

## Comparative evaluation of resilience metrics for water distribution systems using a pressure driven demand-based reliability approach

Abdulrahman A. Bin Mahmoud and Kalyan R. Piratla

### ABSTRACT

Water distribution systems (WDSs) are vital to human survival and the economic prosperity of communities across the globe. The deteriorating infrastructure issues combined with rising number of main breaks is pushing water utilities to keep up with the growing supply reliability challenges. Given the complexity associated with quantifying supply reliability, both of conceptual and computational nature, several surrogate measures which are referred to as resilience metrics were developed and used in the past. This paper presents a comparative evaluation of five such resilience metrics using the minimum cut set reliability approach supported by pressure-driven demand analyses of WDSs. Estimated reliability measures of WDS design solutions obtained using resilience metrics as co-objectives along with cost form the basis for the comparative evaluation presented in this paper. Three benchmark WDSs of different configurations and sizes are used in this study. The results suggest that the network resilience index performed best in the low cost range for smaller networks while the newly proposed probabilistic resilience index performed best in the low to moderate cost range for all networks. The identification of most competent resilience metric will support optimal design and rehabilitation decision making for water distribution systems in a computationally efficient manner.

**Key words** | pressure-driven demand analyses, resilience metrics, water distribution systems

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### INTRODUCTION

Water supply infrastructure plays a crucial role in delivering treated water to residential and industrial consumers thereby enabling healthy living and economic prosperity of communities. Much of this infrastructure in the United States has become old and deteriorated, resulting in not only an increasing number of water main breaks but also an unacceptable amount of leakage (ASCE 2017). As a result, water utilities are increasingly concerned about supply reliability goals and are interested in reliable design alternatives, which would ensure acceptable performance in the face of numerous infrastructure failure contingencies. It is well known that quantifying water supply reliability is a computationally challenging task,

for it requires simulating the water distribution system (WDS) performance in numerous failure contingency scenarios (Al-Zahrani & Syed 2005). Characterizing the WDS performance in a failure scenario is by itself a challenging task due to the lack of readily available pressure-driven demand (PDD)-based network solvers that accurately account for the pressure dependent flow relationship, which is especially useful in pressure-deficient situations. Complicating this challenge further, simulating a large number of possible failure contingencies to estimate the WDS reliability is a time consuming process. Furthermore, reliability assessment also requires that probabilities of various failure contingencies be estimated, but

water utilities often lack quality data to produce these probability estimates.

Previous researchers proposed resilience metrics as surrogate measures of reliability for WDSs. In many such cases, resilience has been characterized as a derivate of the prevailing energy redundancy in the WDS. The rationale is that WDSs lose energy in failure events (i.e. higher demands or component failures) and that any buffer energy available beyond the minimum required would compensate the failure-related energy losses. Previous studies demonstrated the use of these resilience metrics as co-objectives along with cost in the optimal design of WDSs (Tanyimboh & Templeman 2000; Todini 2000; Prasad & Park 2004; Jayaram & Srinivasan 2008). A few studies also comparatively evaluated the resilience metrics (Raad *et al.* 2010; Baños *et al.* 2011; Greco *et al.* 2012; Piratla & Ariaratnam 2013; Creaco *et al.* 2014), but the hydraulic simulation approaches employed in some of those studies have certain limitations that may have affected their findings. Attempting

to address these limitations, this paper presents a new resilience metric and furthermore comparatively evaluates all five resilience metrics using a more accurate non-iterative PDD-based minimum cut-set reliability approach. The identification of most competent resilience metric will support optimal design and rehabilitation decision making for water distribution systems in a computationally efficient manner.

## RESILIENCE METRICS FOR THE DESIGN OF WDSS AND THEIR PRIOR EVALUATION

Several researchers proposed metrics of resilience specifically for WDSs. This study evaluates five WDS resilience metrics which are presented in Table 1. While the first four metrics have been previously studied, the fifth one (i.e. probabilistic resilience index or PRI) is a newly proposed metric in this study. Many of the resilience metrics

**Table 1** | Description of the five resilience metrics studied

Resilience metric	Formulation
Flow Entropy (FE) (Tanyimboh & Templeman 1993)	$FE = \varepsilon_f = \varepsilon_R + \sum_{i=1}^n \frac{Q_i}{Q} \varepsilon_i$ $= - \sum_{k \in R} \frac{q_{R,k}}{Q} \ln \left( \frac{q_{R,k}}{Q} \right) - \frac{1}{Q} \sum_{i=1}^n Q_i \left[ \frac{d_i}{Q_i} \ln \left( \frac{d_i}{Q_i} \right) + \sum_{i \in N_j} \frac{q_{ij}}{Q_i} \ln \left( \frac{q_{ij}}{Q_i} \right) \right]$
Resilience Index (RI) (Todini 2000)	$RI = \frac{\sum_{j=1}^n q_j (ha_j - hr_j)}{(\sum_{r=1}^R Q_r H_r + \sum_{b=1}^B P_b) - \sum_{j=1}^n q_j hr_j}$
Network Resilience Index (NRI) (Prasad & Park 2004)	$NRI = \frac{\sum_{j=1}^n c_j q_j (ha_j - hr_j)}{(\sum_{r=1}^R Q_r H_r + \sum_{b=1}^B P_b) - \sum_{j=1}^n q_j hr_j}$ $c_j = \frac{\sum_{l=1}^{np_j} D_{jl}}{np \cdot \max(D_{jl})}$
Modified Resilience Index (MRI) (Jayaram & Srinivasan 2008)	$MRI = \frac{\sum_{j=1}^n q_j (ha_j - hr_j)}{\sum_{j=1}^n q_j hr_j}$
Probabilistic Resilience Index (PRI)	$PRI = \frac{\sum_{j=1}^n \left( \left( \frac{\sum_{l=1}^{np_j} (1 - P_{f,l,j})}{np} \right) q_j (ha_j - hr_j) \right)}{(\sum_{r=1}^R Q_r H_r + \sum_{b=1}^B P_b) - \sum_{j=1}^n q_j hr_j}$

Parametric description: where  $n$  = number of demand nodes;  $q_j$  = demand at node  $j$ ;  $ha_j$  = head available at node  $j$ ;  $hr_j$  = minimum head required to meet constraints at node  $j$ ;  $R$  = number of reservoirs;  $Q_r$  = flow being supplied to the system by reservoir  $r$ ;  $H_r$  = head at reservoir  $r$ ; and  $P_b$  = power introduced in the system by pump  $b$ ;  $np_j$  = number of pipes connected to node  $j$ ;  $D_{jl}$  = diameter of pipe  $l$  connected to node  $j$ ; and  $P_{f,l,j}$  = probability of failure of pipe  $l$  connected to node  $j$ .

