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Networked Cournot Competition in Platform Markets: Access Control and Efficiency Loss

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Abstract—**This paper studies network design and efficiency loss in online platforms using the model of networked Cournot competition. We consider two styles of platforms: open access platforms and discriminatory access platforms. In open access platforms, every firm can connect to every market, while discriminatory access platforms limit connections between firms and markets in order to improve social welfare. Our results provide tight bounds on the efficiency loss of both open access and discriminatory access platforms. For open access platforms, we show that the efficiency loss at a Nash equilibrium is upper bounded by 3/2. In the case of discriminatory access platforms, we prove that, under an assumption on the linearity of cost functions, a greedy algorithm for optimizing network connections can guarantee the efficiency loss at a Nash equilibrium is upper bounded by 4/3.**

I. Introduction

Online platforms like Uber, Lyft, and Amazon have changed the way entire industries are run. Unlike traditional firms, platforms do not manufacture products or provide a service. Instead, they arrange matches between firms and consumers, facilitating a safe and simple trading process, providing value for all parties involved. Today, platforms like Uber, Amazon, eBay, etc. make up a \$3 trillion market in the US alone [1].

The design and operation of platforms is extremely diverse. For example, Amazon *matches* buyers to sellers in a manner that takes their preferences into account; and online ad exchanges use *pricing* to indirectly control the matching of firms to markets. More recently, some platforms have moved towards directly controlling the allocation of firms to markets, e.g., Uber's explicit matching of drivers to riders [9, 11].

Broadly, there are two common platform designs: (i) *open access*, where the platform provides information about all potential matches, and allows firms and markets to determine their own matching and corresponding allocations [17, 18, 20], or (ii) *discriminatory access*, where the platform restricts the set of markets that each firm is allowed to enter [4, 12, 29]. Examples of open access platforms include eBay and Etsy, and examples of discriminatory access platforms include Amazon's Buy Box. We summarize these two approaches in Figure 1.

Open and discriminatory access designs are contrasting approaches with differing benefits. Open access designs are easy to maintain, completely transparent, and provide natural fairness guarantees [31]. On the other hand, discriminatory access offers the platform additional control to improve upon social welfare, at the expense of complexity, transparency, and fairness. It is, therefore, natural to ask: *to what degree can discriminatory access control improve upon the efficiency of open access platforms?*

A. Contributions of this paper

In this paper we characterize the efficiency of both open access and discriminatory access platform designs, and quantify the improvement in efficiency that discriminatory access designs can provide relative to open access platforms. Concretely, this paper builds on recent work [28], which studies platform design using the model of networked Cournot competition. In the context of this model, this paper makes two main contributions.

First, in Section III, we study the efficiency loss in open access platforms consisting of n firms. We provide a tight upper bound of $\frac{3}{2}(1-1/(3n+6))$ on the efficiency loss in open access platforms in Theorem 4, which improves upon the previously known $16/7$ efficiency loss bound [28]. Additionally, we provide a sharper efficiency loss bound in Proposition 6 that depends not only on the number of firms, but also on a measure of 'asymmetry' between firms' cost functions. In particular, this bound reveals that a reduction in the asymmetry in cost leads to a reduction in efficiency loss.

Second, in Section IV, we illustrate the efficiency improvement discriminatory access platforms provide over open access platforms. Specifically, we consider a setting in which a discriminatory access platform solves an optimal network design problem to maximize the social welfare at Nash equilibrium. This amounts to a mathematical program with equilibrium constraints (MPEC) that is, in general, computationally intractable. Under the simplifying assumption that firms have linear cost functions, we construct a greedy algorithm that is guaranteed to yield an optimal (welfare maximizing) network. Moreover, we provide a tight upper bound on the efficiency loss incurred under the optimal network design. Specifically, we show that discriminatory access platforms designed in this manner yield an efficiency loss that is upper bounded by 4/3 in the worst case, which improves upon the worst case efficiency loss bound of 3/2 for open access platforms.

B. Related work

Our work lies in the intersection of platform design and networked Cournot competition, and contributes to both literatures.

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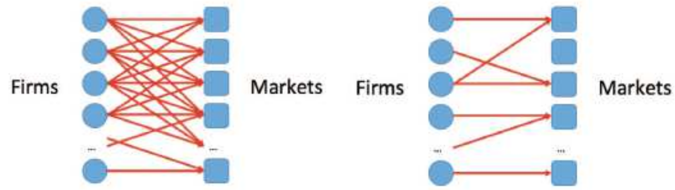

(a) Open access platforms. (b) Discriminatory access platforms.

