Chapter 9

Raman Solitons in Nanoscale Optical Waveguides, with Metamaterials, Having Polynomial Law Nonlinearity Using Collective Variables

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Additional information is available at the end of the chapter

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Abstract

A mathematical analysis is conducted to illustrate the controllability of the Raman soliton self-frequency shift with polynomial nonlinearity in metamaterials by using collective variable method. The polynomial nonlinearity is due to the expanding nonlinear polarization P^{NL} in a series over the field E up to the seventh order. Gaussian assumption is selected to these pulses on a generalized mode. The numerical simulation of soliton parameter variation is given for the Gaussian pulse parameters.

Keywords: Raman solitons, polynomial nonlinearity, collective variables

1. Introduction

Much attention has been devoted to the understanding of metamaterials [1–4]. Through its engineered structures, researchers are able to control and manipulate the electromagnetic fields [5]. Using the freedom of design that metamaterials provide, electromagnetic fields can be redirected at will and propose a design strategy [6]. A general recipe for the design of media that create perfect invisibility within the accuracy of geometrical optics is developed. The imperfections of invisibility can be made arbitrarily small to hide objects that are much larger than the wavelength [7].

Especially, mathematical operations can be performed based on suitably designed metamaterials blocks, such as spatial differentiation, integration, or convolution [8]. Soliton pulse can evolve owning to delicate balance between dispersion and nonlinearity. However, it is

always a challenge to compensate for the loss when engineering these types of waveguide using metamaterials. The strong perturbation of a soliton envelope caused by the stimulated Raman scattering confines the energy scalability preventing the so-called dissipative soliton resonance [9]. It is important to know the limit we can reach expanding the nonlinear polarization P^{NL} in a series over the field E [10]. The fourth-order nonlinear susceptibility $\chi^{(4)}$, the fifth-order nonlinearity $\chi^{(5)}$, and the seventh-order nonlinearity $\chi^{(7)}$ have been measured [11, 12]. The polynomial mode nonlinearity is due to the nonlinear polarization of metamaterials in the power-series expansion form where terms are kept up to the seventh order in the field E [10, 12–15]. This chapter conducts mathematical analysis to illustrate the controllability of the Raman soliton self-frequency shift with polynomial nonlinearity in metamaterials by using collective variable method.

2. Governing model

The dimensionless form nonlinear Schrödinger's equation (NLSE) that governs the propagation of Raman soliton through optical metamaterials, with polynomial law nonlinearity, is given by [16–24].

$$\begin{aligned} i\frac{\partial}{\partial t}\Phi(z,t) + a\frac{\partial^2}{\partial z^2}\Phi(z,t) + \left(c_1|\Phi(z,t)|^2 + c_2|\Phi(z,t)|^4 + c_3|\Phi(z,t)|^6\right)\Phi(z,t) \\ = i\alpha\frac{\partial}{\partial z}\Phi(z,t) + i\lambda\frac{\partial}{\partial z}\left(|\Phi(z,t)|^2\Phi(z,t)\right) + i\nu\frac{\partial}{\partial z}\left(|\Phi(z,t)|^2\right)\Phi(z,t). \end{aligned} \tag{1}$$

In this model, $\Phi(z,t)$ represents the complex valued wave function with the independent variables being z and t that represent spatial and temporal variables, respectively. The first term represents the temporal evolution of nonlinear wave, while the coefficient a is the group velocity dispersion (GVD). The coefficients of c_j for $j = 1, 2, 3$ correspond to the nonlinear terms. Together, they form polynomial mode nonlinearity. The polynomial mode nonlinearity is due to the nonlinear polarization of metamaterials in the power-series expansion form where terms are kept up to the seventh order in the field E [10, 12–15]. It must be noted here that when $c_2 = c_3 = 0$ and $c_1 \neq 0$, the model Eq. (1) collapses to Kerr mode nonlinearity which is due to third-order polarization P^{NL} [15]. However, if $c_3 = 0$ and $c_1 \neq 0$ and $c_2 \neq 0$, one arrives at parabolic mode nonlinearity, and it is from the fifth-order polarization P^{NL} [15, 30]. Thus, polynomial mode stands as an extension version to Kerr and parabolic modes. Actually, the Raman effect is not influenced by the properties of the metamaterials; however, the Raman coefficient combines with the dispersive magnetic permeability of the metamaterials leading to additional higher-order nonlinear terms [10, 12, 14]. The group velocity and self-phase modulation term produce the delicate balance dispersion and nonlinearity that accounts for the formation of the stable soliton. On the right hand side, α describes intermodel dispersion, λ represents the self-steepening term in order to avoid the formation of shocks, and ν is the complex higher-order nonlinear dispersion coefficient.

