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Hybrid Dynamical System Model and Robust Control of a Hybrid Neuroprosthesis Under Fatigue Based Switching

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Abstract—**A hybrid neuroprosthesis controller that integrates a modified PD-based robust controller to compensate for electromechanical delay during functional electrical stimulation and a variable structure controller to control a powered exoskeleton is developed. A hybrid dynamical systems approach is used to model the hybrid neuroprosthesis control and prove the stability. Simulation results demonstrate the model and the control design during a standing task.**

I. Introduction

Functional electrical stimulation (FES) and powered exoskeleton technologies have recently been investigated [1]–[3] to restore lower limb function in persons with spinal cord injury (SCI). FES uses muscles to actuate limb joints, but often results in the rapid onset of muscle fatigue, which limits the duration of its use. Powered exoskeletons, are not limited by muscle fatigue, but could be bulkier to use [4]. A hybrid neuroprosthesis that combines both FES and electrical motors has therefore been proposed [4]–[7] to overcome limitations of FES and powered exoskeleton.

We foresee the following challenges in the control design of a hybrid neuroprosthesis.1) Collaboration between FES and electrical motors should use a control strategy that limits limb joint error as well as muscle fatigue. 2) Multiple control laws are required to address disparate characteristics of FES and electric motors. For example, to ensure system performance and stability, the control of FES, unlike an electric motor control, must compensate for electromechanical delay (EMD), high model uncertainty and nonlinearity of the musculoskeletal system. Therefore, a modeling and control approach that is based on switching or hybrid dynamical system theory [8], [9] seems most pertinent to address these challenges.

In [10], a switching strategy based on fatigue information was designed for a single degree of freedom (DOF) hybrid neuroprosthesis. The controller used feedback linearization and a second order sliding model control for both FES and motors; i.e, the control design neglected the presence of EMD during FES. In our past work, we have shown that improved system performance can be obtained with controllers that compensate for EMD during FES [11]–[13]. This paper

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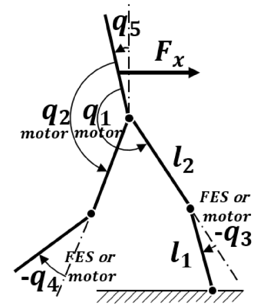

Figure 1. 5 DOF hybrid neuroprosthesis model

integrates two different, robust control laws for FES and motors. Specifically, the control design considers a multi-DOF musculoskeletal system with EMD and model uncertainty. The control laws do not share a common Lyapunov function because the error states are defined differently and separate terms that compensate for EMD are used to control FES. Therefore, the stability proof is non-trivial and a Lyapunov analysis that considers a hybrid dynamical system is required. Main results of this paper are organized as follows. 1) In Section II, a hybrid dynamical system model is formulated. It utilizes two types of control laws: a) a complete powered exoskeleton control b) a mixed FES and motor usage; i.e., motor to control hip joints and FES to control knee joints. The switching between the control laws is based on limb joint error and muscle fatigue state. 2) In section III, the control laws, namely, a PD-based robust controller with an additional variable gain and a delay compensation term and a variable structure controller (VSC), are designed and analyzed for stability. 3) In Section IV, overall stability of the hybrid dynamical system is analyzed via a Lyapunov stability method to obtain a globally uniformly ultimately bounded result.

II. Modeling the Hybrid Neuroprosthesis as a Hybrid Dynamical System

A. Dynamics of the Hybrid Neuroprosthesis

Euler-Lagrange (EL) equation of the 5 degree of freedom (DOF) hybrid neuroprosthesis is based on the 5-link pinned point feet model of walking during swing phase [14], with

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additional terms describing the neuromuscular dynamics and the actuator dynamics,

$$
\begin{aligned}
D^{EL}(q)\ddot{q} + C^{EL}(q,\dot{q})\dot{q} + G^{EL}(q) + M_{ev}^{EL}(q,\dot{q}) + W^{EL} \\
= B_1^{EL}(\xi, u(t-\tau)) + B_2^{EL}(\xi, v) + Q_F^{EL} F_x, \qquad (1)
\end{aligned}
$$

where $t \in \mathbb{R}$ is time, and $q(t) = (q_1, ..., q_5)^T \in \mathbb{R}^5$ are time dependent joint angles, illustrated in Fig. 1. $D^{EL}(q) \in \mathbb{R}^{5\times 5}$, $C^{EL}(q,\dot{q}) \in \mathbb{R}^{5\times 5}$ and $G^{EL}(q) \in \mathbb{R}^5$ are generalized inertia matrix, Coriolis matrix and gravity term, respectively. In (1), $M_{ev}^{EL}(q,\dot{q}) \in \mathbb{R}^5$ is a elastic-viscous vector expressing passive moments introduced by the musculoskeletal system at each joint and $W^{EL} \in \mathbb{R}^5$ is a term containing all the system disturbance. On right hand side of (1), $B_1^{EL}(\xi, u(t-\tau))$ represents joint actuation using FES at knee joints q_3, q_4 and using motors at hip joints q_1, q_2, while $B_2^{EL}(\xi, v)$ represents for actuating q_1, q_2, q_3, q_4 with motors. $u(t-\tau) = (u_1(t-\tau_1), ..., u_4(t-\tau_4))^T \in \mathbb{R}^4$ is the delayed control signal vector in B_1^{EL}, with known constant $\tau_i \in \mathbb{R}_{>0}$ $(i = 3,4)$ due to EMD and $\tau_1 = \tau_2 = 0$ because motors are assumed to have zero delay. $v = (v_1, ..., v_4)^T \in \mathbb{R}^4$ is the control signal vector in B_2^{EL} and $\xi \in \{-1, 1\}$ is a switching signal, which is based on muscle fatigue. A horizontal human assisting force $F_x \in \mathbb{R}$, due to a walker, is introduced to avoid under-actuation.