Fig. 1: The above figures depict (a) open access platforms, where firms can participate in all markets, and (b) discriminatory access platforms, where the platform constrains the markets in which firms can participate. In both platforms, each firm can only access markets that it connects to via the red links, but can choose the exact quantity it allocates to each connected market strategically.

a) Platform design: The recent growth of online platforms has led researchers to focus on identifying design features common to successful platforms. Work in this area has covered a variety of possible design factors, including pricing [32] and competition [6]. Recent empirical studies reveal significant price dispersion in online marketplaces [15], causing platforms to differentiate products in order to create distinct consumer markets [14]. In particular, these results highlight the need to study platforms in using models of *networked competition*.

b) Competition in networked settings: Models of networked competition aim to capture the effects of network constraints on strategic interactions between firms. These models include networked Bertrand competition [5, 10, 19], networked Cournot competition [2, 7, 21], and other networked bargaining games where agents can trade via bilateral contracts over a network that determines the set of feasible trades [3, 16, 27].

Our work relies on a model of networked Cournot competition. A large majority of the literature on networked Cournot competition, e.g., [2, 7, 21], focuses on characterizing and computing Nash equilibria. In a similar spirit to the present paper, [22, 28] provide bounds on the worst case efficiency loss of networked Cournot games, and [8] attempts to understand the impact that system operator governance has on the Nash equilibria that result under network constraints. This paper is the first to provide a tight bound on the efficiency loss of open access platforms, improving on the bounds in [28], and the first to provide an algorithm for optimal network designs with provable efficiency loss guarantees. All proofs are omitted in this version of the paper due to the space constraints.

II. Model and Preliminaries

We describe competition in online platforms according to the *networked Cournot competition* model first introduced by [2] and [7], and later employed by [28] to describe competition in platforms. As a generalization of the classical model of Cournot competition, the networked Cournot model captures the setting in which firms compete to produce a homogeneous good in *multiple markets*, where each market is accessible by a subset of firms. We formally develop the model in the following subsections.

A. Network and Platform Models

The network specifying the connections between firms and markets is described according to a directed bipartite graph $(F, M, \mathcal{E})$. Here, we denote by $F := \{1, \ldots, n\}$ the set of n firms, $M := \{1, \ldots, m\}$ the set of m markets, and $\mathcal{E} \subseteq F \times M$ the set of directed edges connecting firms to markets. That is to say, $(i, j) \in \mathcal{E}$ if and only if firm i has access to market j.

In general, the efficiency of such marketplaces depends on the structure of the underlying graph, which restricts the set of markets to which each firm has access. A crucial role that the platform might, therefore, play in this setting is the selection of markets that are made available to each firm. In what follows, we examine two important classes of platform designs: *open access* platforms and *discriminatory access* platforms.

Open access platforms: An open access platform allows all firms to access all markets. This corresponds to the complete set of directed edges from firms to markets, i.e., $\mathcal{E} = F \times M$. Examples include eBay and Etsy, where every customer is shown every retailer that sells the item she desires.

Discriminatory access platforms: In contrast to open access platforms, a discriminatory access platform can restrict the set of markets that are accessible by each firm. This corresponds to the platform's selection of an edge set $\mathcal{E} \subseteq F \times M$ that may prevent certain firms from accessing certain markets. The goal of this restriction might entail the improvement of producer surplus, consumer surplus, or total social welfare of the system. An example of a discriminatory access platform is Amazon's Buy Box, where Amazon chooses a default seller based on a score that combines pricing, availability, fulfillment, and customer service.

B. Producer Model

Under both the open and discriminatory access platform models, each firm can specify the quantity it produces in each market. Accordingly, we let $q_{ij} \in \mathbb{R}_+$ denote the quantity produced by firm i in market j, and let $q_i := (q_{i1}, \ldots, q_{im}) \in \mathbb{R}^m_+$ denote the supply profile from firm i. We require that $q_{ij} = 0$ for all $(i, j) \notin \mathcal{E}$, and define the set of feasible supply profiles from firm i as:

$$\mathcal{Q}_i(\mathcal{E}) := \left\{ x \in \mathbb{R}^m_+ \mid x_j = 0,\ \forall\ (i, j) \notin \mathcal{E} \right\}.$$

We denote the supply profile from all firms by $q := (q_1, \ldots, q_n) \in \mathbb{R}^{mn}_+$. Accordingly, the set of feasible supply profiles from all firms is given by $\mathcal{Q}(\mathcal{E}) := \prod_{i=1}^n \mathcal{Q}_i(\mathcal{E})$.

