegrity is a self-stressed framework. A framework is a realization of an abstract graph $G(V,E)$ described by a set of vertices V and pairs of vertices E in a d-dimensional space through a set of points in space defined by d coordinates $P=[p_1;p_2;\ldots;p_n]$.

The first step in the design of tensegrity structures is identification of a stable equilibrium configuration under prestress starting from an arbitrary given geometry and with only rigid body motions constrained. This step is well-known as form finding (Adriaenssens et al. 2014). However, most design methods (Zhang and Ohsaki 2006; Bel Hadj Ali et al. 2011) dissociate topology definition and form finding from the desired performance or behavior and thus design. Moreover, few studies have focused on the damage tolerance of tensegrity structures (Ben Kahla and Moussa 2002; Abedi and Shekastehband 2009) with no study providing a general framework for understanding the influence of damage in the self-stress space of a tensegrity structure.

In this paper, a bio-inspired approach for the generation of planar tensegrity structures that combines topology definition and form finding is presented. By defining the self-stress as a control parameter of the form finding, designers can generate tensegrity structures with multiple load paths, adding damage tolerance to the system. This study aims thus to explain damage tolerance in tensegrity structures. The remainder of the paper is organized as follows. Section 2 includes a brief description of the cellular multiplication method. In Section 3, the example of a four-module tensegrity structure is presented. Further discussion and conclusions are presented in Section 4.

THE CELLULAR MULTIPLICATION PROCESS

The cellular multiplication process (Aloui et al. 2018) is a generation method for the combined topology identification and form finding of tensegrity structures inspired by the multiplication process of unicellular organisms. The method is founded on a theorem developed by de Guzmán and Orden (2006) which states that all tensegrity structures are decomposable into elementary tensegrity units called cells. For the two-dimensional case, tensegrity cells are the complete graphs K_4 on four nodes. Figure 1 illustrates the planar tensegrity cells. Planar tensegrity cells have one stable self-stress state and no infinitesimal mechanisms. The analytical solution of the self-stress state for planar tensegrity cells is given in (Aloui et al. 2018).

Figure 1. Elementary tensegrity units (cells) that compose all planar tensegrity structures.

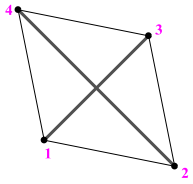
Considering an analogy with the multiplication of unicellular organisms, if no shared elements among cells composing a tensegrity structure are removed, the process is considered as a adhesion with cells being stable and functioning separately. If any elements among cells composing a tensegrity structure are removed after a adhesion step, the process corresponds to fusion with cells functioning as one entity. Fernández and Orden (2011) characterized the dimension of the self-stress space allowing the calculation of the number of self-stress states in a tensegrity structure using the decomposition of the structure into cells. Based on that, it was found (Aloui et al. 2018) that the change in the number of self-stress states during the cellular multiplication process for the planar case is given by $\Delta(\dim(W)) = e_i - 2v_i$, where v_i is the change in the number of nodes, e_i is the change in the number of members. Knowing that each cell has only one self-stress and the change in the number of self-stress states during each step of the cellular multiplication process, a basis for the self-stress space of the structure can be constructed by completing the basis of the structure with any missing vectors calculated using the nodal coordinates of the cells. Consequently, the cellular multiplication process combines topology definition and form finding, enabling designers to decide on the number of self-stress states in a tensegrity structure. Self-stress is the key

parameter in the design of tensegrity structures as it defines the shape and properties of a structure such as damage tolerance.

CELLULAR MULTIPLICATION OF A FOUR-CELL TENSEGRITY STRUCTURE

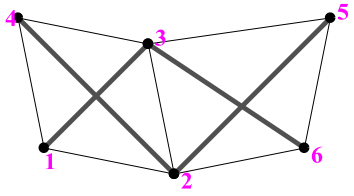
In this section, the cellular multiplication process of a four-cell planar tensegrity structure using cellular adhesion is presented with a detailed description of the changes in the self-stress space. The configuration matrix P stores the nodal coordinates of the structure. Topology (the couples (i,j) that define the connectivity of the elements) is found in the matrix $Link$. The matrix W defines the self-stress states of the structure along the entire process.

