

# A Secure and Verifiable Access Control Scheme for Big Data Storage in Clouds

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**Abstract**—Due to the complexity and volume, outsourcing ciphertexts to a cloud is deemed to be one of the most effective approaches for big data storage and access. Nevertheless, verifying the access legitimacy of a user and securely updating a ciphertext in the cloud based on a new access policy designated by the data owner are two critical challenges to make cloud-based big data storage practical and effective. Traditional approaches either completely ignore the issue of access policy update or delegate the update to a third party authority; but in practice, access policy update is important for enhancing security and dealing with the dynamism caused by user join and leave activities. In this paper, we propose a secure and verifiable access control scheme based on the NTRU cryptosystem for big data storage in clouds. We first propose a new NTRU decryption algorithm to overcome the decryption failures of the original NTRU, and then detail our scheme and analyze its correctness, security strengths, and computational efficiency. Our scheme allows the cloud server to efficiently update the ciphertext when a new access policy is specified by the data owner, who is also able to validate the update to counter against cheating behaviors of the cloud. It also enables (i) the data owner and eligible users to effectively verify the legitimacy of a user for accessing the data, and (ii) a user to validate the information provided by other users for correct plaintext recovery. Rigorous analysis indicates that our scheme can prevent eligible users from cheating and resist various attacks such as the collusion attack.

**Index Terms**—Big data storage, access control, the NTRU cryptosystem, secret sharing, access policy update, cloud computing

## 1 INTRODUCTION

BIG data is a high volume, and/or high velocity, high variety information asset, which requires new forms of processing to enable enhanced decision making, insight discovery, and process optimization [1]. Due to its complexity and large volume, managing big data using on hand database management tools is difficult. An effective solution is to outsource the data to a cloud server that has the capabilities of storing big data and processing users' access requests in an efficient manner. For example in e-health applications, the genome information should be securely stored in an e-health cloud as a single sequenced human genome is around 140 gigabytes in size [2], [3]. However, when a data owner outsources its data to a cloud, sensitive information may be disclosed because the cloud server is not trusted; therefore typically the ciphertext of the data is stored in the

could. But how to update the ciphertext stored in a cloud when a new access policy is designated by the data owner and how to verify the legitimacy of a user who intends to access the data are still of great concerns.

Most existing approaches for securing the outsourced big data in clouds are based on either *attributed-based encryption (ABE)* or *secret sharing*. ABE based approaches [4], [5], [6], [7], [8], [9], [10], [11] provide the flexibility for a data owner to predefine the set of users who are eligible for accessing the data but they suffer from the high complexity of efficiently updating the access policy and ciphertext. Secret sharing [11], [12], [13], [14], [15], [16], [17] mechanisms allow a secret to be shared and reconstructed by certain number of cooperative users but they typically employ asymmetric public key cryptograph such as RSA for users' legitimacy verification, which incur high computational overhead. Moreover, it is also a challenging issue to dynamically and efficiently update the access policies according to the new requirements of the data owners in secret sharing approaches.

As a data owner typically does not backup its data locally after outsourcing the data to a cloud, it cannot easily manage the data stored in the cloud. Besides, as more and more companies and organizations are using clouds to store their data, it becomes more challenging and critical to deal with the issue of access policy update for enhancing security and dealing with the dynamism caused by the users' join and leave activities. To the best of our knowledge, policy update for outsourced big data storage in clouds has never been considered by the existing research [13], [17], [18], [19], [20].

Another challenging issue is how to verify the legitimacy of the users accessing the outsourced data in clouds. Existing

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schemes proposed in [7], [8], [10], [21] do not support user eligibility verification. On the other hand, verifiable secret sharing based schemes rely on RSA [13], [14], [15] for access legitimacy verification. As multiple users need to mutually verify each other using multiple RSA operations, such a procedure has a high computational overhead. Furthermore, the classic asymmetric crypto solutions such as RSA could be broken by quantum computing in the near future. This is not a science fiction as in 2015 IBM brought quantum computing closer to reality [22], making it urgent to exploit new techniques for quantum computing attack resistance. The NTRU cryptosystem is a type of lattice-based cryptography [23], [24], [25], and its security is based on the shortest vector problem (SVP) in a lattice [26]. The major advantages of NTRU are quantum computing attack resistance and lightning fast computation capability. However, NTRU suffers from the problem of decryption failures [27], [28], [29], [30].

The considerations mentioned above motivate us to develop a verifiable access control scheme for securing the big data stored in clouds, tackling the challenges of the following security services:

- *Security.* The proposed scheme should be able to defend against various attacks such as the collusion attack. Meanwhile, access policy update should not break the security of the data storage, disclose sensitive information about the data owner, and cause any new security problem.
- *Verification.* When a user needs to decrypt a stored ciphertext, its access legitimacy should be verified by other participating users and the secret shares obtained from other users must be validated for correct recovery.
- *Authorization.* To reduce the risk of information leakage, a user should obtain authorization from the data owner for accessing the encrypted data.

In this paper, we first propose an improved NTRU cryptosystem to overcome the decryption failures of the original NTRU. Then we design a secure and verifiable scheme based on the improved NTRU and secret sharing for big data storage. The cloud server can directly update the stored ciphertext without decryption based on the new access policy specified by the data owner, who is able to validate the update at the cloud. The proposed scheme can verify the shared secret information to prevent users from cheating and can counter various attacks such as the collusion attack. It is also deemed to be secure with respect to quantum computing attacks due to NTRU. The multi-fold contributions of the paper can be summarized as follows:

- We propose a new NTRU decryption procedure to overcome the decryption failures of the original NTRU without reducing the security strength of NTRU.
- We propose a secure and verifiable access control scheme to protect the big data stored in a cloud. The scheme can verify a user's access legitimacy and validate the information provided by other users for correct plaintext recovery.
- We devise an efficient and verifiable method to update the ciphertext stored in clouds without increasing any risk when the access policy is dynamically changed by the data owner for various reasons.

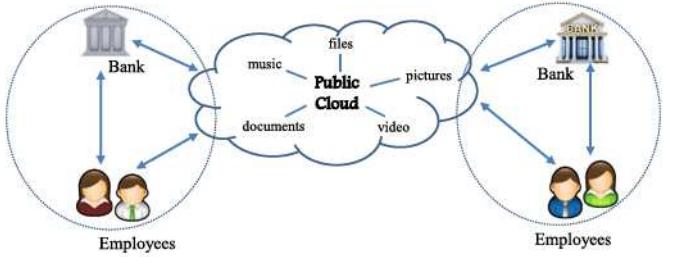


Fig. 1. An example application of big data storage in banking.

- We prove the correctness of the proposed scheme and investigate its efficiency and security strength. Particularly, we demonstrate that our scheme can resist various attacks such as the collusion attack via a rigorous analysis.

The rest of the paper is organized as follows. Section 2 introduces the motivation and the related work. Section 3 presents our system model, security model, and the preliminary knowledge employed by our scheme. Section 4 proposes an improved NTRU cryptosystem to overcome the decryption failures of the original NTRU cryptosystem. Section 5 details our secret sharing based scheme for big data storage in clouds. The correctness, security properties, and performance of our proposed scheme are elaborated in Section 6. Conclusions and future work are discussed in Section 7.

## 2 MOTIVATION AND RELATED WORK

### 2.1 Motivation

In this information era, companies and organizations are facing a challenging problem of effectively managing their complex data. As the development of cloud storage, outsourcing the data to a cloud is an appropriate approach. Generally speaking, clouds can be classified into two major categories: i) public clouds with each being a multi-tenant environment shared with a number of other tenants, and ii) private clouds with each being a single-tenant environment dedicated to a single tenant. For example, the IBM cloud was proposed as a public one for the data management of banking [31]. When a bank stores its data in the cloud server as shown in Fig. 1, only its legal staff members have the rights to access the stored data. Typically the bank system contains many sensitive and private consumer information. In order to reduce the risk of information leakage, the access right of an employee should be properly restricted, and a single employee should not be allowed to reveal the data by itself without obtaining the authorization from other users; that is, recovering the data requires to get the authorization of multiple employees. Moreover, the bank should be able to update the access policies for the stored data in a dynamic and efficient manner. Similarly, military applications can utilize a private cloud to store their complex data as shown in Fig. 2. Since the data is confidential, a military member, who needs to access the data, must pass the verification of its legitimacy and receive the authorization from multiple relevant departments. Besides, the military should be able to dynamically and efficiently update its access policies based on the changing requirements.

Such applications usually require the data to be stored in a cloud in ciphertext format, and the access of the data by a user requires multiple other users to verify the access

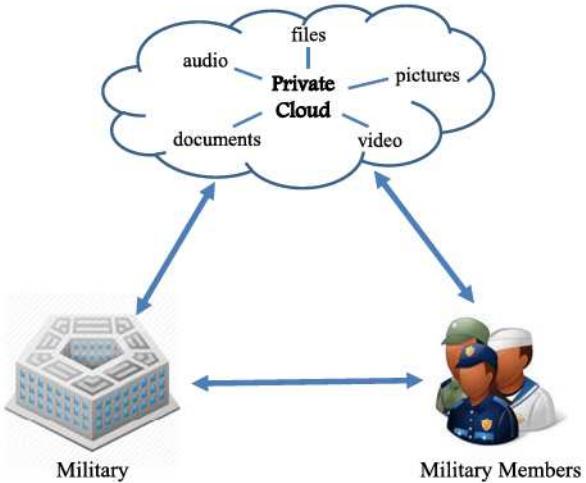


Fig. 2. An example application of big data storage in military.

legitimacy of the user. Therefore in this paper we propose a secure and verifiable access control scheme for big data storage to tackle the following challenges: i) how to securely store the data in a cloud server and distribute the shares of the access right to all legitimate users of the data? ii) how to verify the legitimacy of a user for accessing the data? iii) how to recover the plaintext data when the access right needs to be jointly granted by multiple users? and iv) how to dynamically and efficiently update the ciphertext in the cloud when the access policy of the data is changed by the data owner? To overcome these challenges, we make use of the following techniques in the design of our secure and verifiable access control scheme for big data storage. First, a plaintext data is bound to a secret that is shared by all legitimate users of the data based on  $(t, n)$ -threshold secret sharing, and a message certificate is computed for the data based on the NTRU encryption; the ciphertext is produced from both the shared secret and the message certificate. Second, the legitimacy of a user for accessing the data is verified by both the data owner and at least  $t - 1$  other legal users of the data, and the information provided by other users for the plaintext recovery needs to be validated by the user to prevent against cheating behaviors. Third, the plaintext data can be obtained when at least  $t - 1$  other users participate in the recovery process and provide correct information for the data recovery, based on  $(t, n)$ -threshold secret sharing. Last, the access policy of the data and the secret shares bound to the data can be dynamically changed by the data owner, and the update of the ciphertext is conducted by the cloud server without the need of downloading the previous ciphertext from the cloud to the data owner. Meanwhile, the data owner is able to verify whether the ciphertext stored in the cloud is correctly updated.