We let s_i be the aggregate production quantity of firm $i \in F$. It is given by

$$s_i := \sum_{j=1}^m q_{ij}. \tag{1}$$

The resulting production cost of firm i is defined by $C_i(s_i)$. We assume that the cost function C_i is convex, differentiable on $(0, \infty)$, and satisfies $C_i(s_i) = 0$ for all $s_i \leq 0$.[1] Finally, we define $C := (C_1, \ldots, C_n)$ as the cost function profile.

[1]This family of cost functions represents a generalization of [28], which assumed that all firms have quadratic cost functions.

C. Market Model

As is standard in Cournot models of competition, we model price formation according to an inverse demand function in each market. Similar to [7], we restrict our attention to affine inverse demand functions throughout this paper. Specifically, the price in each market $j \in M$ is determined according to

$$p_j(d_j) := \alpha_j - \beta_j d_j,$$

where d_j denotes the aggregate quantity supplied to market j, given by

$$d_j := \sum_{i=1}^{n} q_{ij}. \tag{2}$$

Here, $\alpha_j > 0$ measures consumers' maximum willingness to pay, and $\beta_j > 0$ measures the price elasticity of demand.

D. Social Welfare

In this paper, we measure the performance (or efficiency) of a platform according to *social welfare*. For platforms, the pursuit of social welfare benefits both buyers and sellers, and in the long run, promotes their expansion. For example, Amazon (in its Buy Box design) believes that welfare measures such as availability, fulfillment, and customer service ultimately lead to increased customer satisfaction, and thereby, promote its growth in the long run [13].

We adopt the standard notion of social welfare defined as aggregate consumer utility less the total production cost. Specifically, the social welfare associated with a supply profile q and a cost function profile C is defined according to

$$\mathrm{SW}(q, C) := \sum_{j=1}^{m} \int_0^{d_j} p_j(z)dz - \sum_{i=1}^{n} C_i(s_i), \tag{3}$$

where s_i and d_j are defined in Eqs. (1) and (2), respectively. We define the *efficient social welfare* associated with an edge set $\mathcal{E}$ and a cost function profile C as:

$$\mathrm{SW}^*(\mathcal{E}, C) := \sup_{q \in \mathcal{Q}(\mathcal{E})} \mathrm{SW}(q, C). \tag{4}$$

A supply profile $q \in \mathcal{Q}(\mathcal{E})$ is said to be *efficient* if it satisfies $\mathrm{SW}(q, C) = \mathrm{SW}^*(\mathcal{E}, C)$. It is straightforward to check that the above supremum can be attained, and that the set of efficient supply profiles is non-empty.

E. The Networked Cournot Game

We describe the equilibrium of the market specified above according to Nash. In particular, we consider profit maximizing firms, where the profit of a firm i, given the supply profiles of all other firms $q_{-i} = (q_1, .., q_{i-1}, q_{i+1}, .., q_n)$, is given by

$$\pi_i(q_i, q_{-i}) := \sum_{j=1}^{m} q_{ij} p_j(d_j) - C_i(s_i). \tag{5}$$

We denote by $\pi := (\pi_1, \ldots, \pi_n)$ the collection of payoff functions of all firms. The triple $(F, \mathcal{Q}(\mathcal{E}), \pi)$ defines a normal-form game, which we refer to as the *networked Cournot game* associated with the edge set $\mathcal{E}$. Its Nash equilibrium is defined as follows.

Definition 1. A supply profile $q \in \mathcal{Q}(\mathcal{E})$ constitutes a *pure strategy Nash equilibrium* of the game $(F, \mathcal{Q}(\mathcal{E}), \pi)$ if for every firm $i \in F$, $\pi_i(q_i, q_{-i}) \geq \pi_i(\overline{q}_i, q_{-i})$, for all $\overline{q}_i \in \mathcal{Q}_i(\mathcal{E})$.

Under the assumptions of convex cost functions and affine inverse demand functions, [2] has shown that the networked Cournot game is an ordinal potential game. Additionally, it admits a unique Nash equilibrium that is the unique optimal solution to a convex program. We summarize the results of [2] in the following lemma.

