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From physics to social interactions: Scientific unification via dynamics

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Abstract

The principle of dynamical similitude—the belief that the same behavior may be exhibited by very different systems—allows us to use mathematical models from physics to understand psychological phenomena. Sometimes, model choice is straightforward. For example, the two-frequency resonance map can be used to make predictions about the performance of multifrequency ratios in physical, chemical, physiological and social behavior. Sometimes, we have to dig deeper into our dynamical toolbox to select an appropriate technique. An overview is provided of other methods, including mass-spring modeling and multifractal analysis, that have been applied successfully to various psychological phenomena. A final demonstration of dynamical similitude comes from the use of the same multifractal method that was used to extract team-level experience from the neurophysiological data of individual team members to the analysis of a large scale economic phenomenon, the stock market index. Continual development of analytical methods that are informed by and can be applied to other sciences allows us to treat psychological phenomena as continuous with the rest of the natural world.

Keywords: Dynamics; Modeling; Nesting; Coordination; Social; Multifractal

1. Introduction

The application of dynamical systems to psychology offers both a means by which to demonstrate the continuity of psychology with the rest of science and the ability to characterize behavior, the subject of psychology, as a stable pattern of change. For the majority of the time that psychology has been a science, psychologists have used analytical methods that reduce the complex behavior exhibited by humans to only one (e.g., a mean) or a few (measures of central tendency) numbers. The implication of that approach is twofold: that behavior was unchanging and could be characterized statically and that observed variation was due to random influence.

Psychologists began to adopt dynamical systems methods during the 1980s by making a straightforward analogy between the rhythmic behavior of the limbs and the rhythmic behavior of pendulums. They adopted coupled oscillator models, meant to capture coordination across two physical oscillators, to better understand stable patterns of bimanual coordination (e.g., Haken, Kelso, & Bunz, 1985; Kelso, 1981, 1984; Kugler & Turvey, 1987). During the 1990s, cognitive, developmental, and social psychologists began to extend the application of dynamical systems thinking and methods to characterize patterns that they observed in social interactions and across development (e.g., Port & Van Gelder, 1995; Thelen & Smith, 1994; Vallacher & Nowak, 1994). The new millennium brought increased exploration of dynamical tools to capture those

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patterns of change, including the identification of fractal processes in reaction time data (Gilden, 1997, 2001; Gilden, Thornton, & Mallon, 1995) and the formulation of a singular model to capture all previously-documented effects in the classic "A-not-B error" in cognitive development (Thelen, Schöner, Scheier, & Smith, 2001). That short story is just a broad overview of the tremendous growth in the dynamical approach to psychology. A detailed review of the history of dynamical systems in psychology could easily fill an entire book.

As psychologists have access to methods of data collection that produce more detailed, continuous data streams—for example, the access to momentary diary data through smart phone apps or neuroimaging techniques—there is an even greater need for preserving in our analyses the processes that are revealed. One approach has been to extend repeated measures analyses common in statistical techniques to accommodate those longer data sets. However, there are assumptions common to traditional statistical techniques—that observed fluctuations are the influence of (random) noise around a true population mean—that become computationally burdensome when scaled up to data sets of hundreds, thousands, or even tens-of-thousands of values. An alternate approach is to treat the observed fluctuation as structured and potentially accommodated by low-dimensional dynamical equations. The focus of the present paper is to present a dynamical approach that captures with few parameters the details of complex human behavior that we wish to study. The principle of dynamical similitude—that the same behavior may be observed across very different systems—allows us to sample from a much broader selection of techniques as our search for new methods extends to fields of science beyond psychology. The emphasis on behavior over structure identifies dynamics as a truly multidisciplinary approach that sees commonalities across the sciences rather than restricting scientific inquiry to phenomena that appear the same structurally.

1.1. Metronomes and people

We often begin with our search for common dynamical principles in the field of physical models, but even those models are motivated by real-world behavior. A great example of dynamical similitude comes in the comparison of two online videos (available on YouTube and other sites): the synchronization of 32 metronomes and gait synchronization during Opening Day of London's Millenium Bridge. In both cases, synchronization occurs across very many different rhythmic processes, but the entities generating those processes—physical objects and people—couldn't look more different. In the first case, 32 metronomes rest on a flexible surface and are set ticking, one after the other. At first, the phasing of the metronomes is completely random, governed by when they were started up by the young YouTuber. There is no cohesive sound to 32 metronomes all beating at roughly the same frequency but not at the same time... it's rather "clackety".

Luckily, it doesn't take long for some of the metronomes to start synchronizing their beats so that the pendula reach the endpoint at the same time. That sounds a little more cohesive to the listener. However, most of the metronomes continue to swing left and right seemingly not in time with the rest or each other. It doesn't take very long before we notice all pendula swinging back and forth together, with the exception of one hold-out, a metronome whose pendulum swings right while the others swing left. Dynamicists call the former pattern *inphase* because the position of those pendulums in their cycles (i.e., their phasing) is the same as that of their neighbors at any given moment. Looking at one pendulum is the same as looking at any one of those other pendulums. The latter pattern is called *antiphase* because the position of that lone pendulum in its movement cycle is exactly opposite to that of its neighbors. At any given moment, that one pendulum looks like the mirror image of any of the other pendulums. Antiphase, in fact, can be rather stable. But the movement of the other metronomes on the flexible platform is too much for that one antiphase metronome, and, eventually, it switches to the same phasing as all of the other metronomes. By the end of the video, all of the metronomes are synchronized inphase and the viewer hears a strong, singular beat given by all metronomes reaching their endpoints at the same exact time.

Why does that synchronization occur? As the pendulum of each metronome moves back and forth, it generates a slight movement of the flexible platform below. That disruption is felt by the other metronomes on the same platform that are also perturbing the platform ever so slightly. Think about it this way: movement of one pendulum to the left disrupts the platform and other metronomes in a direction-specific way, influencing them to behave the same way as it. The coupling medium of the platform serves to connect all of the metronomes so that they can communicate with each other. That bidirectional influence—metronomes influencing and being influenced by each other—provides the conditions for synchronization. An understanding of the behavior of the whole system is not really given by understanding the behavior of one metronome alone but by understanding the relation of the metronome to its entire environment: the other metronomes and the platform on which they rest.

What does this have to do with humans walking across a bridge? The Millenium Bridge is a striking 320-m long steel suspension bridge that spans the River Thames in London. The bridge opened to pedestrians on June 10, 2000 and closed later that same day because of unpredicted sway. As a suspension bridge, a certain amount of sway was expected, but amplitude of that sway generated when the crowds walked across the bridge on opening day was alarming. Like the metronomes on the flexible platform, each person generated a little bit of direction-specific movement that influenced other people to move the same way. That may be hard to imagine not having experienced it first-hand, but think about walking along a rope bridge with just one other person and recognize that your movements would undoubtedly be affected by any movement that they made.

Information was carried across the pedestrians on that suspension bridge in much the same way as information was carried across the metronomes on the flexible platform. Multiply that influence by a large crowd, and the result was a singular crowd dynamics in which everyone's gait pattern was eventually synchronized. From the perspective of a movement scientist, the gait was interesting in that it wasn't just straightforward walking in the forward (anterior-posterior, AP) direction. Instead, people incorporated lateral movements, presumably to increase their stability as they walked. That resulted in a rather funny-looking walk that is discussed further in Steven Strogatz's (2004) TED talk and *Nature* article (Strogatz, Abrams, McRobie, Eckhardt, & Ott, 2005). We have observed that same stabilization strategy in postural control: when asked to extend the feet in the AP direction, as might be seen on beam work in gymnastics, participants angle the feet to the side (mediolateral direction) to increase postural stability (Gibbons, Amazeen, & Likens, 2018).

1.2. Which model: phase locking or frequency locking?

The synchronization discussed so far has focused on phase locking, in which the relative phasing between the two or more processes being coordinated remains constant. There is an enormous literature on relative phase dynamics, starting with Kelso's observation of synchronization across finger movements in 1981 (Kelso, 1981, 1984) and continuing through to the present day. We can imagine that dynamics was adopted into psychology through the area of motor coordination because the physics of pendula were a straightforward fit to the physics of swinging limbs. Scott Kelso claimed that he had the insight to model gait patterns from a phone book jingle (for those who remember phone books)—"Let your fingers do the walking through the Yellow Pages"—and a central paradigm in motor coordination research was born (Kelso, 1995).