## 2.2 Related Work

Because big data frequently contains a huge amount of personal identifiable information, how to securely store the data and how to provide access control over the stored data are two biggest challenges [32]. In this section, we mainly summarize the state-of-the-art of securing the big data stored in clouds.

Outsourcing to clouds is one of the most popular approaches to securing the big data storage [33], [34], [35],

in which the data owners encrypt their data based on cryptographic primitives and store the encrypted data to the clouds. In outsourcing, a secure mechanism should be established between a data owner and a cloud. In order for the cloud to perform operations over the encrypted data, "Fully Homomorphic Encryption" (FHE) [36] was usually adopted, which allows direct addition and multiplication operations over the ciphertexts while preserving decryptability. Homomorphic encryption was also applied to guarantee the security of data storage [37], [38]. Nevertheless, it is an immature cryptosystem, and is extremely inefficient in practice, which renders it hardly applicable in real world applications. Securely outsourcing big data computations to the clouds was also extensively studied [39], [40], [41] but this topic is out of the scope of the paper.

Adequate access control is key to protect the stored data. Access control has traditionally been provided by operating systems or applications restricting access to the information, which typically exposes all the information if the system or application is hacked [42]. A better approach is to protect the information using encryption that only allows decryption by authorized entities. Attribute-Based Encryption (ABE) [4], [7], [8], [10], [11] is one of the most powerful techniques for access control in cloud storage systems. In the past years, quite a few attribute-based access control schemes [19], [20], [43], [44] were proposed, in which the data owners define the access policies based on the attributes required by the data and encrypt the data based on the access policies. By this way the data owners are able to ensure that only the users meeting the access policies can decrypt the ciphertexts. However, it is difficult to update the policies when these ABE based schemes are applied because the data owners do not store the data in their local systems once they outsource the data into the cloud servers. It is also difficult to verify the legitimacy of the downloaded data as the clouds housing the data are not trustworthy. Besides, the operations of encryption and decryption in ABE have a high computational overhead and incur a large energy consumption.

Secret sharing [45] is another powerful technique to protect the big data in cloud storage. The most related work to our proposed scheme are [14] and [15], whose verification procedure can resist potential attacks such as collusion and cheating. In [15], two schemes were proposed, namely Scheme-I and Scheme-II, based on the homogeneous linear recursion and the RSA cryptosystem, in which the homogeneous linear recursion is used to construct the secret share and reconstruct the secret, and RSA is used to verify the users' access legitimacy. The difference between these two schemes lies in that the users in Scheme-I mutually verify each other's legitimacy without seeking help from public values while in Scheme-II the users need the help of public values. In [14], the authors presented a verifiable multi-secret sharing scheme based on cellular automata, which is used to construct the secret share and reconstruct the secret with a linear computational complexity, and the RSA cryptosystem, which is used for verification. In these schemes, as multiple users mutually verify each other using multiple RSA operations, a very high computational overhead occurs. In addition, the classic asymmetric crypto solutions would be broken by quantum computing; that is, these

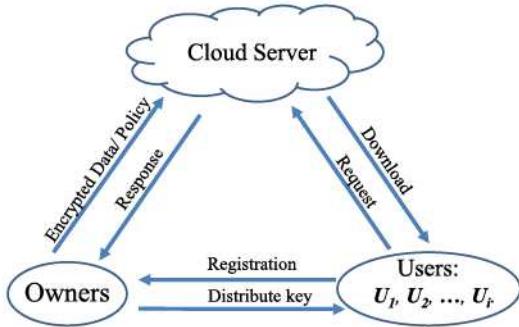


Fig. 3. System model.

traditional verification methods cannot satisfy the verification requirements with respect to quantum computing, which is made closer to reality by IBM in 2015 [22]. Thus we need to seek new verification methods to meet the future requirements. For this purpose, we utilize the NTRU cryptosystem to counter the quantum computing attacks in the design of our proposed scheme.

Delegation is a popular approach for policy update. In [7], a user generates a new private key using its previous private key, and then delegates the new private key to a local authority for access policy update. In [46], a procedure called “ciphertext delegation” was designed for the third party to ‘re-encrypt’ the ciphertext to a more restrictive policy using only public information. These two approaches cannot satisfy our security requirements because they delegate the private key/ciphertext for a new access policy that is more restrictive than the old one—in our perspective, the access policy to a ciphertext might need to be relaxed as time goes for many real world applications. Therefore in our design, we allow the new access policy designated by the data owner to be independent than the previous one, and the ciphertext is updated by the cloud based on the new access policy.

### 3 MODELS AND PRELIMINARIES

#### 3.1 System Model

We consider a cloud storage system that is applicable for both public and private clouds as shown in Fig. 3. It consists of the following three types of entities: cloud server, data owner (owners), and data user (users).

*Cloud Server.* A cloud server provides spaces for data owners to store their outsourced ciphertext data that can be retrieved by the users. It is also responsible for updating the ciphertexts when the data owner changes its access policy.

*Owners.* A data owner designates the access policy for its data, encrypts the data based on the access policy before outsourcing the data to the cloud server, and requests the cloud server to update the encrypted data when a new access policy is adopted. It can also check whether the ciphertext at the cloud server is correctly updated.

*Users.* Each user is assigned with a sub-key for an encrypted data the user is eligible to access. In order to decrypt the ciphertext, the user’s eligibility must be verified by at least  $t - 1$  other users that are also eligible to access the data. The information provided by the  $t - 1$  verifiers must be validated by the user for correct message decryption based on the  $(t, n)$ -threshold secret sharing.

For a piece of data to be stored in a cloud, the data owner generates a public key and privacy key pair, defines an access policy, and computes a sub-key for each potential user based on the policy. Then, the data owner produces a message certificate for the data, and stores the encrypted data with the access policy in the cloud. When a user needs to use the data, it solicits help from other users to recover the data. The cloud server can update the encrypted data with a new policy is designated by the data owner.

The access policy is defined by a  $t - 1$  degree polynomial  $b(x) = b_0 + \sum_{j=1}^{t-1} b_j x^j$  in this paper. Specifically, the data owner splits the access right of an encrypted plaintext into  $n$  pieces for  $n$  users, with one piece for each legitimate user of the data. A user can recover the plaintext data if and only if it obtains the message certificate from the data owner and holds at least  $t$  pieces of the access rights (with the help of at least  $t - 1$  other legal users), where  $t$  is a threshold designated by the data owner for access control based on  $(t, n)$ -threshold secret sharing.

#### 3.2 Adversarial Model and Security Objectives

We assume that the cloud server honestly follows the protocol, i.e., it sends the ciphertext to a user when receiving a request from the user, and updates the ciphertext in its storage when requested by the owner of the data. The data owners do not have incentives to cheat the end users as their goals are to secure the data stored in the cloud server, but one data owner may intend to recover the data belonging to a different data owner. The users do not have to be honest, i.e., they may have different cheating behaviors. For example, a user may provide incorrect information for the decryption of a ciphertext by other users, causing the decryption to fail; multiple users may collude to reveal the message; a user may forge a message certificate; just to name a few.

We intend to achieve the following security objectives when designing our access control scheme for big data storage in clouds:

- Data confidentiality. The outsourced data must be protected from eavesdropping attacks during upload, download, update, and retrieval of the data.
- Verification. The legitimacy of any user to access the data must be verifiable; the information provided by any user for plaintext recovery must be validated.
- Attack resistance. The proposed scheme must be able to counter potential attacks (cheating, collusion attacks, forging attacks, etc.) launched by misbehaving users and adversaries.

#### 3.3 Preliminaries

In this section, we introduce the following two cryptographic primitives: NTRU and secret sharing. For better elaboration, we present the notations and their semantic meanings in Table 1.

##### 3.3.1 The NTRU Cryptosystem

The NTRU cryptosystem is based on the shortest vector problem in a lattice that makes it lightning fast and resistant to quantum computing attacks. It has been proved to be faster than RSA [28], [29]. In this section, we briefly

TABLE 1  
The Notations and Their Semantic Meanings

Notations	means
$D$	The data owner
$U_i$	The $i$ th user, $1 \leq i \leq n$
$id$	The $id$ of a user
$\{H, H_1\}$	Two one-way hash functions
$(h, f)$	A pair of keys, with $h$ being the public key and $f$ being the private key.
$S$	The set of $M$ messages $S = \{S_1, S_2, \dots, S_M\}$
$R$	$R$ is a ring, i.e., $R = \mathbb{Z}[X]/(X^N - 1)$
$p, q$	Two integers in $R$
$f$	A polynomial in $R$
$g$	A polynomial in $\mathbb{Z}[X]/(q, X^N - 1)$
$f_p$	A polynomial in $\mathbb{Z}[X]/(p, X^N - 1)$
$f_q$	A polynomial in $\mathbb{Z}[X]/(q, X^N - 1)$
$L_f$	The set of polynomials in $R$
$L_g$	The set of polynomials in $R$
$AES$	Advanced Encryption Standard

introduce NTRU, and refer the interested readers to [27] and [26] for more details.

NTRU consists of three integer parameters  $(N, p, q)$  and four sets  $L_f, L_g, L_\phi$  and  $L_m$  of polynomials of degree  $N - 1$  with integer coefficients. Note that  $p$  and  $q$  are not required to be prime, but they should simultaneously meet the following two criterions:  $\gcd(p, q) = 1$  and  $q > p$ . NTRU works in the ring  $R = \mathbb{Z}[X]/(X^N - 1)$ , and  $L_f, L_g, L_\phi$  and  $L_m$  are defined in  $R$  whose selection criteria are detailed in [26]. Any element  $f \in L_f$  can be expressed as a polynomial or a vector as follows:

$$f = \sum_{i=0}^{N-1} f_i x^i = [f_0, f_1, \dots, f_{N-1}]. \quad (1)$$

We use  $*$  to denote the convolution multiplication of two polynomials  $f$  and  $g$  in  $R$ , which can be computed via

$$f * g = h \quad \text{with} \quad h_k = \sum_{i=0}^k f_i g_{k-i} + \sum_{i=k+1}^{N-1} f_i g_{N+k-i}, \quad (2)$$

where  $k \in [0, N - 1]$ . Note that when we do a multiplication of two polynomials modulo  $q$ , we mean to reduce the coefficients of the product modulo  $q$ .

NTRU implements the following three basic functions. For better presentation, we use Alice and Bob to represent the two entities for encrypting and decrypting a message.