The multiplication process starts with defining the geometry of the first cell, as this also determines its self-stress state (Aloui et al. 2018). The first step is given by:



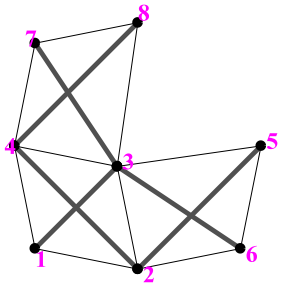
$$P = \begin{bmatrix} 0 & 0 \\ 1 & -0.2 \\ 0.8 & 0.8 \\ -0.2 & 1 \end{bmatrix} \qquad Link = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 1 & 4 \\ 1 & 3 \\ 2 & 4 \end{bmatrix} \qquad W = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

A second cell is then added by choosing the shared nodes {2,3} and specifying the positions of the added nodes {5,6}. Equation (1) gives that the addition of the second cell adds one self-stress state to the system. The state of the second cell is calculated using the nodal coordinates and then the self-stress matrix W is augmented to incorporate the additional self-stress state:



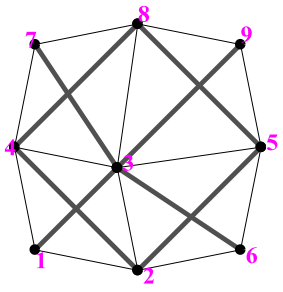
$$P = \begin{bmatrix} 0 & 0 \\ 1 & -0.2 \\ 0.8 & 0.8 \\ -0.2 & 1 \\ 2.2 & 1 \\ 2 & 0 \end{bmatrix} \qquad Link = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 1 & 4 \\ 1 & 3 \\ 2 & 4 \\ 3 & 5 \\ 5 & 6 \\ 2 & 6 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \qquad W = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ -1 & 0 \\ -1 & 0 \\ 0 & 0.7647 \\ 0 & 1.1471 \\ 0 & 1.5 \\ 0 & -1.0833 \\ 0 & -1.0588 \end{bmatrix}$$

Similarly, the addition of a third cell adds one self-stress state to the system:



$$P = \begin{bmatrix} 0 & 0 \\ 1 & -0.2 \\ 0.8 & 0.8 \\ -0.2 & 1 \\ 2.2 & 1 \\ 2 & 0 \\ 0 & 2 \\ 1 & 2.2 \end{bmatrix} \qquad Link = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 1 & 4 \\ 1 & 3 \\ 2 & 4 \\ 3 & 5 \\ 5 & 6 \\ 2 & 6 \\ 2 & 5 \\ 3 & 6 \\ 4 & 7 \\ 7 & 8 \\ 3 & 8 \\ 3 & 7 \\ 4 & 8 \end{bmatrix} \qquad W = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0.7647 & 0 \\ 0 & 1.1471 & 0 \\ 0 & 1.5 & 0 \\ 0 & -1.0833 & 0 \\ 0 & -1.0588 & 0 \\ 0 & 0 & 1.5 \\ 0 & 0 & 1.1471 \\ 0 & 0 & 0.7647 \\ 0 & 0 & -1.0588 \\ 0 & 0 & -1.0833 \end{bmatrix}$$

In the last step, the addition of a fourth cell introduces two states: one that corresponds to the self-stress state of the fourth cell and one that corresponds to the interactions among the four cells. The fourth step thus gives:



$$P = \begin{bmatrix} 0 & 0 \\ 1 & -0.2 \\ 0.8 & 0.8 \\ -0.2 & 1 \\ 2.2 & 1 \\ 2 & 0 \\ 0 & 2 \\ 1 & 2.2 \\ 2 & 2 \end{bmatrix} \qquad Link = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 1 & 4 \\ 1 & 3 \\ 2 & 4 \\ 3 & 5 \\ 5 & 6 \\ 2 & 6 \\ 2 & 5 \\ 3 & 6 \\ 4 & 7 \\ 7 & 8 \\ 3 & 8 \\ 3 & 7 \\ 4 & 8 \\ 5 & 8 \\ 5 & 9 \\ 3 & 9 \\ 8 & 9 \end{bmatrix} \qquad W = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0.5 \\ 0 & 0.764 & 0 & 1 & -0.5 \\ 0 & 1.147 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 & 0 \\ 0 & -1.083 & 0 & 0 & 0.333 \\ 0 & -1.058 & 0 & 0 & 0 \\ 0 & 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1.147 & 0 & 0 \\ 0 & 0 & 0.764 & 1 & -0.5 \\ 0 & 0 & -1.058 & 0 & 0 \\ 0 & 0 & -1.083 & 0 & 0.333 \\ 0 & 0 & 0 & -1.5 & 0.25 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -1.333 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix}$$