For flow entropy:  $\varepsilon_R$  denotes the entropy of the sources (all reservoirs, tanks or external source nodes  $k \in R$ ); where  $n$  is the number of nodes, where  $Q_i$  denotes the total flow reaching node  $i$  and  $\varepsilon_i$  denotes the entropy of node  $i$ , where  $Q$  is the sum of nodal demands, where  $q_{R,k}$  is the inflow from source  $k$ , where  $d_i$  is the demand at node  $i$ , where  $N_j$  denotes the set of all the nodes immediately upstream from and connected to node  $j$ , and  $q_{ij}$  is the flow in the pipe from node  $i$  to node  $j$ .

simply account for the buffer energy available for dissipation in the event of a failure, but do not account for how well connected the nodes with high demands are. How well connected those nodes are depends on the pipe sizes the nodes are connected to and, most importantly, how available those pipes are. Therefore, it would be useful to include the failure probabilities of pipelines in a modified resilience metric, which is proposed in this paper as the probabilistic resilience index (PRI). PRI is an extension of Todini's resilience index wherein it comprises a 'probabilistic nodal connectivity' parameter as a weight to the buffer energy available at each node.

Although some of the resilience metrics identified in the previous section were demonstrated in previous studies, there are only a few studies that attempted to comparatively evaluate their performance. Raad *et al.* (2010) was the first to conduct comparative analysis of resilience index (RI), network resilience index (NRI), flow entropy (FE), and a novel mixed reliability measure. A two-objective optimization algorithm was used to design three benchmark WDSs with cost and resilience measures as objectives, and the resulting solutions were comparatively analyzed for their ability to handle demand uncertainty and pipe failures. They used demand satisfaction as the performance measure and employed OOTEN library with EPANET to carry out pressure-deficient analyses. It was reported that the resilience index performed best in handling demand uncertainty, while network resilience and mixed reliability indices performed better in handling pipe failure contingencies. FE was reported to have the least performance overall.

Baños *et al.* (2011) comparatively evaluated RI, NRI and MRI using only hydraulic uncertainty contingencies. Design solutions obtained from the use of the three resilience indices along with cost as objectives were analyzed for their performance in various demand uncertainty scenarios. They used an objective method of classifying WDS performance in simulated contingencies as satisfactory or unsatisfactory based on just the pressure head values. Network resilience and modified resilience indices were reported to have performed better overall than the resilience index.

Greco *et al.* (2012) and Creaco *et al.* (2014) compared resilience and entropy metrics as indirect measures of network reliability. Greco *et al.* (2012) studied the effects on

network performance, caused by the failure of one or two links, for all the possible network configurations. Creaco *et al.* (2014) investigated the better metric between entropy and resilience indices for an indirect measure of reliability in WDS design. The demand satisfaction rate was used as a performance indicator representing reliability based on a pressure-driven simulation. Results showed that indices such as RI and NRI, which are based on the energy storage, represent a better estimate of reliability than the entropy (Creaco *et al.* 2014). Moreover, it was reported that entropy may not be a useful measure of the network's capability to perform post failure (Greco *et al.* 2012).

In conclusion, previous studies generally concurred that the NRI is a beneficial metric for assessing WDS resilience during its design. Furthermore, only a few previous studies used flow-pressure relationships iteratively to evaluate a more accurate pressure-deficient performance of WDSs (Ang & Jowitt 2006; Suribabu & Neelakantan 2011; Jinesh & Mohan 2012; Gorev & Kodzhesspirova 2013). This study adopts a non-iterative pressure-deficient WDS simulation model for assessing the performance of the resilient design solutions; this model has been proven to accurately represent the functioning of a real-world network in pressure-deficient situations such as those that follow a main break (Pacchin *et al.* 2017). Furthermore, a new resilience metric accounting for the probabilistic nodal connectivity, which characterizes robustness, in addition to buffer energy availability, is also evaluated in this study in comparison with the previously studied WDS resilience metrics.

## STUDY METHODOLOGY

Using three benchmark WDSs, optimal design solutions are first determined using resilience metrics along with cost as objectives. A multi-objective genetic algorithm tool called GANetXL (Savić *et al.* 2011) is used to design the WDSs by minimizing cost and maximizing resilience for the first two case studies, whereas optimization toolbox in MATLAB is used to design the third. The design problem is set up to determine optimal sizes for pipelines and pumps in the WDSs. Various combinations of mutation and crossover rates are used to maximize the chances of global optimality. A conventional reliability assessment approach, namely

minimum cut-set method, is subsequently used to evaluate the reliability of each Pareto-optimal design solution. Minimum cut-sets are ‘a set of system components (e.g. pipelines) which, when failed, cause system failure; and when system failure will not occur if any one of those components does not fail’ (Su *et al.* 1987).

Assuming that a failed pipe or a set of pipes can be isolated from the rest of the system, minimum cut-sets in this study are determined by simulating various combinations of pipeline failures based on a hydraulic simulation model. Conventionally, pressure head deficits estimated using hydraulic solvers such as EPANET 2.0 (Rossman 2000) were commonly used to determine whether a WDS performed satisfactorily or not in any simulated failure state. The problem with such an approach is that EPANET 2.0 by default does not accurately represent pressure-deficient system states. Addressing this limitation, several researchers used EPANET 2.0 to simulate pressure-deficient operating conditions through: (i) executing the algorithm repetitively by adjusting the input/output parameters until convergence is achieved; (ii) modifying the source code to cater for pressure-dependent outflows; or (iii) adding artificial elements, e.g. reservoirs, to the network. Several of those approaches are explicitly iterative where the model converges through multiple runs of EPANET. In this study, a recently proposed non-iterative pressure driven demand (PDD) simulation approach (Sayyed *et al.* 2015) is used in conjunction with EPANET 2.0 for the assessment of WDS performance in failure states. In this approach, emitters are used to simulate pressure-deficient nodal flows. The emitter discharge equation enables the nodal head-flow relationship to be varied to reflect the characteristics of any network (Pacchin *et al.* 2017). This approach addresses the limitations of the previous approaches such as lack of accuracy, high computational time, unsuitability to extended period simulation in the modeling of pressure-dependent nodal flows to better reflect the performance of the nodes with insufficient flow and pressure. The merits of this approach have been illustrated on multiple water distribution networks of different sizes in the literature, one of which is as large as 2465-pipes and the results suggest that the procedure is robust, reliable and fast enough for regular use (Sayyed *et al.* 2015). Shortage of supply at any of the WDS nodes indicates that the corresponding combination

of pipeline failures is a cut-set of the WDS. This procedure is repeated until all the combinations of pipe failures have been considered and subsequently all minimum cut sets of the system are determined. Following the procedures described in the literature, system reliability ( $R_s$ ) is estimated using the following equation (Al-Zahrani & Syed 2005):

$$R_s = 1 - \sum_{i=1}^M \left( \prod_{j=1}^{n_i} P_j \right) (1 - HA_i) \quad (1)$$

where  $HA_i$  is the network hydraulic availability when pipelines in cut-set  $i$  are failed and it is calculated using Equation (2);  $P_j$  is the probability of failure of pipeline  $j$  calculated using Equation (3);  $n_i$  is the number of pipelines in cut-set  $i$ ; and  $M$  is the number of minimum cut-sets of a WDS. It should be noted that the network hydraulic availability ( $HA_i$ ) is calculated (see Equation (2)) differently than in Al-Zahrani & Syed (2005) study. It is calculated as the ratio of summation of all nodal supplies to the summation of all nodal demands, as a way to appropriately account for the partially supplied nodal flows.

$$HA_i = \frac{\sum_{j=1}^n Q_{a,j}}{\sum_{j=1}^n Q_{r,j}} \quad (2)$$

where  $Q_a$  and  $Q_r$  are the actual supply and required flow at each node  $j$ .