- **Key Generation:** To create his public and private keys, Bob randomly chooses two polynomials  $g \in L_g$  and  $f \in L_f$ , in which  $f$  is his private key and is required to have inverse modulo  $q$  and inverse modulo  $p$ . Let  $f_q$  and  $f_p$  respectively denote the inverse modulo  $q$  and the inverse modulo  $p$  of  $f$ , which are formally defined as follows:

$$f_q * f = 1 \pmod{q} \quad \text{and} \quad f_p * f = 1 \pmod{p}. \quad (3)$$

For any properly selected  $f$ , it is easy to compute  $f_q$  and  $f_p$  via the Euclidean algorithm. Then Bob

computes his public key  $h$  according to (4), and publishes his public parameters  $\{p, q, h\}$

$$h = f_q * g \pmod{q}. \quad (4)$$

- **Encryption:** Assume that Alice wants to send a message  $m$  to Bob, she needs to first convert  $m$  into a polynomial in  $L_m$ . For simplicity, we use  $m$  to denote the polynomial representation of  $m$  in  $L_m$  too. Then she randomly chooses a polynomial  $\phi \in L_\phi$ , encrypts  $m$  using Bob's public key  $h$  according to (5), and sends the encrypted message  $e$  to Bob

$$e = p\phi * h + m \pmod{q}. \quad (5)$$

- **Decryption:** Bob receives the encrypted message  $e$  from Alice, and decrypts it using his private key  $f$  and the inverse of  $f$  modulo  $p$ , i.e.,  $f_p$ , via

$$a = e * f \pmod{q}. \quad (6)$$

$$m = a * f_p \pmod{p}. \quad (7)$$

**Remarks.** In NTRU, if the system parameters are properly selected, the probability of correctly recovering the plaintext message is extremely high; otherwise, the probability of decryption failures is extremely high. There are two types of decryption failures: the *Wrap failure* and the *Gap failure*, which will be discussed in Section 4.

### 3.3.2 Secret Sharing Scheme

Secret sharing schemes have been extensively studied [13], [17], ever since its development by Shamir [45] and Blakley [47] in 1979. Generally speaking, secret sharing can be briefly described as follows. In the context of a *dealer* sharing a secret with a number of *users*  $U_1, \dots, U_n$ , a user learns the secret if and only if it can cooperate with at least  $t - 1$  other users (on sharing what they learn from the dealer), where  $t \leq n$  is a pre-determined parameter. The secret to be shared by the dealer and the users is  $s \in GF(p_1)$ , where  $p_1 > N$ . Each user  $U_i$  holds a secret key  $k_i \in GF(p_1)$ , which is only known by  $U_i$  and the dealer.

The dealer follows a two-step process. First, it constructs a polynomial function  $F(x)$  of degree  $t - 1$  shown as follows:

$$F(x) = s + \sum_{j=1}^{t-1} \mu_j x^j, \quad (8)$$

by randomly choosing each  $\mu_j$  i.i.d. with a uniform distribution from  $GF(p_1)$ . Note that all (addition and multiplication) operations used in (8) is modular arithmetic (defined over  $GF(p_1)$ ) as opposed to real arithmetic. Also note that  $s$  is the constant component of  $F(x)$ —i.e.,  $s = F(0)$ . Then, in the second step, the dealer transmits to each  $U_i$  a shared secret  $s_i = F(x_i)$ , where  $x_i$  is a random number selected by  $U_i$  for sharing the secret  $s$  and is sent to the dealer via the secure channel protected by the shared key  $k_i$ .

We now show how  $t$  or more users can cooperate to recover  $s$  by sharing the secrets received from the dealer. Without loss of generality, let  $U_1, \dots, U_t$  be the cooperating

users. These  $t$  users can reconstruct the secret  $s = F(0)$  from  $s_1 = F(x_1), \dots, s_t = F(x_t)$  by computing

$$s = F(0) = \sum_{j=1}^t \left( s_j \prod_{i \in [1, n], i \neq j} \frac{0 - x_i}{x_j - x_i} \right). \quad (9)$$

Note that the cumulative product in (9) is essentially the Lagrange coefficient. The correctness of (9) can be easily verified based on the definition of  $F(x)$ .

In secret sharing, a user may cheat when recovering the secret  $s$ . For example, a user  $U_i$  may provide an incorrect  $s_i$ , making the recovery of  $s$  fail. In order to overcome this problem, verifiable secret sharing schemes [13], [14], [15] have been proposed, which are mainly based on the RSA cryptosystem that incurs high computational overhead. In this paper, we propose a new validation method to prevent a user from submitting fake information for the correct message recovery based on the NTRU cryptosystem.

#### 4 AN IMPROVED NTRU CRYPTOSYSTEM

As reported in Section 3.3.1, there exist two types of decryption failures in NTRU: the Wrap failure and the Gap failure. In other words, the decryption process of NTRU cannot always output a correct decrypted message. To overcome this problem, we propose an improved NTRU cryptosystem in this section. To proceed, we first analyze the reasons of the decryption failures in the original NTRU. When a ciphertext  $e = p\phi * h + m \pmod{q}$  is received, it is decrypted according to the following two steps:

- 1) Compute  $a = e * f \pmod{q}$ ;
- 2) Calculate  $m' = a * f_p \pmod{p}$ .

As analyzed in [48], the NTRU decryption could correctly recover the plaintext  $m$  if  $a = (a_0, a_1, a_2, \dots, a_{N-1})$  remains unchanged after the module  $q$  operation in step 1. This indicates that as long as  $a_i \in (-\frac{q}{2}, \frac{q}{2})$ ,  $i = 0, 1, \dots, N-1$ , before the mod  $q$  operation,  $m$  can be correctly recovered. If there is at least one  $a_i$  with  $|a_i| \geq \frac{q}{2}$ , the Wrap failure could happen, resulting in  $m' \neq m$  with a high probability; if  $\max_{0 \leq i \leq N-1} \{a_i\} - \min_{0 \leq i \leq N-1} \{a_i\} > q$ , the Gap failure could happen, leading to  $m' \neq m$  with a high probability. From the above analysis, one can see that the Gap failure leads to the Wrap failure and that overcoming the Wrap failure can also get rid of the Gap failure.

In order to overcome the Wrap failure, we propose the following improved NTRU decryption algorithm. For  $a = e * f = p\phi * h * f + m * f$ , we first figure out  $\Gamma = \max \{|\max_{0 \leq i \leq N-1} \{a_i\}|, |\min_{0 \leq i \leq N-1} \{a_i\}|\}$  in  $a$ , and then compute  $\tau = \lfloor \frac{\Gamma}{q/2} \rfloor$ . If  $\tau = 0$ , which implies that each  $a_i \in (-\frac{q}{2}, \frac{q}{2})$  in  $a$ , we can decrypt  $e$  to get  $m$  via the traditional approach; otherwise, we adjust  $a$  to obtain  $a' = a - c^{(1)} - c^{(2)} - \dots - c^{(\tau)}$ , where  $c^{(1)}, c^{(2)}, \dots, c^{(\tau)}$  are the adjusting vectors and  $c^{(j)} = (c_0^{(j)}, c_1^{(j)}, \dots, c_{N-1}^{(j)})$  for  $j = 1, 2, \dots, \tau$ .

Next, we present how to select the adjusting vectors  $\{c^{(1)}, c^{(2)}, \dots, c^{(\tau)}\}$ . Consider any  $a_i \in a$ . Let  $\gamma = \lfloor \frac{|a_i|}{q/2} \rfloor$ . There are two cases:

- Case 1: If  $a_i \geq 0$ , it can be denoted by  $a_i = \frac{q-1}{2}\gamma + c_i^{(\gamma+1)}$ , where  $0 \leq c_i^{(\gamma+1)} < \frac{q}{2}$ . There are two sub-cases:
  - Case 1-1: If  $\gamma = 0$ , we have  $a'_i = a_i$ ; then set  $c_i^{(1)} = c_i^{(2)} = c_i^{(3)} = \dots = c_i^{(\tau)} = 0$ .
  - Case 1-2: If  $\gamma \geq 1$ , set  $c_i^{(1)} = c_i^{(2)} = \dots = c_i^{(\tau)} = \frac{q-1}{2}$  and  $c_i^{(\gamma+2)} = \dots = c_i^{(\tau)} = 0$ ; then  $a'_i = a_i - c_i^{(1)} - c_i^{(2)} - \dots - c_i^{(\tau)} = c_i^{(\gamma+1)}$ .
- Case 2: If  $a_i < 0$ , it can be denoted as  $a_i = -\frac{q-1}{2}\gamma + c_i^{(\gamma+1)}$ , where  $-\frac{q}{2} < c_i^{(\gamma+1)} < 0$ . There also exist two sub-cases:
  - Case 2-1: If  $\gamma = 0$ , we have  $a'_i = a_i$ ; set  $c_i^{(1)} = c_i^{(2)} = c_i^{(3)} = \dots = c_i^{(\tau)} = 0$ .
  - Case 2-2: If  $\gamma \geq 1$ , set  $c_i^{(1)} = c_i^{(2)} = \dots = c_i^{(\tau)} = -\frac{q-1}{2}$  and  $c_i^{(\gamma+2)} = \dots = c_i^{(\tau)} = 0$ ; then we have  $a'_i = a_i - c_i^{(1)} - c_i^{(2)} - \dots - c_i^{(\tau)} = c_i^{(\gamma+1)}$ .

After applying the above process to all  $a_i$ 's, we obtain  $a'$  whose elements are guaranteed to be in  $(-\frac{q}{2}, \frac{q}{2})$ . Then the decryption procedure can be formalized by the following four steps:

- 1) Compute  $a = e * f$ ;
- 2) Calculate the adjusting vectors  $c^{(1)}, c^{(2)}, \dots, c^{(\tau)}$  according to the procedure described above;
- 3) Compute  $a' = a - c^{(1)} - c^{(2)} - \dots - c^{(\tau)}$ ;
- 4) Compute  $m' = a' * f_p + \sum_{j=1}^{\tau} c^{(j)} * f_p \pmod{p}$ .

Note that the adjusting vectors  $c^{(1)}, c^{(2)}, \dots, c^{(\tau)}$  can guarantee that any value in  $a'$  is in  $(-\frac{q}{2}, \frac{q}{2})$ ; therefore the modular  $q$  operation in the first step of the original NTRU decryption is not needed. The plaintext message  $m$  can be correctly recovered in the last step because  $a' * f_p = p\phi * h * f * f_p + m * f * f_p - c^{(1)} * f_p - c^{(2)} * f_p - \dots - c^{(\tau)} * f_p = m * f * f_p - c^{(1)} * f_p - c^{(2)} * f_p - \dots - c^{(\tau)} * f_p \pmod{p}$ . The pseudo-code of the improved NTRU decryption is presented in Algorithm 1. Combining with the original NTRU encryption we obtain an improved NTRU cryptosystem that can successfully get rid of the two types of decryption failures.

**Theorem 1.** *The improved NTRU cryptosystem correctly overcomes the Wrap failure and the Gap failure in the decryption procedure of the original NTRU.*

**Proof.** According to [48] and our own analysis mentioned above, one can see that the root cause of the Gap failure and the Wrap failure in the original NTRU decryption is that  $|a_i| \geq q/2$  exists for some  $a_i$  in  $a = e * f$ . Our improved NTRU decryption procedure first computes  $a'$  from  $a$  to ensure that  $-\frac{q}{2} < a'_i < \frac{q}{2}$  for  $\forall a'_i \in a'$ . Then the original message  $m$  is recovered by computing  $a' * f_p + c^{(1)} * f_p + \dots + c^{(\tau)} * f_p \pmod{p}$ .

