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EXPLORING THE OPTIMALITY OF A LIMITED VIEW ANGLE IN THE TWO-DIMENSIONAL VICSEK MODEL

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ABSTRACT

Collective behavior emerges from local interactions in a group, has been observed in many natural systems, and is of significant interests for engineering applications. The Vicsek model is a mathematical tool to study collective alignment in a group of self-propelled particles based on local interaction, which has been well-studied in the literature for its simple algorithm and complex global behaviors. Several studies show that particles reach alignment faster when the directionality of particle interaction is restricted by an optimal view angle. This result seems counterintuitive, since each particle is expected to get more information through omnidirectional interaction. This work seeks to explore the possible causes of this optimal view angle by studying interaction dynamics in Vicsek model with restricted view angle through numerical simulation.

INTRODUCTION

When a group of individuals interact, a group-level pattern may emerge from only local information sharing, even in the absence of any global cues. This group-level complexity is referred as collective behavior and has been observed in many animal systems ranging from cells to humans [1–7]. Such behavior has also been observed in non-living systems, where material particles interact among each other as has been reported in vibrating rods [8] and nematic liquid crystal [9]. Moreover, collective behavior is exploited in the design and control of engineered systems especially in swarm robotics [10]. Therefore, understanding collective behavior can be beneficial to a large range of scientific disciplines. A through review of the previous studies on collective behavior and its applications is published in [11].

There are two main approaches to model collective behavior. In one approach as is implemented in [12], the system is considered as a continuous medium, while the other one is based on defining interaction rules between individuals [13], which is often referred to as agent-based modeling. The so-called Vicsek model is one the most studied agent-based models of collective behavior due to its capability to capture complex behavior using a simple update rule [14]. In this model, self-propelled particles move in two dimensions and every particle aligns its heading direction with an average of the nearby particles, subjected to some random noise. The ability of the particles to align as a group is investigated for different values of noise. As the value of the noise exceeds a *critical noise*, the system shows a *phase transition* from an aligned ordered phase to a random disordered phase [14]. Many variations of Vicsek model can be found in the literature, such as its extension to three dimensions [15], incorporation of a blind spot for interactions [13], addition of attraction and repulsion rules [13], changing the symmetry of the particles [16], and applying different types of noise [17, 18].

Inspired by the restricted angular range of sensing for many animals, collective behavior in the Vicsek model with restricted view angle is investigated in the literature [19–23]. The study reported in [19] shows that, for the two-dimensional Vicsek model with restricted view angle, the critical noise is negligible for view angles less than π and therefore, phase transition exists only when each particle interacts with at least all particles in

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front of it. In another study, the nature of phase transition in a restricted-view-angle Vicsek model is investigated as the view angle increases in the presence of constant noise. The article reports non-monotonicity of an order parameter for the system with changing view angle [20]. This non-monotonicity is also observed in [21] in which they report the existence of an optimal view angle in which the system aligns faster in absence of noise, as compared to when no view angle is imposed. The existence of such an optimal angle is also reported in three-dimensional Vicsek model in [22], even though the interaction rule reported in the paper seems to be two-dimensional. A more recent study adds a random *head motion* to the two-dimensional restricted-view-angle Vicsek model and finds a faster convergence to the ordered system at the optimal angle [23].

Although such an optimal view angle is reported in many studies in the literature, its existence is counter-intuitive since it limits the area each particle perceives by design. As a result, each particle is expected to interact with less neighboring particles when a view angle less than a full circle or sphere is imposed. According to the update rule of the Vicsek model, each particle moves in the averaged direction of its neighbors and therefore less neighboring particles should lead to less averaging and slower alignment. Therefore, alignment should happen slower as the view angle decreases. Moreover, one may expect adding more randomness in terms of head motion may reduce the time to reach alignment. These expectations, however, are in contrast with the results of previous studies that show non-monotonic dependence of convergence time on view angle. Although several independent studies confirm these result, no study investigating the reasons behind these observations exists to the best of our knowledge.

In this paper, we study the two-dimensional Vicsek model with restricted view angle and investigate the correlation between the view angle, convergence time for collective behavior, and the metrics of interaction dynamics between particles using numerical simulation. The results of this work suggest an explanation for the existence of the optimal view angle based on features of the interaction network created by the restricted view angle.

MODELING

The Vicsek model with restricted view angle considers a group of N self-propelled particles moving with constant speed v_0 inside a square of length L with periodic boundary conditions. The heading angle of particle i, θ_i, is defined as the angle between its heading direction and the horizontal line. For each particle, we define the *view region* as a circular sector with radius R and opening angle ϕ such that has the particle's heading direction bisects its opening angle. We refer to the radius R and the opening angle ϕ as the *view range* and *view angle*, respectively. All the particles located in the view region of a specific particle are called its *neighbors*. Figure 1 shows a schematic picture for this model.