The failure probability of a pipeline is calculated using Equation (3) (Goulter & Coals 1986; Su *et al.* 1987):

$$P_j = 1 - e^{-\beta_j L_j} \quad (3)$$

where  $\beta_j$  = prevailing break rate of link  $j$  (# of breaks/year/km);  $L_j$  = length of link  $j$  (km).

The pipeline break rate ( $\beta_j$ ) is assumed to depend on pipe diameter, and  $\beta_j$  values for pipe sizes ranging between 76.2 and 1,625.6 mm are adapted from the literature (Neelakantan *et al.* 2008) after appropriately extrapolating  $\beta_j$  values for diameters greater than 609.6 mm due to lack of data.

In the non-iterative PDD approach employed, a few WDS components are added at each demand node. Specifically, each demand node ( $n$ ) is connected to a dummy node ( $n_d$ ) using a flow control valve (FCV), and the dummy node

is in turn connected to an emitter using a check valve. Emitters are used to estimate the actual supplied flow at each demand node. The generalized equation for the flow at an emitter is (Rossman 2000):

$$q_j^{avl} = C_d (H_j^{avl} - H_j^{min})^\gamma; H_j^{avl} \geq H_j^{min} \quad (4)$$

where  $q_j^{avl}$  is the available flow at demand node  $j$ ,  $H_j^{avl}$  is the available head at demand node  $j$ ,  $H_j^{min}$  is the minimum head at demand node  $j$ ,  $C_d$  is the discharge coefficient and  $\gamma$  is an empirical exponent, both of which are calculated using the following equations (Sayyed *et al.* 2015):

$$C_d = \frac{q_j^{req}}{(H_j^{des} - H_j^{min})^\gamma} \quad (5)$$

$$\gamma = \frac{1}{n_j} \quad (6)$$

where  $H_j^{des}$  is the desired head at demand node  $j$ ,  $n_j$  is a coefficient; a value of 1.5 is used in this study based on recommendations in the literature (Sayyed *et al.* 2015). The base demand at the demand node is set to zero, while the valve setting for FCV is set to the base demand of the corresponding demand node. The elevations of the dummy node and emitter are made equal to the demand node and  $C_d$  is set as the emitter coefficient. Upon completing the hydraulic simulation using EPANET 2.0, actual supplied flow is obtained from the emitter whereas the residual pressure head is obtained from the demand node. More details on this non-iterative PDD approach can be found in Sayyed *et al.* (2015).

## DESIGN ANALYSES

The three benchmark WDSs used in this study, which are depicted in Figure 1(a) (WDS-I), Figure 1(b) (WDS-II), and Figure 1(c) (WDS-III), were originally used in the studies of Costa *et al.* (2000), Ozger & Mays (2003), and Lippai (2005) respectively. These WDSs of different configurations and sizes are deliberately chosen to provide variety in WDS configurations for the comparative evaluation of resilience metrics. The Pareto-optimal solution

fronts obtained from the design of the three WDS networks for all the five resilience metrics are illustrated in Figure 2(a)–(c). Figure 2 depicts solutions only in a curtailed and more practically feasible cost range while the actual cost ranges extends up to about \$28 million for WDS-I, \$16 million for WDS-II, and \$20 million for WDS-III. It can be observed from Figure 2(a)–(c) that FE did not produce many solutions beyond a certain cost range, thereby indicating that greater investment did not necessarily increase FE values. This could be because the FE value increases with pipeline flows and when those flows are uniform across the network. For a given set of nodal demands and pressure constraints, the pipe flows are expected to reach a certain level of uniformity across the network with increased pipe sizes, but would diminish with further increase in pipe sizes.

On the other hand, RI, NRI, MRI and PRI metrics produced greater resilient solutions with increased cost because they all reward surplus nodal pressures resulting from increased pipe sizes. The resulting Pareto-optimal design solutions are comparatively evaluated using the minimum cut-set reliability approach for meaningful analysis of their performances.

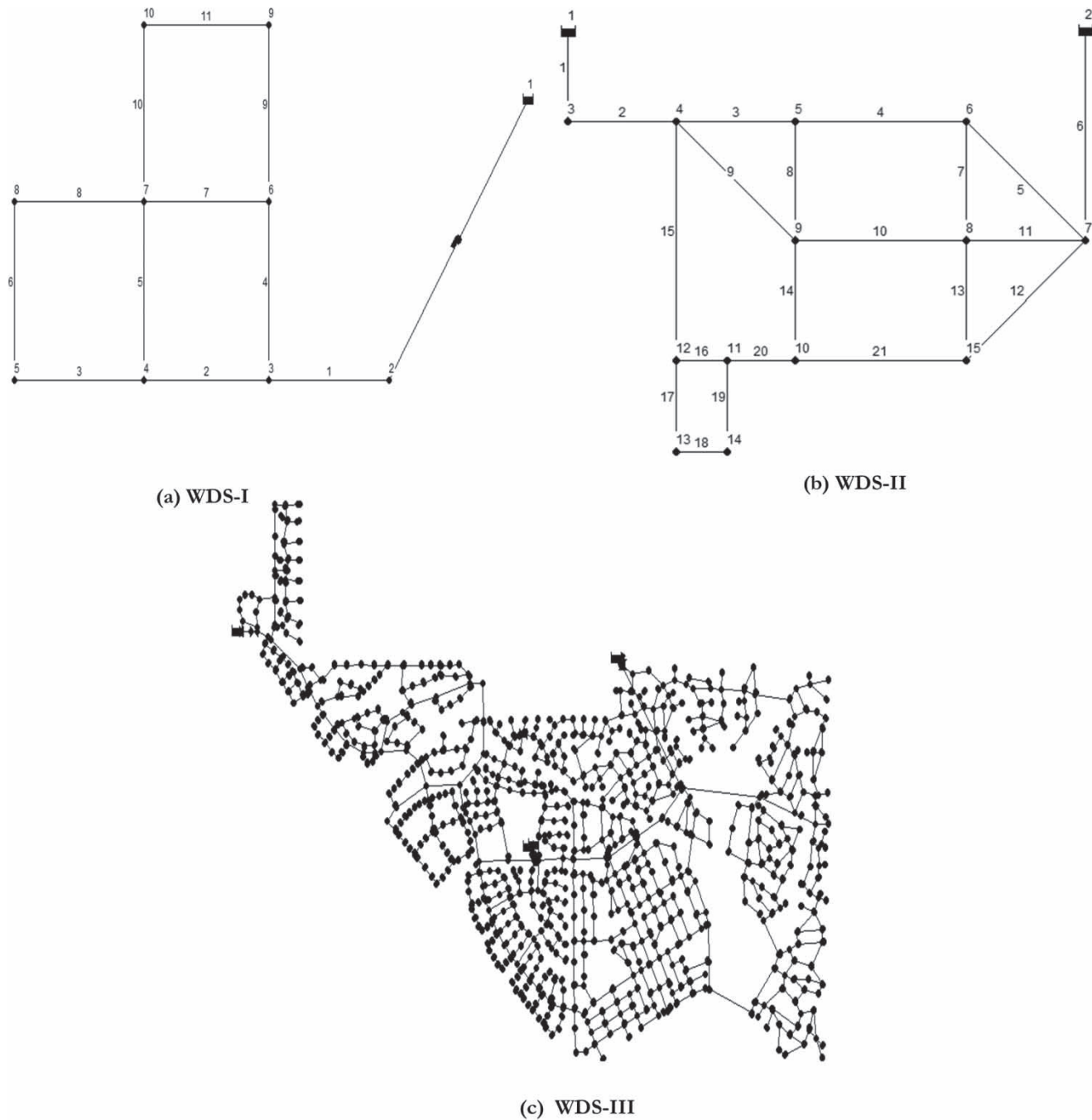
## COMPARATIVE ANALYSIS OF RESILIENCE METRICS