The study of stable patterns of coordination (gaits) in coordinated finger movements gave rise to the adoption of a coupled oscillator model, the Haken et al. (1985; HKB) model, in psychology. For nearly 10 years, dynamicists in psychology studied inphase and antiphase as primary patterns of interest (see summary in Amazeen, Amazeen, & Turvey, 1998). They looked at transitions between them (e.g., Kelso, 1984), patterns across multiple effectors (e.g., Kelso & Jeka, 1992), handedness effects (Treffner & Turvey, 1996), the influence of cognitive effects like the direction of attention (e.g., Amazeen, Amazeen, Treffner, & Turvey, 1997) or the performance of a dual task paradigm (e.g., Temprado, Zanone, Monno, & Laurent, 1999). They studied different relative phase patterns by conducting learning studies (e.g., Amazeen, 2002; Lee, Swinnen, & Verschueren, 1995; Zanone & Kelso, 1992a, 1992b, 1994, 1997) and demonstrated that the same predictions were supported for coordination within and across people (e.g., Amazeen, Schmidt, & Turvey, 1995; Richardson, Marsh, & Schmidt, 2005; Richardson, Marsh, Isenhower, Goodman, & Schmidt, 2007; Schmidt, Bienvenu, Fitzpatrick, & Amazeen, 1998; Schmidt, Carello, & Turvey, 1990).

Phase synchronization has even been extended from the bivariate case, in which phasing is estimated across two rhythmic processes, to the multivariate case, in which it can be estimated across more than two processes (Frank & Richardson, 2010). We've arrived back at the example of the Millenium Bridge. But in all of those examples, the two processes being coordinated moved at approximately the same frequency.

It is the very special case that people coordinate processes that occur at the same exact rate. Instead, across social, business, military, and sport settings, people are able to manage and synchronize concurrent processes that all occur at different rates. Parents coordinate a child's nightly homework assignments with biweekly soccer practices and weekly soccer games; musicians spontaneously coordinate a variety of nested rhythms in jazz; young athletes coordinate jumping patterns between two ropes in the sport of Double Dutch; an employee schedules daily work to meet a goal that is due at the end of the week. In all of those examples, the synchronization across individuals cannot be captured by a singular relative phase value because the component processes are moving at inherently different speeds. Instead of looking for phase locking, we focus on the pattern of frequency locking exhibited. Consider the employee who must coordinate daily activities over the next five working days to accomplish the one bigger goal, due at the end of the week. That pattern can be characterized with a frequency ratio of 5:1, i.e., five working days for one big project. The fact that we can characterize so many of those synchronization examples with these multifrequency ratios, rather than relative phase patterns, makes the multifrequency ratio a better index of coordination for those particular examples and motivates the use of a different model of coordination.

2. A physical model

The two-frequency resonance map is a model of coupled oscillators in which the two oscillators cycle at different frequencies (Bak, 1986; Gilmore & Lefranc, 2002; Hardy & Wright, 1965). Technically, it identifies the phasing of a second cyclic process (Process Q) at some landmark phasing of the first cyclic process (Process P). Consider the work of the employee above: to meet the deadline at the end of the week (Process P), the employee structures the activity of five work days (Process Q). In this model, the boss does not check in on progress during the week but only checks on the status of the project at the end of the week. The two-frequency resonance map works the same way. From the perspective of this two-frequency resonance map, the emphasis is on the delivery (the phase in that project cycle: is it done?) at the end of the week, not the work that is done in the interim.

To be honest, the model assumes that you maintain a steady working frequency during the week, but there's really no way to tell because the strobe is at the end of the week. So how does the model work and what does it predict?

$$\theta_{n+1} = \theta_n + \Omega + \frac{K}{2\pi}\sin(2\pi\theta_n) \quad (1)$$

Eq. (1) predicts the future state θ_{n+1} of Process Q from the previous state θ_n and two control parameters: a ratio that is called the *bare winding number* Ω because it is the ratio that would be observed in the absence of any coupling, and a *coupling function* $\frac{K}{2\pi}\sin(2\pi\theta_n)$ that captures the influence, or coupling strength K, across the two processes. Review of the coordination dynamics literature will reveal that a coupling function of some form is critical for any coordination across multiple processes. We observed the relevance of coupling in both real-world examples discussed so far: the presence of a flexible platform in the metronome example and the fact that the Millenium Bridge was a suspension bridge. Coupling in any system, captured by K in the equation above, promotes communication and eventual synchronization. In other dynamical models of complex behavior, e.g., Agent Based Modeling (e.g., Janssen & Ostrom, 2006), coupling is captured by the rules of influence among individual agents as they interact.

The mathematics underlying the two-frequency resonance model and its parameters are well-documented elsewhere (e.g., Bak, 1986; Bak, Bohr, & Jensen, 1984; deGuzman & Kelso, 1991; Glazier & Libchaber, 1988; González & Piro, 1985; Hardy & Wright, 1965; Hilborn, 1994). Predictions about the frequency ratio between the two processes, Process P and Process Q, are generated by iterating the model for different values of Ω and K. When applied to real systems, that prediction is about the frequency ratio (p:q, $p < q$). There is support for the predictions of the two-frequency resonance map for spatiotemporal patterns in physical, chemical, and biological phenomena, including Rayleigh-Bénard convection, the Belousov-Zhabotinsky (BZ) reaction, periodically-forced semiconductor lasers, cardiac arrhythmias, and drumming patterns (e.g., Glazier & Libchaber, 1988; deGuzman & Kelso, 1991; McGuinness, Hong, Galletly, & Larsen, 2004; Peper, Beek, & van Wieringen, 1991, 1995a, 1995b; Treffner & Turvey, 1993; Winful, Chen, & Liu, 1986).

One can generate predictions by iterating Eq. (1) for all possible combinations of Ω and K or turn to others who have done that before us and summarized the results in intuitive visual displays. The Arnold tongues (Fig. 1a; Arnold, 1983) and Farey tree (Fig. 1b; González & Piro, 1985; Hardy & Wright, 1965), named after their creators,

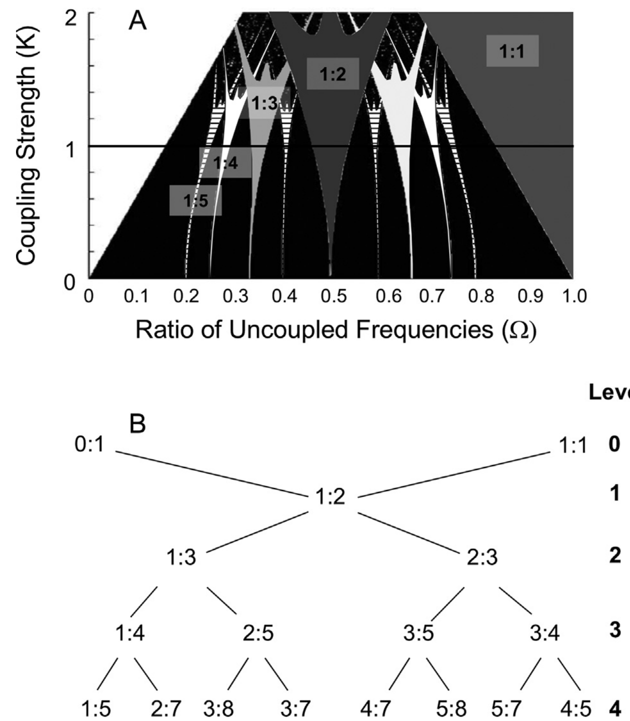

Fig. 1. Predictions from the two-frequency resonance map are depicted in the (A) Arnold tongues and (B) Farey tree.

mathematician Vladimir Arnold and geologist John Farey, Sr., depict the predictions of the two-frequency resonance map. Performed frequency ratios are depicted as tongue-shaped resonance regions in the Arnold tongues of Fig. 1a; for purposes of clarity, only larger tongues are depicted, but there are an infinite number, just as there are an infinite number of p:q ratios. When applied to real-world phenomena, like the BZ reaction or drumming rhythms, tongue width corresponds to performance stability. Examination of Fig. 1a reveals that some ratios (e.g., 1:1, 1:2) are more stable than others (e.g., 1:4, 1:5) and that performance of a ratio becomes more stable as coupling strength increases. Those predictions can be tested empirically.

Comparison of smaller tongues is often difficult. The Farey tree (Fig. 1b) is useful in that regard because it provides a rank ordering of the Arnold tongues by width. Notice that the same ratios that occupied wide tongues in Fig. 1a occupy lower levels of the Farey tree in Fig. 1b. The level of the Farey tree corresponds inversely to ratio stability, so that we would expect to see the ratios that reside at lower levels of the Farey tree performed more often and exhibit lower variability (greater stability) than ratios at higher levels. Fig. 2a depicts predictions for 5 simple ratios (1:1, 2:1, 3:1, 4:1, and 5:1) that occupy five different levels of the Farey tree.

