

Comparative Study of Active Power Curtailment Methods of PVs for Preventing Overvoltage on Distribution Feeders

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Abstract—Overvoltage is one of the major issues on distribution grids with high penetration of photovoltaic (PV) generation. Overvoltage could be prevented through the control of active/reactive power of PVs. However, given the high R/X ratio of low voltage feeders, voltage control by using reactive power would not be as effective as using active power. Therefore, active power curtailment (APC) of PVs, though not desirable, becomes necessary at times to prevent the overvoltage issues. Existing literature is rich in centralized and droop-based methods for APC and/or reactive power control of PVs to prevent overvoltage issues. In this context, this paper revisits the most popular existing methods, and evaluates the performance of droop-based and centralized methods using a typical North American 240 V low voltage feeder with 24 residential homes. In this work, our key findings are: *a*) droop-based methods provided conservative solutions or did not eliminate the overvoltages completely, *b*) power flow sensitivity based droop approach led to 13% more curtailment than the centralized approaches, *c*) centralized approach had 40% less energy curtailed compared with standard droop while no overvoltages were observed, and *d*) operating PVs at non-unity power factor in centralized approach led to 5% less energy curtailment.

Index Terms—Photovoltaic, Power Curtailment, Distribution Grid, Overvoltage, Inverter, Voltage Control.

I. INTRODUCTION

Distributed solar photovoltaic (PV) systems are the fastest growing renewable energy resources being integrated onto distribution grids [1]. Significantly increasing penetration of rooftop PVs has raised a number of concerns to utilities. High PV penetration causes several operational challenges to the distribution grids including overvoltage [2] and power quality [3], [4]. The short circuit ratio of system has altered significantly requiring a review of protection coordination settings [5]. During high PV generation and low load periods, there could be reverse power flow that leads to voltage rise on the low voltage (LV) feeders [6]. Since PV inverters are set at little higher voltage to feed power back into the grid, the aggregate effect of this can lead to overvoltage issues [7]. Overvoltage is one of the main reasons for limiting the capacity of PV that can be connected to LV systems [8].

A recent study performed on a utility showed that hosting capacity of LV feeder is limited by overvoltage during an extreme condition of lowest load and maximum PV generation [7].

Overvoltage on distribution grids could be mitigated through careful consideration of PV control at planning stage [9]. At the operational stage, overvoltage could be mitigated through proper use of energy storage devices [10]. However, the penetration of distributed energy storage systems (ESSs) is not as high as the penetration of rooftop PVs on present distribution grids. Moreover, ESSs are usually expensive and the cost benefit ratio can make them less attractive [6]. The overvoltage due to PVs can also be controlled by controlling online load tap changers (OLTCs) on distribution feeders [11], [12]. However, this approach requires operations of OLTCs be coordinated with the control of PV inverters to prevent frequent switching of the OLTCs. In recent literature and industry practices, active power curtailment (APC) and/or reactive power control are becoming very common approaches for preventing overvoltage on distribution grids with large PV penetration.

The voltage to active power sensitivity, $\partial V/\partial P$, is much larger than voltage to reactive power sensitivity, $\partial V/\partial Q$, for distribution feeders as **R/X** ratio is large [6], [13]. Therefore, APC becomes more effective in controlling overvoltage than reactive power control [14]. The option of APC seems very attractive since it requires minor modifications on the PV inverters control logic [15]. However, from the economic point of view, reactive power control should be used first before APC [16], [17].

A rich literature exists for prevention of overvoltage on distribution system with PVs. The methods are mainly centralized or distributed types for APC and dispatch of reactive power. The centralized approaches solve optimal power flow (OPF) or its variants [18]–[21] to find the curtailment of active power and/or dispatch of reactive power from the PVs. The centralized approaches demand communication infrastructure. However, the distributed approaches are usually droop-based [22]–[24] and avoid the need of communication.

Given the increasing use of droop-based and centralized

approaches for APC and reactive power control to mitigate overvoltage issues on distribution feeder, the main droop-based and centralized methods are revisited and their performance is evaluated in this paper. The rest of the paper is organized as follows. Section II briefly presents the LV feeder used for the simulation. Section III describes the performance of droop-based methods, followed by the generic OPF based APC and reactive power control model for overvoltage prevention in Section IV. Section VI describes the main conclusions drawn from the paper.