Lemma 1. *([2]) The game $(F, \mathcal{Q}(\mathcal{E}), \pi)$ admits a unique Nash equilibrium $q^{NE}(\mathcal{E})$ that is the unique optimal solution to the following convex program:*

$$\underset{q \in \mathcal{Q}(\mathcal{E})}{\text{maximize}} \quad \mathrm{SW}(q, C) - \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{\beta_j q_{ij}^2}{2}. \tag{6}$$

In general, the supply profile at the unique Nash equilibrium differs from the efficient supply profile. We measure this loss of efficiency according the *price of anarchy* of the game [23].[2]

Definition 2. The *price of anarchy* associated with the edge set $\mathcal{E}$, the cost function profile C, and the corresponding networked Cournot game $(F, \mathcal{Q}(\mathcal{E}), \pi)$ is defined as

$$\rho(\mathcal{E}, C) := \frac{\mathrm{SW}^*(\mathcal{E}, C)}{\mathrm{SW}\left(q^{\mathrm{NE}}(\mathcal{E}), C\right)}.$$

We set $\rho(\mathcal{E}, C) = 1$ if $\mathrm{SW}^*(\mathcal{E}, C) / \mathrm{SW}(q^{\mathrm{NE}}(\mathcal{E}), C) = 0/0$.

III. Open Access Platforms

For our first set of results, we provide tight bounds on the price of anarchy of the networked Cournot game in an open access platform, under a variety of assumptions on firms' cost functions. In particular, our tight price of anarchy bounds depend not only on the number of firms, but also on the degree of asymmetry between firms' cost functions. These results improve upon the bounds in [28] and generalize those in [22].

A. Identifying the Worst-case Cost Function Profile

The following technical lemma reveals that the price of anarchy is maximized at a cost function profile consisting of cost functions that are linear over the non-negative reals.

Lemma 2. *Given a cost function profile C, define the cost function profile $\overline{C} = (\overline{C}_1, \ldots, \overline{C}_n)$ according to*

$$\overline{C}_i(s_i) = \left(\partial^+ C_i \left(\sum_{j=1}^{m} q_{ij}^{\mathrm{NE}}(F \times M) \right) \cdot s_i \right)^+$$

for $i = 1, \ldots, n$, where $\partial^+ C_i$ denotes the right-derivative of the function C_i. It holds that $\rho(F \times M, C) \leq \rho(F \times M, \overline{C})$.

Lemma 2 reveals that, given any cost function profile C, it is always possible to construct another cost function profile $\overline{C}$ consisting of (piecewise) linear functions, which has a price of

[2]Implicit in Definition 2 is the fact that the networked Cournot game admits a unique Nash equilibrium. In general, for games with a possible multiplicity of Nash equilibria, the price of anarchy is defined as the ratio of the efficient social welfare over that of the Nash equilibrium with the *worst* social welfare.

anarchy that is no smaller. Therefore, in constructing a price of anarchy bound that is guaranteed to hold for all cost functions belonging to the family specified in Section II-B, it suffices to consider cost functions that are linear on $(0, \infty)$.

B. Efficiency Loss in Open Access Platforms

The characterization of the worst-case cost function profile in Lemma 2 facilitates the derivation of tight upper bounds on the price of anarchy for networked Cournot games. In what follows, we examine the role played by *(a)symmetry* in the cost function profile in determining platform efficiency.

1) Symmetric Cost Functions: We begin by analyzing the setting in which firms have identical cost functions. Under this assumption, we establish a tight upper bound on the price of anarchy in Proposition 3 that is *monotonically decreasing* in the number of firms, and converges to one as the number of firms grows large. This conforms with the intuition that increasing the number of (symmetric) suppliers will manifest in increased competition, and thereby reduce the extent to which any one producer might exert market power.

Proposition 3. *If $C_1 = C_2 = \cdots = C_n$, then the price of anarchy associated with the corresponding open access networked Cournot game $(F, \mathcal{Q}(F \times M), \pi)$ is bounded by*

$$\rho(F \times M, C) \leq 1 + \frac{1}{(n+1)^2 - 1}.$$

Moreover, the bound is tight. That is, for any choice of n, there exists a symmetric cost function profile with a corresponding price of anarchy equal to the upper bound.

