# Productive Mistakes during Design-Based Research as Learning Sites for Prospective Teachers

Productive mistakes occur frequently in engineering research. Some famous mistakes led to the development of products such as sticky notes, penicillin, and rubber tires (Gojak, 2013). Making a mistake often leads one down a reasoning path that otherwise might remain unexplored (Petroski, 1985). Furthermore, making mistakes actually stimulates brain growth, and that growth is greatest when accompanied by a mindset that one can improve with effort (Boaler, 2016). Hence, it is important for teachers to help students see mistakes they make in doing mathematics as learning opportunities rather than avoiding mistakes at all costs (Hiebert et al., 1997). Similarly, teacher educators can help prospective teachers use the mistakes they make when teaching as opportunities for professional development by drawing prospective teachers' attention to the mistakes and using them as prompts to discuss how to improve teaching.

The proposed presentation describes productive mistakes made by prospective teachers within the context of an undergraduate research project that incorporates design-based research. Design-based research is akin to engineering research. It aims to produce empirically tested theory about student learning in tandem with means for supporting student learning (Bakker & van Eerde, 2015). These two products are engineered simultaneously. At the outset of the process, researchers make conjectures about how student learning may occur and design instruction accordingly. They then empirically test the conjectures and the accompanying instructional approach with students, refine the instructional approach and conjectures in light of empirical data, and subject the refined approach and conjectures to further empirical testing with the students. These activities occur in continuous, repetitive cycles (Cobb, Jackson, & Sharpe, 2017).

At the core of design-based research is Simon's (1995) notion of *hypothetical learning trajectory* (HLT), which consists of a goal for student learning, tasks used to promote learning, and hypotheses about how students' learning might develop. Researchers constructing HLTs make initial hypotheses about how students' thinking might develop and then progressively refine them as students' responses to tasks are observed. Researchers design and select subsequent tasks as necessary to help students attain intended learning goals (Steffe & Thompson, 2000).

Along with providing a means for designing theory and instructional materials to support students' learning in a given domain, design-based research offers a potential opportunity for collaboration between practitioners and researchers (e.g., McClain & Cobb, 2001; Smit & van Eerde, 2011). In such collaborative studies, researchers work closely with teachers to construct and re-construct HLTs. The construction and re-construction process involves optimizing instruction by collaboratively designing lessons, analyzing classroom data, making conjectures on how to build students' emergent understanding, and testing the conjectures by introducing instructional materials that embody the conjectures (Cobb, 2000). Engaging in design-based research has great potential for teacher education because of the close attention to classroom data it entails. We conducted 13 such studies with prospective teachers over the course of 4 years.

In design-based research, as in all engineering research, mistakes are inevitable. The number of mistakes is likely to be greater when novice teachers are involved in the process. Prospective teachers in our undergraduate research project made several mistakes which we unpack during the presentation. Focusing on mistakes might, at first glance, seem counter-productive. Certainly, research on teacher learning that uses deficit models to describe teachers' knowledge is of limited value. However, our focus is not simply on describing mistakes, per se, but on how conducting design-based research in collaboration with prospective teachers provided a space for making *productive mistakes*. Examples of productive mistakes we will discuss include failing to probe children's thinking thoroughly, choosing problem contexts largely inaccessible to children, using a series of closed questions while teaching, and assuming children's interpretations of manipulatives were the same as those of the teacher. Productive mistakes were one of the chief means through which prospective teachers had opportunities to learn while conducting design-based research because they created opportunities for self-reflection, dialogue with peers, and dialogue with mentors, leading to changes in teaching practice.

The idea of using productive mistakes as tools for teacher education raises a number of questions for teacher educators that we will pose to the audience for discussion. These questions include: What makes something a mistake in the context of teaching? What makes something a *productive* mistake? How can teacher educators help prospective teachers recognize mistakes? How can prospective teachers be helped to learn from mistakes? How do existing structures for teacher education support or discourage learning from mistakes? As questions of this nature are discussed among mathematics teacher educators, we can more fully conceptualize the notion of "productive mistake" in the context of teacher education and gain better understanding of the roles they can play in mathematics teacher education curricula.

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