2.1. From physics to motor coordination

Application of the two-frequency resonance map to motor and social coordination requires that the two

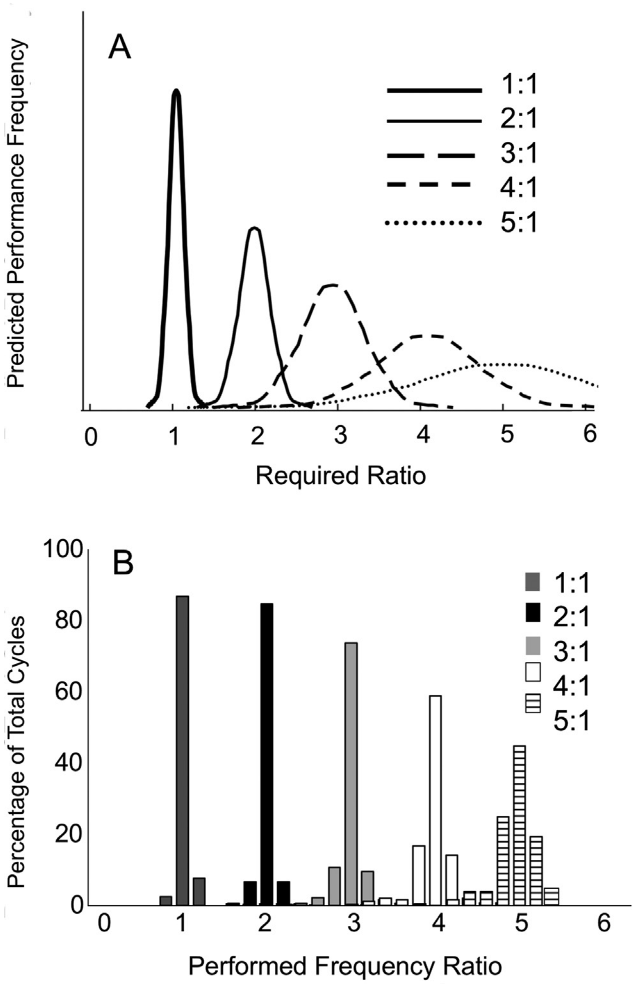

Fig. 2. Frequency distribution of performed frequency ratios as (A) predicted by the two-frequency resonance map and (B) observed in an experiment on motor-respiratory coordination.

parameters Ω and K be operationalized. The ratio of uncoupled frequencies, Ω, can be manipulated by varying differences in the physical properties, and therefore the preferred (eigen-)frequency of the two oscillators; different pacing signals can be provided that drive the oscillators at different frequencies independently of each other; or instructions can be given to perform a required ratio. The prediction is that performance success will be constrained by the dynamics of the two-frequency resonance map. I will not discuss coupling strength in depth in this paper, although it is often manipulated as the overall movement frequency (e.g., Peper et al., 1995b). Additional manipulations include the amount of visual information available about a partner's movements during social coordination (e.g., Gorman, Amazeen, Crites, & Gipson, 2017); and the role of practice in a learning experiment (e.g., Hessler & Amazeen, 2014). The application of the two-frequency resonance map to motor coordination is well-supported by research on bimanual coordination (e.g., drumming patterns, Peper et al., 1991, 1995a, 1995b; also see Treffner & Turvey, 1993).

To understand whether the model might generalize to synchronization across people, we first tested whether predictions hold across two physiological subsystems. A number of years ago, we ran some experiments on motor-respiratory coordination (MRC) to test whether the same model could be used to make predictions about coordination across the motor and respiratory subsystems of the body as occurs when someone synchronizes their breathing with their movements during exercise, for example. We were motivated by the observation that motor-respiratory coordination is widely observed: during both quadrupedal and bipedal locomotion (e.g., walking, van Alphen & Duffin, 1994; running, Bernasconi & Kohl, 1993; Bramble & Carrier, 1983; Lafortuna, Reinach, & Saibene, 1996; cycling, Garlando, Kohl, Koller, & Pietsch, 1985; Paterson, Wood, Morton, & Henstridge, 1986) and during upper-limb locomotion in both animals (e.g., geese, Butler & Woakes, 1980; bats, Suthers, Thomas, & Suthers, 1972) and humans (e.g., wheelchair propulsion, Amazeen, Amazeen, & Beek, 2001; rowing, MacLennan, Silvestri, Ward, & Mahler, 1994; Mahler, Hunter, Lentine, & Ward, 1991; Mahler, Shuhart, Brew, & Stukel, 1991).

Despite large differences in the patterns of coordination produced and the number of joints required, there was remarkably low variability in the ratios observed across those experiments: 1:2 (during rowing only), 1:1, 2:1, 3:1, 4:1, 6:1, 3:2, 5:2 [Note that some researchers do not report integers greater than five (e.g., MacDonald, Kirby, Nugent, & MacLeod, 1992)]. The ratios observed during motor-respiratory coordination are readily apparent in the Arnold tongues and Farey tree of Fig. 1, although the convention in the physiological and sport literature is to report ratios $p > q$, in contrast to the convention of $p < q$ in the model. Both simple (p:1, as in 2:1) and complex ratios ($p \neq q \neq 1$, as in 3:2) are performed in real-world MRC, but it is clear that simple ratios are preferred. We collected data using a simple experimental protocol in which both simple ratios and complex ratios, including novel ratios that had not been observed in the physiological literature, could all be performed without exposing participants to dangers like the potential loss of postural stability during the performance on a treadmill, for example, of some complex ratio like 5:3. In that way, the predictions of the two-frequency resonance map could be fully tested.

Participants sat in a chair and were asked to swing the arm back and forth as they breathed into a mask that measured their respiration. In the first experiment, we asked participants to maintain a simple ratio of 1:1, 2:1, 3:1, 4:1, or 5:1. The instruction was simple enough not to require any additional explanation or training and permitted a comparison of ratios that were predicted to vary in stability. Predicted distributions are depicted in Fig. 2a. Notice that the height of those predicted distributions corresponds directly to the width of the Arnold tongue for that ratio and inversely to the level of the Farey tree at which that ratio resides.

Eight participants performed the ratios in random order, and frequency ratios were calculated per cycle. Group data, which correspond to the predicted data of Fig. 2a, are depicted in Fig. 2b. It is clear that performance variability varied as a function of the ratio performed: ratios occupying larger Arnold tongues and residing at lower levels of the Farey tree were observed more often and performed with greater stability than ratios occupying smaller tongues or residing at higher levels on the Farey tree. Analysis of variance (ANOVA) revealed that both mean frequency ratio and the standard deviation of frequency ratio, a measure of variability, were significantly different across the five ratios, $F(1, 7) = 22{,}131.56$, $p < .001$ and $F(1, 7) = 25.18$, $p < .005$, respectively. We further supported the predictions of the two-frequency resonance map by testing both simple and complex ratios in subsequent experiments (e.g., Hessler & Amazeen, 2014; Hessler, Gonzales, & Amazeen, 2010).

2.2. From physics to social coordination

More recently, we tested the application of the two-frequency resonance map to coordination across dyads. We used a continuation tapping paradigm to allow participants to initiate each trial with a perfect uncoupled frequency ratio Ω but no coupling. Participants closed their eyes and tapped their forefinger to a metronome pacing signal, distinct for each member of the dyad and specifying Ω that was delivered through headphones. After 15 sec, we discontinued the pacing signal but asked the participants, who were seated across from each other, to continue tapping at the same pace as they watched their partner's finger. That was the introduction of coupling K. We tested all 16 ratios (excluding 0:1, which eliminates one process) depicted in the Farey tree of Fig. 1b. A sample time series depicting the performance of 3:2 is depicted in Fig. 3a. Notice that Person 2 completes three full cycles during the time that only two cycles are completed by Person 1.

The results for all 16 ratios are presented as a function of the Farey tree level in Fig. 3b; additional analyses will be presented in a manuscript that is currently being prepared. Notice the correspondence of those results with the results of Fig. 2b. Both figures depict multifrequency coordination across five levels of the Farey tree, but the remarkable difference is that coordination was within a person (and between two physiological subsystems) in Fig. 2b and between two different people (but within the same physiological subsystem) in Fig. 3b. Results are consistent with previous empirical work and support the predictions of the two-frequency resonance map.

