Exploitation of Spatial Coherence for Reducing the Complexity of Acoustic OFDM Systems

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***Abstract*—This paper addresses the question as to how to exploit spatial coherence between receiving elements in multichannel multi-carrier acoustic communication systems in order to reduce the signal processing complexity without compromising the performance. To answer this question, an adaptive pre-combining method is proposed. Without requiring any a priori knowledge about the spatial distribution of received signals, the method exploits spatial coherence between receive channels by linearly combining them into fewer output channels so as to reduce the number of subsequent channel estimators. The algorithm learns the spatial coherence pattern recursively over the carriers, thus effectively achieving broadband beamforming. The reduced-complexity pre-combining method relies on differentially coherent detection which keeps the receiver complexity at a minimum and requires a very low pilot overhead. Using the experimental data transmitted over a 3-7 km shallow water channel in the 10.5-15.5 kHz acoustic band, we study the system performance in terms of data detection mean squared error (MSE) and show that the receiver equipped with the proposed reduced-complexity pre-combining scheme requires three times fewer channel estimators while achieving the same MSE performance as the full-complexity receiver.**

I. Introduction

Multicarrier modulation in the form of orthogonal frequency division multiplexing (OFDM) is an attractive method for data transmission over frequency-selective channels due to its ability to achieve high bit rates at reasonably low computational loads [1]–[4]. This fact motivates the use of OFDM in mobile acoustic communications where the channel exhibits long multipath delays but each narrowband carrier only experiences flat fading, thus eliminating the need for time-domain equalizers. In addition to temporal fluctuations, spatial variability of the underwater channel presents a major problem for single-channel receivers and motivates the use of multiple spatially distributed receivers which offer robustness to fading [1]. However, multichannel processing of high rate underwater acoustic communication signals requires computationally expensive receiver algorithms, and an increase in the number of receiving elements significantly increases the receiver complexity.

In this paper, we propose a pre-combining method that exploits spatial coherence between receiving elements and provides the desired reduction in signal processing complexity. The approach is motivated by time-domain equalization in single-carrier systems [5], which we re-formulate in light of multicarrier systems. The proposed scheme exploits spatial correlation between the input channels by linearly combining them so as to reduce the number of channel estimators in a coherent receiver. The method makes no assumptions about the spatial distribution of signals, relying only on the fact that some coherence will exist between the signals received on different elements when the elements are spaced closely with respect to wavelength. The algorithm learns spatial coherence between channels adaptively, and allows the pre-combiner coefficients to change from one carrier to another, thus effectively accomplishing broadband processing. The adaptive pre-combining method relies on differentially coherent detection which keeps the receiver complexity at minimum and requires only a very low pilot overhead. The technique is demonstrated on experimental data from the Mobile Acoustic Communication Experiment (MACE 2010) showing excellent results. In the MACE'10 experiment, OFDM blocks containing up to 1024 QPSK modulated carriers, which occupy the acoustic frequency range between 10.5 and 15.5 kHz, were transmitted over a long-range (3-7 km) shallow water (about 100 m deep) channel and received over a 12-element vertical array spanning a total linear aperture of 1.32 m. The results lead us to conclude that the proposed reduced-complexity adaptive pre-combining scheme is especially well-suited for implementation in acoustic multicarrier systems.

The rest of the paper is organized as follows. In Sec. II, we introduce the signal and system model. Sec. III details the proposed method of spatial pre-combining. Sec. IV contains the results of experimental data processing. We conclude in Sec.V.

II. System Model

We consider an OFDM system with M receivers and K carriers within a total bandwidth B. Let f_0 and $\Delta f = B/K$ denote the first carrier frequency and the carrier spacing, respectively. The transmitted OFDM block is then given by

$$s(t) = Re\left\{\sum_{k=0}^{K-1} d_k e^{2\pi i f_k t}\right\}, \quad t \in [0,\ T] \tag{1}$$

where $T = 1/\Delta f$ is the OFDM block duration. The data symbol d_k, which modulates the kth carrier of frequency $f_k = f_0 + k\Delta f$, belongs to a unit amplitude phase shift keying alphabet (PSK).