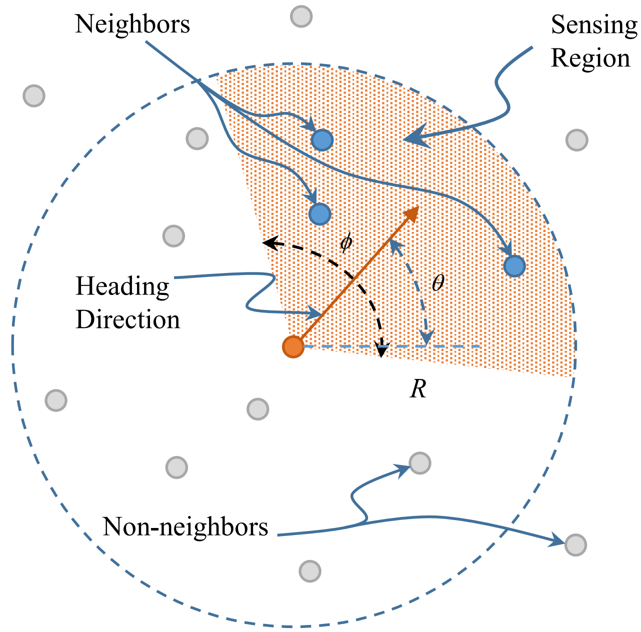

FIGURE 1. A SCHEMATIC OF THE GEOMETRY OF THE MODEL SHOWING THE VIEW REGION, VIEW RANGE R, VIEW ANGLE ϕ HEADING DIRECTION, HEADING ANGLE θ, AND THE NEIGHBORS OF A PARTICLE

The interaction rule between particles is based on averaging the heading direction of neighboring particles. Particle i at time step k has position vector $\mathbf{x}_i(k) \in \mathbb{R}^2$ and heading angle $\theta_i(k)$. Then, the heading direction of this particle at time step $k+1$ is set to be the average of heading directions of its neighbors, disturbed by a noise. In other words,

$$\theta_i(k+1) = \arctan\left(\frac{\langle \sin(\theta_j(k)) \rangle_{\Lambda_i(k)}}{\langle \cos(\theta_j(k)) \rangle_{\Lambda_i(k)}}\right) + \Delta\theta_i(k). \tag{1}$$

where $\Lambda_i(k)$ is the index set of all the neighbors of particle i at time step k, and $\langle . \rangle_A$ denotes the operation of averaging over the set A. The noise $\Delta\theta_i(k)$ is a realization of a random variable with uniform distribution over $\left[-\frac{\eta}{2}, \frac{\eta}{2}\right]$, where η is the noise magnitude. Note that $\Delta\theta_i(k)$ is sampled independently over the particles i and time steps k.

After finding the updated heading angle, the heading direction of the particle can be computed as

$$\mathbf{V}_i(k+1) = \cos(\theta_i(k+1))\,\mathbf{i} + sin(\theta_i(k+1))\,\mathbf{j} \tag{2}$$

where $\mathbf{i}$ and $\mathbf{j}$ are respectively the unit vectors in horizontal and vertical directions. Assuming the particles are self-propelled with common speed v_0, the position of particle i at time step $k+1$ is

$$\mathbf{x}_i(k+1) = \mathbf{x}_i(k) + v_0 \mathbf{V}_i(k+1). \quad (3)$$

The collective behavior generated by the Vicsek model is alignment, which can be captured using *polarization* as order parameter. Polarization is the magnitude of the averaged normalized velocity of the group. In other words, at time step k, the polarization of the group $P(k)$ is equal to

$$P(k) = \frac{1}{N} \left| \sum_{i=1}^{N} \mathbf{V}_i(k) \right|. \quad (4)$$

Therefore, polarization is a number between 0 and 1 and higher values of polarization is associated with better alignment in the group.

For a realization of the model, the polarization time series reaches stationary condition after an initial transient. The mean value of stationary polarization depends of the level of noise η and the view angle ϕ. In case of zero noise and large enough view angle, the group will establish full alignment and polarization reaches to value 1. Therefore, the length of the transient τ captures how fast the group reaches its stationary condition and can be used as a metric to study alignment [21], as we will do in this study. As a comment, the mean value of polarization in stationary condition measures the collective behavior when noise is present, as is used in [14].

INTERACTION METRICS

From the previous research in the literature, we know that the length of the transient depends on the view angle of the particles and that an optimal angle exists which minimizes transient length. In this work, we explore the interaction among the particles during the transient to get some clue about the dynamics of the system that explains the origin of the optimal angle. For this effort, we study the interaction between the particles in a multi-agent system as a dynamic network or graph.