To fully support the principle of dynamical similitude, we brought our testing back into the real-world to study multifrequency ratios generated during Double Dutch jump roping. Double Dutch requires coordination across multiple people: Two rope turners holding the endpoints of two long ropes in their hands rotate the ropes in opposite directions as one or more jumpers jump and perform

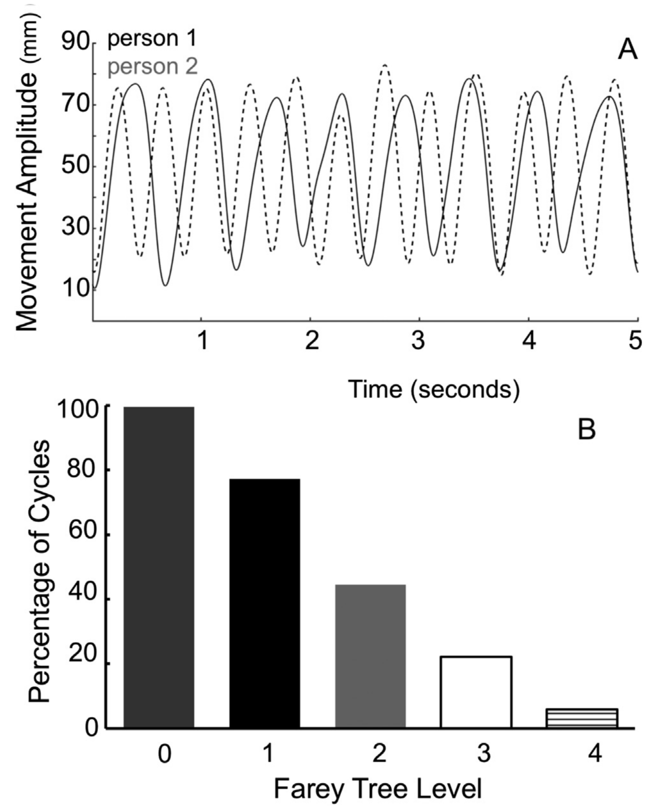

Fig. 3. (A) In one representative dyad, synchronized finger tapping is observed as the maintenance of a 2:3 frequency ratio. (B) Across all dyads, the distribution of frequency ratios performed supports the predictions of the two-frequency resonance map.

acrobatic tricks. Anyone who has observed the sport of Double Dutch can attest to the fact that it involves complex patterns of coordination. We had the opportunity to collect coordination data from elite Double Dutch teams at a summer camp sponsored by the National Double Dutch League. In some conditions, we simply recorded jumping routines and analyzed them for observations of spontaneously-performed multifrequency ratios. In other conditions, we asked teams to explicitly perform particular ratios so that we could examine their stability.

For purposes of illustration, consider the recording of rope turns and footfalls when teams were asked to perform 5:7 (Fig. 4). Blocks of rope turns and footfalls help to identify the performed ratio. Notice that, for every block, the rope hits the ground five times for every seven footfalls so that a 5:7 ratio is maintained. Perhaps more intriguing

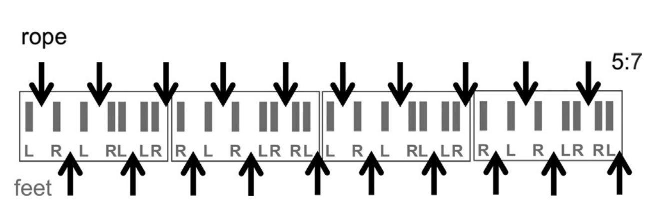

Fig. 4. In the sport of Double Dutch jump roping, maintenance of a 5:7 frequency ratio demonstrates synchronization between the movements of the rope by the rope turner and footfalls of the jumper.

is this team's strategy for performing this particular ratio: with an odd number of footfalls required, there is a higher order structure such that one block starts on the left foot (LRL, RL, LR) and the next block starts on the right (RLR, LR, RL). To maintain a stable complex ratio, teams alternated footfall patterns. Preliminary results are available in a number of papers (Gorman, Amazeen, et al., 2017; Gorman, Dunbar, Grimm, & Gipson, 2017), and full details will be published following additional analyses. For the time being, it is sufficient to note that dynamical similitude is demonstrated in the application of this singular dynamical model to patterns of coordination in physical, chemical, biological, psychological, and social systems.

3. A dynamical systems toolbox

I don't want to leave the reader with the impression that dynamical systems application requires the use of a pre-existing model. In fact, the models identified so far are only two dynamical models out of a whole set of dynamical methods—a *dynamical systems toolbox*—that may be applied in psychology. Dynamical similitude is important for use of this tool box because method selection is based on behavioral, not material, similarity. Therefore, we do not look within a subfield of science or a subfield of psychology for inspiration. Instead, we look across science as a whole to identify appropriate methods for our phenomenon of interest.

We've seen that motor coordination across two rhythmic processes might be characterized as phase-locked or frequency-locked, depending on whether the two processes reside in a common time scale, and are best characterized by their relative phasing, or different time scales, and are best characterized by a frequency ratio. The selection of an appropriate model follows from those data characteristics: relative phase predictions are captured by the HKB model, and frequency ratio predictions are captured by the two-frequency resonance map. From each of those models, we can make predictions about performance accuracy, performance stability, and, relatedly, transitions between patterns as a function of different conditions, including, in both cases, coupling strength.

However, dynamical analysis is not limited to the use of just those two models. Dynamical systems analysis is a toolbox of methods that belong to the general class of time series analyses. Fig. 5 depicts what I playfully call a Hollywood Squares of Dynamical Systems Analysis. Hollywood Squares was a game show where players called on celebrities occupying the squares to answer particular questions. In the best of all worlds, there would be a match between the question asked and the knowledge of the celebrity chosen. That was rarely the case. In the Hollywood Squares of Dynamical Systems Analysis, we choose a dynamical method that helps us to characterize the pattern represented in the box. For example, one might use a logarithmic function to capture the idealized learning curve of Fig. 5d.

Notice that the patterns depicted in Fig. 5 vary considerably: Some patterns, like the stationary data of Fig. 5j, might be characterized with descriptive statistics that do not change over time. A mean calculated over the first third of the data, for example, is not appreciably different from a mean that is calculated over the last third of the data. In contrast, descriptive statistics change over time for nonstationary series, like the phase transition data of Fig. 5h or the learning data of Fig. 5a. The mean for the first third of the series depicted in Fig. 5h is considerably greater than the mean calculated over the last third of those data. In that case, we may want to use a windowing technique where we use different values to characterize the different

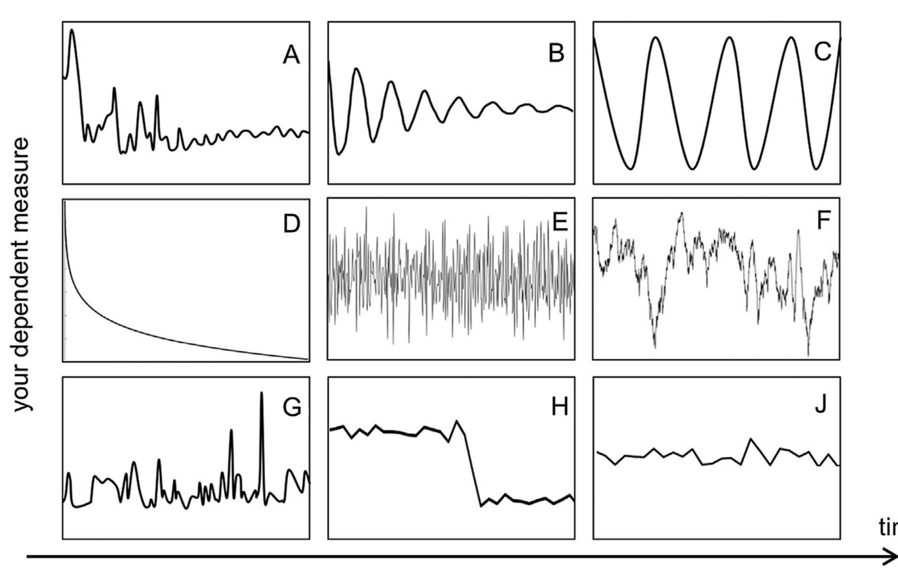

Fig. 5. Dynamical systems toolbox is a collection of dynamical systems methods that can be used to analyze multiple types of patterns, including (A) learning data; (B) damped oscillations; (C) cyclic data; (D) logarithmic data; (E) fractal processes; (F) multifractal processes; (G) data with no apparent pattern (team coordination data from Gorman et al., 2010); (H) phase transitions; and (J) fluctuations about a single mean.

stable regions observed. The size of the window can be varied to capture the transition phenomenon. There are many dependent measures in the field of dynamical systems analysis that can be used to study transitions (Scholz, Kelso, & Schöner, 1987). One option is to estimate transition time to make comparisons of ease of transition across multiple observations.