2) Arbitrary Asymmetric Cost Functions: We now consider the more general setting in which firms have arbitrary asymmetric cost functions satisfying the assumptions in Section II-B. In Theorem 4, we establish a tight upper bound on the price of anarchy that is *monotonically increasing* in the number of firms.

Theorem 4. *The price of anarchy associated with a cost function profile C and the corresponding open access networked Cournot game $(F, \mathcal{Q}(F \times M), \pi)$ is upper bounded by*

$$\rho(F \times M, C) \leq \frac{3}{2}\left(1 - \frac{1}{3n+6}\right).$$

The bound is tight if $\alpha_1 = \alpha_2 = \cdots = \alpha_m$.

The price of anarchy bound established in Theorem 4 is perhaps counterintuitive, in the sense that the efficiency loss at a Nash equilibrium can increase with the number of firms. This seemingly counterintuitive behavior can occur if an expensive firm enters the market. First, note that the entry of this new firm results in an increase in aggregate supply at Nash equilibrium, because of increased 'competition' in the market. However, its entry takes away production from its (cheap) competitors. This manifests in a reduction in social welfare if the increase in production cost exceeds the increase in consumer utility. Such a phenomenon is known as the "excess entry theorem" in the economics literature, and reveals the possibility that a new firm's entry can lead to a reduction in social welfare [24, 26, 30].

Additionally, taking the number of firms $n \to \infty$ yields a price of anarchy bound that is valid for any number of firms, and any number of markets. This recovers the $3/2$ price of anarchy bound first established by Johari and Tsitsiklis [22] for a single market. Moreover, it improves upon the previously known $16/7$ price of anarchy bound for open access networked Cournot games in [28]. We have the following corollary.

Corollary 5. *Open access platforms have a price of anarchy that is at most 3/2.*

3) Linear Cost Functions with Bounds on Asymmetry: The efficiency loss results in Proposition 3 and Theorem 4 appear contradictory. Namely, the price of anarchy bound is decreasing in n if producers have symmetric cost functions. But it is increasing in n if producers are allowed to have asymmetric cost functions. In what follows, we explore how the price of anarchy depends on the asymmetry between firms' cost functions. We restrict ourselves to cost functions that are linear on $(0, \infty)$, and whose slopes lie within $[c_{\min}, c_{\max}] \subseteq \mathbb{R}_+$.

$$\mathcal{L}(c_{\min}, c_{\max}) := \Big\{ C_0 : \mathbb{R} \to \mathbb{R}_+ \;\Big|\; C_0(x) = (cx)^+, \; c \in [c_{\min}, c_{\max}] \Big\}.$$

We write $C \in \mathcal{L}^n(c_{\min}, c_{\max})$ if the cost function profile C satisfies $C_i \in \mathcal{L}(c_{\min}, c_{\max})$ for each firm $i \in F$. It will be convenient to define a non-dimensional parameter γ_j, which measures the degree of asymmetry between firms for each market $j \in M$. Specifically, for each market $j \in M$, define

$$\gamma_j := 1 - \frac{c_{\max} - c_{\min}}{\alpha_j - c_{\min}}.$$

It holds that $\gamma_j \in (-\infty, 1]$ if $c_{\min} < \alpha_j$. Clearly, γ_j is increasing in consumers' maximum willingness to pay α_j, and decreasing in the maximum cost $c_{\max}$. It follows that a value of γ_j close to one implies a small degree of asymmetry between firms' cost functions relative to consumers' maximum willingness to pay in market j.

The following proposition provides a tight bound on the price of anarchy bound when firms have linear cost functions with a bounded degree of asymmetry.

Proposition 6. *Let $C \in \mathcal{L}^n(c_{\min}, c_{\max})$, and assume that $c_{\min} < \max_{j \in M} \alpha_j$. The price of anarchy associated with the corresponding open access networked Cournot game $(F, \mathcal{Q}(F \times M), \pi)$ is upper bounded by*

$$\rho(F \times M, C) \leq \frac{\sum_{j=1}^m \frac{\left((\alpha_j - c_{\min})^+\right)^2}{\beta_j}}{\sum_{j=1}^m \left(\frac{2n+4}{3n+5} + \delta(\gamma_j, n)\right) \frac{\left((\alpha_j - c_{\min})^+\right)^2}{\beta_j}},$$

where the function $\delta(\gamma, n)$ is defined according to

$$\delta(\gamma, n) = \begin{cases} 0 & \text{if } \gamma < \frac{2n+3}{3n+5}, \\ \frac{(n-1)(3n+5)}{(n+1)^2}\left(\gamma - \frac{2n+3}{3n+5}\right)^2 & \text{otherwise}. \end{cases}$$

The bound is tight if $\alpha_1 = \alpha_2 = \cdots = \alpha_m$.