II. SYSTEM DESCRIPTION

For analysis purpose, the 240 V lateral in [6] is modified. It is considered that the feeder is supplied by a 175 kVA, 14.4/0.24 kV transformer as shown in Fig. 1. The feeder supplies 24 houses (labelled node 1 through 24) through service drops. The impedances of lateral sections, i.e., branch 52-53 through 62-63, are assumed identical and equal to $0.0069+j0.0018\Omega$. Similarly, the impedances of the service drops, i.e., branch 52-1 through 63-24, are also assumed identical and equal to $0.011+j0.0017\Omega$. Transformer has an impedance of $0.0143+j0.0357\Omega$ referred to LV side. Each house has different daily load profiles, and the total consumption of the 24 houses for a typical summer day is shown in Fig. 2. The PV inverters are rated 8.4 kW, and in a sunny day total power output of 24 houses is also shown in Fig. 2. The PV output profile of all houses are assumed the same, as the houses are in close neighborhood.

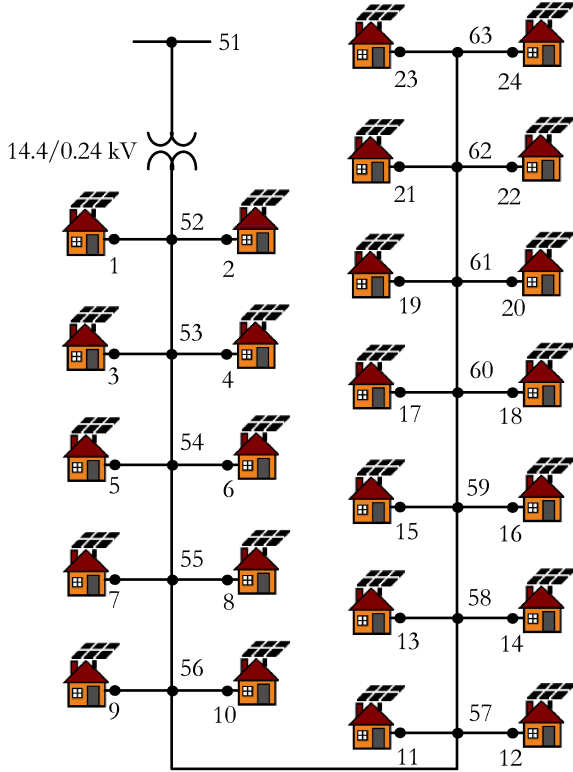


Fig. 1: 24 houses with PVs connected to 240 V network (line lengths not drawn to scale).

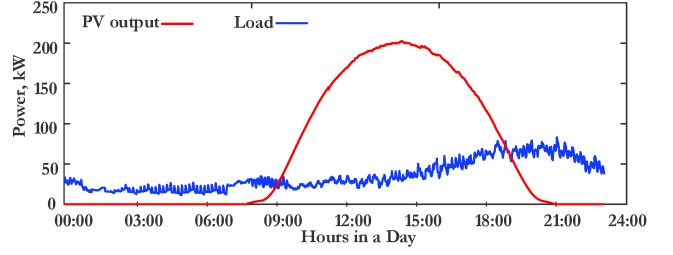


Fig. 2: Load profile and PV generation profile of 24 houses.

Fig. 3 shows the voltage profile at 24-houses, which are obtained by solving power flow using 1-minute resolution load and PV output profiles. Fig. 3 clearly shows during the day time, when the solar insolation is high, houses far away from the transformer experience overvoltage, i.e., voltages exceed extreme operation limit of 1.058 pu. All the houses except House-1 through -4 reach the limits of 1.042 pu for normal operation [6].