Considering the four steps of the improved NTRU decryption, we notice that the first three steps can guarantee that no Gap or Wrap failure occurs during the computation of  $a'$ . The last step recovers  $m$  from  $a'$  and the adjusting vectors  $\{c^{(1)}, c^{(2)}, \dots, c^{(\tau)}\}$ , which does not introduce the Gap failure and the Wrap failure. This completes the proof.  $\square$

**Theorem 2.** *The improved NTRU cryptosystem does not reduce the security strength of the original NTRU.*

**Proof.** Our improved NTRU cryptosystem consists of the original encryption and the proposed improved decryption algorithm. Thus, proving this theorem is equivalent to showing that our improved decryption cannot reduce the security strength of the original NTRU. This can be analyzed from the following two aspects. First, similar to the original NTRU, our improved NTRU is also based on the shortest vector problem in a lattice. Second, the improved decryption gets rid of the Gap failure and the Wrap failure to correctly recover the original message  $m$  without revealing any sensitive information as the decryptor computes the adjusting vectors and keeps them to itself, which implies that the improved decryption procedure is as secure as the original NTRU decryption. Therefore we claim that the improved NTRU cryptosystem does not reduce the security strength of the original NTRU.  $\square$

An instance of decryption failure of the original NTRU cryptosystem is shown in Appendix. We also demonstrate that the improved NTRU cryptosystem can successfully decrypt the ciphertext for the same example.

---

#### Algorithm 1. The Improved NTRU Decryption

---

```

1: Input: cipher text  $e$ , secret key  $\{f, f_p\}$ .
2: Output: plaintext  $m$ ;
3: The decryptor computes  $a = e * f$ ;
4:  $\Gamma = \max\{|\max_{0 \leq i \leq N-1}\{a_i\}|, |\min_{0 \leq i \leq N-1}\{a_i\}|\}$ ;
5:  $\tau = \lfloor \frac{\Gamma}{q/2} \rfloor$ ;
6: If  $\tau = 0$ 
7:    $m = a * f_p \pmod{p}$ .
8: Else
9:   For  $0 \leq i \leq N-1$ ,
10:    Compute  $\gamma = \lfloor \frac{|a_i|}{q/2} \rfloor$ ;
11:    If  $\gamma = 0$ 
12:       $a'_i = a_i$  and  $c_i^{(1)} = c_i^{(2)} = \dots = c_i^{(\tau)} = 0$ ;
13:    Else If  $a_i \geq 0$ 
14:       $a'_i = a_i - \frac{q-1}{2}\gamma$ ;
15:       $c_i^{(1)} = c_i^{(2)} = \dots = c_i^{(\gamma)} = \frac{q-1}{2}$ ;
16:       $c_i^{(\gamma+1)} = a'_i$ ;
17:       $c_i^{(\gamma+2)} = \dots = c_i^{(\tau)} = 0$ ;
18:    Else
19:       $a'_i = a_i + \frac{q-1}{2}\gamma$ ;
20:       $c_i^{(1)} = c_i^{(2)} = \dots = c_i^{(\gamma)} = -\frac{q-1}{2}$ ;
21:       $c_i^{(\gamma+1)} = a'_i$ ;
22:       $c_i^{(\gamma+2)} = \dots = c_i^{(\tau)} = 0$ ;
23:    EndIf
24:  EndFor
25:   $m' = a' * f_p + c^{(1)} * f_p + \dots + c^{(\tau)} * f_p \pmod{p}$ ;
26: EndIf
27: Output plaintext  $m'$ .

```

---

## 5 THE PROPOSED SCHEME

Let  $B$  be the set of users that are eligible to access the outsourced sensitive data in the cloud. Assume that  $U_i \in B$  is a user who needs to use the data. According to the access policy, at least  $t - 1$  other users in  $B$  should participate in the processes of verifying  $U_i$ 's eligibility and obtaining the data for  $U_i$ . To achieve this objective, we propose a secure and verifiable access control scheme for the big

data storage in a cloud server in this section. Our scheme consists of the following four stages: i) Setup: the data owner initializes the system to generate the public and private keys via the improved NTRU cryptosystem; ii) Construction: the data owner generates a sub-key for each user in  $B$ , produces a message certificate for each message, and stores the data securely in the cloud server; iii) Reconstruction: the user  $U_i$  and  $t - 1$  other users in  $B$  mutually verify each other and work together to help  $U_i$  reconstruct the data; iv) Policy update: the cloud server instead of the data owner updates the encrypted data with a new policy if needed.

### 5.1 Setup

A data owner has a set of messages  $S = \{S_1, S_2, \dots, S_M\}$  for cloud storage, with  $M$  being the total number of messages in  $S$ . At the setup stage, the data owner generates its public key  $h$  and private key  $f$  according to the improved NTRU cryptosystem and then initializes the system according to the process presented in Algorithm 2.

---

#### Algorithm 2. Initialization

---

```

1: Chooses three integer parameters  $(N, p, q)$  satisfying
    $\text{gcd}(p, q) = 1$  and  $q > p$ ;
2: Chooses four sets  $L_f, L_g, L_\phi, L_m$  of polynomials of degree
    $N - 1$  in  $R$  with integer coefficients;
3: Generates the key pair  $(f, h)$  according to the improved
   NTRU cryptosystem, where  $f$  is the private key and
    $h$  is the corresponding public key;
4: Selects two one-way hash functions  $H_1$  and  $H$ ;
5: Publishes  $\{p, q, h\}$  and the selection criteria of  $\{L_\phi, L_m\}$ .

```

---

### 5.2 Construction

The data owner constructs a sub-key for each legitimate user in  $B$ , generates a certificate for each message in  $S$ , and stores the encrypted data into the cloud server.

#### 5.2.1 Sub-Key Construction

The data owner randomly generates  $t$  different integers  $b_0, b_1, \dots, b_{t-1}$ , where  $b_j \in Z[X]/(X^N - 1)$  for  $j = 0, 1, \dots, t - 1$ , and uses them as coefficients to construct the following  $t - 1$  degree polynomial  $b(x)$

$$b(x) = b_0 + \sum_{j=1}^{t-1} b_j x^j. \quad (10)$$

Each user  $U_i$  chooses a random integer  $r_i$ , encrypts  $r_i$  using the data owner's public key  $h$  to get the ciphertext  $v_i = p\phi * h + r_i \pmod{q}$ , and then sends  $\{id_i, H(r_i), v_i\}$  to the data owner.

When the data owner receives the ciphertext  $v_i$ , it decrypts it using its private key  $f$  to get the plaintext  $r'_i$ . Then the data owner checks the following two conditions: i)  $H(r'_i) = H(r_i)$  for the user  $U_i$ ; and ii)  $r_i \neq r_\sigma$  for the user  $U_i$  and any user  $U_\sigma$  who has received a sub-key from the data owner. If condition i) cannot hold, the message is falsified during transmission and it could be sent again; if condition ii) cannot hold, the data owner requests the user  $U_i$  to choose a different secret number and repeat the procedure to compute a sub-key for  $U_i$ .

If the two conditions hold true for  $U_i$ , the data owner calculates  $H(id_i||r_i)$  and generates the sub-key  $x_i$  for  $U_i$  using (11). Then the data owner securely broadcasts  $\{id_i, x_i, H(id_i||r_i)\}$  to all the users in  $B$

$$x_i = b(r_i) = \sum_{j=0}^{t-1} b_j \cdot (r_i)^j. \quad (11)$$

### 5.2.2 Message Certificate Construction

In order to validate the users in  $B$  for their legitimacy of accessing the data in  $S$  and to prevent against the cheating behaviors of the users, the data owner computes a certificate for each message in  $S$  according to the following procedure: the data owner first randomly chooses the parameters  $\phi \in L_\phi$  and  $e = \{e_1, \dots, e_j, \dots, e_M\}$ , where  $e_j \in Z[X]/(X^N - 1)$ ; then it generates the certificate  $(e_j, d_j)$  for message  $S_j \in S$  by (12), where  $1 \leq j \leq M$ ,

$$d_j = p\phi * f + e_j \pmod{q}, \quad (12)$$

and finally, the data owner publishes  $e = \{e_1, e_2, \dots, e_j, \dots, e_M\}$  to all users in  $B$ .

### 5.2.3 Data Encryption

Before outsourcing the data to the cloud server, the data owner computes a set of ciphertexts  $K = \{k_1, k_2, \dots, k_j, \dots, k_M\}$  for the messages in  $S = \{S_1, S_2, \dots, S_j, \dots, S_M\}$  according to (13), where  $k_j$  is the ciphertext of  $S_j$

$$k_j = S_j \oplus H(b_0 * d_j). \quad (13)$$

Then the data owner keeps  $H_1(S_j)$  to itself and stores  $k_j$ ,  $j = 1, 2, \dots, M$ , in the cloud server.

The construction operations in Sections 5.2.1, 5.2.2, and 5.2.3 are shown in Algorithm 3.

## 5.3 Message Reconstruction

Assume that a user  $U_i \in B$  needs to get the message/data  $S_j$ . It downloads  $k_j$  from the cloud server and then seeks help from other users in  $B$  to decrypt  $k_j$ . This procedure consists of the following three stages: i) Exchange certificate computation:  $U_i$  gets  $S_j$ 's message certificate from the data owner and computes an exchange certificate. ii) Certificate verification: other users in  $B$  and  $U_i$  mutually validate each other. iii) Message reconstruction:  $U_i$  reconstructs the message  $S_j$ .

### 5.3.1 Exchange Certificate Computation

$U_i$  sends a request to the data owner to obtain  $S_j$ 's message certificate  $d_j$ . Upon receiving this request, the data owner encrypts  $d_j$  using  $U_i$ 's secret number  $r_i$  based on AES as shown in follows:

$$C_{d_j} = AES_{r_i}(d_j). \quad (14)$$

Upon receiving the ciphertext  $C_{d_j}$  from the data owner,  $U_i$  first decrypts  $C_{d_j}$  to obtain  $d_j$ . Then, it uses its sub-key  $x_i$  to compute the exchange certificate  $W_{ij}$  via (15) and sends  $W_{ij}$  to other users in  $B$

$$W_{ij} = x_i * d_j. \quad (15)$$

### Algorithm 3. Construction

---

```

1: The data owner generates a polynomial  $b(x)$  according to
   (10) in Section 5.2.1;
2: For each user  $U_i$  in  $B$ 
3:   Encrypts its selected number  $r_i$  using the data owner's
      public key  $h$  to get  $v_i = p\phi * h + r_i \pmod{q}$ ;
4:   Computes  $H'' = H(r_i)$  and sends  $\{id_i, H'', v_i\}$  to the data
      owner;
5: The data owner decrypts  $v_i$  to get  $r_i$  using Algorithm 1;
6: If  $H'' = H(r_i)$ 
7:   If  $r_i \neq r_\sigma$  for any  $U_\sigma$  that has received a sub-key  $x_\sigma$ 
8:     The data owner generates the sub-key  $x_i$  using (11);
9:     The data owner broadcasts  $\{id_i, H(id_i||r_i), x_i\}$  to all
   users in  $B$ ;
11: Else
12:   The data owner requests  $U_i$  to choose a different
      random number  $r_i$  and then go back to step 3;
14: EndIf
15: Else
16:   The message is falsified;
17:   Retransmits  $\{id_i, H'', v_i\}$  to the data owner;
18:   Go back to step 5;
19: EndIf
20: EndFor
21: The data owner chooses parameters  $\phi \in L_\phi$  and
    $e = (e_1, e_2, \dots, e_j, \dots, e_M)$ ;
22: For each message  $S_j \in S$ 
23:   The data owner generates a message certificate  $(e_j, d_j)$ 
      for  $S_j$  by (12);
24:   The data owner encrypts the data  $S_j$  by (13) to get  $k_j$ 
      and computes  $H_1(S_j)$ ;
25:   The data owner publishes  $e_j$  and stores  $k_j$  in the cloud
      server.
26: EndFor