Define error signal $e = (e_1, ..., e_4)^T \in \mathbb{R}^4$, $r = (r_1, ..., r_4)^T \in \mathbb{R}^4$ and $e_c = (e_{c,1}, ..., e_{c,4})^T \in \mathbb{R}^4$, with $e_{c,i} = \int_{t-\tau_i}^{t} u_i(\theta)d\theta$ $(i = 1,2)$, $e_{c,i} = \int_{t-\tau_i}^{t} \operatorname{sgn}(r_i(\theta))\, u_i(\theta)d\theta$ $(i = 3,4)$, such that $\forall i = 1...4$,

$$e_i = q_{d,i} - q_i, \qquad (2)$$

$$r_i = \dot{e}_i + \alpha e_i - \frac{\xi+1}{2}\beta e_{c,i}, \qquad (3)$$

where $\alpha, \beta \in \mathbb{R}_{>0}$ are constants to be designed and $q_d = (q_{d,1}, ..., q_{d,4})^T \in \mathbb{R}^4$ is the desired trajectory for joint q_i, $(i = 1...4)$. Upper body position angle q_5, instead of following a precise trajectory, is maintained within an approximate range by human assisting force F_x. Therefore, $q_{d,5}$ can be directly taken as $q_{d,5} = q_5$ and it is unnecessary to define e_5. Open loop error dynamics regarding e, r can then be rewritten as

$$
\begin{aligned}
D(q)\left(\ddot{q}_d + \alpha r - \alpha^2 e - \dot{r} + \frac{\xi+1}{2}\beta\left(\alpha e_c - \dot{e}_c\right)\right) \\
+C(q,\dot{q})\left(\dot{q}_d + \alpha e - r - \frac{\xi+1}{2}\beta e_c\right) \\
+G(q) + M_{ev}(q,\dot{q}) + D_5(q)\ddot{q}_{d,5} + C_5(q,\dot{q})\dot{q}_{d,5} \\
+W = B_1(\xi, u(t-\tau)) + B_2(\xi, v) + Q_F F_x, \qquad (4)
\end{aligned}
$$

where G, M_{ev}, W, B_1, B_2, Q_F are first 4 rows of G^{EL}, M_{ev}^{EL}, W^{EL}, B_1^{EL}, B_2^{EL}, Q_F^{EL}, respectively, while $D \in \mathbb{R}^{4\times 4}$, $D_5 \in \mathbb{R}^4$ and $C \in \mathbb{R}^{4\times 4}$, $C_5 \in \mathbb{R}^4$, are corresponding submatrices of D^{EL} and C^{EL}, respectively.

B. Actuator Dynamics

Detailed expressions of $B_1(\xi, u(t-\tau))$, $B_2(\xi, v)$, $Q_F F_{bx}$ in (4) will be derived in this subsection.

When joint q_i $(i = 3,4)$ is actuated by FES, torque exerted via corresponding quadriceps-hamstring (Q-H) muscle pair, can be written as

$$
\begin{aligned}
T_{FES,i} = \frac{1+\operatorname{sgn}(r_i)}{2}\frac{\mu_{Q,i}\eta_{Q,i}\left(u_{act,Q,i} - U_{t,Q,i}\right)}{U_{s,Q,i} - U_{t,Q,i}} \\
-\frac{1-\operatorname{sgn}(r_i)}{2}\frac{\mu_{H,i}\eta_{H,i}\left(u_{act,H,i} - U_{t,H,i}\right)}{U_{s,H,i} - U_{t,H,i}}, \qquad (5)
\end{aligned}
$$

where, Q and H muscles are activated in turn determined by $\operatorname{sgn}(r_i)$ signals. $\eta_{Q,i}, \eta_{H,i} \in \mathbb{R}_{>0}$ are unknown q dependent nonlinear functions containing torque-length/torque-velocity relationship, while $U_{t,Q,i}$,$U_{t,H,i} \in \mathbb{R}_{>0}$ and $U_{s,Q,i}$,$U_{s,H,i} \in \mathbb{R}_{>0}$ are threshold and saturation stimulation levels, respectively [15]. By defining $n \in \{Q, H\}$, $i \in \{3,4\}$, the actual input stimulation amplitude $u_{act,n,i}$ is assumed to always operate beneath saturation level and is designed as

$$u_{act,n,i} = K_{B,i}u_i + U_{t,n,i}, \qquad (6)$$

where $K_{B,i} \in \mathbb{R}_{>0}$ is a variable gain to be designed. $\mu_{Q,i} \in [\varsigma_{Q,i}, 1]$, $\mu_{H,i} \in [\varsigma_{H,i}, 1]$ in (5) are muscle fatigue states [10] modeled as the following when EMD exists,

$$
\begin{aligned}
\dot{\mu}_{n,i} &= \Gamma_{n,i}(\mu_{n,i}, r_i(t-\tau_i), u_i(t-\tau_i), t) \qquad (7)\\
&= \frac{1}{T_{f,n,i}}(\varsigma_{n,i} - \mu_{n,i})a_{n,i} + \frac{1}{T_{r,n,i}}(1-\mu_{n,i})(1-a_{n,i}).
\end{aligned}
$$

$a_{Q,i} = \frac{1+\operatorname{sgn}(r_i(t-\tau_i))}{2}\frac{K_{B,i}u_i(t-\tau_i)}{U_{s,Q,i}-U_{t,Q,i}}$, $a_{H,i} = \frac{1-\operatorname{sgn}(r_i(t-\tau_i))}{2}\frac{K_{B,i}u_i(t-\tau_i)}{U_{s,H,i}-U_{t,H,i}}$. $T_{f,n,i}, T_{r,n,i} \in \mathbb{R}_{>0}$ are fatiguing and recovery constants while $\varsigma_{n,i} \in (0,1)$ is minimum fatigue level.

When joint q_i $(i = 1...4)$ is actuated by electrical motor, torque generated depends on the motor constant $K_{m,i} \in \mathbb{R}_{>0}$. Since B_1 and B_2 in (4) utilize different control signals, motor torque $T_{motor,i}^{B_1} (i = 1,2)$ and $T_{motor,i}^{B_2}$ $(i = 1...4)$ will be modeled differently, as

$$T_{motor,i}^{B_1} = K_{m,i}K_{B,i}u_i, \quad T_{motor,i}^{B_2} = K_{m,i}v_i, \qquad (8)$$

where $K_{B,i}$, u_i and v_i all preserve their previous definitions.