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3. Mathematical formulation

The pulse may not only be able to translate as a whole entity, but it may also execute more or less complex internal vibrations depending on the type of the perturbations in the system. This particle-like behavior has led to the formulation of the collective variable (CV) techniques [25]. The basic idea is that the soliton solution depends on a collective of variables, called CVs, symbolically $Z_j(j = 1, \ldots N)$, which represent pulse width, amplitude, chirp, frequency, and so on [25–28]. To this end, the original field is decomposed into two components, say $\Phi(z, t)$ at position z in the metamaterials and at time t, in the following way:

$$\Phi(z,t) = f(Z_1, Z_2, \ldots, Z_N, t) + q(z,t), \tag{2}$$

where the first component f constitutes soliton solution and the second one q represents the residual radiation that is known as small amplitude dispersive waves. Introduction of these N CVs increases the phase space of the dynamical system.

In order for the system to remain in the original phase space and best fit for the static solution, the CV method is obtained by configuring the function $f(Z_1, Z_2, \ldots, Z_N, t)$ and minimizes residual free energy (RFE) E, where

$$E = \int_{-\infty}^{\infty} |q|^2 dt = \int_{-\infty}^{\infty} |\Phi(z,t) - f(Z_1, Z_2, \ldots, Z_N, t)|^2 dt. \tag{3}$$

The approximation of neglecting the *residual field* is called "bare approximation" in condensed matter physics [27].

Let CVs evolve only in a particular direction to minimize the PFE in the dynamical system with the following simple way:

$$C_j = \frac{\partial E}{\partial Z_j} = \frac{\partial}{\partial Z_j}\left(\int_{-\infty}^{\infty} |q|^2 dt\right) = \int_{-\infty}^{\infty} \left(\frac{\partial q}{\partial Z_j} q^* + \frac{\partial q^*}{\partial Z_j} q\right) dt. \tag{4}$$

The rate of change of C_j with respect to the normalized distance is defined as.

$$\dot{C}_j = \frac{dC_j}{dz} = 2\Re\left(\frac{d}{dz}\left(\int_{\infty}^{\infty} \frac{\partial q^*}{\partial Z_j} q dt\right)\right), \tag{5}$$

where $\Re$ stands for the real part. Here, the weak equality indicates that the constraints C_j need not be exactly zero [28].

Then, we define a second set of constraints:

$$\frac{dC_j}{dz} \approx 0. \tag{6}$$

Through Eqs. (2)–(6), it leads to the equations of motion:

$$\dot{C}_j = -2\Re \sum_{k=1}^{N} \left(\int_{-\infty}^{\infty} \frac{\partial f^*}{\partial Z_j} \frac{\partial f}{\partial Z_k} dt - \int_{-\infty}^{\infty} \frac{\partial^2 f^*}{\partial Z_j \partial Z_k} q dt \right) \frac{dZ_k}{dt} + R_j, \tag{7}$$

where

$$R_j = 2\Re \int_{-\infty}^{\infty} \frac{\partial f^*}{\partial Z_j} \frac{d\Phi}{dz} dt, \tag{8}$$

for $1 \le j \le N$.

The set of Eqs. (5)–(8) is equivalent to the matrix equation:

$$\dot{C} = \frac{\partial C}{\partial z} \dot{z} + R, \tag{9}$$

where

$$z = \begin{pmatrix} Z_1 \\ Z_2 \\ \dots \\ Z_N \end{pmatrix}, \tag{10}$$

$$R = \begin{pmatrix} R_1 \\ R_2 \\ \dots \\ R_N \end{pmatrix}, \tag{11}$$

while the $N \times N$ Jacobian is given by

$$\frac{\partial C}{\partial z} = \frac{\partial (C_1, C_2, \dots, C_N)}{\partial (Z_1, Z_2, \dots, Z_N)} = \left(\frac{\partial C_j}{\partial Z_k} \right)_{N \times N}, \tag{12}$$

with

$$\frac{\partial C_j}{\partial Z_k} = -2\Re \left(\int_{-\infty}^{\infty} \frac{\partial f^*}{\partial Z_j} \frac{\partial f}{\partial Z_k} dt - \int_{-\infty}^{\infty} \frac{\partial^2 f^*}{\partial Z_j \partial Z_k} q dt \right), \tag{13}$$

for $1 \le j, k \le N$.