Reliability estimates for the Pareto-optimal design solutions are separately discussed for WDS-I, WDS-II, and WDS-III.

### WDS-I: pump-driven WDS

Figure 3(a) presents the comparison of reliability estimates for the Pareto-optimal solutions of the resilience metrics over the entire cost range. It can be observed from Figure 3(a) that RI and MRI have clearly performed poorly with relatively smaller reliability values compared to other metrics over most of the cost range. Figure 3(b) presents the comparison of reliability values in a more practically feasible cost range of less than \$10 million. It can also be observed from Figure 3(b) that NRI produced the best reliability values in the cost range of less than \$7.4 million, while PRI fared the best in the remainder of the cost range. FE's performance was comparable to

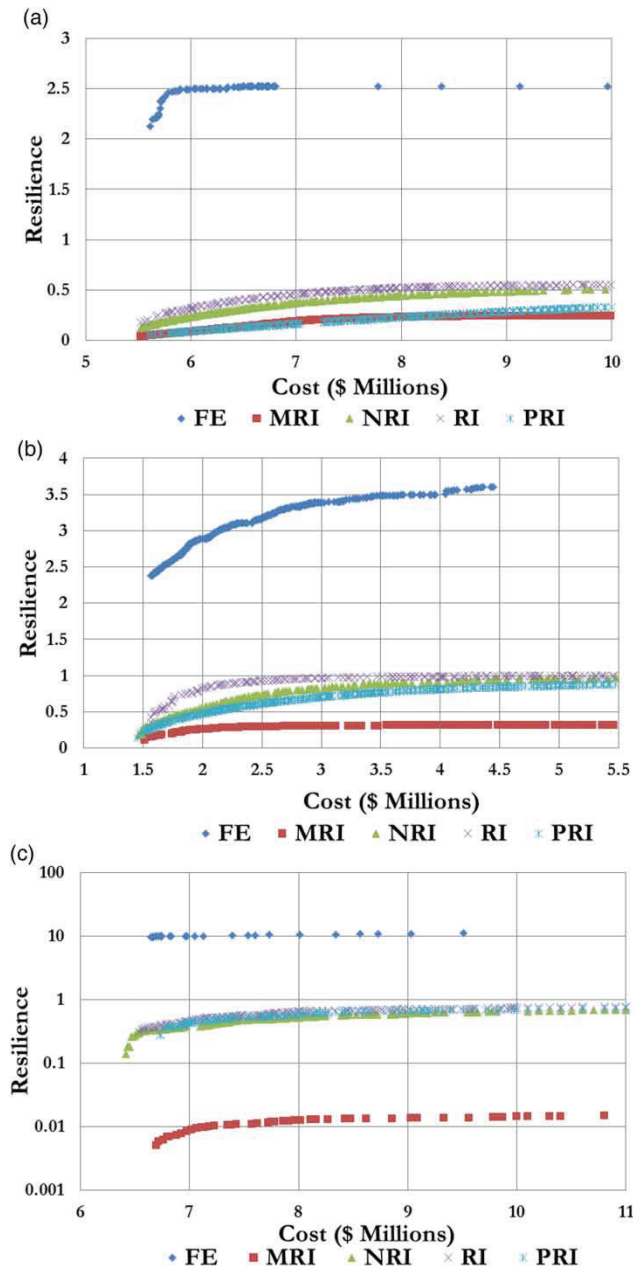




**Figure 1** | Layout of: (a) WDS-I (adapted from Costa *et al.* (2000)); (b) WDS-II (adapted from Ozger & Mays (2003)); and (c) WDS-III (adapted from Lippai (2005)).

NRI up until about \$6.2 million. In the high extreme of the cost range, all the resilience metrics produced similar solutions of large diameter pipelines and the corresponding reliability values are therefore similar and convergent, as can be observed from Figure 3(a). FE and

NRI metrics drive the WDS to have uniform flows and pipe sizes, respectively, and such uniformity seemed to have helped the WDS in handling mechanical WDS failures, especially in the low cost range with smaller diameter pipelines.



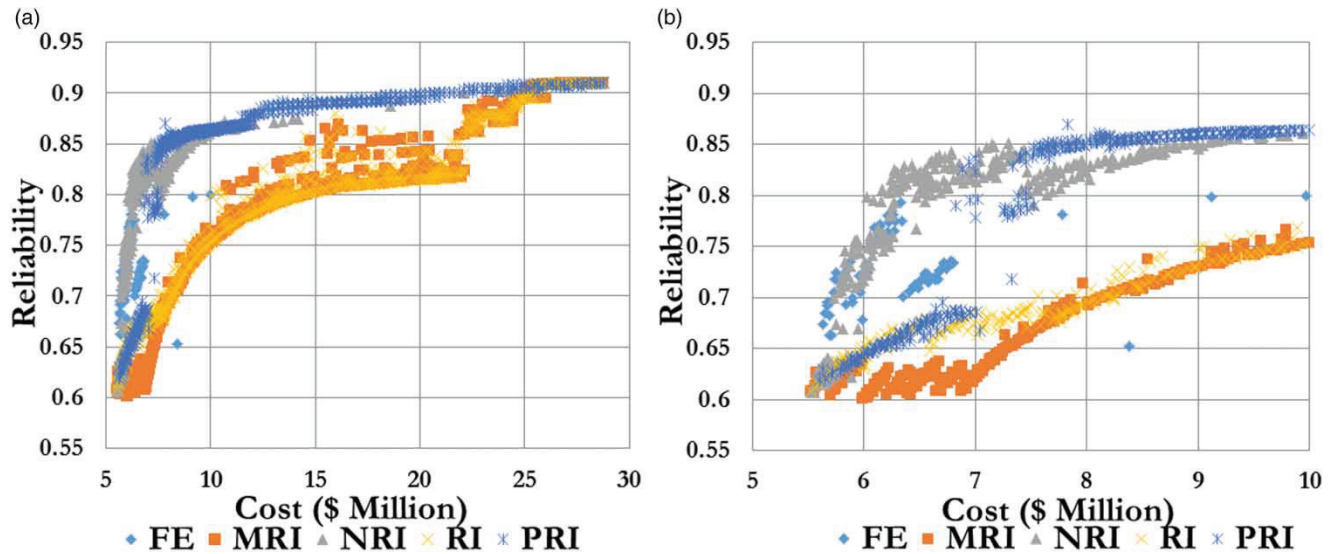
**Figure 2** | Optimal design solutions for: (a) WDS-I; (b) WDS-II; and (c) WDS-III.