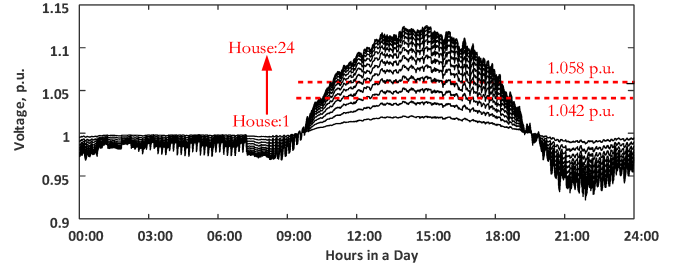


Fig. 3: Voltage profile in a typical day in Summer. House-1 to House-4 do not experience overvoltage issues, while House-5 to House-24 experience voltage over 1.058 pu during day time when solar insolation is high.

III. DROOP-BASED METHODS

A. Uniform Droop

Droop-based method is becoming popular for active power curtailment due to its simplicity. However, selection of droop parameter impacts the amount of energy curtailed and the voltage profile. A higher droop value may ensure feasible voltage for all loading and PV output profiles, but it leads to higher APC; whereas, a small droop value may not be able to ensure feasible voltage profiles at all the times.

We follow the similar procedures as in [6] to find one droop value for all 24 houses in Fig. 1. Then, we use the following relation to find the amount of power curtailed.

$$p^{cur} = K^D (V - V^{cr}) \quad \text{for } V > V^{cr} \quad (1)$$

Then, the following limits are applied to ensure curtailment does not exceed PV generation.

$$p^{cur} = p^{PV} \quad \text{if } p^{cur} > p^{PV} \quad (2)$$

In the above formulations, K^D represents droop parameter, p^{cur} represents active power curtailed, p^{PV} represents PV

generation, V represents nodal voltage, and V^{cr} represents voltage beyond which APC becomes effective. To find K^D (in kW/pu), a series of power flow are run with net injection varied from 0 to 8.4 kW in each node with PVs. Then looking at the extreme node, i.e., the House-24, and using the slopes of the $V(P^{PV})$ plots, the droop parameter K^D is calculated. Fig. 4 (black solid line) shows the voltage response of House-24 when net PV injection is varied.

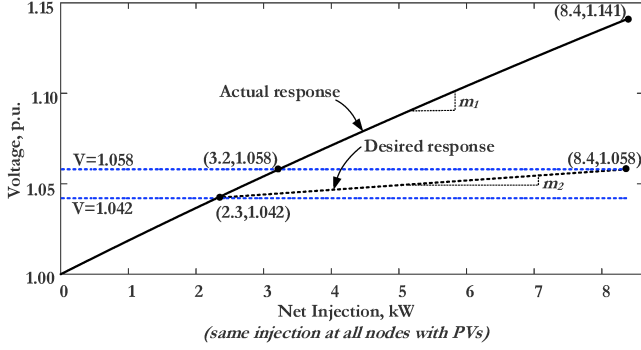


Fig. 4: Droop calculation using the voltage profile of the most distant house (from service transformer).

The desired voltage response after curtailment is also shown in Fig. 4 (black dotted line). Active power of PVs begin to curtail when the node voltage exceeds V^{cr} (which is 1.042 pu). At the peak PV injection, i.e., at 8.4 kW, nodal voltage should be below the extreme operation limit of 1.058 pu. Therefore, we need to bring the slope m_1 to m_2 after APC. Therefore, the droop should be set at [6],

$$K^D = \frac{1}{m_1 - m_2} \quad (3)$$

Using the procedure discussed above, we found K^D to be 73.5 kW/pu. Using this droop value for all 24 houses, APC is obtained. Fig. 5 shows the curtailed energy, which is approximately 401.11 kWhr for 24 houses, while the total available energy is 1583.48 kWhr. Table I shows energy curtailment of each house. There is no curtailment for first four houses since the voltage never exceeded 1.042 p.u. The curtailed energy is higher as we move along the feeder. The small difference on the energy curtailment of two houses which are at the same distance from the transformer (e.g., House-23 and -24) is due to the difference on the base loads.