At this stage, through Eq. (6) we can solve Eq. (9) by the following CV equations of motion:

$$\dot{X} = \left(\frac{\partial C}{\partial z} \right)^{-1} R. \tag{14}$$

The set of Eqs. (4)–(14) represents the complete CV treatment for the generalized NLSE Eq. (1).

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4. Computational results

In this part the adiabatic parameter dynamics of solitons in optical metamaterials with polynomial nonlinearity will be obtained by CV method. A Gaussian is given by

$$f(Z_1, Z_2, Z_3, Z_4, Z_5, Z_6; t) = Z_1 \exp\left[-(t - Z_2)^{2m/X_3^2} + i\frac{Z_4}{2}(t - Z_2)^2 + iZ_5(t - Z_2) + iZ_6\right], \quad (15)$$

where Z_1 is the soliton amplitude, Z_2 is the center position of the soliton, Z_3 is the inverse width of the pulse, Z_4 is the soliton chirp, Z_5 is the soliton frequency, and Z_6 is the soliton phase. Also, m is the Gaussian parameter, where $m > 0$.

In this case, with $N = 6$:

$$\frac{\partial C}{\partial z} = \begin{pmatrix} \frac{\partial C_1}{\partial Z_1} & \frac{\partial C_1}{\partial Z_2} & \frac{\partial C_1}{\partial Z_3} & \frac{\partial C_1}{\partial Z_5} & \frac{\partial C_1}{\partial Z_5} & \frac{\partial C_1}{\partial Z_6} \\ \frac{\partial C_2}{\partial Z_1} & \frac{\partial C_2}{\partial Z_2} & \frac{\partial C_2}{\partial Z_3} & \frac{\partial C_2}{\partial Z_5} & \frac{\partial C_2}{\partial Z_5} & \frac{\partial C_2}{\partial Z_6} \\ \frac{\partial C_3}{\partial Z_1} & \frac{\partial C_3}{\partial Z_2} & \frac{\partial C_3}{\partial Z_3} & \frac{\partial C_3}{\partial Z_5} & \frac{\partial C_3}{\partial Z_5} & \frac{\partial C_3}{\partial Z_6} \\ \frac{\partial C_4}{\partial Z_1} & \frac{\partial C_4}{\partial Z_2} & \frac{\partial C_4}{\partial Z_3} & \frac{\partial C_4}{\partial Z_5} & \frac{\partial C_4}{\partial Z_5} & \frac{\partial C_4}{\partial Z_6} \\ \frac{\partial C_5}{\partial Z_1} & \frac{\partial C_5}{\partial Z_2} & \frac{\partial C_5}{\partial Z_3} & \frac{\partial C_5}{\partial Z_5} & \frac{\partial C_5}{\partial Z_5} & \frac{\partial C_5}{\partial Z_6} \\ \frac{\partial C_6}{\partial Z_1} & \frac{\partial C_6}{\partial Z_2} & \frac{\partial C_6}{\partial Z_3} & \frac{\partial C_6}{\partial Z_5} & \frac{\partial C_6}{\partial Z_5} & \frac{\partial C_6}{\partial Z_6} \end{pmatrix}, \quad (16)$$

$$z = \begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{pmatrix}, \quad (17)$$

$$\mathcal{R} = \begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \end{pmatrix}, \quad (18)$$

where

$$\mathcal{R}_1 = -aZ_1(Z_4 + X_1Z_5)\frac{\Gamma\left(\frac{1}{2m}\right)}{m\left(\frac{2}{Z_3^2}\right)^{\frac{1}{2m}}} - \left(\frac{2aZ_1^2X_4}{m} + \frac{12\lambda Z_1^3}{Z_3^2}\right)\frac{\Gamma\left(\frac{1}{m}\right)}{\left(\frac{4}{Z_3^2}\right)^{\frac{1}{m}}} - 2\alpha Z_1 + 4Z_1^3\nu, \tag{19}$$