Therefore, with $T_{FES,i}$ $(i = 3,4)$, $T_{motor,i}^{B_1}$ $(i = 1,2)$ contributing to B_1 when $\xi = 1$ and $T_{motor,i}$ $(i = 1...4)$ contributing to B_2 when $\xi = -1$, considering EMD τ_i $(i = 1...4)$ in B_1, it can be written explicitly that

$$B_1(\xi, u(t-\tau)) = \frac{1+\xi}{2}K_B\widehat{B}_1 u(t-\tau), \qquad (9)$$

$$B_2(\xi, v) = \frac{1-\xi}{2}\left(K_m v + \widetilde{B}_2\right). \qquad (10)$$

$K_B = \operatorname{diag}(K_{B,1}, ..., K_{B,4})$, $K_m = \operatorname{diag}(K_{m,1}, ..., K_{m,4})$, $\widehat{B}_1 = \operatorname{diag}(K_{m,1}, K_{m,2}, \Upsilon_3, \Upsilon_4)$, $\Upsilon_i = \frac{1+\operatorname{sgn}(r_i(t-\tau_i))}{2}\frac{\mu_{Q,i}\eta_{Q,i}}{U_{s,Q,i}-U_{t,Q,i}} - \frac{1-\operatorname{sgn}(r_i(t-\tau_i))}{2}\frac{\mu_{H,i}\eta_{H,i}}{U_{s,H,i}-U_{t,H,i}}$ $(i = 3,4)$. $\widetilde{B}_2 \in \mathbb{R}^4$ is the remaining actuation from B_1 due to EMD after switching, which can be assumed bounded and will certainly disappear after time $t^* + \max_{i=1...4}\{\tau_i\}$, where t^* is the time instant when ξ switches from 1 to -1.

Finally, Q_F is used to transfer F_x to generalized force and is expressed as $Q_F = (l_1\cos(q_1+q_3+q_5) + l_2\cos(q_1+q_5), 0, l_1\cos(q_1+q_3+q_5), 0)^T$.

C. Fatigue Based Switching Logic

The torque generated by FES varies with muscle fatigue and recovery, as in (7). Therefore, to obtain a consistent joint torque a fatigue based switching logic determines the value of $\xi \in \{-1, 1\}$, which further determines activation of either (9) or (10). The fatigue based switching logic is designed as

$$\xi^+ = -\xi^- \quad (e^{-^T}, r^{-^T}, \mu^{-^T}, \xi^-)^T \in \mathcal{D} \ , \tag{11}$$

where $(\cdot)^-$and $(\cdot)^+$ denote the value just before and after the switch, respectively. Define $\mu = (\mu_{Q,3}, \mu_{H,3}, \mu_{Q,4}, \mu_{H,4})^T$, $y = (e^T, r^T, \mu^T, \xi)^T$, universe $\mathcal{U} = \mathbb{R}^8 \times [\varsigma_{Q,3}, 1] \times [\varsigma_{H,3}, 1] \times [\varsigma_{Q,4}, 1] \times [\varsigma_{H,4}, 1] \times \{-1, 1\}$ and set $\mathcal{D}$ is given by

$$\begin{aligned} \mathcal{D} = &\Big\{ y \in \mathcal{U} : \xi = 1, \exists n \in \{Q, H\}, i \in \{3, 4\} \text{ s.t.} \mu_{n,i} \leq \underline{\mu} \Big\} \\ &\cup \mathcal{D}_{f\xi} \cup \mathcal{D}_{er\xi} \cup \bigg(\mathcal{D}_f \cap \mathcal{D}_{er_j} \cap \Big\{ y \in \mathcal{U} : \xi = -1, \\ &\forall n \in \{Q, H\}, i \in \{3, 4\} \text{ s.t.} \mu_{n,i} \geq \overline{\mu} \Big\} \bigg), \end{aligned} \tag{12}$$

where $\underline{\mu}, \overline{\mu} \in [\max_{i=3,4} \{\varsigma_{Q,i}, \varsigma_{H,i}\}, 1]$, $\underline{\mu} < \overline{\mu}$ are constants describing fatiguing and recovery limits to enable switching. $\mathcal{D}_{f\xi}$, $\mathcal{D}_{er\xi}$, $\mathcal{D}_f$ and $\mathcal{D}_{er_j}$ are additional states dependent conditions to be designed to address stability issues. Note that $\dot{\xi} = 0$ when $(e^T, r^T \mu^T, \xi)^T \in \partial\mathcal{D} \cup \mathcal{D}^c$, where $\partial\mathcal{D}$ denotes boundary of $\mathcal{D}$ while $\mathcal{D}^c$ denotes the complement set regarding universe $\mathcal{U}$. (11), (12) indicates that ξ changes from 1 (or -1) to -1 (or 1) once certain criteria mainly determined by fatigue states μ of Q-H muscle pairs are met. After one of the fatigue states drops below the designed threshold, system will utilize actuation (10) instead of (9) until all of the fatigue states recover to the designed values.

D. Tracking Error Based Hybrid Dynamical System Model

According to definition in [8] and using (2), (3), (4), (7), (9), (10), (11), (12), the error based hybrid neuroprosthetic system, under fatigue based switching, tracking q_d can be modeled as

$$\mathcal{H}_{q_d} : \begin{cases} \dot{y} = \mathcal{F}(y, u, v) & y \in \mathcal{C} \\ y^+ = \mathcal{G}(y^-) & y^- \in \mathcal{D} \end{cases}, \tag{13}$$

where state vector $y = (e^T, r^T, \mu^T, \xi)^T$. Jump set $\mathcal{D}$ is given by (12) while flow set is $\mathcal{C} = \partial\mathcal{D} \cup \mathcal{D}^c$. Flow map $\mathcal{F}$ and jump map $\mathcal{G}$ are given by

$$\begin{aligned} \mathcal{F} &= (r - \alpha e + \frac{\xi + 1}{2} \beta e_c, \mathcal{F}_r, \mathit{\Gamma}, 0)^T, \\ G &= (e^-, r^-(-\xi^-), \mu^-, -\xi^-)^T, \end{aligned} \tag{14}$$

where $\mathcal{F}_r = \ddot{q}_d + \alpha r - \alpha^2 e + \frac{\xi+1}{2} \beta \left(\alpha e_c - \dot{e}_c\right) + D^{-1} \big(C \big(\dot{q}_d + \alpha e - r - \frac{\xi+1}{2} \beta e_c \big) + G + M_{ev} + D_5 \ddot{q}_{d,5} + C_5 \dot{q}_{d,5} + W - B_1(\xi, u(t - \tau)) - B_2(\xi, v) - Q_F F_x \big)$ and $\mathit{\Gamma} = (\mathit{\Gamma}_{Q,3}, \mathit{\Gamma}_{H,3}, \mathit{\Gamma}_{Q,4}, \mathit{\Gamma}_{Q,4})$.

In the end of this section, additional assumptions will be made as the following: (A1) q_d, $q_{d,5}$ and their derivatives are known and bounded. F_x is estimated as $|F_x| \leq \bar{F}_x$, $\bar{F}_x \in \mathbb{R}_{>0}$. Q_F is bounded because it consists of linear combinations of $\sin(\cdot)$. Disturbance W is assumed to be bounded. (A2) $M_{ev}(q, \dot{q})$, $\eta_{n,i}$, $n \in \{Q, H\}$, $i \in \{3, 4\}$, are bounded. (A3) Parameters in (12) can be designed so that time between two switches are always greater than $\max_{i=1...4} \{\tau_i\}$.