The superior performance of PRI in the \$7.4–10 million cost range, as can be seen from Figure 3(b), can be attributed to its solutions having larger and uniform pipe diameters with smaller pump sizes compared to NRI which produced smaller and uniform pipe diameters with larger pump sizes. Table 2 presents a summary of NRI and PRI solutions in three cost ranges: (a)  $\leq 6.8$  million; (b) \$6.8–7.4 million; and (c) \$7.4–8.5 million. It can be seen from Table 2 that

all PRI solutions had smaller pump sizes of 4, 5 or 6, whereas NRI solutions had a wider range of pump sizes from 4 to 10. Also for comparable pump sizes, the average pipe diameters of PRI are greater than NRI, as expected. The choice of larger pipe diameters with PRI is likely due to the inclusion of a robustness parameter  $\left( \frac{\sum_{l=1}^{np} (1 - P_{f,l,j})}{np} \right)$  in its numerator (see Table 1), which penalizes smaller diameter pipes due to greater failure probability values. Furthermore, the average standard deviation of WDS pipe diameters for PRI solutions is much greater than that of NRI solutions in cost ranges (a) and (b), but comparable or lower in cost range (c). It can also be observed from Table 2 that NRI produced a greater number of solutions than PRI in cost ranges (a) and (b), but not in cost range (c). This could be because smaller pump sizes selected with PRI have enabled a greater choice of larger pipe sizes to enhance resilience with more investment, as opposed to NRI where additional investment went into the selection of larger pumps leaving a limited choice of smaller pipe sizes. It can be further noted from Table 2 that the standard deviation for PRI and NRI solutions has generally decreased with increased pump sizes, and therefore comparable standard deviation values for PRI and NRI solutions in cost range (c), despite PRI leading to smaller pump sizes, signifies more uniformity in its pipes' sizes. It is therefore reasonable to interpret that larger and more uniform pipe sizes led to the superior performance of PRI compared to NRI in the \$7.4–10 million cost range.

## WDS-II: two-reservoir WDS

Figure 4(a) illustrates the comparison of reliability values over the entire cost range, while Figure 4(b) presents the same comparison in a more practically feasible cost range of less than \$5 million. It can be observed from Figure 4 that PRI and NRI seemed to have consistently outperformed other metrics with PRI having better reliability values than NRI over most of the cost range, except for \$1.7–2.4 million. This variation in performances of PRI and NRI is further investigated using three smaller cost ranges: (a) \$0–1.7 million, (b) \$1.7–2.4 million, and (c) \$2.4–5 million. It can be observed from Figure 4(b) that



**Figure 3** | Reliability vs. cost tradeoff for WDS-I over: (a) entire cost range; and (b) smaller cost range.

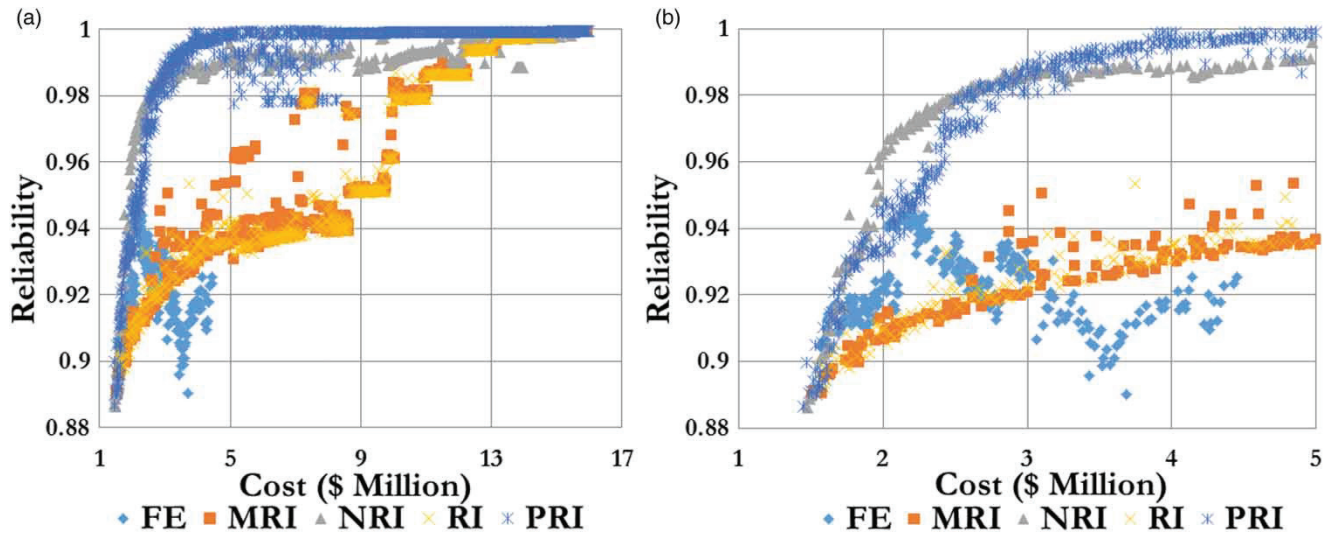
**Table 2** | Summary of PRI and NRI solutions for WDS-I in the cost range of  $\leq \$8.5$  million