```

---

### 5.3.2 Certificate Verification

When a user  $U_\sigma$  in  $B$  receives  $W_{ij}$ , it uses the public  $e_j$  to verify  $W_{ij}$  according to (16). If (16) holds,  $U_i$  is verified by  $U_\sigma$ , which implies that  $U_i$  is a valid user of the message  $S_j$  in  $U_\sigma$ 's perspective. Then  $U_\sigma$  provides its  $\{id_\sigma, r_\sigma\}$  to  $U_i$

$$W_{ij} = x_i * e_j. \quad (16)$$

When  $U_i$  receives  $U_\sigma$ 's  $\{id_\sigma, r_\sigma\}$ , it computes  $H' = H(id_\sigma||r_\sigma)$ . Since  $U_i$  holds the  $H(id||r)$  values of all users in  $B$ , it can verify whether the  $r_\sigma$  value provided by  $id_\sigma$  is correct. If  $r_\sigma$  does not pass the verification,  $U_\sigma$ 's sub-key cannot be used by  $U_i$  for the reconstruction of  $S_j$ , which implies that  $U_\sigma$  has a cheating behavior in the perspective of  $U_i$ .

At least  $t - 1$  users in  $B$  should be able to verify the legitimacy of  $U_i$  for accessing  $S_j$  and these users must be verifiable by  $U_i$  in order for  $U_i$  to use their provided sub-keys for the reconstruction of  $S_j$ .

### 5.3.3 Message Reconstruction

When the user  $U_i$  obtains authorizations from  $t - 1$  legal users in  $B$ , it recovers the message  $S_j$  according to

$$S_j = k_j \oplus H \left( \left( \sum_{U_i \in B} x_i \prod_{U_\sigma \in B, U_\sigma \neq U_i} \frac{r_\sigma}{r_\sigma - r_i} \right) * d_j \right). \quad (17)$$

The processes of reconstruction and verification are summarized in Algorithm 4.

#### Algorithm 4. Reconstruction and Verification

---

```

1: The user  $U_i$  downloads  $k_j$  from the cloud server, and sends
   a request to the data owner to obtain  $d_j$ ;
2: The data owner encrypts  $d_j$  with  $r_i$  to obtain ciphertext
    $C_{d_j} = AES_{r_i}(d_j)$ , and then sends  $C_{d_j}$  to  $U_i$ ;
3:  $U_i$  decrypts  $C_{d_j}$  to get  $d_j$  using its secret number  $r_i$ ;
4:  $U_i$  computes the exchange certificate  $W_{ij}$  via (15), and sends
    $W_{ij}$  to other users in  $B$ ;
5: For each user  $U_\sigma$  in  $B$  Do
6:   Upon receiving  $W_{ij}$ ,  $U_\sigma$  verifies  $W_{ij}$  by (16);
7:   If (16) holds
8:      $U_\sigma$  sends its  $\{id_\sigma, r_\sigma\}$  to  $U_i$ ;
9:     Upon receiving  $\{id_\sigma, r_\sigma\}$ ,  $U_i$  computes  $H(id_\sigma || r_\sigma)$ 
10:    to verify  $\{id_\sigma, r_\sigma\}$ ;
11:    If  $H(id_\sigma || r_\sigma)$  passes the verification
12:       $U_\sigma$  participates in the reconstruction of  $S_j$ ;
13:    EndIf
14:  EndIf
15: EndFor
16: If at least  $t - 1$  other users in  $B$  are able to participate in the
   recovery of  $S_j$ 
17:    $U_i$  can reconstruct  $S_j$  via (17);
18: EndIf

```

---

#### 5.4 Policy Update

The data owner can dynamically control the access of its data by updating the access policy defined for the data. Intuitively, as the ciphertext of the data is stored in the cloud server, the data owner needs to download the ciphertext, decrypt it to get the plaintext, and then re-encrypt the plaintext according to the new access policy. This is the approach adopted by all existing research. In this section, we consider to update the encrypted data at the cloud server based on the new access policy, which differs significantly than the traditional approaches.

The new access policy is defined by  $b'(x) = b'_0 + \sum_{j=0}^{t'-1} b'_j x_j$ , where  $b'_0, b'_1, \dots, b'_{t'-1}$  are the  $t'$  coefficients of the new polynomial  $b'(x)$  selected by the data owner. The data owner needs to execute the procedures in Sections 5.2.1 and 5.2.2 to generate a new sub-key  $x'_i$  for each user  $U_i \in B$ , and compute a new message certificate  $d'_j$  for each  $S_j \in S$ . Then the data owner generates the intermediate value  $k'_{iv}$  based on the values of  $b_0, d_j, b'_0$ , and  $d'_j$  by

$$k'_{iv} = H(b_0 * d_j) \oplus H(b'_0 * d'_j). \quad (18)$$

The data owner  $D$  sends  $k'_{iv}$  to the cloud server, which will update the encrypted data  $k_j$  via

$$k'_j = k_j \oplus k'_{iv}. \quad (19)$$

After that the cloud server sends  $k'_j$  back to the data owner to verify whether the server has successfully update the ciphertext constructed from the new access policy. This procedure can be done by the data owner according to

$$H_1(S_j) = H_1(k'_j \oplus H(b'_0 * d'_j)). \quad (20)$$

**Remark.** The cloud server can update the ciphertext when a new access policy is specified by the data owner, and this update can be verified by the data owner; meanwhile, each user in  $B$  can easily update its secret number and the corresponding sub-key.

## 6 ANALYSIS OF THE PROPOSED SCHEME

In this section, we conduct a rigorous analysis on the correctness, security strengths, and computational complexity of the proposed scheme.

### 6.1 Correctness

**Theorem 3.** A user  $U_i \in B$  who needs the data  $S_j$  and each of the other users in  $B$  can verify each other to help  $U_i$  recover the message  $S_j$ .

**Proof.** After receiving  $d_j$  from the data owner,  $U_i$  generates its exchange certificate  $W_{ij} = x_i * d_j$  and sends  $W_{ij}$  to the other users in  $B$ . Any user  $U_\sigma \in B$  can verify the exchange certificate  $W_{ij}$  by computing  $x_i * e_j$  and checking whether it is equal to  $W_{ij}$ . If  $W_{ij}$  does equal  $x_i * e_j$ ,  $U_\sigma$  concludes that  $U_i$  holds the correct message certificate  $d_j$ , which implies that  $U_i$  is the legitimate user of  $S_j$ . Such a verification process is supported by the following derivation:

$$\begin{aligned} W_{ij} &= x_i * d_j \\ &= x_i * (p\phi * f + e_j) \bmod p \\ &= x_i * e_j. \end{aligned}$$

If  $U_\sigma$  verifies  $U_i$  to be a legitimate user of  $S_j$ , it sends its  $\{id_\sigma, r_\sigma\}$  to  $U_i$ . After receiving  $\{id_\sigma, r_\sigma\}$ ,  $U_i$  calculates  $H' = H(id_\sigma || r_\sigma)$  and checks against its local copy of  $H(id_\sigma || r_\sigma)$ —recall that the data owner broadcasts the  $H(id || r)$  value of each user in  $B$  to all other users in  $B$  during the sub-key construction stage. If they are equal,  $U_\sigma$  provides the correct  $r_\sigma$  for the recovery of  $S_j$ ; otherwise,  $U_\sigma$  cheats and its  $r_\sigma$  value will not be used for the reconstruction of  $S_j$ .  $\square$

**Theorem 4.** A user  $U_i$  in  $B$  can reconstruct a message  $S_j$  via (17) if there exist at least  $t - 1$  other users in  $B$  satisfying the following two conditions: i) each of the  $t - 1$  users can verify that  $U_i$  is a legitimate user of  $S_j$ ; and ii) each of the  $t - 1$  users can be verified by  $U_i$  to provide the correct sub-key information.

**Proof.** Without loss of generality, we denote by  $U_1, U_2, \dots, U_{t-1}$  the  $t - 1$  users with each being able to verify the legitimacy of  $U_i$  and each being verified by  $U_i$  for providing the correct  $r$  values. This implies that  $U_i$  holds the correct message certificate  $d_j$  and the correct  $\{id_\sigma, r_\sigma\}$  of user  $U_\sigma$ , where  $\sigma = 1, 2, \dots, t - 1$ . Thus  $U_i$  can obtain the following result:

$$\begin{aligned} &\left( \sum_{U_i \in B} x_i \prod_{U_\sigma \in B, U_\sigma \neq U_i} \frac{r_\sigma}{r_\sigma - r_i} \right) * d_j \\ &= \left( \sum_{U_i \in B} x_i \prod_{U_\sigma \in B, U_\sigma \neq U_i} \frac{r_\sigma}{r_\sigma - r_i} \right) * d_j \\ &= \left( \sum_{U_i \in B} b(r_i) \prod_{U_\sigma \in B, U_\sigma \neq U_i} \frac{r_\sigma}{r_\sigma - r_i} \right) * d_j. \end{aligned}$$

Since

$$b_0 * d_j = b(0) * d_j = \left( \sum_{U_i \in B} b(r_i) \prod_{U_\sigma \in B, U_\sigma \neq U_i} \frac{r_\sigma}{r_\sigma - r_i} \right) * d_j.$$

We have  $b_0 * d_j = (\sum_{U_i \in B} x_i \prod_{U_\sigma \in B, U_\sigma \neq U_i} \frac{r_\sigma}{r_\sigma - r_i}) * d_j$ . Thus  $S_j = k_j \oplus H(b_0 * d_j)$ , which implies that the message  $S_j$  can be correctly reconstructed by  $U_i$  via (17).  $\square$

**Theorem 5.** *The cloud server instead of the data owner can correctly update the access policy of a message without any decryption process. The data owner can verify the correctness of the update procedure.*

**Proof.** According to Section 5.4, the data owner re-generates the  $t'$  integers  $\{b'_0, b'_1, \dots, b'_{t'-1}\}$  that are used as the coefficients of the polynomial  $b'(x)$ , the new access policy, and computes the intermediate value  $k_{iv}^j = H(b_0 * d_j) \oplus H(b'_0 * d'_j)$  based on the values of  $b_0, d_j, b'_0$ , and  $d'_j$  by (18). Then the data owner sends the intermediate value  $k_{iv}^j$  to the cloud server, which updates the encrypted data in the cloud via (19) as follows:

$$\begin{aligned} k'_j &= k_j \oplus k_{iv}^j \\ &= S_j \oplus H(b_0 * d_j) \oplus k_{iv}^j \\ &= S_j \oplus H(b_0 * d_j) \oplus H(b_0 * d_j) \oplus H(b'_0 * d'_j) \\ &= S_j \oplus H(b'_0 * d'_j). \end{aligned}$$

Then, the cloud server sends  $k'_j$  back to the data owner who can verify the correctness of the policy update result via (20).  $\square$

## 6.2 Security Analysis

The security of our scheme is based on the NTRU public key cryptosystem [26] and the Shamir's secret sharing scheme [45]. In this section, we analyze the security strength of our proposed scheme by examining how it can defend against several major attacks.