III. Control Development

In this section, based on (13), stabilization of cases: $\xi = 1$ and $\xi = -1$ are considered as separate problems and two different controllers are individually designed based on Lyapunov stability analysis.

A. $\xi = 1$, Knee Joint Angle Actuated by FES

When $\xi = 1$, each knee joint is actuated by a Q-H muscle pair using FES. Due to the main challenge induced by EMD, a PD-based controller with additional variable gain K_B and a delay compensation term will be designed. Firstly, applying feedback control law

$$u(t) = R K_u r(t). \tag{15}$$

Constant $K_u \in \mathbb{R}_{>0}$, $R = \text{diag}\left(1, 1, \text{sgn}\left(r_3\right), \text{sgn}\left(r_4\right)\right)$. Secondly, define $\Phi = \frac{1}{2} \dot{D} r + e + D \left(\ddot{q}_d + \alpha \dot{e} \right) + C \dot{q} + G + M_{ev} + D_5 \ddot{q}_{d,5} + C_5 \dot{q}_{d,5}$, $\Phi_d = D(q_d) \ddot{q}_d + C(q_d, \dot{q}_d) \dot{q}_d + G(q_d) + M_{ev}(q_d, \dot{q}_d) + D_5(q_d) \ddot{q}_{d,5} + C_5(q_d, \dot{q}_d) \dot{q}_{d,5}$, so that $\|\Phi - \Phi_d\| \leq \rho(\|z\|) \|z\|$, $z = (e^T, r^T, e_c^T)^T$, which can be proved by mean value theorem [16], with $\rho(\|z\|)$, a positive, globally invertible nondecreasing real function. Further, $\|\Phi_d + W\| \leq \Psi$ and $\|Q_F F_x\| \leq \bar{Q}_F \bar{F}_x$, $\Psi, \bar{Q}_F \in \mathbb{R}_{>0}$, according to (A1), (A2). Finally, by defining $R_\tau = \text{diag}\left(1, 1, \text{sgn}\left(r_3(t - \tau_3)\right), \text{sgn}\left(r_4(t - \tau_4)\right)\right)$, closed loop error dynamics is derived as

$$\begin{aligned} D\dot{r} = &-\frac{1}{2} \dot{D} r - e + \Phi - \Phi_d + \Phi_d + W - Q_F F_x \\ &- D\beta K_u r + \Big(D\beta R_\tau - K_B \widehat{B}_1 \Big) R_\tau K_u r(t - \tau). \end{aligned} \tag{16}$$

The following properties and definitions will be used during stability analysis.

$$\sigma_1 \|r\|^2 \leq r^T D r \leq \sigma_2 \|r\|^2. \tag{17}$$

$$\begin{aligned} \lambda_1 \|y_{er}\|^2 \leq \frac{1}{2} e^T e + \frac{1}{2} r^T D r \leq \lambda_2 \|y_{er}\|^2, \\ y_{er} = (e^T, r^T)^T. \end{aligned} \tag{18}$$

$$\|B^* r(t - \tau)\| \leq \sigma_3 \|r(t - \tau)\|, \ |\sigma_3| \leq \bar{\sigma}_3. \tag{19}$$

$$\beta \|e\| \|e_c\| \leq \frac{\beta^2 \epsilon^2}{4} \|e\|^2 + \frac{1}{\epsilon^2} \|e_c\|^2. \tag{20}$$

$$\|r\| \|r(t - \tau)\| \leq \frac{\varepsilon^2}{2} \|r(t - \tau)\|^2 + \frac{1}{2\varepsilon^2} \|r\|^2. \tag{21}$$

$$-\tau_i \int_{t - \tau_i}^{t} u_i^2(\theta) d\theta \leq -e_{c,i}^2. \tag{22}$$

Remark 1. (i) In (19), $B^* = D - \beta^{-1} K_B \widehat{B}_1 R_\tau$, $\sigma_3 = \max\left\{ \sqrt{\text{eig}(B^{*^T} B^*)} \right\}$, where $\text{eig}(B^{*^T} B^*) = \{\sigma_{3,1} ... \sigma_{3,4}\}$ are eigenvalues of $B^{*^T} B^*$. Variable gain K_B introduced in (9) provides some flexibility to manipulate σ_3 to reach desired ranges with bounded perturbations, therefore $|\sigma_3| \leq$

$\bar{\sigma}_3 \in \mathbb{R}_{>0}$. (ii) In (17), $\sigma_1, \sigma_2 \in \mathbb{R}_{>0}$ are minimal and maximal eigenvalues of $D(q)$, therefore $\sigma_1 = \sigma_1(e, q_d)$, $\sigma_2 = \sigma_2(e, q_d)$. σ_1, σ_2 are upper and lower bounded due to property of inertia matrix. In (18), $\lambda_1, \lambda_2 \in \mathbb{R}_{>0}$ are constants. (20) and (21) are obtained by Young's inequality and constants $\epsilon^2, \varepsilon^2 \in \mathbb{R}_{>0}$. (22) is obtained by Cauchy-Schwarz inequality.

A Lyapunov function candidate can be designed as

$$V_{B_1} = \frac{1}{2}e^T e + \frac{1}{2}r^T D r + \sum_{i=1}^{4}\left(P_{1,i} + P_{2,i}\right), \tag{23}$$

where $P_{1,i} = \omega_i \int_{t-\tau_i}^{t}\left(\int_s^t u_i^2(\theta)d\theta\right)ds$, $P_{2,i} = \frac{\bar{\sigma}_3\beta K_u\varepsilon^2}{2}\int_{t-\tau_i}^{t} r_i^2(\theta)d\theta$, $\omega_i \in \mathbb{R}_{>0}$, $i = 1...4$.