Cost range	Resilience metric	Pump size	# of solutions	Avg. cost (\$)	Norm. mean pipe size (mm)	Avg. std. dev. of pipe diameters (mm)
(a) \$0–6.8 M	NRI	4	57	5.74 M	280.1	159.7
		5	78	6.3 M	316.3	142.7
		6	19	6.64 M	317.0	124.3
	PRI	4	82	6.21 M	313.4	186.3
		5	13	6.53 M	324.9	181.0
(b) \$6.8–7.4 M	NRI	6	13	6.99 M	344.2	144.7
		7	27	7.14 M	339.9	137.3
		8	5	7.16 M	329.7	118.8
	PRI	4	13	6.92vM	365.9	196.0
		5	20	7.16 M	373.4	195.6
(c) \$7.4–8.5 M	NRI	7	10	7.54 M	369.9	147.1
		8	20	7.72 M	369.9	148.7
		9	23	7.98 M	369.9	137.9
		10	14	8.35 M	380.7	139.5
	PRI	5	59	7.74 M	425.0	159.0
		6	30	8.28 M	440.2	138.4

the performance of PRI and NRI are comparable in cost range (a) while NRI performance is superior in cost range (b) and PRI performance is superior in cost range (c). The variation in the relative performance of NRI and PRI can be explained by the normalized (by length) pipe sizes and their uniformity in the solutions produced by these two metrics. Table 3 summarizes the NRI and PRI solutions produced in each of the cost ranges for WDS-II. It can be observed from Table 3 that the average standard

deviation in pipe sizes for PRI solutions is slightly greater than that of NRI solutions in cost range (a) while the average WDS normalized pipe sizes are comparable. On the other hand, average standard deviation for PRI solutions is considerably larger than that of NRI solutions in cost range (b) while the average WDS normalized pipe sizes are comparable. In cost range (c), the average standard deviation for PRI solutions is considerably smaller than that of NRI solutions and the normalized mean WDS





**Figure 4** | Reliability vs. cost tradeoff for WDS-II over: (a) entire cost range; and (b) smaller cost range.

**Table 3** | Summary of PRI and NRI solutions for WDS-II in the cost range of  $\leq \$5$  million

Cost range	Resilience metric	# of solutions	Avg. cost (\$)	Norm. mean pipe size (mm)	Avg. std. dev. of pipe sizes (mm)
(a) \$0–1.7 M	NRI	20	1.6 M	249.6	221.5
	PRI	34	1.59 M	246.1	229.7
(b) \$1.7–2.4 M	NRI	60	~2 M	309.8	230.4
	PRI	92	~2 M	307.3	250.5
(c) \$2.4–5 M	NRI	99	3.54 M	466.9	296.0
	PRI	215	3.51 M	479.4	270.5

pipe sizes are larger. It can therefore be inferred from these results that the metric producing more uniform (i.e. smaller standard deviation) and larger normalized pipe sizes tends to perform better in terms of reliability. Furthermore, the poor performance of MRI and RI can also be attributed to the high standard deviation in the pipe sizes obtained using these metrics compared to other metrics, as illustrated in Figure 5.

Interestingly, reliability values for FE have considerably diminished beyond about \$2.2 million of budget, as can be seen in Figure 4(b). It was observed that sizes of non-critical pipelines and other pipelines connected to nodes of lower demands have increased with greater investment in the case of FE metric and as a result the supply reliability has not improved for such design solutions. To demonstrate this fact, correlation between system cost and pipe sizes is investigated for three critical and three non-critical pipelines, as shown in Figure 6. The system cost vs. pipe size

correlation values for pipes 3 (closer to a reservoir and one of the longest), 6 (directly connected to the reservoir) and 7 (connecting two nodes with greatest demands), which can be classified as critical pipelines, are calculated to be  $-0.05$ ,  $-0.89$  and  $-0.13$ , respectively. On the other hand, correlation values for pipes 16, 17 and 20, which can be classified as non-critical pipelines because they connect nodes with very low to zero demands, are  $0.94$ ,  $0.95$  and  $0.93$ , respectively. In other words, sizes of non-critical pipes have increased with cost while those of critical pipelines have decreased. It is very likely that larger sizes of non-critical pipelines have not helped in increasing the reliability of FE solutions beyond a certain cost range.

### WDS-III: large WDS

Figure 7(a) presents the comparison of reliability values over the entire cost range, while Figure 7(b) presents the same

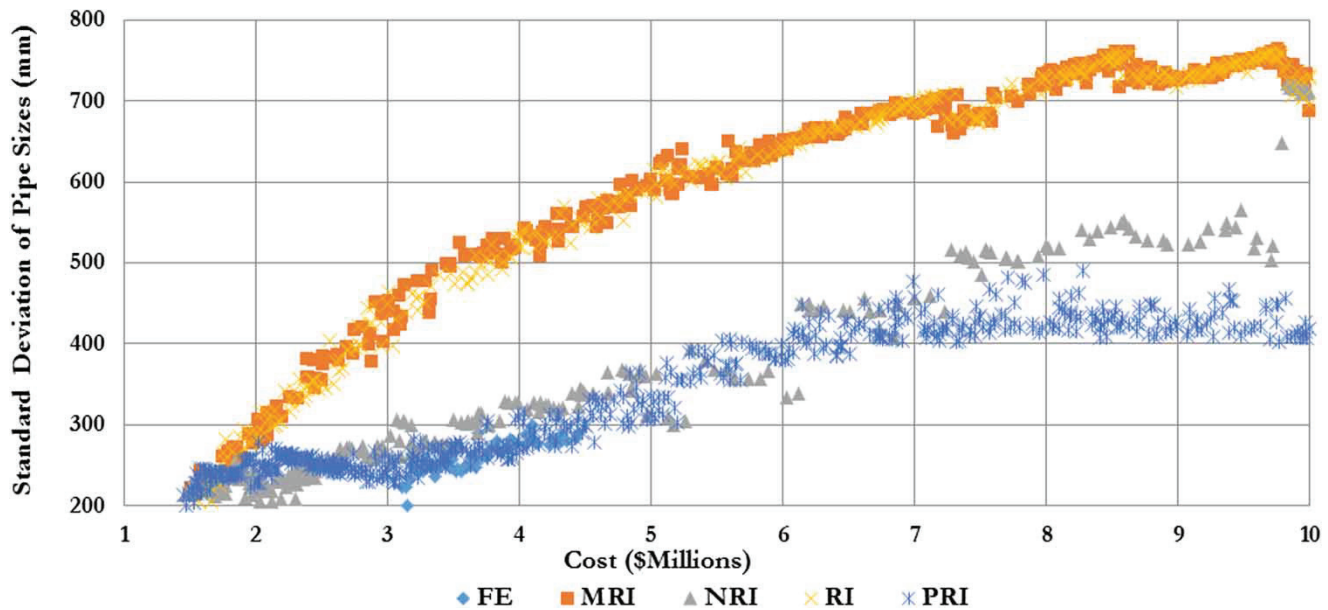


Figure 5 | Variation in standard deviation of pipe sizes with cost for design solutions of WDS-II.