Notice in Fig. 5 that some of the changing patterns appear to be systematic: the learning data of Fig. 5a approach some accuracy value and become less variable over time (practice). Other systematicities—we might call them *patterns of change*—include the cyclic data of Fig. 5c and damped cycling in Fig. 5b. In those situations, we can identify a pattern that repeats itself over time either precisely or with some systematic transformation like damping. Finally, there are those panels that appear incredibly noisy (e.g., Fig. 5e) or whose patterning, if it exists, is not visually accessible or easily described (Fig. 5g). All of the patterns depicted in Fig. 5 have been observed in psychological experiments, and there are dynamical analyses that can characterize each of them. The key to applying these methods is making an appropriate selection for your data.

Consider the damped oscillations of Fig. 5b. We observed patterns like this in diary data from a month-long study on patients with rheumatoid arthritis (Finan et al., 2010). The dependent measure of interest was the accuracy with which these patients predicted their daily pain because errors in either direction have negative consequences for the patient's lifestyle and coping (Rachman & Arntz, 1991). Traditional studies that focused on aggregating the data ended up with conclusions that treated patients as static: they were overpredictors (Arntz, van Eck, & Heijmans, 1990; Rachman & Lopatka, 1988) or underpredictors (e.g., Arntz & Peters, 1995; Finan, Zautra, & Tennen, 2008; McCracken, Gross, Sorg, & Edmands, 1993). In those studies, fluctuations were characterized as random (Arntz et al., 1990; Finan et al., 2008; Rachman & Lopatka, 1988).

Were the observed fluctuations truly random or was there some systematicity that could be characterized dynamically? Review of the raw time series data for all 170 patients revealed two consistencies: nearly all patients exhibited cycling and damping. Instead of consistently overpredicting their pain or underpredicting it, as the literature had suggested, patients cycled between overprediction and underprediction during the course of the month. They also became more accurate as they engaged in the prediction process, an observation that was consistent with previous literature (Crombez, Vervaet, Baeyens, Lysens, & Eelen, 1996; Rachman & Arntz, 1991).

The appropriate model choice, then, was *mass-spring models,* where estimation of parameters like stiffness, damping, and nonlinear escapements serve to characterize the size and shape of the cycles (e.g., Beek & Beek, 1988; Beek, Schmidt, Morris, Sim, & Turvey, 1995; Butner, Amazeen, & Mulvey, 2005). Mass-spring models have the very boring origin of being used to characterize—get this—the behavior of a mass resting on a surface (with particular friction characteristics) attached to a wall by a spring (of particular stiffness). It is through the modeling of those friction and stiffness parameters that we estimate observed damping and cycling, respectively. Nonlinear parameters allow us to capture nonlinearities in the cycling pattern. It turns out, though, that mass-spring models, while being developed for that simple physical system, characterize the behavior of very many types of systems that exhibit cycling.

We combined mass-spring modeling with multilevel modeling to extract group-level patterns of behavior as well as identify the effects of individual differences, like pain control, on the patterns observed. As a group, we found support for our initial observations of cycling and damping. One particularly interesting finding was that patients who differed in pain control (using an 11-point scale found in Tennen, Affleck, & Zautra, 2006) differed in the manner in which they approached accuracy: patients who were low on the pain control index cycled toward accuracy in a linear fashion. That means that their cycling pattern didn't change as a function of where they were in their cycle. In contrast, patients who were high in pain control cycled in a nonlinear fashion, lingering more in regions of accuracy than in regions of inaccuracy. In addition to being useful for diagnostics, one can imagine that knowledge being relevant for the timing of therapeutic interventions.

Mass-spring models can also be coupled, allowing for the analysis of how two cyclic processes influence each other over time (e.g., Butner et al., 2005; Hessler, Finan, & Amazeen, 2013). That method would be useful for social application. Although the equations are slightly different, those models are used in social psychology to characterize patterns in everything from weekly cycles in emotions (Chow, Ram, Boker, Fujita, & Clore, 2005) to a wide range of other clinical and social phenomena (see summary in Boker, Staples, & Hu, 2016).

A discussion of all of the tools represented in the Hollywood Squares of Dynamical Systems Analysis is not within the scope of this paper. In the interest of providing one more example of a tool that can be used across physical, physiological, cognitive, and social systems, we will consider the fractal data of Fig. 5e and f in the next section.

4. Fractal methods for nested data structures

In the absence of conducting our psychological research in a vacuum, we recognize that we can never completely isolate a phenomenon from everything that influences it. Instead, the measurements that we make inherit the stamp of all of the other effects, both above and below the level of analysis, that have some influence. Consider the very practical example of having your heart rate recorded at a doctor's office. The doctor analyzes heart rate not because she is only interested in the activity of your heart but because that recording acts as an index for the overall health of the body. That is because when we record heart rate, we are picking up on the very many other interacting physiological subsystems of the body. It has been known for some time that breathing patterns, for example, influence the cardiac signal: heart rate increases during inhalation and decreases during expiration, a phenomenon known as respiratory sinus arrhythmia (Hirsch & Bishop, 1981; Melcher, 1976; Weiss & Salzano, 1971). The effects on heart rate of cognitive and social factors, like psychological stress and social isolation, are well-documented (Horsten et al., 1999; Schnall, Dobson, & Landsbergis, 2016; Steenland et al., 2000).

Consider the schematic drawing of Fig. 6, inspired by Iberall (1987) and Van Orden, Holden, and Turvey (2003). The ordering of the processes identified in Fig. 6 is not critical. What is important is the identification of activities both above and below the level of measurement that might influence the measurement that is taken. For example, social interactions can range in time from short social encounters at a grocery counter to long-lasting friendships and family relationships. The study of a relationship, for example will necessarily be influenced by slower-moving cycles, like the annual (seasonal) cycles that dictate holiday celebrations, the academic year, sports seasons, and taxes, and by faster-moving cycles that influence each individual in the relationship, like weekly emotional cycles (Chow et al., 2005), cognitive processes like shifts in the direction of attention, level of motivation and engagement, and physiological processes. On any given day, a relationship may flourish because of some of these influences and suffer because of others. A measurement taken in a psychological experiment, like a daily diary data study, is a necessary product of all of those influences and demonstrates the integration of dynamics across scales of behavior.

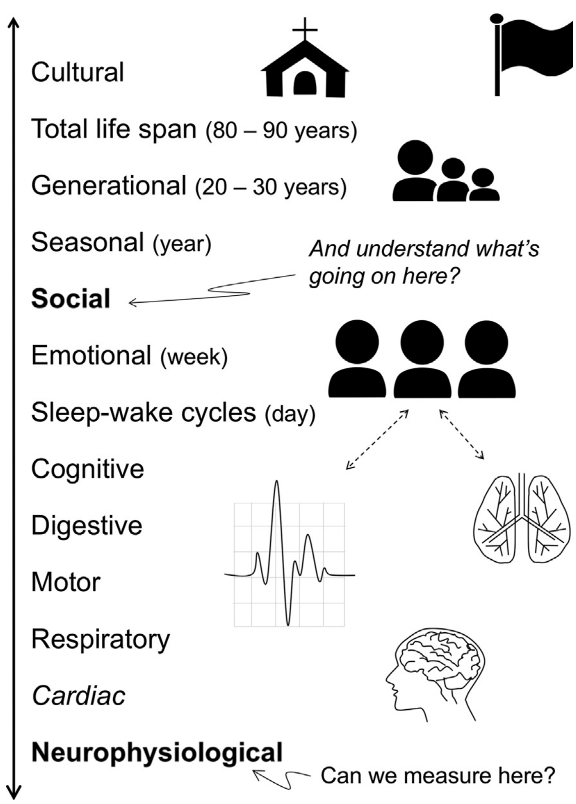

Fig. 6. Schematic drawing of the very many nested processes that, through their interactions, give rise to fractal behavior.

Additional influences both above and below the level of the social group are presented in Fig. 6. Importantly, the influence across each of those processes is bidirectional, so that fluctuations in individual behavior, for example, influence the social relationship, but fluctuations in the social relationship also influence the individual's behavior. Specifically, an individual's cognitive reasoning, affected by a lack of sleep, may help to create friction with his partner. At the same time, miscommunications within the couple can affect the individual's cognitive state and sleep pattern. That bidirectional influence will form the basis for the analysis of the empirical data presented next.