### 6.2.1 Attack Resistance

**Attack 1.** *A plotter  $E$  (not in  $B$ ) may try to reveal the message  $S_j$  from  $k_j$  without knowing any information about  $t$  users.*

**Analysis.** If the plotter  $E$  intends to recover the message  $S_j$  from  $k_j$  by computing  $k_j \oplus H(\sum_{U_i \in B} x_i \prod_{U_\sigma \in B, U_\sigma \neq U_i} \frac{r_\sigma}{r_\sigma - r_i} * d_j)$ , it has to (i) get  $d_j$  from the data owner of  $S_j$  and pass the verification by at least  $t$  users in  $B$ ; and (ii) obtain the secret numbers  $\{r_1, r_2, \dots, r_t\}$  from the  $t$  users. To get  $d_j$  from the data owner,  $E$  needs a shared secret with the data owner to prove that it is a legal user of  $S_j$ ; to get  $r_\sigma$  from  $U_\sigma$ ,  $E$  needs to pass the verification by  $U_\sigma$ , which again needs  $d_j$  for the computation of the exchange certificate. As sharing a secret with the data owner is a proof of legitimacy, it is impossible for  $E$  to get  $d_j$ ; thus the plotter  $E$  cannot recover  $S_j$ .

**Attack 2.** *The users in  $B$  may cheat to fail data recovery.*

**Analysis.** The cheating behaviors can be prevented during data recovery according to Theorem 3. Let  $U_i$  be the user who would like to get  $S_j$ . (i) If  $U_i$  cheats, it provides a wrong exchange certificate  $W_{ij}$ , which can be identified by any

other honest user in  $B$  when verifying the legitimacy of  $U_i$  on  $S_j$ ; (ii) If  $U_\sigma$  provides an incorrect secret number  $r_\sigma$  to a legal user  $U_i$  for the recovery of  $S_j$ , it can be detected because  $U_i$  holds a local copy of the correct  $H(id_\sigma || r_\sigma)$ , which is broadcast by the data owner during the sub-key construction process. Therefore, our proposed scheme can prevent users from cheating.

**Attack 3.** *The users in  $B$  may collude.*

**Analysis.** Our scheme can resist the following type of collusion attack:  $t - 1$  or less number of users in  $B$  collude to recover the message  $S_j$ . From (13), we know that the users should obtain  $b_0$  and  $d_j$  in order to decipher the message. Nevertheless, since the security of our proposed scheme is guaranteed by  $(t, n)$ -threshold secret sharing, it is impossible for any  $t - 1$  or less number of users to obtain  $b_0$ .

**Attack 4.** *An attacker intends to forge  $W_{ij}$ .*

**Analysis.** In order to forge  $W_{ij}$ , which is defined to be  $x_i * (p\phi * f + e_j)$  according to (12) and (15), the attacker has to obtain  $f$ , the private key of the data owner. However, it is impossible for the attacker to get  $f$  as  $f$  is generated according to the NTRU cryptosystem, which is proved to be secure.

**Theorem 6.** *The data owner can only decrypt its own ciphertexts stored in the cloud server.*

**Proof.** Although a data owner can obtain any ciphertext from the cloud server, it cannot decrypt the ciphertext encrypted by other data owners. This claim can be proved as follows.

Without loss of generality, assume the data owner  $D_i$  downloads the ciphertext  $k'_{D_j}$  generated by the data owner  $D_j$ , where  $k'_{D_j} = S'_{D_j} \oplus H(b_0^{D_j} * d_j^{D_j})$ . To obtain the plaintext  $S'_{D_j}$ ,  $D_i$  needs to compute  $H(b_0^{D_j} * d_j^{D_j})$ . However, the values of  $b_0^{D_j}$  is securely chosen by  $D_j$ , which can be recovered only by  $t$  or more legal users of  $S'_{D_j}$  based on secret-sharing, and the value of  $d_j^{D_j}$  is computed by  $D_j$  from  $e_j^{D_j}$ , which can be securely obtained from  $D_j$  only when a legal user of  $S'_{D_j}$  requests from  $D_j$  and proves its legitimacy to  $D_j$  via the shared secret between the user and  $D_j$ ; therefore  $D_i$  cannot figure out the correct  $b_0^{D_j}$  and  $d_j^{D_j}$  as it cannot prove its legitimacy with respect to the data  $S'_{D_j}$ . Thus a data owner can only decrypt its own ciphertexts stored in the cloud server.  $\square$

## 6.3 Performance Analysis

### 6.3.1 Computational Complexity

In this section, we analyze the computational complexity of our proposed scheme and compare it with those of the three schemes proposed in [15] and [16]. Note that our analysis will be focused on the processes of certificate verification and message reconstruction as other processes incur very low computational overhead that can be ignored. The two schemes Scheme-I and Scheme-II proposed in [15] and the scheme presented in [16] are briefly introduced in Section 2.2.

**Certificate Verification.** The verification processes of Scheme-I and Scheme-II in [15] and the scheme in [16] are based on the RSA cryptosystem. Since the computational

TABLE 2  
The Comparisons Among [15]'s Scheme-I and Scheme-II,[16] Scheme and Our Scheme

Properties	Scheme-I [15]	Scheme-II [15]	[16]	Our scheme
Prevent users from cheating by providing fake information	Yes	Yes	Yes	Yes
Prevent the collusion of $t - 1$ or fewer users in message recovery	Yes	Yes	Yes	Yes
Methods for establishing a secure channel	RSA	RSA	RSA	AES
Resist quantum computing attacks	No	No	No	Yes
Resist collusion attack	Yes	Yes	Yes	Yes
Dynamically update the access policy	No	No	No	Yes
Protect previously encrypted data	No	No	No	Yes
Dynamically update the ciphertext for user join and leave activities	No	No	No	Yes
Computational complexity of Verification	$O(N^3)$	$O(N^3)$	$O(N^3)$	$O(N \log N)$
Computational complexity of Reconstruction	$O(N)$	$O(N)$	$O(N)$	$O(N \log^2 N)$
NP-Hard problems for security guarantee	DFL	DFL	DFL	SVP

complexity of the RSA encryption is  $O(N^3)$ , the computational complexities of these three schemes are also  $O(N^3)$ . Our proposed scheme is based on the NTRU cryptosystem whose security is based on the SVP hardness in lattice. Its encryption process only requires add and shift operations without involving any multiplication. By employing the Fast Fourier transform (FFT), the encryption and decryption of NTRU can be performed within  $O(N \log N)$ .

*Message Reconstruction.* In [15] and [16], recovering a plaintext message involves a linear computational complexity of  $O(N)$ . In our scheme, the message is recovered by a Lagrange interpolation polynomial with a computational complexity of  $O(N \log^2 N)$ , which is slightly larger than those of the schemes in [15] and [16].

**Remark.** The schemes in [15] and [16] and our proposed scheme all require a secure channel to transmit the messages. However, the secure channels of [15] and [16] are established by the asymmetric cryptosystem RSA, while our scheme employs the symmetric cryptosystem AES, which has a much lower computational overhead.

### 6.3.2 Dynamic Update

Our proposed scheme allows the dynamic update (refresh, insert, delete) of the information regarding the users and the messages.

- *Insert a New Message.* When a data owner needs to store a new message  $S_{new}$  into the cloud server, it generates the certificate  $(e_{new}, d_{new})$  for  $S_{new}$  by (12), encrypts  $S_{new}$  to obtain the ciphertext  $k_{new}$  via (13), computes  $H_1(S_{new})$ , and then stores  $k_{new}$  in the cloud server.
- *Delete an Old Message.* When a data owner needs to delete a message  $S_{del}$ , it sends an authenticated request to the cloud server. Upon receiving the request, the cloud server deletes the ciphertext of  $S_{del}$ .
- *A New User Joins.* When a new user  $U_{new}$  joins the system, the data owner first verifies its legitimacy of accessing its data; then it computes the sub-key  $x_{new}$  for the legal user  $U_{new}$  according to (11), and publishes  $x_{new}$  to all users in  $B$ .
- *Remove a User.* When a data owner wants to prevent a user in  $B$  from accessing its data, it updates its access policy according to the policy update procedure proposed in Section 5.4 and computes a new sub-key for each of the other users in  $B$ .

### 6.3.3 Summary

In Table 2, we present a comprehensive comparison study over Scheme-I and Scheme-II in [15], the scheme in [16], and our proposed scheme in terms of attack resistance, security, policy update, and so on. From Table 2, one can see that the significant advantages of our scheme over the other three schemes include: quantum computing attack resistance, dynamic policy update, and lower computational complexity.

## 6.4 Discussions

In our scheme, a data owner maintains the access right to its encrypted data. The advantages of such a policy are listed as follows:

- The data owner splits the access right of the encrypted data into  $n$  pieces, with each legitimate user holding one piece of the access right. This can effectively reduce the risk of information leakage in big data storage.
- If a user  $U_i$  wants to decrypt the data owner's encrypted data  $S_j$ , its legitimacy must be verified by the data owner (by securely and successfully retrieving the message certificate  $d_j$  from the data owner) and at least  $t - 1$  other legitimate users of the data (via the exchange certificate); the information provided by the  $t - 1$  other users for the message recovery also must be verified by  $U_i$  to prevent the user from providing fake information. This verification process can ensure the legality of  $U_i$  to access  $S_j$ .
- The cloud sever only stores the outsourced ciphertext data—it cannot obtain the data owner's original plaintext data. In addition, the data owner updates the access policy while the cloud server updates the encrypted data. No data owner can access the plaintext data outsourced by other data owners in the cloud server.

Compared with RSA, at an equivalent cryptographic strength, NTRU performs the costly encryption and decryption operations at a much faster speed, and its implementations in both hardware and software are easier and require a smaller size of memory; therefore NTRU can be employed in applications involving devices with limited computation power and storage such as mobile devices and smart-cards. On the other hand, when our scheme is utilized in real world applications, an adequate threshold  $t$  needs to be

selected by the data owner before deployment because an unsuitable threshold  $t$  would decrease the security of the data if  $t$  is too small or have a harmful effect on the flexibility of access control if  $t$  is too large. Nevertheless, the selection of a right value for  $t$  is not easy. One possible remedy approach is to pick up an initial value of  $t$  based on experience and adjust it via access policy update as time goes.