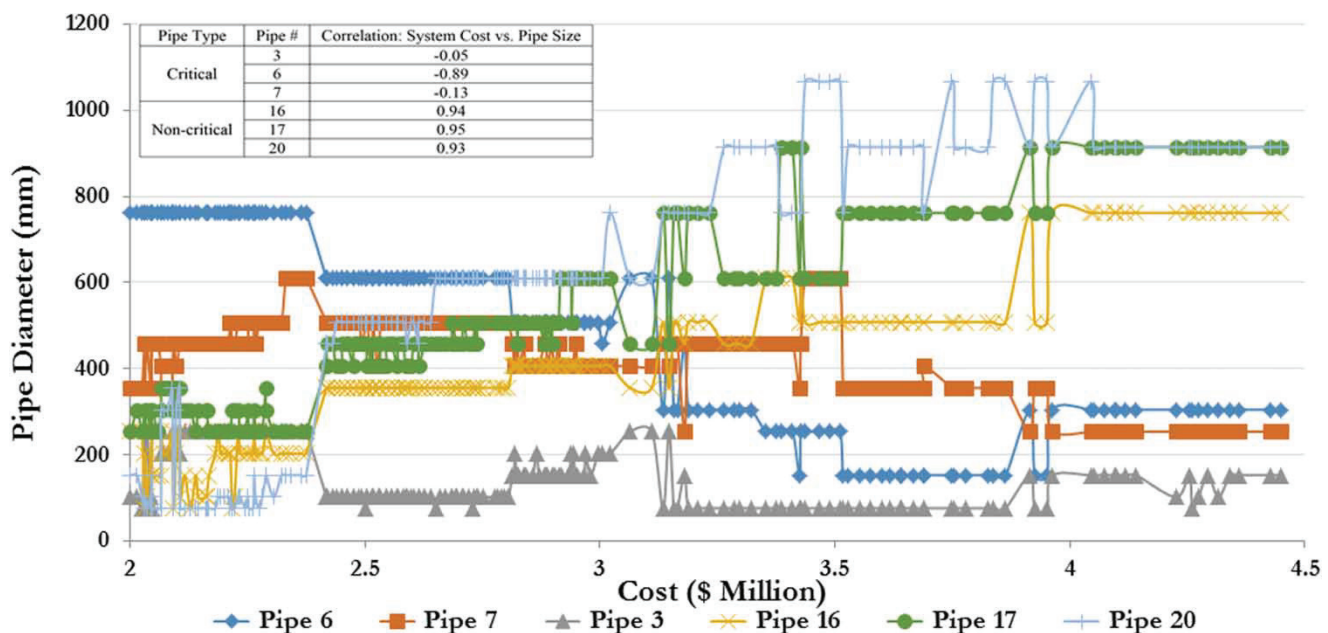
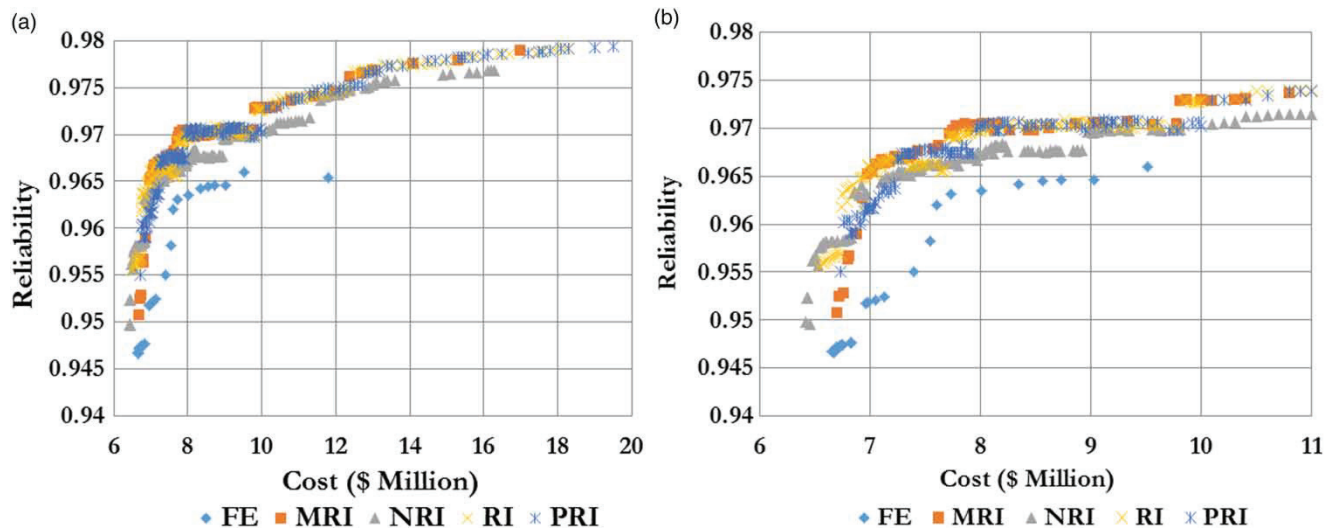


Figure 6 | Variation in pipe sizes of FE solutions with cost for WDS-II: three critical and three non-critical pipelines.

comparison in a more practically feasible cost range of less than \$11 million. It can be observed from Figure 7 that FE clearly performed worse than other metrics and that other metrics' performances are not clearly distinguishable. In contrast to the previous two WDSs, RI and MRI seemed

to have performed well in comparison with other metrics over most of the cost range, as can be seen from Figure 7. Furthermore, PRI has shown improvement and performed on a par with RI and MRI beyond \$8 million, but NRI has underperformed over the majority of the cost range. The



**Figure 7** | Reliability vs. cost tradeoff for WDS-III over: (a) entire cost range; and (b) smaller cost range.

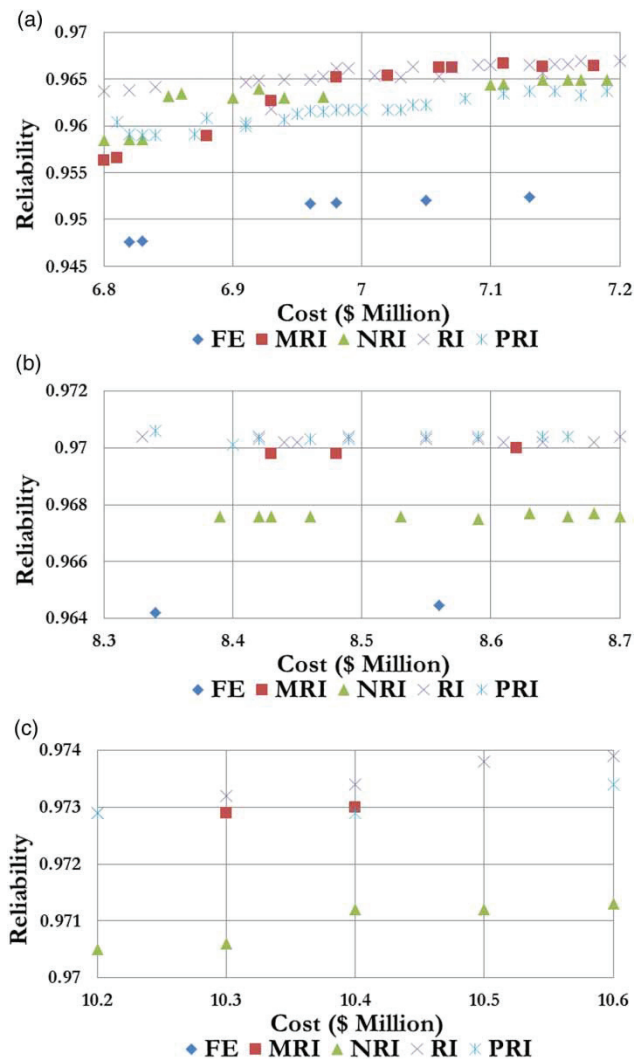
trend of PRI improving over the lower cost range and becoming the top performing metric in the medium cost range is consistent with the other two WDSs.