After synchronization,[1] carried out using the method proposed in [3], and overlap-and-add procedure (or cyclic-prefix removal), the lowpass equivalent received signal on the mth receiving element is modeled as

$$v_m(t) = \sum_{k=0}^{K-1} H_k^m d_k e^{2\pi i k\Delta f t} + w_m(t), \quad t \in [0, T] \tag{2}$$

where H_k^m is the channel frequency response at the kth carrier of the mth receiving element, and $w_m(t)$ is the additive zero-mean complex baseband noise.

Conventional coherent receivers apply channel estimation to the post-FFT observations

$$y_k^m = \frac{1}{T}\int_T v_m(t) e^{-2\pi i k\Delta f t} dt \tag{3}$$

Using maximum ration combining (MRC), the data symbols are then detected as

$$\hat{d}_k = \frac{\hat{\mathbf{H}}_k^H \mathbf{y}_k}{\hat{\mathbf{H}}_k^H \hat{\mathbf{H}}_k} \tag{4}$$

where $\mathbf{y}_k$ and $\hat{\mathbf{H}}_k$ are the vectors of post-FFT observations y_k^m and channel estimates $\hat{H}_k^m$, respectively. If there is no spatial coherence,

[1] Synchronization includes frame synchronization, initial resampling and frequency offset compensation.

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the M channel estimates are formed independently. However, if there exists a correlation between the M channels, this correlation can be exploited to reduce the receiver complexity.

III. Reduced Complexity Receiver

In this section, we propose a pre-combining method for multichannel OFDM systems. The proposed technique reduces the complexity of coherent OFDM receivers through linearly combining the M input channels into $Q \leq M$ output channels, so as to reduce the number of channel estimators needed. The technique makes no assumption about the spatial distribution of signals, and relies on differentially coherent detection which keeps the receiver complexity at a minimum. Fig. 1 shows the block diagram of the receiver.

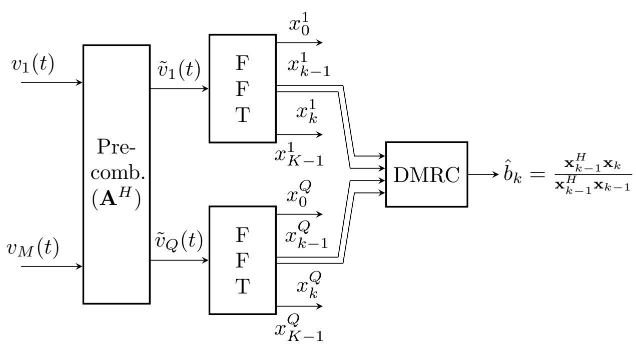


Fig. 1. Block diagram of the receiver with pre-combining. The pre-combiner $\mathbf{A}$ consisting of columns $\boldsymbol{\alpha}_q$, $q = 1, \ldots, Q$, linearly combines the M input channels into $Q \leq M$ output channels. From the second equality in (5), it follows that pre-combining can also be performed after FFT demodulating the M input channels. The differentially-encoded data symbols are then estimated by applying differential maximum ratio combining (DMRC) after FFT demodulation.

Let $\boldsymbol{\alpha}_q$ denote the pre-combiner weight for the q-th output channel, and let the vector $\mathbf{v}(t)$ contain the M input signals $v_m(t)$. FFT demodulation then yields

$$x_k^q = \frac{1}{T}\int_T \boldsymbol{\alpha}_q^H \mathbf{v}(t) e^{-2\pi i k \Delta f t} dt = \boldsymbol{\alpha}_q^H \mathbf{y}_k \quad (5)$$

where the vector $\mathbf{y}_k, k = 0, \ldots, K-1$, contains the M FFT outputs corresponding to the M input channels. Expression (5) above shows that the receiver structure of Fig. 1, which employs pre-FFT combining, is equivalent to one that would employ post-FFT combining by applying the Q weights $\boldsymbol{\alpha}_q$ to the FFT outputs $\mathbf{y}_k$. We use this fact to develop a method for computing the weights recursively over the carriers.