Theorem 2. *Provided σ_1, λ_1, $\bar{\sigma}_3$, ϵ^2, ε^2 from Remark 1, $\bar{\omega}_\tau = \max_i\{\omega_i\tau_i\}$ $(i = 1...4)$, control gains α, β, K_u and initial conditions $(e_0^T, r_0^T)^T$ of system (16) satisfying $\alpha - \frac{\beta^2\epsilon^2}{4} > 0$, $f(e^T, r^T, q_d, K_B) = \sigma_1 - \frac{\bar{\sigma}_3}{2\varepsilon^2} - \frac{\bar{\sigma}_3\varepsilon^2}{2} > 0$, $\sigma_1\beta K_u - \left(\frac{\bar{\sigma}_3}{2\varepsilon^2} + \frac{\bar{\sigma}_3\varepsilon^2}{2}\right)\beta K_u - \bar{\omega}_\tau K_u^2 - K_1^* - K_2^* > 0$, $(e_0^T, r_0^T)^T \in \Omega_0$, $\Omega_0 = \big\{(e^T, r^T)^T : e, r \in \mathbb{R}^4, \lambda_2\left\|(e^T, r^T)^T\right\|^2 + \sum_{i=1}^{4}\left(P_{1,i} + P_{2,i}\right) \le \lambda_1 \min_{i=1...4}\{1, \frac{\bar{\sigma}_3\beta\varepsilon^2}{2\tau_i^2 K_u\lambda_1}\}\rho^{-2}\left(2\sqrt{K_1^*\chi}\right) - \frac{\lambda_2\left(\Psi + \bar{Q}_F\bar{F}_x\right)^2}{4\lambda_3 K_2^*} - \delta_1\big\}$, where $K_1^*, K_2^* \in \mathbb{R}_{>0}$ are constants, $K_u > K_1^* + K_2^*$, Lyapunov function (23) converges semi-globally from $V_{B_{1_0}}$ according to $V_{B_1} \le V_{B_{1_0}}e^{-\varrho t} + \Theta\left(1 - e^{-\varrho t}\right)$. Constant $\delta_1 \in \mathbb{R}_{>0}$ can be designed arbitrary small. $\chi, \varrho, \Theta \in \mathbb{R}_{>0}$ are constants derived from subsequent stability analysis.*

Proof: Take time derivative of V_{B_1} with control law (15) and close loop error dynamics (16) substituted into the equation and further apply property (17), (19), as well as previously discussed bounded characteristics of $\|\Phi - \Phi_d\|$, $\|\Phi_d + W\|$ and $\|Q_F F_x\|$,

$$\begin{aligned}\dot{V}_{B_1} \le& -\alpha\|e\|^2 - \beta K_u\sigma_1\|r\|^2 + \bar{\sigma}_3\beta K_u\|r\|\|r(t-\tau)\| \\ &+ \beta\|e\|\|e_c\| + \|r\|\rho(\|z\|)\|z\| + \|r\|\left(\Psi + \bar{Q}_F\bar{F}_x\right) \\ &+ \sum_{i=1}^{4}\omega_i\tau_i K_u^2 r_i^2 + \sum_{i=1}^{4}\frac{\bar{\sigma}_3\beta K_u\varepsilon^2}{2}\left(r_i^2 - r_i^2(t-\tau_i)\right) \\ &- \sum_{i=1}^{4}\omega_i\int_{t-\tau_i}^{t}u_i^2(\theta)d\theta.\end{aligned}$$

Further, by defining a set $\mathcal{T} = \{i|\,\tau_i \ne 0, i = 1...4\}$ to deal with the singularity issue when $\tau_i = 0$ and using (20) and (21) , $\dot{V}_{B_1}$ can then be derived as

$$\begin{aligned}\dot{V}_{B_1} \le& -\left(\alpha - \frac{\beta^2\epsilon^2}{4}\right)\|e\|^2 - \sum_{i\in\mathcal{T}}\frac{1}{\tau_i}\left(\omega_i - \kappa_i - \frac{\tau_i}{\epsilon^2}\right)e_{c,i}^2 \\ &- \left(\sigma_1\beta K_u - \left(\frac{\bar{\sigma}_3}{2\varepsilon^2} + \frac{\bar{\sigma}_3\varepsilon^2}{2}\right)\beta K_u - \bar{\omega}_\tau K_u^2\right)\|r\|^2 \\ &+ \|r\|\rho(\|z\|)\|z\| + \|r\|\left(\Psi + \bar{Q}_F\bar{F}_x\right) \\ &- \sum_{i\in\mathcal{T}}\left((\kappa_i - \gamma_i)\int_{t-\tau_i}^{t}u_i^2(\theta)d\theta + \gamma\int_{t-\tau_i}^{t}u_i^2(\theta)d\theta\right),\end{aligned}$$

where constants $\kappa_i, \gamma_i, \epsilon^2 \in \mathbb{R}_{>0}$ are always able to choose such that $\kappa_i > \gamma_i, \omega_i - \kappa_i - \frac{\tau_i}{\epsilon^2} > 0$. By completing squares, applying (22) and considering the fact that $\int_{t-\tau_i}^{t}\left(\int_s^t u_i^2(\theta)d\theta\right)ds \le \tau_i \sup_{t-\tau_i\le s\le t}\int_s^t u_i^2(\theta)d\theta = \tau_i\int_{t-\tau_i}^{t}u_i^2(\theta)d\theta$, $\dot{V}_{B_1}$ will become

$$\begin{aligned}\dot{V}_{B_1} \le& -\left(\chi - \frac{\rho^2(\|z\|)}{4K_1^*}\right)\|z\|^2 + \frac{\left(\Psi + \bar{Q}_F\bar{F}_x\right)^2}{4K_2^*} \\ &- \sum_{i\in\mathcal{T}}\Bigg(\frac{(\kappa_i - \gamma_i)}{\tau_i\omega_i}\omega_i\int_{t-\tau_i}^{t}\left(\int_s^t u_i^2(\theta)d\theta\right)ds \\ &+ \frac{2\gamma_i K_u^2}{\bar{\sigma}_3\beta K_u\varepsilon^2}\frac{\bar{\sigma}_3\beta K_u\varepsilon^2}{2}\int_{t-\tau_i}^{t}r_i^2(\theta)d\theta\Bigg).\end{aligned}$$