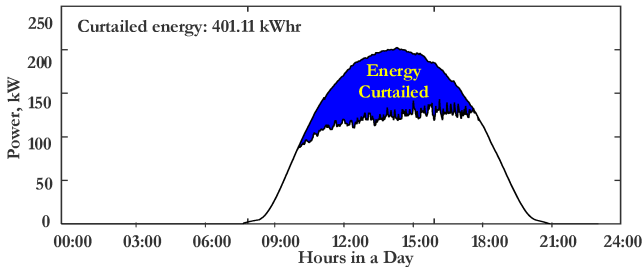


Fig. 5: Power curtailment of 24 houses using constant droop of $K^D=73.5$ kW/pu.

TABLE I: PV Energy Curtailed (in kWhr) from each House using Uniform Droop for all Houses.

House	E Curt.	House	E Curt.	House	E Curt.
H1	0	H2	0	H3	0
H4	0	H5	1.8201	H6	1.7494
H7	6.4467	H8	6.5613	H9	11.7081
H10	11.7247	H11	16.4723	H12	16.4346
H13	20.7766	H14	20.7744	H15	24.5323
H16	24.4992	H17	27.4789	H18	27.4040
H19	29.6191	H20	29.6350	H21	31.1880
H22	31.1954	H23	32.0026	H24	32.0146
				Total	401.11

Voltage profile at all houses for the entire day is shown in Fig. 6. Using the droop value of 73.5kW/pu, voltages are within the limit of 1.058 p.u. However, looking at the voltage profile, it is possible that APC is non-optimal as less power could have been curtailed so that the maximum voltage hits exactly 1.058 p.u.

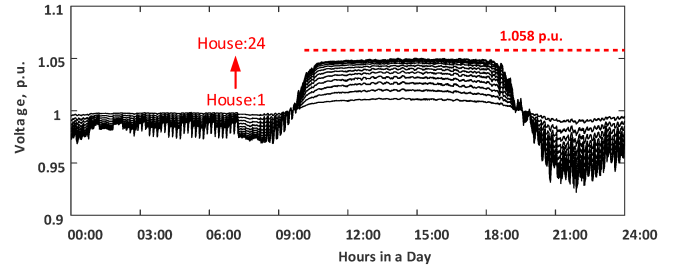


Fig. 6: Voltage profile of 24 houses for an entire day using a constant droop for all houses.

B. Nodal Droop

Droop can be set differently for each node. A systematic way to design nodal droop would be to look at the sensitivities of voltage to active power and reactive power using the elements of power flow Jacobian. The sensitivity matrix can be defined as,

$$S = \begin{bmatrix} 1 & 0 \\ 0 & V \end{bmatrix} \begin{bmatrix} \frac{\partial P}{\partial \delta} & V \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & V \frac{\partial Q}{\partial V} \end{bmatrix}^{-1} \approx \begin{bmatrix} \frac{\Delta \delta}{\Delta P} & \frac{\Delta \delta}{\Delta Q} \\ \frac{\Delta V}{\Delta P} & \frac{\Delta V}{\Delta Q} \end{bmatrix} \quad (4)$$

For APC, $\Delta V/\Delta P$ are used. Matrix $\Delta V/\Delta P$ can be further expanded as,

$$\frac{\Delta V}{\Delta P} = \begin{bmatrix} \frac{\Delta V_1}{\Delta P_1} & \frac{\Delta V_1}{\Delta P_2} & \dots & \frac{\Delta V_1}{\Delta P_n} \\ \frac{\Delta V_2}{\Delta P_1} & \frac{\Delta V_2}{\Delta P_2} & \dots & \frac{\Delta V_2}{\Delta P_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\Delta V_n}{\Delta P_1} & \frac{\Delta V_n}{\Delta P_2} & \dots & \frac{\Delta V_n}{\Delta P_n} \end{bmatrix} \quad (5)$$

Then, the nodal droops are set at the inverse of diagonal elements (that ignores the dependency of voltage rise on one node due to active power injection at the rest of the nodes on the feeder) as,

$$\mathbf{K}^D = \begin{bmatrix} \frac{\Delta P_1}{\Delta V_1} & \frac{\Delta P_2}{\Delta V_2} & \dots & \frac{\Delta P_n}{\Delta V_n} \end{bmatrix}^T \quad (6)$$

For reactive power based voltage control $\Delta V/\Delta Q$ can be used to calculate the nodal droops, which follows the similar procedures as APC droops.