Interactions among cells are reflected by the presence of virtual cells: collateral stable sub-structures with a single self-stress state composed of elements from different cells, but which do not use all of the elements from an existing cell. In this example, the virtual cell is identified as the

wheel structure that develops around node 3 (Figure 2). The analytical solution for the self-stress of a wheel structure can be found in (Aloui et al. 2018).

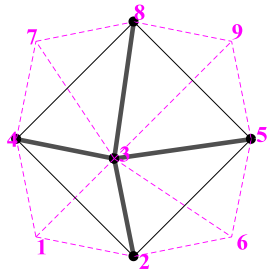


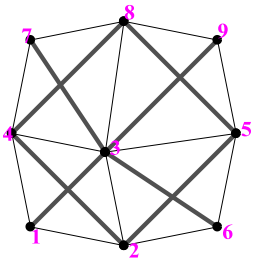
Figure 2. Virtual cell identified in the four-cell tensegrity structure.

ELEMENT DAMAGE ON A FOUR-MODULE TENSEGRITY STRUCTURE

In this section, element damage on a four-cell planar tensegrity structure is analyzed with a detailed description of the changes in the self-stress space, with member damage modeled as a member removal. Under this assumption, the effect of the element damage in a structure depends on the number of self-stress states (number of unicellular sub-structures composing the system) and whether the damage engenders a mechanism. Thus, two cases can be distinguished: the resulting structure is infinitesimally rigid or the resulting structure comprises infinitesimal mechanisms. The first case is the focus of this study. Since the resulting structure is infinitesimally rigid, the configuration of the damaged structure will remain the same as the configuration of the undamaged structure, resulting in a redistribution of the self-stress of the damaged element to the rest of the structure. The difference between the damaged and undamaged structure lies in their topology. However, maintaining the same configuration allows one to use the self-stress space of the undamaged structure as a starting point to obtain the self-stress space of the damaged system. A damaged element can thus be thought of as the result of fusion if the element belongs to multiple unicellular sub-structures or the result of necrosis (death of a cell) if the element belongs to one cell.

The example of the four-cell planar tensegrity structure is used to illustrate how cellular multiplication can be used to assess damage on a tensegrity structure. The effect of the damage depends on the element being removed, with two cases to consider: i. damage of an element that belongs to one cell (boundary elements) and ii. damage of an element that belongs to two or more cells (Table 1). In this study, one element from each group of elements sharing the same number of cells is considered, as the results can be extended to other elements of the same group based on the topological symmetry of the system.

Table 1: Element classification according to the number of cells they belong to.

	Elements belonging to one cell	Elements belonging to two or more cells
	{(1,2), (1,4), (1,3), (2,6), (5,6), (3,6), (4,7), (7,8), (3,7), (5,9), (8,9), (3,9)}	{(2,4), (2,5), (4,8), (5,8), (2,3), (3,4), (3,5), (3,8)}

