Cognitive Agency and Computer-Based Tasks

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The Problem

As computer-focused policies and trends become more popular in schools, more students access math curriculum online. Computer-based programs may be responsive to input but their algorithmic basis can make it more difficult for them to be as prepared for student thinking as teachers.

- How do students approach the process of mathematics while using online materials, especially in terms of engaging in original thought?
- How might features of lessons that incorporate online materials function to allow students to become interested in the mathematics and pursue their own ideas, rather than only seeking results as directed by a handout or teacher?

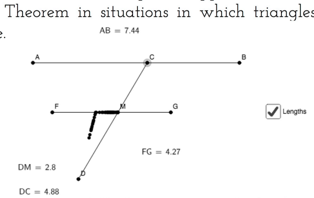
The Lesson

The lesson takes place in an honors geometry class in a public high school in the New England region of the US. Students were seated in groups of 3-4. The lesson was video-taped from multiple angles and each group was audio-recorded. All names are pseudonyms.

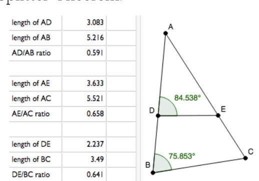
Content goals: The lesson topic was the side splitter theorem. Students already knew the midpoint theorem.

Lesson structure:

- **Part 1 - 7 minutes, groups:** Students work on a handout that they begun the previous day. Handout questions include directions to access 3 Geogebra applets (one is shown below), each designed by their teacher, Mr. Lincoln. Students explore applications of the Midpoint Theorem in situations in which triangles are not visible.



- **Part 2 - 5 minutes, full class:** Mr. Lincoln leads a full-class discussion in which answers to the handout are shared by students, verified by Mr. Lincoln, and discussed.
- **Part 3 - 18 minutes, groups:** Students work in groups on a new handout, which directs them to 2 more original Geogebra applets (one is shown below). Students are asked to find equivalent ratios within various triangles and to attempt to state their findings as a first draft of the Side-Splitter Theorem.



- **Part 4 - 15 minutes, full class:** Mr. Lincoln leads a full-class discussion reviewing answers to the questions on the handout. The class concludes with Mr. Lincoln presenting a version of the Side-Splitter Theorem and noting that the Midpoint Theorem is a special case of this broader theorem.

Approach to Analysis

After transcribing the student groups' conversations, sections in which students appeared to shift between bushwhacking and trail-taking were identified.

Analysis involved interpreting identified sections as interactions between the students and the mathematical objects, using the concepts of bushwhacking and trail-taking to describe student behavior.

Conceptual Framework: Trail-taking and Bushwhacking

While **trail-taking**, students follow an established approach.

Trail-taking relates to Pickering's (1995) concept of disciplinary agency, wherein the mathematics "leads [them] through a series of manipulations" (p. 115). Here, the series is seen as a trail that a student may follow. Trail-taking may exist at different grain sizes: a student may repeat steps from a textbook example, or a student may follow a problem-solving heuristic.

Bushwhacking refers to actions a student takes of their own invention.

It is possible that, unknown to the student, these actions have been taken before by others. The concept draws on Pickering's (1995) human agency, which he describes as involving "choice and discretion" (p. 117). Bushwhacking may be brief, such as when a student selects a formula, or extended, as when students create original methods in solving novel problems.

Findings: Student Interactions

Reconsidering and Changing Set Approaches

Ava and Brenna are working on the following question during Part 1 of the lesson:

7. What if *M* is <u>not</u> a midpoint? In the Geogebra diagram*, *M* is about 1/3 of the way from *D* to *C*.
   a. Make a <u>prediction</u> about how the length of the path traced by *M* compares to *AB* if you dragged *C* back and forth.
   b. Drag *C* back and forth. What do you notice? (Click "Lengths" for segment measurements.) Was your prediction correct?
   c. Now move *M*. Then drag *C*. What invariant relationships can you find between the segment lengths that are shown?

** See the diagram beneath Part 1 within "The Lesson" at left.*

Requests for explanation can lead students to reconsider previously established trails.

Ava asks Brenna to explain her answer to 7C. Brenna complies by describing the steps that she took as a trail, but has difficulty and reconsiders several individual steps in the trail:

> Brenna: Um, so then this length, ov- you divide by this. Alright. Um, which is, which is like the scale… like the factor divided by the- *[laughing]*
> Ava: Oh, sorry. So, this length.
> Brenna: Oh, no. Yeah. Yeah. Divided by *MC*. Wait, no. Wait *DC*… so that makes, so you know… *[Silent for 40 seconds]* Oh there is an F…

Automatic measurements or calculations can prompt students to leave trails.

Brenna regains confidence in her approach, but is quickly alerted to an error when she divides numbers automatically displayed by the program. She abandons her trail.

> Brenna: And then um, *AB* over *FG*, which is like length, *AB* over *FG* which is 7.44… Well <u>that</u> was… not correct. Ha ha! *[Ava and Brenna laugh]* Let's try it the other way. *[erasing]* Let's see.
> Ava: So if you did AB…

Making Conjectures about Interactive Diagrams

Danielle and Erica are working on the following question during the third section of the lesson:

1e. This time, *DE* is <u>not</u> parallel to *BC*. The spreadsheet is calculating values for some of the ratios you considered earlier. Are any of the ratios equal to each other if *DE* is not parallel to *BC*?

** See the diagram beneath Part 3 within "The Lesson" at left. This is the first diagram with only drag-able points.*

Drag-able elements of diagrams can support students in making conjectures even when not required to do so.

Erica, who has been relatively quiet, probes the reason behind the answer to 1e, which is not required and therefore an instance of bushwhacking. The drag-ability of the points in the diagram is key to her thinking:

> Erica: Wouldn't they not be the same anyways if you move it?
> Erica: You can move D so the lengths are going to be different anyways.