Stacking the FFT outputs x_k^q into a column vector $\mathbf{x}_k$, the differentially-encoded data symbols $b_k = d_{k-1}^* d_k$ are estimated by differential maximum ratio combining (DMRC) over the Q channels as

$$\hat{b}_k = \frac{\mathbf{x}_{k-1}^H \mathbf{x}_k}{\mathbf{x}_{k-1}^H \mathbf{x}_{k-1}} \quad (6)$$

Differentially coherent detection (8) introduces a particular type of non-linearity into the data estimation problem, namely a quadratic dependence on the pre-combiner weights. Such a dependence prevents one from using a standard (linear) recursive algorithm such as least mean squares (LMS) or recursive least squares (RLS). This problem can be solved by treating $\mathbf{x}_{k-1}$ as independent of the pre-combiner weights, and retaining only the dependence of $\mathbf{x}_k$ on the pre-combiner weights [6]. Such an approach leads to linear MMSE estimation, and also allows for explicit amplitude normalization.

The estimated data symbol is now expressed as

$$\hat{b}_k = \frac{\sum_q (x_{k-1}^q)^* x_k^q}{\sum_q |x_{k-1}^q|^2} = \sum_q (\bar{x}_{k-1}^q)^* \boldsymbol{\alpha}_q^H \mathbf{y}_k \quad (7)$$

where $\bar{x}_{k-1}^q = x_{k-1}^q / \sum_q |x_{k-1}^q|^2$. The expression (7) can be written in a vector form as

$$\hat{b}_k = \underbrace{\begin{bmatrix} \boldsymbol{\alpha}_1^H & \cdots & \boldsymbol{\alpha}_Q^H \end{bmatrix}}_{\mathbf{a}^H = (\text{vec}(\mathbf{A}))^H} \underbrace{\begin{bmatrix} (\bar{x}_{k-1}^1)^* \mathbf{y}_k \\ \vdots \\ (\bar{x}_{k-1}^Q)^* \mathbf{y}_k \end{bmatrix}}_{\mathbf{u}_k = \bar{\mathbf{x}}_{k-1}^* \otimes \mathbf{y}_k} = \mathbf{a}^H \mathbf{u}_k \quad (8)$$

where the vec($\mathbf{A}$) operator creates a column vector $\mathbf{a}$ from the matrix $\mathbf{A}$ by stacking columns of $\mathbf{A}$ one below another, $\bar{\mathbf{x}}_k = \begin{bmatrix} \bar{x}_k^1 & \cdots & \bar{x}_k^Q \end{bmatrix}^T$, and $\otimes$ denotes the Kronecker product.