4.1. From motor to social coordination

Fig. 6 allows us to start considering the very many nested cycles in a psychological measurement. Now let us consider concretely some real empirical data, specifically, electroencephalogram (EEG) data from individual team members (Stevens, Galloway, et al., 2013; Stevens, Galloway, Wang, & Berka, 2012; Stevens, Gorman, Amazeen, Likens, & Galloway, 2013) that formed the basis for a multifractal analysis of neural signatures of team (social) coordination (Likens, Amazeen, Stevens, Galloway, & Gorman, 2014). That analysis will be discussed shortly. Mimicking the structure of Fig. 6, we see that the team resides at the social level but may be constrained by both slower processes at levels above the level of the group and smaller, faster processes. EEG signals came from team members who were engaged in a Submarine Piloting And Navigation (SPAN) simulation task at the Naval Training Academy in Groton, CT. That means that team behavior observed during the training session was influenced by more slowly-changing processes like the training procedures of the Naval Training Academy and the broader mission and directives of the U.S. Navy. At the same time, team behavior was influenced by individual characteristics of team members, i.e., their cognitive, motor, respiratory, cardiac, and neurophysiological states.

A significant feature of this data set was the assumption of bidirectional influence: For example, teams engaged in rhythmic tasks, like the "taking of Rounds", which consisted of a highly scripted reporting procedure that recurred every 3 min. It is highly likely that the presence of that recurring team-level task caused the neurophysiological states of individual team members to be correlated in some important way during the execution of that task. Their neurophysiological states may not have reflected the same level of coordination during times when they were not engaged in that common task. That assumption allowed us to expect that analysis of EEG signals at the level of measurement would reflect the state of the team at the social level.

Let us consider briefly the multifractal analysis that was conducted. Fifty years ago, Mandelbrot (1967, 1975) coined the term *fractal* to refer to phenomena whose interesting features did not exist at one privileged level of analysis but were distributed across all scales; that is, there is a single unifying scaling principle. This concept may be illustrated with a simple method for discovering a fractal scaling law called power spectral density (PSD). Any simple (e.g., simple cycling/sinusoid) or complex time series (e.g., music) can be decomposed via the Fourier transform into component frequency signals. The presence of a single peak in the resulting power spectrum identifies the contribution of a single frequency, as expected in a simple sine wave; the implication is that all observed fluctuations are accounted for by that one frequency. The presence of multiple peaks identifies the contribution of multiple frequencies, as in music. Fractal structure is identified by logarithmic decay across those spectral components; that is, all frequencies contribute to the observed series, but the energy accounted for by each frequency is inversely related—via a logarithm—to the size of that frequency. When transformed to logarithmic coordinates, the slope of the decay is -1; thus, the label 1/f noise. That is one way of estimating a scaling exponent.

In addition to PSD, many more sophisticated methods have been developed to characterize the fractal nature of a time series (see summary in Eke, Herman, Kocsis, & Kozak, 2002). All of them have as their output some variant of the scaling exponent discussed above that characterizes the relation of fluctuations across scales of analysis. Those methods allow us to quantify in empirical data Mandelbrot's scaling principle, the distribution of important features across all scales. Decades of research have contributed to our understanding that fractals exist broadly in the natural, psychological and cognitive sciences (e.g., Mandelbrot, 1982; Wagenmakers, Farrell, & Ratcliff, 2004). Some particularly interesting examples include the fractal structure of geographical distance and timing between earthquakes as related to the fractal structure of fault lines (e.g., Abe & Suzuki, 2003; Bak, Christensen, Danon, & Scanlon, 2002; Bak & Tang, 1989); the fractal structure of a healthy heart (Goldberger et al., 2002; Ivanov et al., 1996; Solé & Goodwin, 2000); coordination across people (e.g., Delignières, Almurad, Roume, & Marmelat, 2016; Fine, Likens, Amazeen, & Amazeen, 2015); and performance on a wide range of cognitive tasks, including simple response time, word naming, choice decision, and interval estimation (Gilden, 2001; Ihlen & Vereijken, 2010; Thornton & Gilden, 2005; Van Orden et al., 2003).

It is the very special case that real systems exhibit a single scaling principle across all levels of analysis. Real phenomena, including some of the examples mentioned above, exhibit multiple scaling regions, where each scaling exponent characterizes relations across a range of scales or across a range of time (e.g., Ihlen, 2012; Ihlen & Vereijken, 2010). Consider fractal structures within the human body, including the lungs and neural pathways, which exhibit different scaling relations and motivate the use of multifractal analysis.

It was the insight of Likens et al. (2014) that multifractals could be used to extract meaningful structures like events from empirical data. Pre-processing steps used to convert 54 channels of EEG data (nine channels per team member) to the Shannon entropy time series of Fig. 7a are presented in detail in Likens et al. (2014) and Stevens et al. (2012), but I will present a general outline here: EEG data were converted to one Engagement time series per participant using the B-Alert® EEG system. Those six individual time series were further reduced to a single time series of neurodynamic symbol patterns, labeled 1–25, that corresponded approximately to activity level across team members from least to greatest (see Stevens et al., 2012). A transition matrix tracked the changes in those patterns over time by mapping the current symbol onto the preceding symbol. The distribution of activity across that matrix was captured with Shannon entropy, calculated in 100-sec sliding windows ("Epochs" in Fig. 7a). In this context, entropy was inversely related to team organization: low values of entropy indicated high levels of organization, rigidity in the extreme, and high values indicated disorganization or transitioning.

Just as there are many methods for estimating fractals, there are many methods for estimating multifractals. One

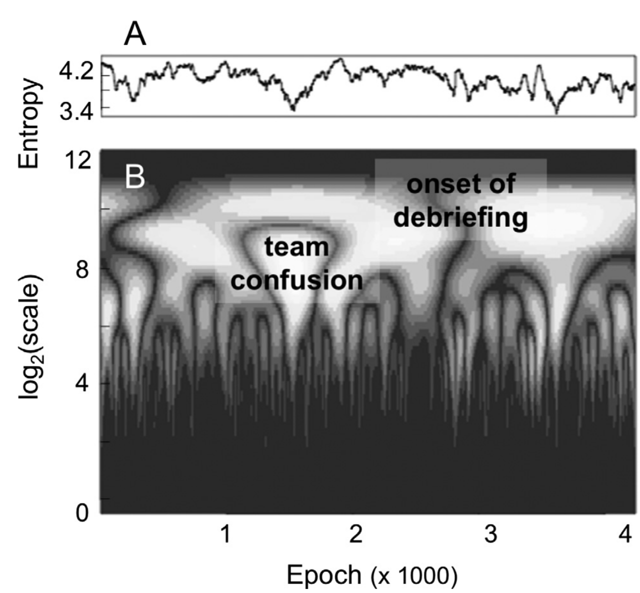

Fig. 7. Multifractal analysis of (A) an entropy signal derived from EEG data revealed (B) regions of organization that corresponded to team-level experiences in Likens et al. (2014). Brightness corresponds to the correspondence of the analyzing wavelet to features of the time series in (A) at a particular time (horizontal axis) and scale (vertical axis).

popular method, called multifractal detrended fluctuation analysis (MFDFA, Ihlen, 2012), estimates the spectrum (range) of scaling exponents. Likens et al. (2014) used another multifractal method called wavelet transform modulus maxima (WTMM; Mallat, 1999; Muzy, Bacry, & Arneodo, 1993; Percival & Walden, 2000; Struzik, 2001) because of its ability to localize those exponent changes. WTMM replaces the spectral components used in PSD and other fractal analyses with analyzing wavelets that permit customization to characteristics of the signal being analyzed and promote the identification of singularities, or transition points. That allows for wavelet analysis to be used to locate meaningful events in both time and scale of analysis.

In the first step, a continuous wavelet transform is generated by translating a wavelet of a particular size across a time series to estimate wavelet coefficients, or correlations of the wavelet with the series, over time. The process is repeated with wavelets of different sizes to derive the same estimates across a range of scales. The output is a matrix of wavelet coefficients that can be depicted as a heat map with brightness used to refer to the value of the wavelet coefficient, or correlation of the analyzing wavelet to features in the original signal. In Fig. 7b, brighter points identify strong positive correlations.

Interpretation is most meaningful when we consider regions of color/brightness rather than each individual point, for it is those regions that correspond to events. Coordinates of those events are given by time (“Epoch”) on the horizontal axis and scale (“$\log_2$(scale)”), or the size of the analyzing wavelet, on the vertical axis. Those coordinates can be used to locate meaningful events in both time and scale of analysis. Branching indicates nesting of smaller, faster-moving events inside larger, slower-moving events that mimics the nested structure (Fig. 6) that we expect gives rise to the fluctuations we observe in our measurements of human behavior.