Provided gain and initial conditions in Theorem 2 satisfied, χ is some constant such that $\chi \le \min\{\alpha - \frac{\beta^2\epsilon^2}{4}, \varPi_1, \varPi_2\}$, $\chi - \frac{\rho^2(\|z\|)}{4K_1^*} > 0$, where $\varPi_1 = \sigma_1\beta K_u - \left(\frac{\bar{\sigma}_3}{2\varepsilon^2} + \frac{\bar{\sigma}_3\varepsilon^2}{2}\right)\beta K_u - \bar{\omega}_\tau K_u^2 - K_1^* - K_2^*$, $\varPi_2 = \frac{1}{\tau_i}\left(\omega_i - \kappa_i - \frac{\tau_i}{\epsilon^2}\right)$. Using (18) and the fact $-\|z\| \le -\|y_{er}\|$, as well as $P_{1,i} = P_{2,i} = 0$ when $\tau_i = 0$, defining constant $\lambda_3 \in \mathbb{R}_{>0}$ such that $\lambda_3 \le \min_{i\in\mathcal{T}}\{\chi - \frac{\rho^2(\|z\|)}{4K_1^*}, \frac{\lambda_2(\kappa_i - \gamma_i)}{\tau_i\omega_i}, \frac{2\lambda_2\gamma_i K_u}{\bar{\sigma}_3\beta\varepsilon^2}\}$, $\dot{V}_{B_1}$ can be further derived as

$$\begin{aligned}\dot{V}_{B_1} \le& -\left(\chi - \frac{\rho^2(\|z\|)}{4K_1^*}\right)\|y_{er}\|^2 + \frac{\left(\Psi + \bar{Q}_F\bar{F}_x\right)^2}{4K_2^*} \\ &- \sum_{i\in\mathcal{T}}^{4}\frac{1}{\lambda_2}\left(\frac{\lambda_2(\kappa_i - \gamma_i)}{\tau_i\omega_i}P_{1,i} + \frac{2\lambda_2\gamma_i K_u}{\bar{\sigma}_3\beta\varepsilon^2}P_{2,i}\right) \\ \le& -\frac{\lambda_3}{\lambda_2}\left(\frac{1}{2}e^T e + \frac{1}{2}r^T D r + \sum_{i=1}^{4}\left(P_{1,i} + P_{2,i}\right)\right) \\ &+ \frac{\left(\Psi + \bar{Q}_F\bar{F}_x\right)^2}{4K_2^*}.\end{aligned}$$

Considering (23), $\dot{V}_{B_1} \le -\frac{\lambda_3}{\lambda_2}V_{B_1} + \frac{\left(\Psi + \bar{Q}_F\bar{F}_x\right)^2}{4K_2^*}$ can be obtained and solved as

$$V_{B_1} \le V_{B_{1_0}}e^{-\frac{\lambda_3}{\lambda_2}t} + \frac{\lambda_2\left(\Psi + \bar{Q}_F\bar{F}_x\right)^2}{4\lambda_3 K_2^*}\left(1 - e^{-\frac{\lambda_3}{\lambda_2}t}\right). \tag{24}$$

■

B. $\xi = -1$, *Knee Joint Angle Actuated by Motor*

When $\xi = -1$, all the joints are actuated by electrical motors. Following the same procedure as when $\xi = 1$, the open loop error dynamics can be expressed as

$$\begin{aligned}D\dot{r} =& -\frac{1}{2}\dot{D}r - e - \widehat{B}_2 v + \Phi' - \Phi'_d, \\ &+ \Phi'_d - \widetilde{B}_2 - Q_F F_x + W,\end{aligned} \tag{25}$$

where $\Phi' = \frac{1}{2}\dot{D}r + e + D\left(\ddot{q}_d + \alpha\dot{e}\right) + C\dot{q} + G + M_{ev} + D_5\ddot{q}_{d,5} + C_5\dot{q}_{d,5}$, $\Phi'_d = D(q_d)\ddot{q}_d + C(q_d, \dot{q}_d)\dot{q}_d + G(q_d) + M_{ev}(q_d, \dot{q}_d) + D_5(q_d)\ddot{q}_{d,5} + C_5(q_d, \dot{q}_d)\dot{q}_{d,5}$. According to (A1), (A2), $\|\Phi' - \Phi'_d\| \le \rho'(\|y_{er}\|)\|y_{er}\|$, $\left\|\Phi'_d + W - \widetilde{B}_2\right\| \le \Psi'$ and $\|Q_F F_x\| \le \bar{Q}_F\bar{F}_x$, where $y_{er} = (e^T, r^T)^T$, $\Psi' \in \mathbb{R}_{>0}$ and

$\rho'(\|y_{er}\|)$ is real, positive, globally invertible, nondecreasing. To facilitate the stability analysis, a Lyapunov function candidate is chosen as

$$V_{B_2} = \frac{1}{2}e^T e + \frac{1}{2}r^T Dr. \quad (26)$$

Theorem 3. *Lyapunov function (26) of system (25) converges exponentially from any initial value, provided control law* $v = K_m^{-1}\Big(\frac{r}{\|r\|}\left(\rho'(\|y_{er}\|)\|y_{er}\| + \Psi' + \bar{Q}_F\bar{F}_x\right) + K_v r\Big)$, *where control gain* $K_v \in \mathbb{R}_{>0}$ *is a constant.*

Proof: Take time derivative of V_{B_2}, substitute the error dynamics (25) into the equation with input designed according to Theorem 3 and finally use property (18),

$$\begin{aligned}\dot{V}_{B_2} =& e^T(r-\alpha e) + \frac{1}{2}r^T\dot{D}r + r^T\Big(-\frac{1}{2}\dot{D}r - e - K_m v \\ &+ \Phi' - \Phi'_d + \Phi'_d - \widetilde{B}_2 - Q_F F_x + W\Big) \\ \le& -\alpha\|e\|^2 + \|r\|\left(\rho'(\|y_{er}\|)\|y_{er}\| + \Psi' + \bar{Q}_F\bar{F}_x\right) \\ &- r^T\Big(\frac{r}{\|r\|}\left(\rho'(\|y_{er}\|)\|y_{er}\| + \Psi' + \bar{Q}_F\bar{F}_x\right) + K_v r\Big) \\ \le& -\alpha\|e\|^2 - K_v\|r\|^2 \\ \le& -\frac{\lambda_4}{\lambda_2}V_{B_2},\end{aligned}$$

where $\lambda_4 = \min\{\alpha, K_v\}$. With initial value $V_{B_{2_0}}$, V_{B_2} can be solved as

$$V_{B_2} \le V_{B_{2_0}} e^{-\frac{\lambda_4}{\lambda_2}t} \quad (27)$$

■

IV. Stability Analysis of the Hybrid Dynamical System

Considering (13), (14) with controllers designed according to Theorem 2,3, all the states vary continuously within flow set $\mathcal{C}$ and r immediately jumps with $\xi \in \{-1, 1\}$ changing its sign when reaching jump set $\mathcal{D}$. Zeno behavior never occurs because every jump immediately drives system states from $\mathcal{D}$ to the interior of $\mathcal{C}$ with finite nonzero distance to any boundary points, so that there must exist a lower bounded time duration that system states will spend flowing back to $\mathcal{D}$ for next jump. Therefore, state vector $y(t) = (e^T, r^T, \mu^T, \xi)^T$ behaves in a piecewise continuous manner affected by ξ. Instead of analyzing system behavior around some equilibrium points, stability of sets is motivated when exploring hybrid dynamical systems [8]. In this section, design of the switching logic (12) will be completed and it can be shown that state vector of (13) converges to a set in finite time.