Algorithm 1 Adaptive pre-combining algorithm

1: **Input:** Number of carriers K, number of blocks N_b, forgetting factor λ, initial variance σ^2, post-FFT observations $\mathbf{y}_k(n), \forall k = 0, \ldots, K-1, n = 1, \ldots, N_b$, pilot set $\mathcal{K}_p(n)$.
2: **Initialization:** Initialize $\mathbf{A}_0(1)$ to the Q maximally spaced receiving elements, initialize the inverse covariance matrix $\mathbf{P}_0(1) = \sigma^{-2}\mathbf{I}_{QM}$ where $\mathbf{I}_{QM}$ is a $QM \times QM$ identity matrix
3: $\mathbf{x}_0(1) = \mathbf{A}_0(1)^H \mathbf{y}_0(1)$
4: $\mathcal{C}_1 = \{1, 2, \ldots, K-1\}$, $\mathcal{C}_{-1} = \{K-2, K-3, \ldots, 0\}$
5: **for** $n = 1, \ldots, N_b$ **do**
6: $\quad d = (-1)^{n-1}$ ($d = \pm 1$ depending on the direction of iteration)
7: $\quad$ **for** $k \in \mathcal{C}_d$ **do**
8: $\quad\quad \mathbf{u}_k(n) = \frac{\mathbf{x}_{k-d}^*(n)}{\|\mathbf{x}_{k-d}(n)\|^2} \otimes \mathbf{y}_k(n)$
9: $\quad\quad \mathbf{a}_{k-d}(n) = \text{vec}(\mathbf{A}_{k-d}(n))$
10: $\quad\quad \hat{b}_k(n) = \mathbf{a}_{k-d}^H(n)\mathbf{u}_k(n)$
11: $\quad\quad \tilde{b}_k(n) = \begin{cases} b_k(n) & k \in \mathcal{K}_p(n) \\ \text{dec}(\hat{b}_k(n)) & \text{o.w.} \end{cases}$
12: $\quad\quad e_k(n) = \tilde{b}_k(n) - \hat{b}_k(n)$
13: $\quad\quad \mu_k(n) = (\lambda + \mathbf{u}_k^H(n)\mathbf{P}_{k-d}(n)\mathbf{u}_k(n))^{-1}$
14: $\quad\quad \mathbf{a}_k(n) = \mathbf{a}_{k-d}(n) + \underbrace{\mu_k(n)\mathbf{P}_{k-d}(n)\mathbf{u}_k(n)}_{\mathbf{G}_k(n)} e_k^*(n)$
15: $\quad\quad \mathbf{P}_k(n) = \lambda^{-1}(\mathbf{I}_{QM} - \mathbf{G}_k(n)\mathbf{u}_k^H(n))\mathbf{P}_{k-d}(n)$
16: $\quad\quad \mathbf{x}_k(n) = \mathbf{A}_k^H(n)\mathbf{y}_k(n)$
17: $\quad$ **end for**
18: $\quad \mathbf{A}_k(n+1) = \mathbf{A}_k(n)$
19: $\quad \mathbf{x}_k(n+1) = \mathbf{A}_k^H(n)\mathbf{y}_k(n)$
20: $\quad \mathbf{P}_k(n+1) = \sigma^{-2}\mathbf{I}_{QM}$
21: **end for**

The combiner coefficients $\mathbf{a}$ are computed recursively over the carriers as

$$\mathbf{a}_k = \mathbf{a}_{k-1} + \mathcal{A}(\mathbf{u}_k, e_k) \quad (9)$$

where $\mathcal{A}(\mathbf{u}_k, e_k)$ represents a particular algorithm's increment computed for the input $\mathbf{u}_k$ and the error $e_k = b_k - \mathbf{a}^H \mathbf{u}_k$. For instance, if LMS is used, then $\mathcal{A}(\mathbf{u}_k, e_k) = \mu \mathbf{u}_k e_k^*$, where μ is the LMS step size, and if RLS is used, then $\mathcal{A}(\mathbf{u}_k, e_k)$ is the RLS increment given in Algorithm 1, line 14. Clearly, the choice of the adaptive algorithm always presents a trade-off between computational complexity and speed of convergence. In the RLS algorithm, the stochastic gradient $e_k^* \mathbf{u}_k$ (line 14) is premultiplied by an estimate of the inverse of the covariance matrix $\mathbf{P}_{k-1}$, which has the effect of decorrelating the inputs to the adaptive filter. This decorrelation, along with the learning rate μ_k (line 13), enhances the convergence rate of the algorithm, thus requiring fewer pilots then the LMS algorithm at the cost of more computations. $O((QM)^2)$ operations are required for each weight update of the RLS whereas only $O(QM)$ are necessary with LMS.

The algorithm forms the error using the pilot data symbols $b_k, k \in \mathcal{K}_p = \{0, \ldots, K_p - 1\}$ in the training mode. Thereafter, it switches into decision-directed mode where the decisions are made on the composite estimate (8). Note that the process can continue into the next block, where recursion will evolve in reverse order (from the highest carrier to the lowest) and require fewer (or no) pilots [2]. Algorithm 1 summarizes the differentially-aided RLS-based pre-combining method.