What is often lost in empirical reports is the story of how the analysis unfolded. What I find most remarkable about this particular analysis is the fact that we did not have access to the transcripts while we completed the multifractal analysis. We identified what we considered relevant features and our colleagues, who had the transcripts, looked up the times we identified to provide a match to our observations. What is immediately clear is that larger, brighter regions at the largest scales of Fig. 7b are filled with branching that distinguishes events at smaller scales. At approximately Epoch 2800, the onset of debriefing is identified as a dark line that separates the largest scale events of Experimental Session and Debriefing. The impact of that debriefing pervades all scales of analysis, from the largest “session identity” to the smallest, momentary interactions. I mentioned previously the taking of Rounds, a recurring event for the team. The band of branching at $\log_2$scale 6 corresponds to that rhythmic event. Finally, we notice an encapsulated event at approximately Epoch 1500 and $\log_2$scale 8. Consultation of the transcript revealed a period of confusion for the team that resulted from a near-collision with another ship under conditions of reduced visibility.

Thinking back to the nested processes of Fig. 6, we can imagine that one of the reasons we can see this team-level experience in what started as neurophysiological data is due to the bidirectional influence of the very many nested processes inherent in social behavior. We used multifractal analysis to integrate neurophysiological and social scales of behavior. Moving forward, we must recognize that the fluctuations that we observe in our measurements in any experiment may reflect fluctuations in the process that we intend to measure as well as fluctuations of many other processes that reside both above and below the level of measurement. Instead of treating those fluctuations as negative, we can harness them to identify larger-scale and smaller-scale events of significance.

The next step is to characterize lagged influences across both spatial and temporal scale to identify the influence of one process on another at both concurrent and later points in time. That method will enable us to identify precise timing and size of influences across scales of behavior. Methods that identify historical (lagged) influence across bivariate or multivariate processes include vector autoregressive models (Hamilton, 1994; Sims, 1980), recurrence quantification analysis (RQA) and its variants (e.g., CRQ, JRQ) (e.g., Eckmann, Kamphorst, & Ruelle, 1987; Marwan, Romano, Thiel, & Kurts, 2007; Romano, Thiel, & von Bloh, 2004; Wallot, Roepstorff, & Mønster, 2016; Webber & Zbilut, 1994), and spectral and wavelet coherence analysis (e.g., Grinsted, Moore, & Jerejeva, 2004; Mandel & Wolf, 1976). Other methods characterize multiscale but contemporaneous influences across two processes (e.g., Kristoufek, 2015; Podobnik & Stanley, 2008; Podobnik, Wang, Horvatic, Grosse, & Stanley, 2010).

We recently submitted a paper that introduces a technique called Multiscale Lagged Regression Analysis (MLRA) that accomplishes both of those tasks (Likens and Amazeen, submitted for publication). Following the precedent of Kristoufek (2015), we apply detrended fluctuation analysis (DFA), a multiscale method, across two data streams whose lagged relationship is varied systematically. In this way, we are able to determine the influence of one process on another across both spatial and temporal scale. Using the example above, this would enable us to measure both neurophysiological and social states and estimate directly the influence of each one on the other at different scales (momentary vs long-term) and over time (immediate vs delayed). The future direction of this research program is to track influences across both time and scale in the multivariate case, that is, for the very many nested processes (e.g., Fig. 6) that contribute to the richness and complexity of human behavior.

4.2. From social coordination to large scale social phenomena

Is multifractal analysis limited to team coordination? Of course not. Dynamical similitude means that we can find those same dynamical patterns across many different systems. Because wavelets are particularly well-suited for spatial recognition, they have been used across a wide variety of tasks in the physical (cloud formations, Arneodo, Audit, Decoster, Muzy, & Vaillant, 2002); biological (DNA sequencing, Arneodo et al., 2002); neural (detection of event-related potentials, Aniyan, Philip, Samar, Desjardins, & Segalowitz, 2014); and psychological (facial recognition, van der Lubbe, Szumska, & Fajkowska, 2016; hand-mouse coordination, Nie, Dotov, & Chemero, 2011) sciences. Let's scale our question to the extremely large scale social level of an economic indicator: the Dow Jones Industrial Average (Dow Jones, for short). I must caution readers that this analysis is presented to illustrate the potential for this technique and is not rigorous enough to create conclusions about what events are and are not significant economically. The main point is to illustrate the generalizability of the technique used above for human teams and to point to potential for future development and application.

Consider temporal fluctuations in a stock market index (e.g., Mandelbrot & Hudson, 2004; Solé & Goodwin, 2000). The stock market, like the cardiac signal considered earlier, does not exhibit a characteristic frequency but is composed of fluctuations of all sizes. Like the social behavior considered in Fig. 6, the stock market index is influenced from above and below by slower- and faster-moving processes. Consider the fact that the value of the Dow Jones, a stock market index, fluctuates as a function of the behavior of the 30 companies of which it is composed. However, the behavior of those companies is constrained from above by federal laws governing business practices and the overall state of the world economy and influenced from below by the productivity of its employees, the cost of its materials, and the successes and failures of other companies with which it does business. We can think about those processes as bidirectional: the behavior of the company is both influenced by and influences the behavior of the employees. Success of the company as a whole might be reflected in larger bonuses, greater happiness, less stress (or more stress!); likewise, how the employees feel wealth-wise, happiness-wise, stress-wise, can affect the success of the company. Modeling the analysis of Likens et al. (2014), we can use multifractal analysis to integrate those very different scales of behavior.

We can use WTMM to extract information from the Dow Jones closing prices not just about the success of those 30 companies, but about larger-scale, impactful business and world events. Fig. 8a depicts closing prices of the Dow Jones from opening day in 1885 to the present day; the trendline has been removed to keep it from dominating the analysis. Notice that, even with the trend line removed, there are considerable fluctuations in the market. Most notable is the sharp decrease during late October 1929 that marked the beginning of the Great Depression. Sometimes, landmark events like that are obvious in the original time series. However, other events are temporally extended so that visual detection is less obvious.

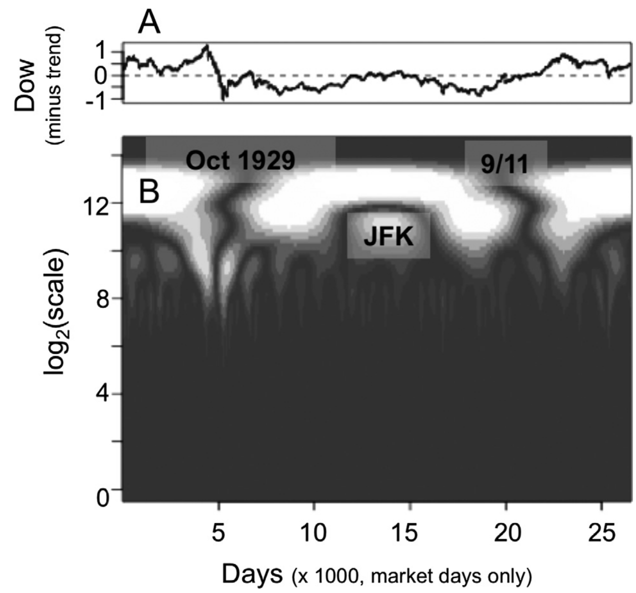

Fig. 8. Multifractal analysis of (A) the closing prices of the Dow Jones Industrial Average (growth trend removed) revealed (B) regions of organization that corresponded to significant historical events. Brightness corresponds to the correspondence of the analyzing wavelet to features of the time series in (A) at a particular time (horizontal axis) and scale (vertical axis).

Multifractal analysis allows us to extract from that signal major events in time that are not just economic events but signify important national and international events as well. In effect, the output of WTMM, depicted in Fig. 8b, expands into two dimensions the one-dimensional time series of Fig. 8a. The level of brightness in Fig. 8b corresponds to the value of the wavelet coefficient, with bright regions indicative of positive correlations between the analyzing wavelet and features in the time series. Transition points are identified as dark lines separating bright regions; in the analysis, they correspond to a lack of correlation with the analyzing wavelet. As demonstrated earlier, the coordinates of "Days" and scale ("$\log_2$(scale)"), or the size of the analyzing wavelet, can be used to locate, in time and size, meaningful events in the Dow Jones time series. Branching indicates nesting of smaller, faster-moving events inside larger, slower-moving events that mimics the nested structure (Fig. 6) that we expect gives rise to the fluctuations we observe in our measurements of human behavior.