Firstly, a Lyapunov function candidate for hybrid dynamical system [8] is designed as

$$V = \frac{1}{2}e^T e + \frac{1}{2}r^T Dr + \frac{1+\xi}{2}\sum_{i=1}^{4}\left(P_{1,i} + P_{2,i}\right), \quad (28)$$

where all notations preserve their definitions. To describe the piecewise switching behavior, define sequence $sw = \{sw_j\}_{j=1}^{j_m}$ along state vector y, where $j_m \in \mathbb{Z}_{>0}$, $sw_j = (t_j, \xi_j, y_j, V_j, V_{0,j}, V_{e,j}) \in \mathbb{R}_{\ge 0}\times\{-1,1\}\times\mathcal{U}\times\mathbb{R}_{\ge 0}\times\mathbb{R}_{\ge 0}\times\mathbb{R}_{\ge 0}$. j refers to the piece between $(j-1)th$ and jth jump. $t_j \in [t_{e,j-1}, t_{e,j}]$, $j \in \mathbb{Z}_{>0}$, $t_{e,0} = 0$. $t_{e,j-1}, t_{e,j}$ are time instants when jump occurs and $y_j = (e_j^T, r_j^T, \mu_j^T, \xi_j)^T = y(t_j), V_j = V(t_j), V_{0,j} = V(t_{e,j-1}, \xi_j), V_{e,j} = V(t_{e,j}, \xi_j)$. Without loss of generality, y is assumed initially inside flow set $\mathcal{C}$ with $\xi = 1$ so that $\xi_j = (-1)^{j+1}$. Secondly, basing on (24), choose ultimate bound of V_{B_1} as $\Omega_u = \delta_2 + \frac{\lambda_2\left(\Psi + \bar{Q}_F\bar{F}_b\right)^2}{4\lambda_3 K_2^*}$ (δ_2 arbitrarily small constant) and further design $\mathcal{D}_{er\xi}$, $\mathcal{D}_{er_j}$, $\mathcal{D}_{f\xi}$ in (12) as

$$\begin{aligned}\mathcal{D}_{er\xi} &= \Big\{(e^T, r^T, \mu^T, \xi)^T \in \mathcal{U} : \xi = 1, (e^T, r^T)^T \in \Omega_0'\Big\}, \\ \mathcal{D}_{er_j} &= \Big\{(e_j^T, r_j^T, \mu_j^T, \xi_j)^T \in \mathcal{U} : (e_j^T, r_j^T)^T \in \Omega_0 \cap \Omega_{1_j}\Big\}, \\ \mathcal{D}_{f\xi} &= \Big\{(e^T, r^T, \mu^T, \xi)^T \in \mathcal{U} : \xi = 1, \forall K_B, \text{s.t.} f \le \delta_3\Big\}, \\ \mathcal{D}_f &= \Big\{(e^T, r^T, \mu^T, \xi)^T \in \mathcal{U} : \exists K_B, \text{s.t.} f \ge \delta_4\Big\}. \quad (29)\end{aligned}$$

$\Omega_0' = \big\{(e^T, r^T)^T : e, r \in \mathbb{R}^4, \lambda_2\|(e^T, r^T)^T\|^2 + \sum_{i=1}^{4}\left(P_{1,i} + P_{2,i}\right) \ge \lambda_1 \min_{i=1\ldots4}\{1, \frac{\bar{\sigma}_3\beta\varepsilon^2}{2\tau_i^2 K_u\lambda_1}\}\rho^{-2}\left(2\sqrt{K_1^*\chi}\right) - \frac{\lambda_2\left(\Psi + \bar{Q}_F\bar{F}_x\right)^2}{4\lambda_3 K_2^*} - \delta_1'\big\}$, $\delta_1' \in \mathbb{R}_{>0}$, $\delta_1' < \delta_1$. δ_1, Ω_0 is given in Theorem 2 and $\Omega_{1_j} = \big\{(e_j^T, r_j^T)^T : e_j, r_j \in \mathbb{R}^4, \frac{1}{2}e_j^T e_j + \frac{1}{2}r_j^T Dr_j \le \max\{\Omega_u, V_{e,j-1} - \delta_5\}, V_{e,0} = V_{0,1}\big\}$. f is given in Theorem 2. δ_3, δ_4 are constants and $0 < \delta_3 < \delta_4$. Constant $\delta_5 \in \mathbb{R}_{>0}$ can be chosen arbitrarily small. Finally, stability results of (13) are stated in Theorem 4.

Theorem 4. *Provided* $y(t) = (e^T, r^T, \mu^T, \xi)^T$ *is well defined on* $\mathcal{U}$, $\mathcal{D}_{er\xi}$, $\mathcal{D}_{er_j}$, $\mathcal{D}_{f\xi}$ *and* $\mathcal{D}_f$ *in (12) are designed as (29), controllers are designed according to Theorem 2,3, then tracking error* $y_{er} = (e^T, r^T)^T$ *of (13) is globally uniformly ultimately bounded, i.e.,* $y \in \mathcal{U}$ *converges from any initial value to a set* $\Omega_{\mathcal{H}} = \big\{(e^T, r^T, \mu^T, \xi)^T \in \mathcal{U} : \|(e^T, r^T)^T\| \le \Omega_{er}\big\}$ *in finite time, where constant* $\Omega_{er} \in \mathbb{R}_{>0}$ *is given by subsequent analysis.*

Proof: Considering (23), (24), (26), (27), (28) and definition of sw_j, when conditions in Theorem 4 are satisfied, following properties can be obtained.