The initial value $\mathbf{a}_0 = \text{vec}(\mathbf{A}_0)$ needs to be chosen carefully. A possible choice corresponds to selecting Q out of M channels and passing them through the pre-combiner intact. For example, the initial channels can be chosen as equally spaced, starting with the first channel. It is important to set the initial conditions such that the algorithm is allowed enough freedom to form the best Q outputs that have as little correlation as possible.

Recursive computation of the combiner coefficients across carriers requires M FFTs but allows the pre-combiner to change from one carrier to another, thus effectively accomplishing broadband processing. If the channels are slowly-varying, where the pre-combiner computed in one block could be preserved for use in the next block, pre-combining could be implemented before FFT demodulation, thus simultaneously reducing the total number of channel estimators as well as the total number of FFT operations.

Relying on differentially coherent detection allows the algorithm not only to be used as stand-alone, but also to be used in a coherent receiver where the pre-combined FFT outputs x_k^q are used to estimate the Q "channel" coefficients $\tilde{H}_k^q$, subsequently yielding the data symbols estimates

$$\hat{d}_k = \frac{\tilde{\mathbf{H}}_k^H \mathbf{x}_k}{\tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k} \tag{10}$$

IV. Experimental Result

To asses the system performance, we focus on the experimental data from the Mobile Acoustic Communication Experiment (MACE'10) which took place off the coast of Martha's Vineyard, Massachusetts, in June 2010. The experimental signals, whose parameters for blocks with $K = 512$ and 1024 carriers are given in Table I, were transmitted using the acoustic frequency range between 10.5 kHz and 15.5 kHz.

TABLE I
MACE'10 Signal Parameters. The guard interval is $T_g = 16$ msec. The total bandwidth is $B = 5$ kHz and the lowest carrier frequency is $f_0 = 10.5$ kHz. The bandwidth efficiency is calculated assuming 36 pilots in the first block and no pilot from the second block and on. The result excludes the pilot overhead used for frequency offset compensation and channel estimation which is 136 pilots per block.

number of carriers K	512	1024
number of blocks per frame N_b	16	8
carrier spacing Δf [Hz]	9.8	4.9
bit rate [kbps]	8.6	9.3
bandwidth efficiency [bps/Hz]	1.73	1.85

The receiver array of 12 equally-spaced elements spanning a total linear aperture of 1.32 m was deployed at the depth of 40 m, and the transmitter was towed at the depth of 40-60 m. The water depth was approximately 100 m, and the transmission distance varied between 3 km and 7 km. More details about the experiment can be found in [2].

The experiment consisted of multiple repeated transmissions, each containing all the OFDM signals listed in Table I. There was a total of 54 transmissions spanning 3.5 hours of recording. During this time, the transmitting station moved away and towards the receiving station, at varying speeds ranging from 0.5 m/s to 1.5 m/s. The results provided in this section are obtained from all 54 transmissions.

We compare the performances of coherent and differentially coherent receivers equipped with the proposed pre-combining scheme with those of the conventional coherent and differentially coherent receivers. We quantify the performance in terms of data detection mean-squared error (MSE). Coherent receivers use the path identification (PI) channel estimation method proposed in [4]. The PI channel estimator is based on a physical model of multipath propagation and iteratively estimates the channel path delays and gains in a greedy manner. The data symbols are then estimated using (4) or (10), for full-complexity, or spatially pre-combined methods, respectively. We also report on the estimated cumulative density function (CDF) of the MSE measured in each signal frame.