Case i: damage of an element that belongs to one cell

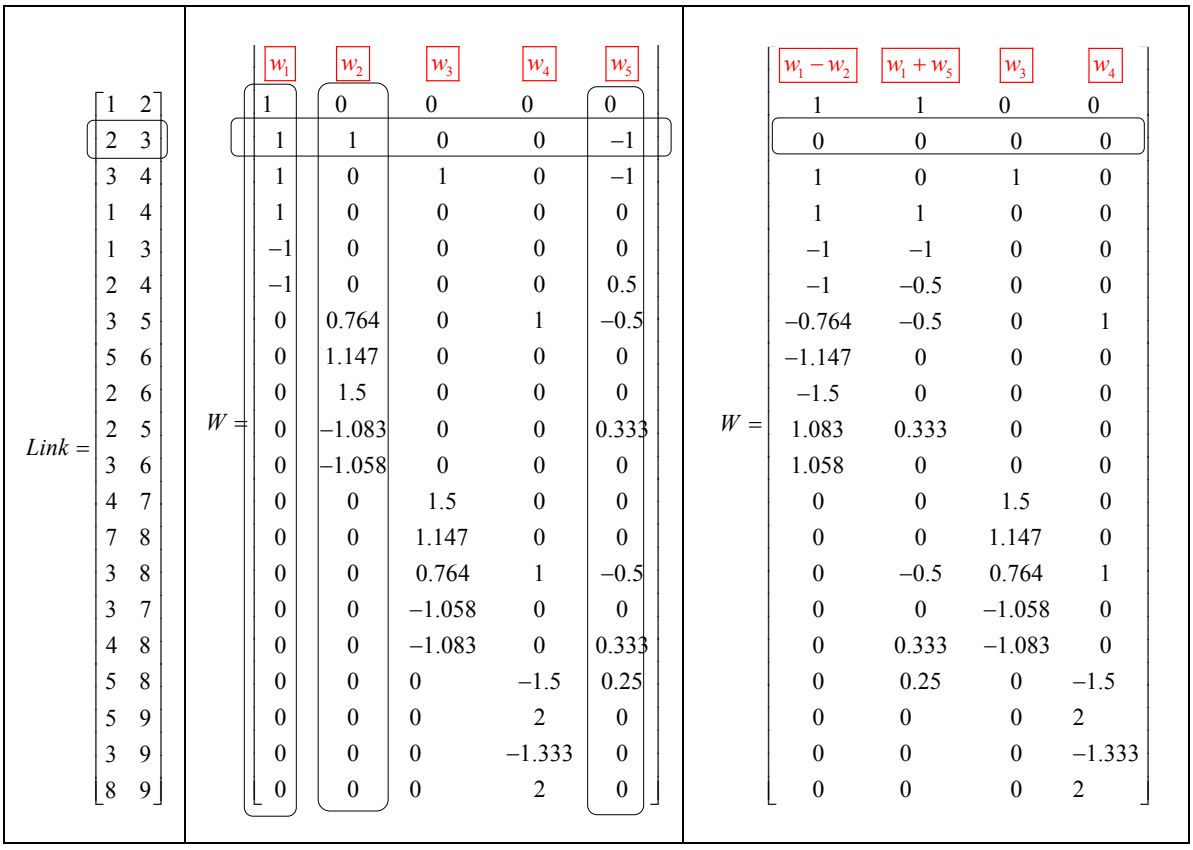
Assume damage occurs on element (1,2) that belongs to one cell only. The removal of the element results in node 1 having only two incident members. Since the position of the node cannot change, the node is no longer in equilibrium. To guarantee the equilibrium of the entire structure, damage has to propagate to elements (1,3) and (1,4) resulting in the necrosis of cell {1,2,3,4}. Although cell {1,2,3,4} is no longer available, elements (2,3), (3,4) and (2,4) remain active as members of other unicellular sub-structures. If the self-stress states are obtained through cellular multiplication, identifying the effect of damage on the self-stress space can be obtained by simply removing the state corresponding to cell {1,2,3,4} and setting the values on the rows corresponding to elements (1,2), (1,3) and (1,4) equal to zero.

$$Link = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 1 & 4 \\ 1 & 3 \\ 2 & 4 \\ 3 & 5 \\ 5 & 6 \\ 2 & 6 \\ 2 & 5 \\ 3 & 6 \\ 4 & 7 \\ 7 & 8 \\ 3 & 8 \\ 3 & 7 \\ 4 & 8 \\ 5 & 8 \\ 5 & 9 \\ 3 & 9 \\ 8 & 9 \end{bmatrix} \quad W = \begin{bmatrix} w_1 & w_2 & w_3 & w_4 & w_5 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0.5 \\ 0 & 0.764 & 0 & 1 & -0.5 \\ 0 & 1.147 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 & 0 \\ 0 & -1.083 & 0 & 0 & 0.333 \\ 0 & -1.058 & 0 & 0 & 0 \\ 0 & 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1.147 & 0 & 0 \\ 0 & 0 & 0.764 & 1 & -0.5 \\ 0 & 0 & -1.058 & 0 & 0 \\ 0 & 0 & -1.083 & 0 & 0.333 \\ 0 & 0 & 0 & -1.5 & 0.25 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -1.333 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix} \quad W = \begin{bmatrix} w_2 & w_3 & w_4 & w_5 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.5 \\ 0.764 & 0 & 1 & -0.5 \\ 1.147 & 0 & 0 & 0 \\ 1.5 & 0 & 0 & 0 \\ -1.083 & 0 & 0 & 0.333 \\ -1.058 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 1.147 & 0 & 0 \\ 0 & 0.764 & 1 & -0.5 \\ 0 & -1.058 & 0 & 0 \\ 0 & -1.083 & 0 & 0.333 \\ 0 & 0 & -1.5 & 0.25 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & -1.333 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