Just as landmark events revealed in Fig. 7b were clarified by reference to team training transcripts, so the economic analysis presented here is clarified by reference to external information. Because this analysis is illustrative, I will refrain from commenting on smaller features and will identify just three important events: Most interesting is the observation that two events—the onset of the Great Depression in 1929 and the terrorist attacks of 9/11/2001—seem to form boundaries for some large-scale economic period that spans WWII and so very many other economic and social events. Within that period is a large encapsulated region that could correspond to any number of events around the time of John F. Kennedy's assassination; that might include important events in the Civil Rights movement, including Martin Luther King's "I Have a Dream" speech. Follow-up analyses that focus on that time period would be better able to localize the effect in time. The dark regions along the bottom of Fig. 8b are non-interpretable in this analysis. Again, follow-up analyses that focus on the smaller scales of analysis may reveal additional structure.

The significant contribution of this analysis is the identification of patterns that are not otherwise obvious. I think back to the discussion over these past 10 years about when the Great Recession (2008) ended. The National Bureau of Economic Research (NBER, 2010) defined June 2009 as the end of the recession based on the presence of a trough in the Gross Domestic Product and Gross Domestic Income, but the reporting committee acknowledged that other economic indicators at different levels of analysis had not improved. Both impact and recovery differed across areas of the country, socioeconomic groups, and industries (e.g., automobile, housing markets) (Reid, Carroll, & Ye, 2013). Areas that had experienced recent housing booms were hit particularly hard (Charles, Hurst, & Notowidigdo, 2015). At the more local level, families that experienced job loss delayed educational opportunities for their children (Charles et al., 2015). Some economists point to "economic scarring", that is, long-lasting effects of nutritional and educational disruptions that persist across future generations through their impact on health and wage earning potential (Irons, 2009). Analyses such as this one would enable an answer to that question across levels of analysis, from the largest national or international scale to smaller scales that might be indicative of the housing market or consumer confidence.

The analysis presented here is merely illustrative and limited due to a number of factors. To track events more accurately, we have to recognize that the data are closing prices for the Dow Jones for all of the days that the market was open. The fact that weekends and holidays exist within the same time frame means that the time steps are uneven along the horizontal axis. Recall that data were analyzed in Epochs in the previous example and here, the data are treated individually. The choice of step size, as well as the range of scales considered, will affect the structures that are visible. In Likens et al. (2014), follow-up analyses were conducted to provide quantitative evaluations of the structures that we observe visually. Finally, surrogate analyses, in which the analysis is repeated multiple times on shuffled copies of the same data set, provide a test of the null hypothesis that observed structure is spurious.

5. Lessons learned and future directions

Modern cognitive science laboratories have access to all types of continuous data streams, thanks to the affordability of modern-day eye trackers, physiological measurement devices, and a wide range of neuroimaging techniques. At the same time, the computational power of modern computers reduces processing of very large data sets to reasonable amounts of time (minutes or hours, depending on the analysis). The Dynamical Systems Toolbox offers cognitive scientists a collection of techniques to explore the information in fluctuations that accompany time series data. Studying cognition in this way acknowledges the primacy of behavior over (static) structure.

Psychology has a rich history of contrast between structure and function. Wilhelm Wundt and his student, Edward Titchener, promoted a structuralist approach whose influence persists to this day. Titchener (1898) modeled psychology after the science of biology, promoting reduction of a phenomenon to its parts to understand structure. He believed that an understanding of function (behavior) would logically follow. The structuralist tradition persisted through the 1900s and continues in some cognitive science laboratories today with researchers who treat the content of cognition as static mental constructs upon which operations are performed or who use neural imagery primarily to catalogue the neural structures responsible for different aspects of cognition. With the goal of looking for constancy, the standard approach is to heavily filter out what a dynamicist would consider potentially meaningful fluctuations and reduce data sets to single data points that imply punctate events.

The functionalist approach came from the branch of psychology that originated in America. Its founder, William James, believed in the primacy of function, or behavior, over structure. His famous treatment of a stream of thought identified cognition as a continuous process, with the implication that the memory of a past event was fundamentally different from the contemporaneous experience of that event (James, 1890). The application of dynamical systems to cognitive science is a natural extension of the Jamesian vision of cognition as a dynamic, constantly evolving process. It is the functionalist approach that allows for the principle of dynamical similitude to be used to understand behavior across systems that are structurally very different. It is the functionalist approach that allows us to borrow and adapt dynamical methods from very different scientific fields.

Two of my favorite examples of the contrast between function and structure come from modern-day treatments of team coordination and the brain: Gorman, Cooke, and colleagues use dynamical systems to study team coordination as a flexible, constantly evolving process rather than a shared mental model (Gorman, 2017; Gorman, Amazeen, & Cooke, 2010; Gorman, Cooke, & Amazeen, 2010). Buzsáki (2006) likewise treats the brain as constantly active and changing in his book, *Rhythms of the Brain*. That dynamic weakens the argument for subtraction techniques that are commonly used to compare multiple brain scans, for example, during baseline and cognitive task performance trials.

5.1. Implications for cognitive science

The current status of dynamics in psychology is a continuation of the trajectory that was begun in the 1980s with the first application of dynamical systems models to psychology in the field of bimanual coordination. Advances follow technological upgrades in measurement devices, development of analytical methods, and improved accessibility to equipment and knowledge through price reductions and electronic information sharing. It would be impossible to provide an overview of all of the latest developments. In this section is presented just some of the most recent research that extends the methods reported in this paper.

Some of the most recent research continues to broaden our understanding of the range of applicability of well-established models. There is continued interest in the study of relative phase and multifrequency (motor) coordination both within and across two people in both laboratory and applied settings (e.g., Bingham, Snapp-Childs, & Zhu, 2018; Gorman, Amazeen, et al., 2017; Pickavance, Azmoodeh, & Wilson, 2018). Researchers are taking advantage of technological advances in artificial intelligence to generate artificial teammates to perturb team coordination dynamics (e.g., Demir, Amazeen, McNeese, Likens, & Cooke, 2017). Those studies provide a crossover to human factors research questions about the dynamics between human and machine systems. There continues to be a lot of interest in the dynamics of teams, and researchers are scaling up group size to study the creation of nested dynamical patterns, for example, the creation of subgroups within larger teams (Zhang, Kelso, & Tognoli, 2018).

Analytical developments, particularly in the area of multifractal methods, are being used to understand the interdependence of perception, action, and cognition. Researchers are using a variety of methods to study how complexity matching facilitates both coordination and communication across individuals (Delignières et al., 2016; Fine et al., 2015; Ramirez-Aristizabal, Médé, & Kello, 2018). They are studying postural adjustments, estimated from multiple body sites, to understand how participants correct their actions in response to the visual distortions created by prism goggles (Carver, Bojovic, & Kelty-Stephen, 2017) and kinematic and physiological measures to understand how perceptions are formed (Hajnal, Clark, Doyon, & Kelty-Stephen, 2018; Waddell & Amazeen, 2017, 2018).

Dynamical modeling also allows for simulation work that promotes new insights: Recall that wavelets were particularly well-suited to detect features in time series. A recent study reported the development of a wavelet-based algorithm for the automatic detection of lying from EEG data (Xiong, Gao, & Chen, 2018). One of my favorite applications of dynamics to cognition was the use of entropy to track the emergence of insight in a problem-solving task (Stephen, Dixon, & Isenhower, 2009). Recent work models the emergence of innovation as a series of random walks on a complex topology of ideas and concepts (Iacopini, Milojevic, & Latora, 2018).

5.2. Summary and conclusion

The principle of dynamical similitude implies that the same method may be used to capture the same basic behavior pattern across systems whose physical appearance seems very different. In this paper, I presented examples that characterize two distinct approaches to using dynamical similitude to understand human behavior: The first entailed the use of models that characterize patterns across patterns. That is, they serve to unify behavioral patterns through parameterization of a single dynamical model. Models of both phase locking and frequency locking were derived from physical systems but apply to psychological and social systems. Those models can be used to make predictions about the patterns that we see as well as patterns that we have not yet observed and, importantly, the relation that bridges them all.

The second approach entailed the use of dynamical methods to characterize observed patterns. Mass-spring models and multifractal analysis, as well as other methods represented in the Dynamical Systems Toolbox, have been applied successfully across physical, chemical, biological, psychological, social, and economic systems. I would argue that this latter approach characterizes most of the use of dynamics in psychology today. However, to lose sight of the power and generativity of the first approach would be a mistake. Our goal as psychologists should be to continue to search for unifying dynamical models that focus on behavior over structure to ensure the continuity of psychological phenomena with the rest of the natural world and to ensure an impact of our science beyond our own disciplinary boundaries.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.cogsys.2018.07.033.

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