Danielle directly contradicts the question's premise and explores the probable impact of parallel lines within the diagram, making active use of the fact that she can manipulate the diagram:

> Danielle: Unless you make this parallel to the bottom but I don't think you… yes you can. Just make it the same angle… *[pause]* I don't think it is, guys.

Returning to the non-parallel premise of 1e, Danielle attempts to create a geometric explanation of the relationship between non-parallel lines and the ratios within the triangle; again, this is not required by the handout:

> Danielle: Maybe… *AD* to… No. Because the thing is it's not a parallel,so it's not like they're both being divided by the same amount. So the ratios wouldn't, because the ratio of the side would-
> Erica: No, none of them are the same.
> Danielle: It just makes sense like that.

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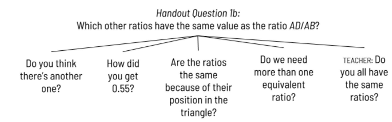
Bushwhacking Behaviors

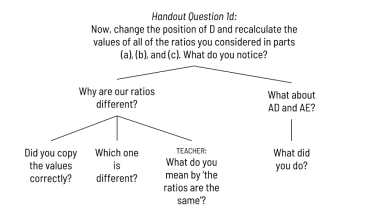
The following behaviors were related to bushwhacking:

- **Asking a new question**: When students ask a new question, they are likely setting off into unknown territory. Even if the act of asking is prompted externally, answering a self-generated question is likely to require at least some bushwhacking.
- **Making a conjecture**: Proposing a rationale for an observed phenomenon, or making a prediction, requires students to make connections for themselves.
- **Reconsidering an approach**: Students who are currently trail-taking may have cause to adapt or leave their trail, both of which require at least momentary bushwhacking.

Focus on Questions

Since student questions frequently appeared in bushwhacking episodes, student questions (explicit and implicit) that stemmed from handout problems were identified and mapped. Two question maps from Danielle and Erica's group are below. A question is shown as the offshoot of a previous question if it is asked as students strive to answer the previous question. This structure may be able to assist in pointing to which questions lead to extended sessions of bushwhacking.





Factors related to Bushwhacking

In these two excerpts, **prompts to explain results**, **automatic values**, and **interactive diagrams** played important roles in episodes in which students transitioned to bushwhacking.

When Ava asked Brenna to **explain** her answer, Brenna briefly bushwhacking when she had difficulty sharing her process as a trail. She did so again when **automatically generated values** alerted her to an error. Ava's request was not a feature of the diagram, but it may have been enabled by the fact that students were working in groups and had access to the same diagrams.

Danielle and Erica both bushwhacked by conjecturing about the reasons behind the handout questions, both incorporating ideas closely related to the **interactive** elements of the diagram in their thinking.

Reference

Pickering, A. (1995). The Mangle of practice: Time, agency, and science. Chicago, US: University of Chicago Press.

COGNITIVE AGENCY AND COMPUTER-BASED TASKS

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As computer-focused policies and trends become more popular in schools, more students access math curriculum online. While computer-based programs may be responsive to some student input, their algorithmic basis can make it more difficult for them to be prepared for divergent student thinking, especially in comparison to a teacher. Consider programs that assess student work by judging how well it matches pre-set answers. Unless designed and enacted in classrooms with care, computer-based curriculum materials might encourage students to think about mathematics in pre-determined ways. How do students approach the process of mathematics while using online materials, especially in terms of engaging in original thought?

Drawing on Pickering's (1995) dance of agency and Sinclair's (2001) conception of students as path-finders or track-takers, I define two modes of mathematical behavior: trail-taking and bushwhacking. While trail-taking, students follow an established approach, often relying on Pickering's (1995) *disciplinary agency*, wherein the mathematics "leads [them] through a series of manipulations" (p. 115). The series of manipulations can be seen as a trail that a student may choose to follow. Bushwhacking, on the other hand, refers to actions a student takes of their own invention. It is possible that, unknown to the student, these actions have been taken before by others. In bushwhacking, the student possesses agency, which Pickering (1995) describes as active (rather than passive) and as hallmarked by "choice and discretion" (p. 117).

In this study, students worked in several dynamic geometric environments (DGEs) during a geometry lesson about the midline theorem. The lesson was originally recorded as part of a larger study designing mathematically captivating lessons. Students accessed both problems and online addresses for corresponding DGEs via a printed packet. Students interacted with the DGEs on individual laptops, but were seated in groups of three or four. Passages of group conversations in which students transitioned between trail-taking and bushwhacking were selected for closer analysis, which involved identifying evidence of each mode and highlighting the curricular or social forces that may have contributed to shifts between modes.

Of particular interest were episodes in which students asked one another to share results, which led to students reconsidering previously set approaches, and episodes in which students interacted with DGEs containing a relatively high proportion of drag-able components, which corresponded to some students working in bushwhacking mode, spontaneously suggesting and revising approaches for manipulating the DGE (e.g., "unless you make this parallel to the bottom, but I don't think you… yes you can."). Both types of episodes were found in multiple groups' conversations. Further analysis of student interactions with tasks, especially with varying levels of student control and sharing, could serve to inform future computer-based task design aimed to encourage students to productively engage in bushwhacking while problem-solving.

References

Pickering, A. (1995). *The Mangle of Practice: Time, Agency, and Science*. Chicago, UNITED STATES: University of Chicago Press. Retrieved from http://ebookcentral.proquest.com/lib/bu/detail.action?docID=648147

Sinclair, N. (2001). The aesthetic IS relevant. *For the Learning of Mathematics*, *21*(1), 25–32.