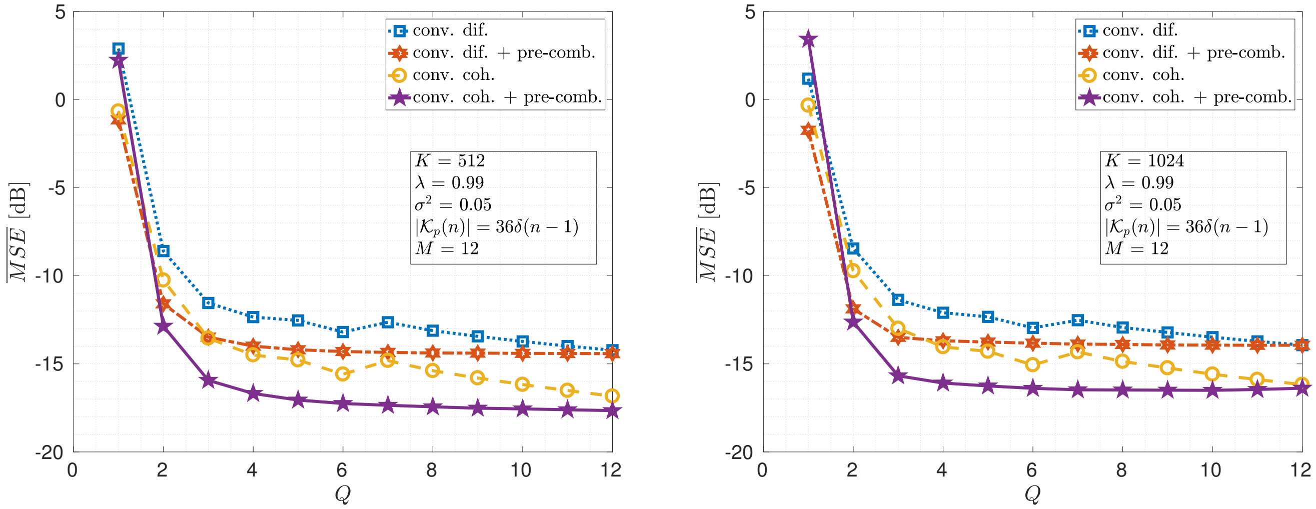
Fig. 2 illustrates the average MSE of the pre-combining scheme, which is used in both differentially coherent and coherent receivers, as a function of the number of pre-combiner outputs Q varying from $Q = 1$ to $Q = M = 12$. Clearly, there exists a form of saturation in performance; with $Q = 4$ the average MSE reaches a value that remains almost constant with further increase in Q. While an increase in Q from 1 to 4 improves the performance dramatically, changing the number of pre-combiner output channels from 4 to 12 results in a total fluctuation of the average MSE of less than 1 dB which is insignificant for the overall receiver performance at the given values of the average MSE. However, the corresponding change in complexity is considerable. Using a Q-channel configuration instead of the full-complexity configuration reduces the total number of channel estimators by M/Q. This feature brings a significant reduction in complexity when the value of Q at which saturation is reached is low.

Fig. 3 illustrates the estimated cumulative density function of the MSE per block for $K = 512$ carriers and different detection methods. The results include the 54 frames, transmitted over 3.5 hours. The results shown in this figure demonstrate that the receivers with reduced-complexity pre-combining achieve the same level of reliability as the conventional receivers with full-complexity. Fig. 3a shows that differentially coherent receivers equipped with the 4-channel pre-combiner deliver MSE below -12 dB for 93% the OFDM blocks. Coherent receivers with the 4-channel pre-combiner deliver MSE below -15 dB for 91% of the OFDM blocks as shown in Fig. 3b.