The price of anarchy bound specified in Proposition 6 depends on the degree of asymmetry between firms' cost functions only through the terms $\delta(\gamma_j, n)$ for $j = 1, \ldots, m$. In particular, as $\delta(\gamma, n)$ is non-decreasing in γ, a reduction in the degree of asymmetry between firms' cost functions manifests in a reduction in the price of anarchy bound.

IV. Discriminatory Access Platforms

While many early platforms relied on an open access model, more recent platforms have begun to exercise control over the set of markets to which each firm has access. In the setting of our model, such access control corresponds to the specification of the edge set of the bipartite graph that connect firms to markets, with the goal of maximizing the social welfare at the unique Nash equilibrium of the resulting networked Cournot game.

In what follows, we first show that the problem of choosing the optimal edge set that maximizes the social welfare at Nash equilibrium amounts to a mathematical program with equilibrium constraints (MPEC), and is, in general, computationally intractable. Under the simplifying assumption that each firm's cost function is linear on $(0, \infty)$, we present a greedy algorithm that is guaranteed to generate an optimal solution to the MPEC. Moreover, we present a tight price of anarchy bound for the networked Cournot game that results under the optimal network design. The bound reveals the reduction in efficiency loss achievable through discriminatory access platforms.

A. Network Design

The optimal network design problem amounts to the selection of an edge set $\mathcal{E}$, which maximizes the social welfare at the unique Nash equilibrium of the resulting networked Cournot game. Formally, Lemma 1 provides a characterization of the supply profile at the unique Nash equilibrium of the game $(F, \mathcal{Q}(\mathcal{E}), \pi)$ as the unique optimal solution to a convex program. Therefore, the *optimal network design problem* admits a formulation as the following MPEC:

$$\begin{aligned} \text{maximize} \quad & \mathrm{SW}(q, C) \\ \text{subject to} \quad & \mathcal{E} \subseteq F \times M \\ & q \in \arg\max_{x \in \mathcal{Q}(\mathcal{E})} \left\{ \mathrm{SW}(x, C) - \sum_{j=1}^{m} \sum_{i=1}^{n} \frac{\beta_j x_{ij}^2}{2} \right\} \end{aligned} \tag{7}$$

Here, the decision variables are the edge set $\mathcal{E}$ and the supply profile q. The challenge in solving problem (7) stems from the equilibrium constraint[3] on q, and the presence of the discrete decision variable $\mathcal{E}$. In what follows, we show that, under the simplifying assumption of linear cost functions, problem (7) can be solved using a simple greedy algorithm.

B. Greedy Algorithm for Optimal Network Design

In this section, we restrict ourselves to cost functions that are linear on $(0, \infty)$. Specifically, we assume that the cost function of each firm $i \in F$ satisfies $C_i(s_i) = (c_i s_i)^+$, where $c_i \geq 0$. Leveraging on this assumption, we propose a greedy algorithm for solving the optimal network design problem (7) in Algorithm 1. For each market $j \in M$, the greedy algorithm visits firms in ascending order of marginal cost, and provides each firm it visits access to market j if its inclusion in that market increases social welfare.

Algorithm 1 The Greedy Algorithm

Require: $c_1 \leq \cdots \leq c_n$.
Initialize edge set $\mathcal{E} \leftarrow \emptyset$.
for $j = 1$ to m **do**
  Initialize firm index $i \leftarrow 1$.
  Initialize edge set $\widetilde{\mathcal{E}} \leftarrow \mathcal{E}$.
  repeat
    Update edge set $\mathcal{E} \leftarrow \widetilde{\mathcal{E}}$.
    if $i \leq n$ **then**
      Set edge set $\widetilde{\mathcal{E}} \leftarrow \mathcal{E} \cup (i, j)$.
      Set firm index $i \leftarrow i + 1$.
    end if
  until $\mathrm{SW}(q^{\mathrm{NE}}(\widetilde{\mathcal{E}}), C) \leq \mathrm{SW}(q^{\mathrm{NE}}(\mathcal{E}), C)$.
end for
return $\mathcal{E}$.