$$\begin{aligned}\mathcal{R}_2 &= 2aZ_1^2((m-1)Z_4 + Z_1Z_5) + (1-2m)\left(\frac{aZ_1^2Z_5}{2m}\left(\frac{2}{Z_3^3}\right)^{\frac{1}{2m}} + \left(\frac{\alpha Z_1^2}{16} - \frac{\nu Z_1^5}{8m}\right)\left(\frac{4}{Z_3^2}\right)^{\frac{1}{2m}}\right)\Gamma\left(-\frac{1}{2m}\right)\\ &= -\left(\frac{Z_1^2Z_4}{m}\left(c_1\left(\frac{Z_3^2}{4}\right)^{\frac{1}{m}} + c_2\left(\frac{Z_3^2}{6}\right)^{\frac{1}{m}} + C_3\left(\frac{Z_3^2}{8}\right)^{\frac{1}{m}}\right) - \frac{4\lambda Z_1^4Z_4Z_5}{\left(\frac{6}{Z_3^2}\right)^{\frac{1}{m}}}\right)\Gamma\left(\frac{1}{m}\right)\\ &\quad + \left(\frac{\alpha Z_1^2Z_5}{m\left(\frac{2}{Z_3^2}\right)^{\left(\frac{1}{2m}\right)}} - \frac{\lambda Z_1^4Z_5^2}{m\left(\frac{6}{Z_3^2}\right)^{\left(\frac{1}{2m}\right)}}\right)\Gamma\left(\frac{1}{2m}\right) + \frac{(m-1)\lambda Z_1^4Z_3^2}{3m}\left(\frac{6}{Z_3^2}\right)^{\left(\frac{1}{2m}\right)}\Gamma\left(-\frac{1}{m}\right),\end{aligned} \tag{20}$$

$$\begin{aligned}\mathcal{R}_3 &= \frac{\nu Z_1^5}{Z_3} - \frac{3aZ_1^3Z_3\left(\frac{2}{m}-1\right)}{m}Z_4\Gamma\left(\frac{1}{m}\right) - \frac{2\alpha Z_1^2}{X_3} - \frac{3\lambda Z_1^4}{2Z_3}\\ &\quad + \left(\frac{c_1Z_1^4}{2Z_3\left(\frac{4}{Z_3^2}\right)^{\left(\frac{1}{2m}\right)}} - \frac{aZ_1^2(Z_4 + Z_1Z_5)}{\left(\frac{2}{Z_3^2}\right)^{\left(\frac{1}{m}\right)}} + \frac{c_2Z_1^6}{3Z_3\left(\frac{6}{Z_3^2}\right)^{\left(\frac{1}{2m}\right)}} + \frac{c_3Z_1^8}{4Z_3\left(\frac{8}{Z_3^2}\right)^{\left(\frac{1}{2m}\right)}}\right)\frac{\Gamma\left(\frac{1}{2m}\right)}{2m^2},\end{aligned} \tag{21}$$

$$\begin{aligned}\mathcal{R}_4 &= \left(\frac{aZ_1^2(1-2m)}{\left(\frac{2}{Z_3^2}\right)^{\left(\frac{1}{2m}\right)}}\right)\frac{\Gamma\left(\frac{1}{2m}\right)}{4m} - \left(\frac{aZ_1^3}{\left(\frac{2}{Z_3^2}\right)^{\left(\frac{1}{m}\right)}} - \frac{\lambda Z_1^4Z_5}{\left(\frac{4}{Z_3^2}\right)^{\left(\frac{1}{m}\right)}}\right)\frac{\Gamma\left(\frac{1}{m}\right)}{m} + \left(\frac{\alpha Z_1^2Z_4}{2\left(\frac{2}{Z_3^2}\right)^{\left(\frac{2}{m}\right)}} + \frac{2\lambda Z_1^5}{\left(\frac{4}{Z_3^2}\right)^{\left(\frac{2}{m}\right)}}\right)\frac{\Gamma\left(\frac{2}{m}\right)}{m}\\ &\quad - Z_1^2\left(\frac{c_1Z_1^2}{\left(\frac{4}{Z_3^2}\right)^{\left(\frac{3}{2m}\right)}} + \frac{c_2Z_1^4}{\left(\frac{6}{Z_3^2}\right)^{\left(\frac{3}{2m}\right)}} + \frac{c_3Z_1^6}{\left(\frac{8}{Z_3^2}\right)^{\left(\frac{3}{2m}\right)}} - \frac{\alpha Z_5}{\left(\frac{2}{Z_3^2}\right)^{\left(\frac{3}{2m}\right)}}\right)\frac{\Gamma\left(\frac{3}{2m}\right)}{m},\end{aligned} \tag{22}$$