Using the aforementioned procedure, we found $\mathbf{K}^D$ for 24 houses. The droops are computed assuming the PV injection is at peak from each house. $\mathbf{K}^D$ in kW/pu are listed in Table II. Fig. 7 shows the energy curtailed of 24 houses using the nodal droops. Table III shows energy curtailed from each house. TABLE II: Droop Setting of each House using Sensitivities.

House	$\mathbf{K}^D$	House	$\mathbf{K}^D$	House	$\mathbf{K}^D$
H1/H2	246.86	H3/H4	134.80	H5/H6	96.64
H7/H8	77.62	H9/H10	66.39	H11/H12	59.13
H13/H14	54.18	H15/H16	50.71	H17/H18	48.28
H19/H20	46.63	H21/H22	45.60	H23/H24	45.10

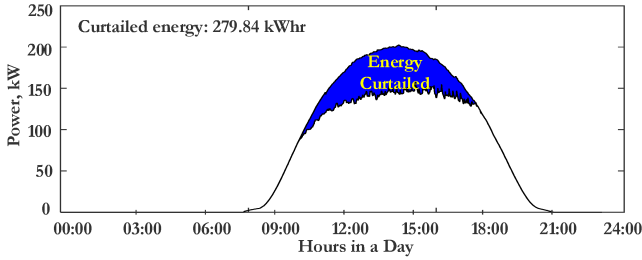


Fig. 7: Energy curtailment of 24 houses using nodal droops.

TABLE III: PV Energy Curtailed (in kWhr) from each House using nodal Droops.

House	E Curt.	House	E Curt.	House	E Curt.
H1	0	H2	0	H3	0
H4	0	H5	2.3933	H6	2.3003
H7	6.8087	H8	6.9297	H9	10.5914
H10	10.5914	H11	13.2529	H12	13.2225
H13	15.3166	H14	15.315	H15	16.927
H16	16.9042	H17	18.0515	H18	18.0024
H19	18.7925	H20	18.8027	H21	19.3509
H22	19.3555	H23	19.6386	H24	19.6459
Total				279.84	

Compared to the uniform droop, energy curtailed is less using the nodal droop. However, looking at the voltage profile shown in Fig. 8, the nodal droops are not able to fix the overvoltage during high PV output. This is due to a) approximation made in Jacobians in finding the droop, where the sensitivity of voltage of a node with respect to active power injection at all other nodes are ignored, and b) the Jacobians are calculated assuming PV injection at all node is at maximum capacity, i.e., 8.4 kW.

IV. CENTRALIZED APPROACH

For the centralized approach, OPF model can be solved to minimize APC while maintaining operating limits, and power balance equations. A generic OPF model [25] for this purpose can be formulated as,

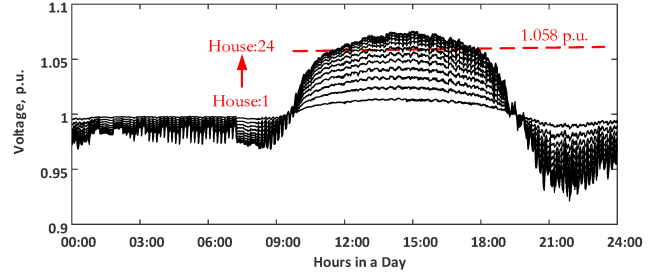


Fig. 8: Voltage profile of 24 houses for an entire day using nodal droops for all houses. This shows the droop based method could lead to voltage infeasibility, i.e., the overvoltage is not completely mitigated.