Case ii: damage of an element that belongs to two or more cells

Element (2,4) belongs to two unicellular sub-structures: cell {1,2,3,4} and the virtual cell {2,3,4,5,8}. The self-stress state of cell {1,2,3,4} corresponds to the first column of matrix W while the self-stress state of virtual cell {2,3,4,5,8} corresponds to the 5th column. The damage of this element can thus be described by the fusion of the two cells, which algebraically corresponds to finding a linear combination that sets the self-stress component of element (2,4) equal to zero. Similarly, the damage of element (2,3) which belongs to three unicellular sub-structures (cells {1,2,3,4} and {2,3,5,6}, as well as virtual cell {2,3,4,5,8}) can be described by the fusion of the three cells. It should be noted that multiple combinations can be considered and are possible. However, the resulting bases will describe the same space although they comprise different states.

$$Link = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 1 & 4 \\ 1 & 3 \\ 2 & 4 \\ 3 & 5 \\ 5 & 6 \\ 2 & 6 \\ 2 & 5 \\ 3 & 6 \\ 4 & 7 \\ 7 & 8 \\ 3 & 8 \\ 3 & 7 \\ 4 & 8 \\ 5 & 8 \\ 5 & 9 \\ 3 & 9 \\ 8 & 9 \end{bmatrix} \quad W = \begin{bmatrix} w_1 & w_2 & w_3 & w_4 & w_5 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0.5 \\ 0 & 0.764 & 0 & 1 & -0.5 \\ 0 & 1.147 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 & 0 \\ 0 & -1.083 & 0 & 0 & 0.333 \\ 0 & -1.058 & 0 & 0 & 0 \\ 0 & 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1.147 & 0 & 0 \\ 0 & 0 & 0.764 & 1 & -0.5 \\ 0 & 0 & -1.058 & 0 & 0 \\ 0 & 0 & -1.083 & 0 & 0.333 \\ 0 & 0 & 0 & -1.5 & 0.25 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -1.333 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix} \quad W = \begin{bmatrix} w_1 + 2w_5 & w_2 & w_3 & w_4 \\ 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0.764 & 0 & 1 \\ 0 & 1.147 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0.666 & -1.083 & 0 & 0 \\ 0 & -1.058 & 0 & 0 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 1.147 & 0 \\ -1 & 0 & 0.764 & 1 \\ 0 & 0 & -1.058 & 0 \\ 0.666 & 0 & -1.083 & 0 \\ 0.5 & 0 & 0 & -1.5 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -1.33 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$



At this point, the existence of a valid self-stress state should be investigated. A valid self-stress must respect the typology of the remaining elements as well as the principle of conservation of total energy. Two extreme cases are considered in this study with: a) the potential energy in the system after damage remaining the same (energy conservation), and b) the potential energy in the system after damage decreasing by the same amount of potential energy released from the damaged element (energy dissipation). In the first case, the damaged structure is assumed to have the same potential energy as the undamaged structure. The strain energy of the damaged element is thus distributed to the other members of the structure. In the second case, the remaining potential energy is distributed throughout the system through a valid self-stress. Let m be the number of elements in the structure, N_i the axial force in the element, and k_i the stiffness of the element. The potential energy E_p in the structure is given by:

$$E_p = \frac{1}{2}\sum_{i=1}^{m}\frac{N_i^2}{k_i} \qquad (1)$$

In matrix form it becomes:

$$E_p = \frac{1}{2}N^T C N \qquad (2)$$

where N is $[N_1, N_2, \ldots, N_m]^T$ and C is a diagonal m×m matrix where $C_{ii}=1/k_i$. Let L be an m×m diagonal matrix where the i^{th} diagonal entry is the length of member i. Using the force density vector w, N =Lw and the equation for the potential energy E_p becomes:

$$E_p = \frac{1}{2} w^T L^T C L w \quad (3)$$

Now let α be a vector of s linear combination coefficients, where s is the number of the self-stress states so that $w=W\alpha$. The potential energy E_p is expressed by:

$$\begin{aligned} E_p &= \frac{1}{2} \alpha^T W^T L^T C L W \alpha \\ &= \frac{1}{2} \alpha^T \overline{C} \alpha \end{aligned} \quad (4)$$

After damage, let W' be the new self-stress basis and α' the new combination vector of a valid self-stress. In addition to the energy equation, α' should also respect the typology of the remaining elements. Consequently, the space of valid combinations is given by the system:

$$\begin{cases} E_p = \frac{1}{2} \alpha'^T \overline{C}' \alpha' \\ W'_{cable} \alpha' > 0 \end{cases} \quad (5)$$

where W'_{cable} are the rows of the self-stress basis that correspond to cable elements.

Assuming that the four-cell planar tensegrity structure treated previously has an initial prestress of $10^6 \cdot W \cdot [1\ 1\ 1\ 1\ 0.5]^T$ N/m and considering that the same material is used for cables and bars with a Young's modulus of 100 GPa and an area of 1 cm^2 for the cables and 10 cm^2 for the bars, the total initial potential energy in the structure is equal to 1710 KJ. Table 2 shows a valid self-stress solution for the system for the damage of elements (1,2), (2,4) and (2,3).

Table 2: Examples of self-stress solutions for different damaged cases.

	Element damaged (1,2)		Element damaged (2,4)		Element damaged (2,3)	
	Energy conservation	Energy dissipation	Energy conservation	Energy dissipation	Energy conservation	Energy dissipation
E_p (KJ)	1710	1607	1710	1696	1710	1606.8
Solution for α	[0.55, 1.1, 1.1, -0.1]·10^6	[0.35, 1.1, 1.1, -0.1]·10^6	[1.647, 1.1, 1.1, 1]·10^6	[1.603, 1.1, 1.1, 1]·10^6	[-0.3, 0.5, 1.58, 0.8]·10^6	[-0.3, 0.5, 1.53, 0.8]·10^6
Members	w_1 (10^6N/m)	w_2 (10^6N/m)	w_3 (10^6N/m)	w_4 (10^6N/m)	w_5 (10^6N/m)	w_6 (10^6N/m)
(1,2)	0	0	1.0000	1.0000	0.5000	0.5000
(2,3)	0.6530	0.4500	0.6470	0.6320	0	0
(3,4)	1.2000	1.2000	0.1000	0.1000	0.2000	0.2000
(1,4)	0	0	1.0000	1.0000	0.5000	0.5000
(1,3)	0	0	-1.0000	-1.0000	-0.5000	-0.5000
(2,4)	-0.0500	-0.0500	0	0	-0.1000	-0.1000
(3,4)	1.5725	1.4174	1.3583	1.3468	1.4062	1.3562
(5,6)	0.6343	0.4014	1.8891	1.8719	0.3441	0.3441
(2,6)	0.8295	0.5250	2.4705	2.4480	0.4500	0.4500
(2,5)	-0.6322	-0.4123	-1.1177	-1.1015	-0.0585	-0.0585
(3,6)	-0.5851	-0.3703	-1.7425	-1.7267	-0.3174	-0.3174
(4,7)	1.6500	1.6500	1.6500	1.6500	0.7500	0.7500
(7,8)	1.2617	1.2617	1.2617	1.2617	0.5735	0.5735
(3,8)	1.9904	1.9904	0.9404	0.9404	1.5590	1.5090
(3,7)	-1.1638	-1.1638	-1.1638	-1.1638	-0.5290	-0.5290
(4,8)	-1.2246	-1.2246	-0.5253	-0.5253	-0.2751	-0.2751
(5,8)	-1.6750	-1.6750	-1.1500	-1.1500	-2.1655	-2.0905
(5,9)	2.2000	2.2000	2.2000	2.2000	3.1540	3.0540
(3,9)	-1.4663	-1.4663	-1.4663	-1.4663	-2.1021	-2.0355
(8,9)	2.2000	2.2000	2.2000	2.2000	3.1540	3.0540

Table 2 shows that for each damage case a valid solution that respects the energy assumptions and element typology can be determined by solving system (5). These solutions were identified by trial-and-error since the problem as presented in this paper is under defined and additional constraints are required to confine the solution space into a unique self-stress.