$$\begin{aligned}\mathcal{R}_5 &= 2a(1-2m)Z_1^2 - \left(\frac{aZ_1^3}{\left(\frac{2}{Z_3^2}\right)^{\left(\frac{1}{2m}\right)}}\right)\frac{\Gamma\left(\frac{1}{2m}\right)}{m} - 2\left(\frac{\alpha Z_1^2Z_4}{\left(\frac{2}{X_3^2}\right)^{\left(\frac{3}{2m}\right)}} - \frac{\lambda Z_1^4Z_4}{\left(\frac{4}{X_3^2}\right)^{\left(\frac{3}{2m}\right)}}\right)\frac{\Gamma\left(\frac{3}{2m}\right)}{m}\\ &\quad - 2\left(\frac{\alpha Z_1^2X_5}{\left(\frac{2}{Z_3^2}\right)^{\left(\frac{1}{m}\right)}} - \frac{\lambda Z_1^4Z_5}{\left(\frac{4}{X_3^2}\right)^{\left(\frac{1}{m}\right)}}\right)\frac{\Gamma\left(\frac{1}{m}\right)}{m} - \frac{aZ_1^3\Gamma\left(\frac{1}{2m}\right)}{m\left(\frac{2}{Z_3^2}\right)^{\left(\frac{1}{2m}\right)}} + 2a(1-2m)aZ_1^2,\end{aligned} \tag{23}$$

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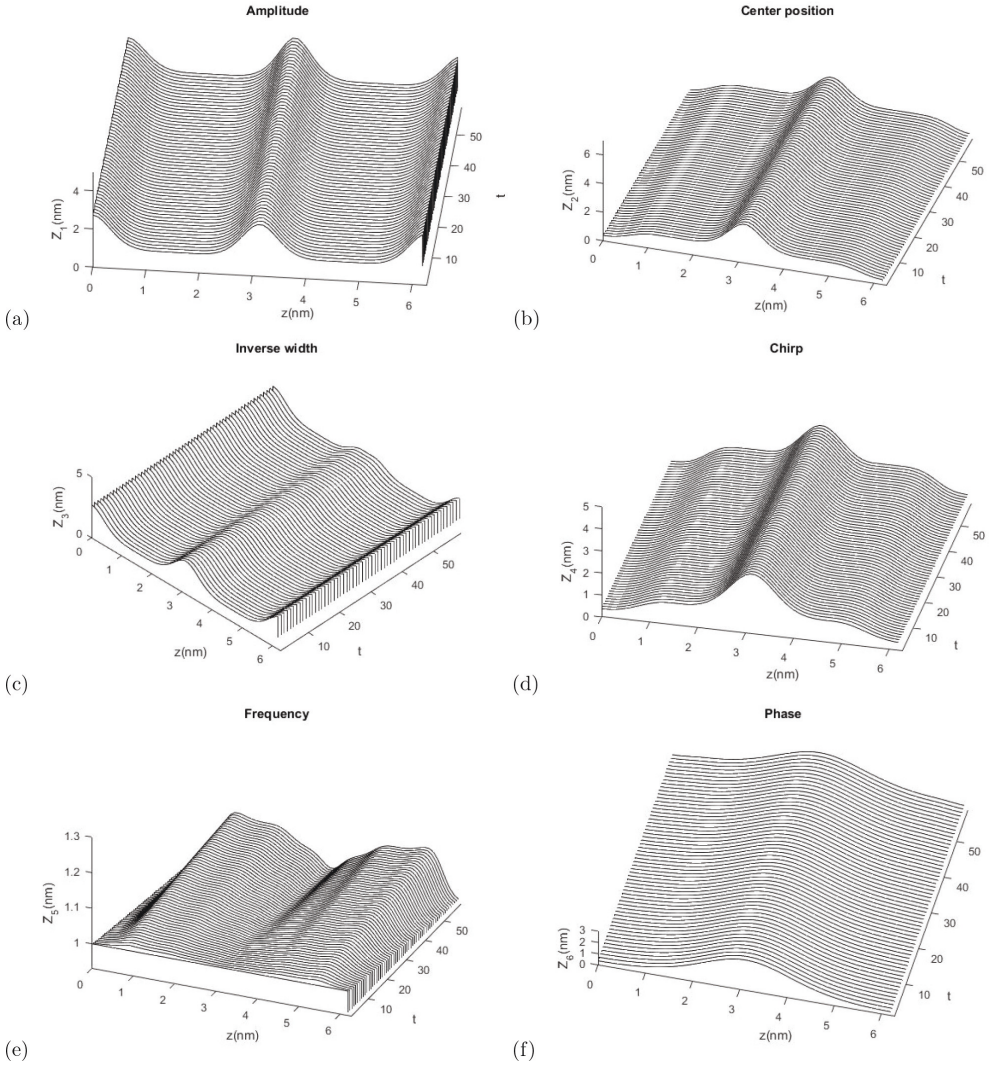