$$\text{Min: } E^{\text{cur}} = \sum_{m \in N, t \in T} P_{m,t}^{\text{cur}} \Delta t \quad (7)$$

subject to:

$$V_{j,t} = \sum_{k \in N} Y_{j,k} I_{k,t} \quad \forall j \in N, t \in T \quad (8)$$

$$P_{j,t}^{\text{PV}} - P_{j,t}^{\text{cur}} - P_{j,t}^L = \text{Real} \left(V_{j,t} I_{j,t}^* \right) \quad \forall j \in N, t \in T \quad (9)$$

$$Q_{j,t}^{\text{PV}} - Q_{j,t}^L = \text{Imag} \left(V_{j,t} I_{j,t}^* \right) \quad \forall j \in N, t \in T \quad (10)$$

$$|V_{m,t}| \leq V^{\text{max}} \quad \forall m \in N, t \in T \quad (11)$$

$$Q_{m,t}^{\text{PV}} \leq \sqrt{\left(S_{m,t}^{\text{PV}} \right)^2 - \left(P_{m,t}^{\text{PV}} - P_{m,t}^{\text{cur}} \right)^2} \quad \forall m \in N, t \in T \quad (12)$$

$$Q_{m,t}^{\text{PV}} \geq -\sqrt{\left(S_{m,t}^{\text{PV}} \right)^2 - \left(P_{m,t}^{\text{PV}} - P_{m,t}^{\text{cur}} \right)^2} \quad \forall m \in N, t \in T \quad (13)$$

Nomenclature used for OPF Formulation.

j, k	set of nodes. $\{j, k \in N\}$
m	set of nodes with PVs and loads. $\{m \in N\}$
t	Time index. $\{t \in T\}$
Δt	Time interval
Y	Y-bus Matrix
P^{PV}	PV Generation
P^L	Active Power of Load
Q^{PV}	Reactive Power of PV
Q^L	Reactive Power of Load
P^{cur}	Active Power Curtailment of PVs
E^{cur}	Curtailed Energy
S^{PV}	Inverter Rating
V	Nodal Voltage
I	Nodal Current Injection
V^{max}	Nodal Voltage Limit

The OPF-based Optimal APC (OAPC) is modeled in GAMS and solved using KNITRO solver. Assuming that the inverters are operating at unity power factor, the amount of curtailed energy is 243.68 kWhr, and voltages are all within the limit of 1.058 pu (Fig. 9). The reduction in energy curtailment accounts for 40% compared to the uniform droop-based approach, and 13% compared to the droop calculated from power flow sensitivities. If PVs inject reactive power, then the net energy curtailment could be further reduced by 5%. The power factor of inverters are not fixed in the optimization; rather it can take

any value based on active power generation and the inverter capability. Therefore, each PVs may be operating in different power factor. Please note that, the OPF model used is non-convex; therefore, the optimal curtailment solution could be a local optimal.

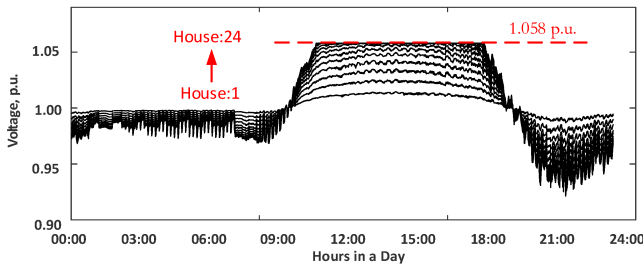


Fig. 9: Voltage profile of 24 houses for an entire day using centralized approach. Each PVs are operating at unity power factor.

V. CONCLUSION

In this paper, three commonly used active power curtailment methods of PVs were implemented for overvoltage protection on distribution feeders, and their performance were analyzed using a LV feeder with 24 houses. The uniform droop based approach is simple to implement; however, depending on the droop setting the curtailment solution could be conservative and non-optimal. Setting the droop parameter high can ensure overvoltage prevention but at the cost of curtailing more energy from PVs. The power flow sensitivity based approach for setting nodal droop is also popular; however, through the case studies we have demonstrated that this approach may not completely eliminate the overvoltage issues. The centralized approach, which bases on optimal power flow model, yields the optimal curtailment solution and also ensures the overvoltages are mitigated at all times. However, the centralized approach requires communication infrastructure to be implemented. Based on the case studies carried out, the centralized approach yielded 40% less energy curtailment compared to uniform droop based approach, and this was reduced further by 5% when PVs were operated at non-unity power factor.

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