$$V_j \le V_{0,j}e^{-\varphi_j t_j} + S_j\left(1 - e^{-\varphi_j t_j}\right), \quad (30)$$

$$V_{e,2k} = V_{0,2k+1},\ k \in \mathbb{Z}_{>0}, \quad (31)$$

$$|V_{0,2k} - V_{e,2k-1}| = \Delta|_{t=t_{e,2k-1}} \in \mathcal{L}_\infty,\ k \in \mathbb{Z}_{>0}, \quad (32)$$

where $\varphi_j, S_j \in \mathbb{R}_{>0}$ are determined by (24), (27) and $S_{2k} = 0$, $S_{2k-1} = \frac{\lambda_2\left(\Psi + \bar{Q}_F\bar{F}_b\right)^2}{4\lambda_3 K_2^*}$, $k \in \mathbb{Z}_{>0}$. (31) holds because $P_{1,i} = P_{2,i} = e_{c,i} = 0$, $i = 1 \ldots 4$ at time $t_{e,2k}$ due to assumption A3. $\Delta = \big|\sum_{i=1}^{4}\left(P_{1,i} + P_{2,i}\right) + \frac{1}{2}\left(\dot{e} + \alpha e - \beta e_c\right)^T D\left(\dot{e} + \alpha e - \beta e_c\right) - \frac{1}{2}\left(\dot{e} + \alpha e\right)^T D\left(\dot{e} + \alpha e\right)\big|$ and $(\cdot) \in \mathcal{L}_\infty$ denotes boundedness. Due to fatigue dependent $\mathcal{C}$, $\mathcal{D}$ and the assumption that y initially flows in $\mathcal{C}$ with $\xi = 1$, as well as (30), (31) and (32), for any finite j, $V_{0,j}$ and $t_{e,j} - t_{e,j-1}$ are also finite. Hence, two cases are discussed:

(i) When $j_m \to \infty$ as $t \to \infty$, consider subsequence $\{sw_{2k-1}\}_{k=1}^{\infty}$ and $\{sw_{2k}\}_{k=1}^{\infty}$, $k \in \mathbb{Z}_{>0}$. Due to (31) and Ω_1

in (29), forcing $V_{0,2k+1} = V_{e,2k} \le \max\{\Omega_u, V_{e,2k-1} - \delta_5\}$, there must exist some finite integer $k^* > \frac{V_{e,1}-\Omega_u}{\delta_5}$, such that $\forall k \ge k^*$, $V_{0,2k+1} \le \Omega_u$. Besides, because of boundedness property at odd jump indices given by (32) and monotonic behavior of V_j given by (30) during flow, it can be obtained that $\forall k \ge k^*$, $V_{0,2k+2} \le \Delta|_{t=t_{e,2k+1}} + V_{e,2k+1} \le \Delta|_{t=t_{e,2k+1}} + V_{0,2k+1} \le \max_{\Omega_V}\{\Delta\} + \Omega_u$, where $\Omega_V = \{(e^T, r^T)^T : e, r \in \mathbb{R}^4, \frac{1}{2}e^T e + \frac{1}{2}r^T D r + \sum_{i=1}^{4}(P_{1,i} + P_{2,i}) \le \Omega_u\}$. Therefore, $\forall j \ge 2k^*+1$, $V_{0,j} \le \max_{\Omega_V}\{\Delta\}+\Omega_u$.Using (30), it can be obtained that $V_j \le (V_{0,j}-S_j)e^{-\varphi_j t_j}+S_j \le \max\{V_{0,j}, S_j\} \le \max_{\Omega_V}\{\Delta\}+\Omega_u$. This means that $\forall t \ge \sum_{j=1}^{2k^*}(t_{e,j}-t_{e,j-1})$ tracking error $\|y_{er}(t)\| \le \sqrt{V/\lambda_1} \le \sqrt{(\max_{\Omega_V}\{\Delta\} + \Omega_u)/\lambda_1}$ and uniform ultimate boundedness result is therefore guaranteed.

(ii) When j_m is finite as $t \to \infty$, the proof is trivial because there is no switch after time instant t_{e,j_m-1}. Therefore, $\forall t \ge \sum_{j=1}^{j_m-1}(t_{e,j} - t_{e,j-1})$, V decays continuously according to $V(t) \le V_{0,j_m}e^{-\varphi_{j_m}t} + S_{j_m}(1 - e^{-\varphi_{j_m}t})$ so that $\|y_{er}(t)\|$ has a uniform ultimate bound. ∎

To sum up, (13) under designed controllers and switching logic yields global uniform ultimate boundedness. It should be noted that assuming y initially inside $\mathcal{C}$ with $\xi = 1$ does not reduce generality because according to $\mathcal{F}$, $\mathcal{G}$, $\mathcal{C}$ and $\mathcal{D}$, y of any other initial conditions will enter $\mathcal{C}$ with $\xi = 1$ within finite time and jumps and exactly same procedure of proof can be applied after that.

V. Simulation Results

In this section, simulation is conducted in a standing scenario, where system is reduced to 3 DOF by assuming both legs have exactly the same behavior. As a result, q_2, q_4 of the swing leg are removed and q_1, q_3 will represent for position angles of both stance legs. Torso position angle q_5 is controlled by human. Results illustrated in Fig. **2** show that the standing posture described by q_1, q_3 are regulated from $q_1 = 185°$, $q_3 = -10°$ to $q_1 = 190°$, $q_3 = -30°$ by using FES with 70 ms EMD at knee joints, motors at knee and hip joints. FES and motors collaborate according to switching logic and designed control laws, which not only guarantee stability of the hybrid dynamical system, but also allow muscle fatigue states to decrease and recover between desired values $\underline{\mu} = 0.6$ and $\overline{\mu} = 0.85$.

VI. Conclusion

In this paper, a multi-DOF hybrid neuroprosthesis, integrating a modified PD-based robust controller and a VSC to control FES and motors, is developed to address problems associated with muscle fatigue, EMD and model uncertainty. The hybrid dynamical system model is formulated to design switching logic and analyze stability. Globally uniformly ultimately bounded tracking is proven, with demonstration in the simulation to accomplish a standing task. Future studies will include optimal trajectory planning, input saturation and output constraint problems, so that simulations and experiments can be conducted in more complicated scenario.

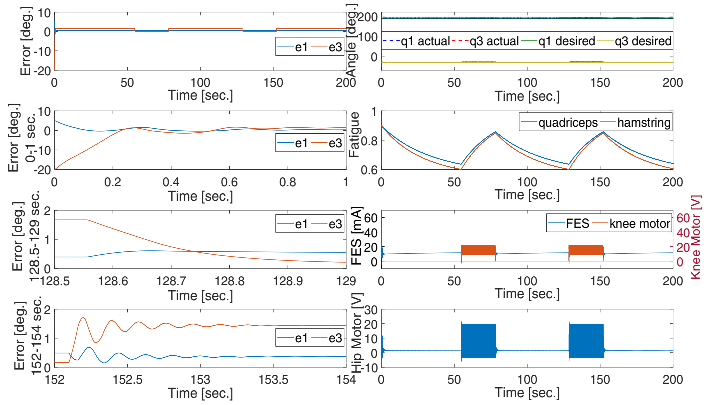

Figure 2. Simulation result in a standing scenario

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