## 7 CONCLUSION AND FUTURE WORKS

In this paper, we first propose an improved NTRU cryptosystem to overcome the decryption failures of the original NTRU and then present a secure and verifiable access control scheme based on the improved NTRU to protect the outsourced big data stored in a cloud. Our scheme allows the data owner to dynamically update the data access policy and the cloud server to successfully update the corresponding outsourced ciphertext to enable efficient access control over the big data in the cloud. It also provides a verification process for a user to validate its legitimacy of accessing the data to both the data owner and  $t - 1$  other legitimate users and the correctness of the information provided by the  $t - 1$  other users for plaintext recovery. The security of our proposed scheme is guaranteed by those of the NTRU cryptosystem and the  $(t, n)$ -threshold secret sharing. We have rigorously analyzed the correctness, security strength, and computational complexity of our proposed scheme.

Designing a secure, privacy preserving, and practical scheme for big data storage in a cloud is an extremely challenging problem. In our future research, we will further improve our scheme by combining the  $(t, n)$ -threshold secret sharing with attribute-based access control, which involves an access structure that can place various requirements for a user to decrypt an outsourced ciphertext data in the cloud. Meanwhile, we will investigate the security problems when a data owner outsources its data to multi-cloud servers and consider an attribute-based access structure that can be dynamically updated, which is more applicable for practical scenarios in big data storage.

## APPENDIX

In this appendix, we give an instance of the NTRU decryption failure of the original NTRU cryptosystem. Meanwhile, we demonstrate how our improved NTRU can correct the failure. The NTRU system parameters are  $N = 107, p = 3, q = 64$ .

*Secret key:*

$$f = \begin{bmatrix} -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & -1 & -1 \\ -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$f_p = \begin{bmatrix} 2 & 2 & 1 & 1 & 1 & 0 & 2 & 0 & 2 & 0 \\ 2 & 1 & 0 & 2 & 2 & 0 & 0 & 2 & 2 & 1 \\ 1 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 2 & 2 \\ 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 2 & 2 & 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 2 & 2 & 2 & 2 & 1 & 0 & 0 & 2 & 2 \\ 2 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 2 & 1 & 1 & 0 & 2 & 2 & 1 & 0 \\ 1 & 0 & 1 & 2 & 0 & 0 & 2 & 0 & 0 & 1 \\ 1 & 1 & 2 & 2 & 2 & 2 & 1 & & & \end{bmatrix}$$

$$f_q = \begin{bmatrix} 59 & 5 & 41 & 10 & 34 & 28 & 31 & 13 & 5 & 36 \\ 0 & 21 & 36 & 28 & 18 & 41 & 46 & 13 & 38 & 5 \\ 31 & 7 & 1 & 49 & 26 & 37 & 27 & 12 & 14 & 58 \\ 60 & 9 & 23 & 7 & 17 & 41 & 6 & 29 & 38 & 12 \\ 54 & 39 & 52 & 9 & 36 & 19 & 47 & 29 & 44 & 26 \\ 40 & 30 & 52 & 60 & 25 & 52 & 10 & 61 & 7 & 7 \\ 48 & 21 & 14 & 46 & 55 & 51 & 7 & 16 & 49 & 29 \\ 27 & 3 & 60 & 10 & 12 & 8 & 24 & 61 & 7 & 39 \\ 2 & 54 & 47 & 34 & 4 & 47 & 52 & 17 & 41 & 60 \\ 25 & 44 & 20 & 62 & 55 & 33 & 52 & 58 & 10 & 63 \\ 63 & 7 & 58 & 42 & 52 & 44 & 49 & & & \end{bmatrix}.$$

*Public Key:*

$$h = \begin{bmatrix} 28 & 63 & 13 & 41 & 31 & 6 & 30 & 54 & 45 & 37 \\ 63 & 41 & 33 & 58 & 32 & 62 & 13 & 62 & 18 & 59 \\ 44 & 25 & 12 & 40 & 49 & 7 & 41 & 8 & 2 & 31 \\ 10 & 7 & 10 & 10 & 34 & 47 & 4 & 7 & 27 & 47 \\ 32 & 46 & 58 & 54 & 36 & 48 & 38 & 51 & 7 & 45 \\ 26 & 22 & 35 & 45 & 53 & 9 & 19 & 63 & 50 & 38 \\ 5 & 52 & 14 & 25 & 60 & 61 & 50 & 37 & 20 & 2 \\ 36 & 11 & 20 & 36 & 18 & 11 & 54 & 23 & 36 & 6 \\ 45 & 26 & 11 & 20 & 41 & 45 & 59 & 31 & 21 & 13 \\ 9 & 8 & 48 & 25 & 9 & 57 & 45 & 0 & 29 & 60 \\ 12 & 50 & 14 & 22 & 1 & 55 & 33 & & & \end{bmatrix}.$$

*Plaintext m:*

$$m = \begin{bmatrix} 1 & 2 & 1 & 2 & 0 & 2 & 0 & 0 & 2 & 0 \\ 1 & 2 & 0 & 2 & 2 & 2 & 0 & 2 & 1 & 0 \\ 0 & 2 & 2 & 1 & 2 & 2 & 2 & 2 & 0 & 2 \\ 2 & 2 & 1 & 2 & 2 & 2 & 2 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 2 & 2 \\ 2 & 1 & 0 & 2 & 0 & 0 & 2 & 1 & 0 & 2 \\ 2 & 2 & 0 & 0 & 0 & 2 & 0 & 2 & 2 & 0 \\ 2 & 2 & 1 & 0 & 2 & 1 & 2 & 0 & 0 & 0 \\ 2 & 0 & 0 & 2 & 2 & 2 & 2 & 1 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 0 & 0 & 2 & 2 & 2 \\ 1 & 2 & 2 & 0 & 2 & 0 & 2 & & & \end{bmatrix}$$

Polynomial  $\phi$ :

$$\phi = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Ciphertext  $e$ :

$$e = \begin{bmatrix} 29 & 45 & 0 & 46 & 9 & 58 & 40 & 23 & 3 & 13 \\ 26 & 11 & 62 & 31 & 23 & 4 & 9 & 18 & 12 & 22 \\ 41 & 46 & 24 & 0 & 21 & 63 & 43 & 39 & 32 & 1 \\ 27 & 29 & 29 & 47 & 39 & 10 & 32 & 60 & 19 & 48 \\ 32 & 22 & 42 & 28 & 19 & 15 & 10 & 18 & 54 & 50 \\ 29 & 42 & 35 & 57 & 57 & 29 & 44 & 26 & 28 & 57 \\ 38 & 58 & 33 & 3 & 47 & 28 & 21 & 29 & 41 & 1 \\ 23 & 4 & 62 & 53 & 17 & 57 & 45 & 47 & 59 & 39 \\ 42 & 22 & 18 & 23 & 61 & 60 & 11 & 54 & 31 & 56 \\ 36 & 32 & 58 & 5 & 59 & 25 & 12 & 47 & 26 & 8 \\ 22 & 46 & 56 & 49 & 20 & 60 & 51 & & & \end{bmatrix}.$$

The decryption result of the original NTRU cryptosystem  $m'$ :

$$m' = \begin{bmatrix} 2 & 0 & 1 & 2 & 1 & 2 & 0 & 0 & 2 & 2 \\ 2 & 1 & 2 & 0 & 0 & 2 & 2 & 0 & 0 & 2 \\ 0 & 2 & 0 & 2 & 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 2 & 2 & 1 & 0 & 1 \\ 2 & 0 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 0 & 2 & 2 & 2 & 2 & 1 & 1 \\ 2 & 1 & 0 & 2 & 1 & 2 & 0 & 0 & 2 & 0 \\ 1 & 1 & 0 & 1 & 0 & 2 & 0 & 2 & 1 & 1 \\ 1 & 2 & 2 & 2 & 0 & 2 & 0 & 1 & 0 & 1 \\ 2 & 0 & 0 & 2 & 2 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 2 & 2 & 0 & & & \end{bmatrix}.$$

Obviously,  $m' \neq m$ , which indicates that a decryption failure occurs in the original NTRU decryption. Next we analyze the cause of the decryption failure. The process of decryption based on the original NTRU is as follows:

$$a^{\text{exf}} = \begin{bmatrix} 1 & 11 & 1 & -4 & 5 & 2 & -5 & -4 & 6 & 10 \\ 12 & 0 & 4 & -10 & 1 & -3 & -7 & -1 & 3 & -1 \\ 1 & 6 & -5 & -9 & -9 & 1 & 6 & 1 & 3 & 8 \\ 3 & -15 & 2 & 2 & -1 & 14 & 1 & -4 & -15 & -2 \\ -7 & -6 & 3 & -8 & -6 & -8 & 6 & 1 & 12 & 5 \\ -2 & -5 & -4 & 8 & 0 & -8 & 0 & -10 & 8 & -9 \\ 0 & -5 & -5 & -6 & 2 & 7 & -6 & 10 & 2 & -2 \\ 11 & -3 & 12 & 7 & 8 & -8 & 8 & 33 & 8 & 7 \\ 15 & 6 & 5 & -2 & -3 & -1 & 4 & -3 & 10 & -6 \\ 1 & 11 & 10 & 1 & 6 & 6 & 5 & 6 & 4 & -1 \\ -2 & 6 & -4 & 5 & 10 & 10 & -4 & & & \end{bmatrix}.$$

One can see that one element of  $a$  is 33, which is larger than  $\frac{q}{2} = 32$  and causes the decryption failure. Now we utilize the improved NTRU cryptosystem to decrypt  $e$ . One can obtain  $a'$  as follows:

$$a' = \begin{bmatrix} 1 & 11 & 1 & -4 & 5 & 2 & -5 & -4 & 6 & 10 \\ 12 & 0 & 4 & -10 & 1 & -3 & -7 & -1 & 3 & -1 \\ 1 & 6 & -5 & -9 & -9 & 1 & 6 & 1 & 3 & 8 \\ 3 & -15 & 2 & 2 & -1 & 14 & 1 & -4 & -15 & -2 \\ -7 & -6 & 3 & -8 & -6 & -8 & 6 & 1 & 12 & 5 \\ -2 & -5 & -4 & 8 & 0 & -8 & 0 & -10 & 8 & -9 \\ 0 & -5 & -5 & -6 & 2 & 7 & -6 & 10 & 2 & -2 \\ 11 & -3 & 12 & 7 & 8 & -8 & 8 & 31 & 8 & 7 \\ 15 & 6 & 5 & -2 & -3 & -1 & 4 & -3 & 10 & -6 \\ 1 & 11 & 10 & 1 & 6 & 6 & 5 & 6 & 4 & -1 \\ -2 & 6 & -4 & 5 & 10 & 10 & -4 & & & \end{bmatrix}.$$

The adjusted vector  $c$  is shown below

$$c = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

And the decrypted result is

$$m'' = a' \times f_p + c \times f_p = \begin{bmatrix} 1 & 2 & 1 & 2 & 0 & 2 & 0 & 0 & 2 & 0 \\ 1 & 2 & 0 & 2 & 2 & 2 & 0 & 2 & 1 & 0 \\ 0 & 2 & 2 & 1 & 2 & 2 & 2 & 2 & 0 & 2 \\ 2 & 2 & 1 & 2 & 2 & 2 & 2 & 2 & 0 & 2 \\ 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 2 & 2 \\ 2 & 1 & 0 & 2 & 0 & 0 & 2 & 1 & 0 & 2 \\ 2 & 2 & 0 & 0 & 0 & 2 & 0 & 2 & 2 & 0 \\ 2 & 2 & 1 & 0 & 2 & 1 & 2 & 0 & 0 & 0 \\ 2 & 0 & 0 & 2 & 2 & 2 & 2 & 1 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 0 & 0 & 2 & 2 & 2 \\ 1 & 2 & 2 & 0 & 2 & 0 & 2 & & & \end{bmatrix}.$$

The decrypted result shows that  $m'' = m$ , which implies that our improved NTRU cryptosystem can successfully correct the decryption failure while the original NTRU can not.