V. Conclusion

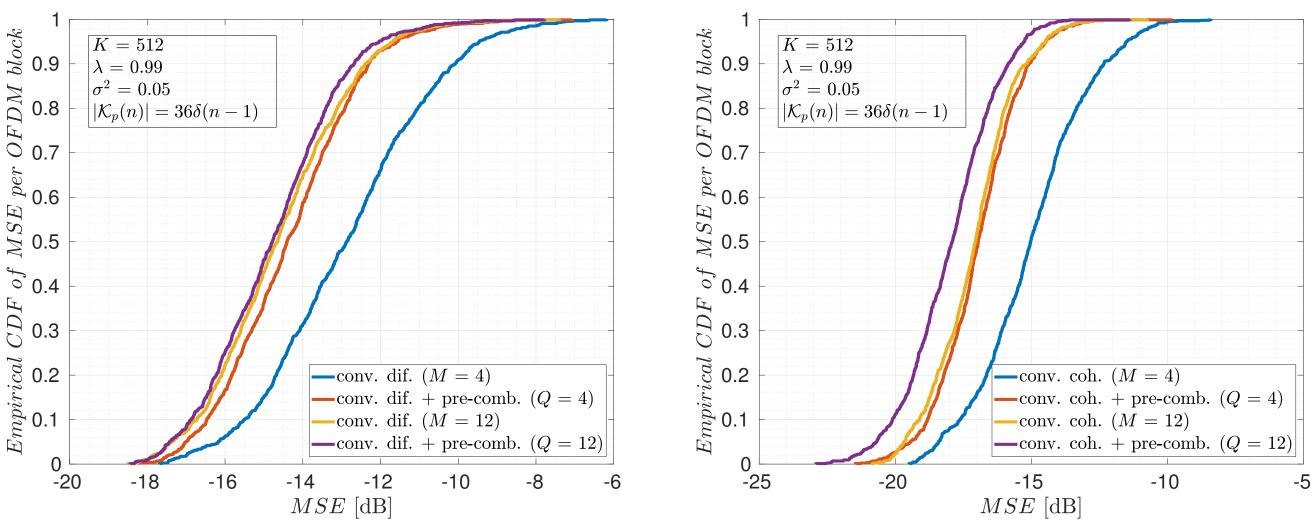
We presented a pre-combining scheme based on differentially coherent detection for acoustic multichannel OFDM systems. Without requiring a priori knowledge about the spatial distribution of signals, the scheme extracts spatial coherence between receiving elements (if such exists) by linearly combining the M available channels into Q channels, so as to reduce the number of subsequent channel estimators. The algorithm learns spatial coherence of signal recursively, allowing the pre-combiner coefficients to change from one carrier to another, thus effectively achieving broadband processing.

We demonstrated the performance of the proposed technique using experimental signals recorded over a mobile acoustic channel. Our results show that pre-combining the 12 input channels into 4 for subsequent coherent processing yields the same performance in terms of average data detection MSE at the cost of very few additional pilots. We also showed that the 4-channel configuration delivers an average MSE below -15 dB for 91% of OFDM blocks.



(a) $K = 512$. (b) $K = 1024$.

Fig. 2. Average MSE as a function of the number of pre-combiner outputs Q for differentially coherent and coherent receivers with and without the proposed pre-combining scheme.The two plots (a) and (b) refer to the cases with $K = 512$ and $K = 1024$ carriers, respectively. The conventional receivers use Q maximally spaced receivers to detect the data symbols. The forgetting factor of the RLS algorithm is $\lambda = 0.99$ and the initial variance σ^2 is set to 0.05 in all of the cases. The pre-combining algorithm uses the first 36 carriers of the first block as pilots for training, and thereafter switches to the decision-directed mode for the entire frame.



(a) Differential coherent detection. (b) Coherent detection.

Fig. 3. Estimated CDF of the MSE for the adaptive pre-combining algorithm used in four OFDM systems. In all the cases, the CDFs reflect the 54 transmissions during MACE'10. In (b), the coherent receiver with 4-channel pre-combiner algorithm deliver MSE below -15 dB for 91% of the transmitted OFDM blocks. The conventional receivers using four maximally equi-spaced elements achieve the same level of reliability while delivering MSE below -12.2 dB. In this case, the full-complexity receiver with pre-combining has slightly better performance than does the conventional full-complexity receiver.

Future research will focus on jointly estimating spatial coherence and channel and examining the receiver performance in terms of bit error rate.

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