The following result establishes optimality of the greedy algorithm when firms' cost functions are linear over $(0, \infty)$.

Theorem 7. *Assume that each firm's cost function is linear over $(0, \infty)$. If $\mathcal{E}^*$ is the edge set generated by the greedy algorithm, then $\left(\mathcal{E}^*, q^{\mathrm{NE}}(\mathcal{E}^*)\right)$ is an optimal solution to (7).*

Clearly, Algorithm 1 yields an edge set $\mathcal{E}^*$, whose corresponding Nash equilibrium has a social welfare that is no smaller than that of the open access platform. In the following theorem, we quantify the resulting improvement in social welfare via a tight bound on the price of anarchy in discriminatory access networked Cournot games.

Theorem 8. *Let $C \in \mathcal{L}^n(c_{\min}, c_{\max})$, and assume that $c_{\min} < \max_{j \in M} \alpha_j$. If $\mathcal{E}^*$ is the edge set generated by the greedy algorithm, then the efficient social welfare associated with the edge set $\mathcal{E}^*$ satisfies*

$$\mathrm{SW}^*(\mathcal{E}^*, C) = \mathrm{SW}^*(F \times M, C).$$

Moreover, the price of anarchy associated with the discriminatory access networked Cournot game $(F, \mathcal{Q}(\mathcal{E}^*), \pi)$ is upper bounded by

$$\rho(\mathcal{E}^*, C) \leq \frac{\sum_{j=1}^{m} \frac{\left((\alpha_j - c_{\min})^+\right)^2}{\beta_j}}{\sum_{j=1}^{m} \max\limits_{k \in \{1,..,n\}} \left\{\frac{2k+4}{3k+5} + \delta(\gamma_j, k)\right\} \frac{((\alpha_j - c_{\min})^+)^2}{\beta_j}}.$$

The above bound is tight if $\alpha_1 = \alpha_2 = \cdots = \alpha_m$.

Theorem 8 reveals the advantage discriminatory access platforms have over open access ones in reducing the efficiency loss at Nash equilibrium. Namely, when the edge set is chosen to be an optimal solution of the network design problem (7), the discriminatory access platform is guaranteed to have a tight bound on the price of anarchy that is no larger than that of the open access platform. Moreover, this price of anarchy bound is guaranteed to be non-increasing in the number of firms n.

[3] An equilibrium constraint requires that a vector be an optimal solution to a optimization problem. In general, this leads to a nonconvex and disconnected feasible region for MPECs. See [25] for a more detailed discussion.

Additionally, choosing the number of firms $n = 1$ yields a price of anarchy bound of $4/3$ for (optimized) discriminatory access platforms with any number of firms and markets. It improves upon the $3/2$ price of anarchy bound for open access platforms established in Corollary 5. The result is formally stated as follows.

Corollary 9. *Assume that each firm's cost function is linear over* $(0, \infty)$. *Discriminatory access platforms have a price of anarchy of at most* $4/3$.

V. Concluding Remarks

This paper examines the design and efficiency loss of open and discriminatory access platforms. Open access platforms offer transparency to market participants, while discriminatory access platforms provide additional control that might be leveraged on to improve market efficiency. For open access platforms, we establish a tight upper bound on the price of anarchy that is decreasing in the number of firms, when costs are symmetric. On the other hand, when costs are asymmetric, we derive a tight upper bound on the price of anarchy that is increasing in the number of firms, and show that open access platforms have a price of anarchy of at most 3/2.

Our second set of results contrast this bound against the case of discriminatory access platforms. We formulate the optimal network design problem for discriminatory access platforms as a mathematical program with equilibrium constraints (MPEC), which is computationally intractable, in general. Under the assumption that the firms' costs are linear, we propose and prove the optimality of a greedy algorithm, recovering the optimal network design for discriminatory access platforms in networked Cournot games. In this setting, we show that the price of anarchy bound shrinks to 4/3, thereby improving upon the worst-case efficiency loss of open access platforms.

Our work builds on a growing literature studying networked Cournot competition, including [2, 7, 8, 21, 28]. While this literature is maturing, there are still a wide variety of important open questions that remain. For example, the formulation of the optimal network design as an MPEC highlights that they are, in general, difficult to solve. The problem of constructing approximation algorithms with provable bounds on performance is an interesting direction for future research.

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