The variable performance of different resilience metrics in the case of WDS-III is further investigated using the following three smaller cost ranges: (a) \$6.8–7.2 million, (b) \$8.3–8.7 million, and (c) \$10.2–10.6 million. Table 4 presents a summary of all the solutions in each of these three cost ranges, including average pipe flows and average nodal pressure heads. Figure 8 presents the reliability comparison of solutions in each of the three smaller cost ranges

separately. Unlike in the cases of WDS I and II, it can be observed from Table 4 that there are no clear trends of correlation between reliability and normalized mean pipe diameter or standard deviation. It is interesting to note that both FE and NRI, which performed worse than other metrics, produced lower average nodal pressures, can be seen from Table 4. Subsequently, statistical correlation between reliability and average nodal pressure heads is determined using all the individual solutions and it was found to be as high as 0.92 in the case of WDS-III, whereas it was 0.54 and 0.58 for WDS-I and WDS-II, respectively. The lower

**Table 4** | Summary of resilience metrics solutions for WDS-III at three small ranges in the cost range of  $\leq \$11$  million

Cost range	Resilience metric	# of Solutions	Avg. cost (\$)	Avg. std. dev. of pipe sizes (mm)	Norm. mean pipe size (mm)	Avg. reliability	Avg. pipe flows (GPM)	Avg. nodal pressure head (m)
\$6.8–7.2 M	NRI	15	\$6,984,000	37.44	206.45	0.963	99.03	23.66
	PRI	25	\$6,985,200	38.15	206.20	0.961	99.28	24.42
	FE	6	\$6,961,667	38.68	206.15	0.951	110.43	19.46
	MRI	11	\$6,998,182	42.72	206.60	0.963	99.68	24.20
	RI	24	\$7,017,083	39.04	207.01	0.965	100.22	24.46
\$8.3–8.7 M	NRI	10	\$8,549,000	30.81	248.01	0.968	99.98	28.08
	PRI	9	\$8,505,556	37.90	245.14	0.970	100.8	29.33
	FE	2	\$8,450,000	39.80	244.12	0.964	123.70	24.11
	MRI	3	\$8,510,000	42.44	244.30	0.970	99.20	29.60
	RI	11	\$8,536,364	40.28	245.31	0.970	100.52	29.26
\$10.2–10.6 M	NRI	5	\$10,400,000	33.02	288.19	0.971	100.41	30.42
	PRI	3	\$10,400,000	39.42	286.44	0.973	101.40	30.99
	MRI	2	\$10,350,000	42.77	285.19	0.973	100.34	31.21
	RI	5	\$10,400,000	41.94	286.99	0.973	100.65	31.09



**Figure 8** | Reliability vs. cost tradeoff for WDS-III over three smaller cost ranges: (a) \$6.8–7.2 million; (b) \$8.3–8.7 million; and (c) \$10.2–10.6 million.

nodal pressures resulted with NRI solutions and led to its poor performance. The lower nodal pressures with NRI are likely due to the fact that NRI solutions resulted in the largest normalized mean pipe diameters and greater uniformity (i.e. lowest standard deviation) for cost ranges (b) and (c). Furthermore, it can be noticed from Table 4 that the mean pipe flow is considerably larger and mean nodal pressures are considerably smaller for FE's solutions in comparison with those of other metrics and therefore FE's poor performance can also be attributed to these facts. Moreover, at cost range (a) RI has outperformed other metrics with larger normalized mean pipe diameter and larger average nodal pressure

head. In cost ranges (b) and (c), RI, PRI, and MRI have more or less similar performance measures in terms of normalized mean pipe diameter and average nodal pressures.

Another noteworthy observation is that uniformity of pipe sizes has not made much difference in the case of WDS-III, which is a considerably larger network. Such large networks may actually benefit from diversity in pipe sizes, especially when the demands are low and very similar across the WDS. Overall, normalized mean pipe diameters and average nodal pressure heads proved to be crucial for WDS-III, whereas uniformity in pipe diameter did not seem significant.

Several similarities and differences are observed in the results of WDS-I, WDS-II and WDS-III. They are: (a) NRI and FE metrics have generally performed better than others in the very low cost range for WDS-I and WDS-II, while PRI performed better in the low to moderate cost range for all WDSs; (b) performance of FE did not necessarily improve with cost except in the very low cost range for WDS-I and WDS-II while it has performed worse over the entire cost range for WDS-III; (c) RI and MRI metrics performed poorly for WDS-I and WDS-II; (d) reliability values of WDS-II and WDS-III are generally high (i.e. in the range of 0.89–1) as opposed to those of WDS-I that range between 0.6 and 0.91, and this is likely due to the redundant layouts of WDS-II and WDS-III comprising multiple reservoirs and multiple loops; and (e) reliability values for WDS-I and WDS-II seemed sensitive to uniformity of pipe sizes but this is not true in the case of WDS-III, and as a result RI and MRI performed better for WDS-III than NRI unlike in the cases of WDS-I and II.

## CONCLUSIONS AND RECOMMENDATIONS

The comparative performance of five resilience metrics is evaluated in this study for their ability to produce reliable designs of water distribution systems (WDSs). NRI and PRI metrics performed better than others with NRI being more suitable for the very low cost range and PRI more suitable in the low to moderate cost range. However, NRI's performance in the large-scale WDS was not as high as it is in the smaller WDSs. FE also performed well in the very low cost range, but its performance declined with further investment



thereafter for WDS-I and WDS-II, it has the worst performance in the case of the large WDS-III. Larger normalized mean pipe sizes and uniformity in pipe sizes are two design features that are observed to have enabled superior reliability performance of the two smaller WDS-I and WDS-II. However, for the large network WDS-III, greater normalized mean pipe sizes and greater nodal pressures resulted in higher reliability values. The contributions of this study to the body of knowledge include: (a) formulation of the new PRI metric for the design of WDSs; (b) demonstration of a revised minimum cut-set reliability quantification approach where non-iterative PDD analysis is used to more appropriately estimate the pressure-deficient performance of WDSs; (c) evidence that NRI metric is more suitable for reliable design of smaller WDSs in the low cost range, while PRI should be preferred in the low to moderate cost range; and (d) RI or MRI are better metrics to use on large-scale networks as buffer nodal pressures seemed more crucial than uniformity of pipe sizes. The approach and findings presented in this paper will support optimal design and rehabilitation decision making for WDSs in a computationally efficient manner. One of the limitations of this study that may be addressed in the future is the exclusion of pump failure contingencies in the reliability assessment of WDSs.

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