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## REFERENCES

- [1] M. A. Beyer and D. Laney, *The Importance of 'Big Data': A Definition*. Stamford, CT, USA: Gartner, 2012.

[2] V. Marx, "Biology: The big challenges of big data," *Nature*, vol. 498, no. 7453, pp. 255–260, 2013.

[3] G. P. Consortium, et al., "A map of human genome variation from population-scale sequencing," *Nature*, vol. 467, no. 7319, pp. 1061–1073, 2010.

[4] A. Sahai and B. Waters, "Fuzzy identity-based encryption," in *Advances in Cryptology*. Berlin, Germany: Springer, 2005, pp. 457–473.

[5] C. Hu, F. Zhang, X. Cheng, X. Liao, and D. Chen, "Securing communications between external users and wireless body area networks," in *Proc. 2nd ACM Workshop Hot Topics Wireless Netw. Secur. Privacy*, 2013, pp. 31–36.

[6] C. Hu, H. Li, Y. Huo, T. Xiang, and X. Liao, "Secure and efficient data communication protocol for wireless body area networks," *IEEE Trans. Multi-Scale Comput. Syst.*, vol. 2, no. 2, pp. 94–107, Apr.–Jun. 2016.

[7] V. Goyal, O. Pandey, A. Sahai, and B. Waters, "Attribute-based encryption for fine-grained access control of encrypted data," in *Proc. 13th ACM Conf. Comput. Commun. Secur.*, 2006, pp. 89–98.

[8] B. Waters, "Ciphertext-policy attribute-based encryption: An expressive, efficient, and provably secure realization," in *Proc. 14th Int. Conf. Practice Theory Public Key Cryptography*, 2011, pp. 53–70.

[9] C. Hu, N. Zhang, H. Li, X. Cheng, and X. Liao, "Body area network security: A fuzzy attribute-based signcryption scheme," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 9, pp. 37–46, Sep. 2013.

[10] A. Lewko and B. Waters, "Decentralizing attribute-based encryption," in *Advances in Cryptology*. Berlin, Germany: Springer, 2011, pp. 568–588.

[11] C. Hu, X. Cheng, Z. Tian, J. Yu, K. Akkaya, and L. Sun, "An attribute-based signcryption scheme to secure attribute-defined multi-cast communications," in *Proc. 11th EAI Int. Conf. SecureComm*, 2015, pp. 418–435.

[12] A. Shamir, "Identity-based cryptosystems and signature schemes," in *Advances in Cryptology*. Berlin, Germany: Springer, 1985, pp. 47–53.

[13] M. Dehkordi and S. Mashhadi, "An efficient threshold verifiable multi-secret sharing," *Comput. Standards Interfaces*, vol. 30, no. 3, pp. 187–190, 2008.

[14] Z. Eslami and J. Z. Ahmadabadi, "A verifiable multi-secret sharing scheme based on cellular automata," *Inf. Sci.*, vol. 180, no. 15, pp. 2889–2894, 2010.

[15] M. H. Dehkordi and S. Mashhadi, "New efficient and practical verifiable multi-secret sharing schemes," *Inf. Sci.*, vol. 178, no. 9, pp. 2262–2274, 2008.

[16] J. Zhao, J. Zhang, and R. Zhao, "A practical verifiable multi-secret sharing scheme," *Comput. Standards Interfaces*, vol. 29, no. 1, pp. 138–141, 2007.

[17] C. Hu, X. Liao, and X. Cheng, "Verifiable multi-secret sharing based on LFSR sequences," *Theoretical Comput. Sci.*, vol. 445, pp. 52–62, 2012.

[18] C. Hu, X. Liao, and D. Xiao, "Secret image sharing based on chaotic map and Chinese remainder theorem," *Int. J. Wavelets Multiresolution Inf. Process.*, vol. 10, no. 3, 2012, Art. no. 1250023.

[19] K. Yang, X. Jia, K. Ren, B. Zhang, and R. Xie, "DAC-MACS: Effective data access control for multiauthority cloud storage systems," *IEEE Trans. Inf. Forensics Secur.*, vol. 8, no. 11, pp. 1790–1801, Nov. 2013.

[20] K. Yang and X. Jia, "Expressive, efficient, and revocable data access control for multi-authority cloud storage," *IEEE Trans. Parallel Distrib. Syst.*, vol. 25, no. 7, pp. 1735–1744, Jul. 2014.

[21] J. Bethencourt, A. Sahai, and B. Waters, "Ciphertext-policy attribute-based encryption," in *Proc. IEEE Symp. Secur. Privacy*, 2007, pp. 321–334.

[22] A. Córcoles, et al., "Demonstration of a quantum error detection code using a square lattice of four superconducting qubits," *Nature Commun.*, vol. 6, pp. 1–10, 2015.

[23] O. Regev, "New lattice-based cryptographic constructions," *J. ACM*, vol. 51, no. 6, pp. 899–942, 2004.

[24] D. Micciancio, "Lattice-based cryptography," in *Post-Quantum Cryptography*. Berlin, Germany: Springer, 2009, pp. 147–191.

[25] C. Peikert, "Lattice cryptography for the internet," in *Post-Quantum Cryptography*. Berlin, Germany: Springer, 2014, pp. 197–219.

[26] J. Hoffstein, J. Pipher, and J. Silverman, "NTRU: A ring-based public key cryptosystem," in *Proc. 3rd Int. Symp. Algorithmic Number Theory*, 1998, pp. 267–288.

[27] *The NTRU Public Key Cryptosystem—A tutorial*, NTRU Cryptosystems, Wilmington, MA, USA, 1998.

[28] A. Langlois, D. Stehlé, and R. Steinfield, "GGH Lite: More efficient multilinear maps from ideal lattices," in *Advances in Cryptology*. Berlin, Germany: Springer, 2014, pp. 239–256.

[29] S. Garg, C. Gentry, and S. Halevi, "Candidate multilinear maps from ideal lattices," in *Advances in Cryptology*. Berlin, Germany: Springer, 2013, pp. 1–17.

[30] European Telecommunications Standards Institute, "Quantum safe cryptography and security—An introduction, benefits, enablers and challenges," Eur. Telecommun. Standards Inst., *White Paper*, Jun. 2015. [Online]. Available: <https://www.etsi.org/images/files/ETSIWhitePapers/QuantumSafeWhitepaper.pdf>

[31] IBM, "IBM storage solutions for banking," (2015). [Online]. Available: <http://www-03.ibm.com/systems/storage/solutions/industries/banking.html>

[32] A. Mora, et al., "Expanded top ten big data security and privacy challenges," Cloud Secur. Alliance, Seattle, WA, USA, Tech. Rep. 20130616, 2013.

[33] L. Wei, et al., "Security and privacy for storage and computation in cloud computing," *Inf. Sci.*, vol. 258, pp. 371–386, 2014.

[34] H. Cheng, C. Rong, K. Hwang, W. Wang, and Y. Li, "Secure big data storage and sharing scheme for cloud tenants," *China Commun.*, vol. 12, no. 6, pp. 106–115, 2015.

[35] J. Baek, Q. H. Vu, J. K. Liu, X. Huang, and Y. Xiang, "A secure cloud computing based framework for big data information management of smart grid," *IEEE Trans. Cloud Comput.*, vol. 3, no. 2, pp. 233–244, Apr.–Jun. 2015.

[36] C. Gentry, "A fully homomorphic encryption scheme," Ph.D. dissertation, Dept. Comput. Sci., Stanford Univ., Stanford, CA, USA, 2009.

[37] S. Kamara and K. Lauter, "Cryptographic cloud storage," in *Proc. 14th Int. Conf. Financial Cryptography Data Secur.*, 2010, pp. 136–149.

[38] A. Hamlin, N. Schear, E. Shen, M. Varia, S. Yakoubov, and A. Yerukhimovich, "Cryptography for big data security," in *Big Data: Storage, Sharing, and Security*, F. Hu, Ed. Boca Raton, FL, USA: CRC Press, 2016, ch. 10, pp. 241–288.

[39] G. Zhuo, Q. Jia, L. Guo, M. Li, and P. Li, "Privacy-preserving verifiable set operation in big data for cloud-assisted mobile crowdsourcing," *IEEE Internet Things J.*, vol. PP, no. 99, p. 1, 2016, Doi: 10.1109/JIOT.2016.2585592.

[40] L. Guo, Y. Fang, M. Li, and P. Li, "Verifiable privacy-preserving monitoring for cloud-assisted mHealth systems," in *Proc. IEEE Conf. Comput. Commun.*, 2015, pp. 1026–1034.

[41] S. Salinas, X. Chen, J. Ji, and P. Li, "A tutorial on secure outsourcing of large-scale computations for big data," *IEEE Access*, vol. 4, pp. 1406–1416, Apr. 2016.

[42] V. C. Hu, D. Ferraiolo, and D. R. Kuhn, "Assessment of access control systems," US Dept. Commerce, Nat. Inst. Standards Technol., Gaithersburg, MD, USA, Tech. Rep. 7316, 2006.

[43] K. Yang and X. Jia, "Attributed-based access control for multi-authority systems in cloud storage," in *Proc. IEEE 32nd Int. Conf. Distrib. Comput. Syst.*, 2012, pp. 536–545.

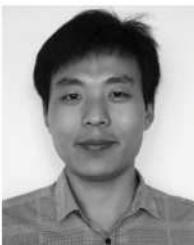
[44] T. Jung, X.-Y. Li, Z. Wan, and M. Wan, "Privacy preserving cloud data access with multi-authorities," in *Proc. IEEE INFOCOM*, 2013, pp. 2625–2633.

[45] A. Shamir, "How to share a secret," *Commun. ACM*, vol. 22, no. 11, pp. 612–613, 1979.

[46] A. Sahai, H. Seysioglu, and B. Waters, "Dynamic credentials and ciphertext delegation for attribute-based encryption," in *Advances in Cryptology*. Berlin, Germany: Springer, 2012, pp. 199–217.

[47] G. Blakley, "Safeguarding cryptographic keys," in *Proc. Nat. Comput. Conf.*, 1979, vol. 48, pp. 313–317.

[48] J. H. Silverman and W. Whyte, "Estimating decryption failure probabilities for NTRUEncrypt," *NTRUEncrypt*, Tech. Rep. #018, Version 1, May 2003. [Online]. Available: <http://grouper.ieee.org/groups/1363/lattPK/submissions/NTRUTech018.pdf>



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