Figure 1. (a) Surface plot of soliton amplitude, (b) surface plot of soliton center position, (c) surface plot of soliton inverse width of the pulse, (d) surface plot of soliton chirp variation, (e) surface plot of soliton frequency, and (f) surface plot of soliton phase.

$$\mathcal{R}_6 = 2\left(\frac{\alpha Z_1 Z_4}{\left(\frac{2}{Z_3^2}\right)^{\left(\frac{1}{m}\right)}} + \frac{\lambda Z_1^4 Z_4}{\left(\frac{4}{Z_3^2}\right)^{\left(\frac{1}{m}\right)}}\right)\frac{\Gamma\left(\frac{1}{m}\right)}{m} + \left(\frac{\alpha Z_1 Z_5}{\left(\frac{2}{Z_3^2}\right)^{\left(\frac{1}{2m}\right)}} + \frac{\lambda Z_1^4 Z_5}{\left(\frac{4}{Z_3^2}\right)^{\left(\frac{1}{2m}\right)}}\right)\frac{\Gamma\left(\frac{1}{2m}\right)}{m} + \frac{aZ_1(2m-1)}{m}\left(\frac{Z_3^2}{2}\right)^{\left(\frac{1}{2m}\right)}\Gamma\left(-\frac{1}{2m}\right) - 2aZ_1^2. \tag{24}$$

5. Numerical simulation

The nonlinear dynamical system discussed in the previous section is plotted to illustrate the collective variables numerically; see **Figure 1**. The parameter values are as follows: $m = 1$, $a = 9.9 \times 10^{-2}$, $\lambda = 9.8 \times 10^{-2}$, $\alpha = 1 \times 10^{-1}$, $c_1 = 9.9 \times 10^{-1}$, $c_2 = -8 \times 10^{-2}$, $c_3 = -8 \times 10^{-3}$, and $\nu = 1.01 \times 10^{-1}$ [10, 12, 15, 30].

This continuous surface plot shows the dynamical relationship between the time and collective variables, in **Figure 1**. It shows soliton amplitude, center position, inverse width of the pulse, chirp, and phase keeping the original shape as time goes by. This is because group velocity and self-phase modulation term produce the delicate balance dispersion and nonlinearity that accounts for the formation of the stable soliton. It also describes Stokes Raman scattering that is due to transmitted wave at higher frequency and anti-Stokes Raman scattering where transmitted wave is at lower frequency by **Figure 1(e)** [29]. These results are consistent with Raman soliton scattering effect.

6. Conclusion

This chapter gives Raman soliton solutions in optical metamaterials that is studied with polynomial nonlinearity. The polynomial mode nonlinearity is due to expanding the nonlinear polarization P^{NL} in a series over the field E up to the seventh order [13–15]. The polynomial mode nonlinearity is an extension of the Kerr and parabolic mode nonlinearity, which are from third- and fifth-order polarization P^{NL} [15, 30], respectively. The analytical results are supplemented with numerical simulation by collective variables. The continuous surface plot shows the dynamical relationship between the time and collective variables. It shows soliton amplitude, center position, inverse width of the pulse, chirp, and phase keeping the original shape as time goes by since group velocity and self-phase modulation term produce the delicate balance dispersion and nonlinearity. It also describes Stokes Raman scattering that is due to transmitted wave at higher frequency and anti-Stokes Raman scattering where transmitted wave is at lower frequency by **Figure 1(e)** [8, 29].

In the future, the set of plot with $m \neq 1$ will be plotted, and third-order dispersion (TOD) and fourth-order dispersion (FOD) will be included [15]. Nonlinear polarization of medium in the form of a power-series expansion, keeping the terms up to the ninth order, will be explored [10].

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