CONCLUSIONS

The analogy between elementary tensegrity units and biological unicellular organisms, which is at the core of the cellular multiplication process, induces a novel way of designing tensegrity structures that provides control over their self-stress states and allows one to easily assess their damage behavior. The proposed calculation method promotes this idea, with each vector of the self-stress matrix containing only the self-stress components that stabilize the associated cell. This facilitates the identification of a suitable linear combination of the vectors by the designer so that the conglomeration of tensegrity cells can be designed to preserve the integrity of the whole system while maintaining each cell's own identity. Finally, by identifying damage-critical elements,

preventive measures that can turn a physical structure more damage tolerant and avoid total failure by stopping the propagation of the problem can be sought.

ACKNOWLEDGEMENTS

This material is based upon work supported by the National Science Foundation under grant no. 1638336.

REFERENCES

Abedi, K., and Shekastehband, B. (2009). "Effects of member loss on the structural integrity of tensegrity systems." *Symposium of the International Association for Shell and Spatial Structures*. Editorial Universitat Politècnica de València, Valencia, Spain, 2274-2285.

Adriaenssens, S., Block, P., Veenendaal, D., and Williams, C. (2014). *Shell structures for architecture: form finding and optimization*, Routledge, London, UK.

Aloui, O., Orden, D., and Rhode-Barbarigos, L. (2018). "Generation of planar tensegrity structures through cellular multiplication." *Int. J. Appl. Math.* (under review)

Bel Hadj Ali, N., Rhode-Barbarigos, L. and Smith, I.F.C. (2011). "Analysis of clustered tensegrity structures using a modified dynamic relaxation algorithm", *Int. J. Solids Struct.*, 48(5), 637-647.

Ben Kahla, N. and Moussa, B. (2002). "Effect of a Cable Rupture on Tensegrity Systems." *Int. J. Solids Struct*., 17(1), 51-65.

Châtelet, V. (2007). "Interactive Cities, Anomalie Digital_Arts no6." Chapter 2 in *Moving towards control tensegrity*, Valerie Châtelet ed. HYX, Orléans, France, 36–75.

de Guzmán, M. and Orden, D. (2006). "From graphs to tensegrity structures: geometric and symbolic approaches." *Publicacions Matemàtiques*, 279-299.

Fernández, J. D. and Orden, D. (2011). "On the atomic decomposition length of graphs and tensegrities." *Avances en Matemática Discreta en Andalucía,* 2, 27–34.

Ingber, D.E., Wang, N., and Stamenovic D. (2014). "Tensegrity, cellular biophysics, and the mechanics of living systems." *Rep. Prog. Phys.* 77(4):046603.

Motro, R. (2003). *Tensegrity: structural systems for the future*, Elsevier, Amsterdam, Netherlands.

Rhode-Barbarigos, L., Bel Hadj Ali, N., Motro, R., and Smith, I.F.C. (2012). "Design aspects of a deployable tensegrity-hollow-rope footbridge." *Int. J. Solids Struct.*, 27(2), 81-96.

Sabelhaus, A.P., Bruce, J., Caluwaerts, K., Manovi, P., Fallah Firoozi, R., Dobi, S., Agogino, A.M., and SunSpiral, V. (2015). "System design and locomotion of SUPERball an untethered tensegrity robot." *Proc., 2015 IEEE Int. Conf. on Robotics and Automation (ICRA)*, IEEE Press, Piscataway, NJ, 2867-2873.

Zhang, J.Y., Ohsaki, M., and Kanno, Y. (2006). "A direct approach to design of geometry and forces of tensegrity systems." *Int. J. Solids Struct.*, 43(7), 2260-2278.