Mathematically, a (directed) *graph* G is defined as an ordered pair of sets $G = (V, E)$ in which elements of E are the ordered pairs of elements of V. The elements of V and E are called *vertices* and *edges* of the graph G, respectively. We say that the vertex v_j is *adjacent* or *neighbor* to vertex v_i if (v_i, v_j) is an edge of the graph. For each graph, the *adjacency matrix* $\mathbf{A}$ summarizes the adjacency relationship between vertices of the graph. The element a_{ij} of the adjacency matrix $\mathbf{A}$ is equal to 1 if (v_i, v_j) is an edge of the graph, otherwise it is equal to 0.

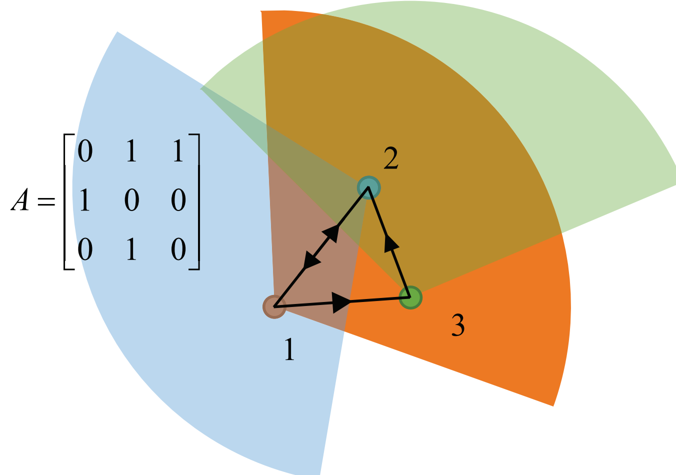

FIGURE 2. A SCHEMATIC OF SMALL GROUP OF PARTICLES ALONG WITH THEIR VIEW REGIONS AND CORRESPONDING INTERACTION GRAPH AND ADJACENCY MATRIX

For a multi-agent system, at each time step, we can define an interaction graph G_k based on the neighborhood relationship between the particles. We assign a node v_i to the i^{th} particle and we can define and edge (v_i, v_j) if particle j is a neighbor of particle i. We can also define the adjacency matrix $\mathbf{A}_k$ at each time step. Figure 2 shows an example of an interaction graph as well as the adjacency matrix for a small group of particles.

In order to study the interaction between the particles during the transient time, we choose two different metrics. The first one is the total number of the interactions averaged over the group, calculated during the transient time. This metric is chosen to investigate if increasing view angle leads to more interaction in the group. Knowing the adjacency matrix at each time step, we can define I, the total number of averaged interaction, as

$$I = \left\langle \sum_{k=1}^{K_t} \mathbf{A}_k \mathbf{1}_{N\times 1} \right\rangle_{\mathcal{N}} \quad (5)$$

where $\mathcal{N} = \{1, 2, \ldots, N\}$ is the index set of the particles and K_t is the normalized number of time steps in the transient.

The second metric we use to study interaction among particles is the total number of distinct neighbors averaged over the group calculated during the transient time. This metric aims to measure how much *mixing* is happening between the particles. If the total number of distinct neighbors averaged over the group is high, particles are more inclined to change neighbors which shows high mixing inside the group. On the other hand, if this metric is low, each particle is only interacting with a small set of particles and therefore the system in essence more *rigid*. Sin the adjacency matrix is known at each time step, the total numb of distinct neighbors averaged over the particles can be found

$$D = \left\langle \text{sign}\left(\sum_{k=1}^{K_t} \mathbf{A}_k\right) \mathbf{1}_{N\times 1} \right\rangle_{\mathcal{N}} \quad ($$

where $\text{sign}(\cdot)$ returns the sign of a matrix argument.

Since the length of transient parts are different for differe view angles, it can affect the total number of interactions a total number of distinct neighbors during the transient part. ' compensate this effect, one can scale the time steps for differe view angles to ensure the transient happens at the same numb of time steps.

NUMERICAL SIMULATION

To investigate the correlation between the particle intera tions during the transient time and its length, we use numeric simulation on a two-dimensional Vicsek model with restricted view angle. The parameters used in this simulation are listed in table 1, which are the same parameters as used in [21] for comparison. Since there is no noise in the system and the view angles are large enough, polarization will reach value 1 in the steady-state condition. The length of transient τ is calculated as the time step in which polarization reaches 0.99 for the first time, in line with the analysis performed in [21]. The simulation is run for 1400 time steps to ensure we capture the entire transient part for all considered view angles.

The initial position and heading angles of all the particles are randomly assigned for each view angle. After finding the length of the transient, the step size of simulation is scaled for each view angle to make sure that in all cases, the transients have the same number of time steps. In other words, we normalized each simulation length with the length of its transient. This will allow us to

TABLE 1. SIMULATION PARAMETERS

Variable	Symbol	Value
Simulation time steps	K	1400
Normalized transient time steps	K_t	1000
Domain side length	L	10
Number of particles	N	400
Speed of particles	v_0	0.04
Noise amplitude	η	0
View range	R	0.6

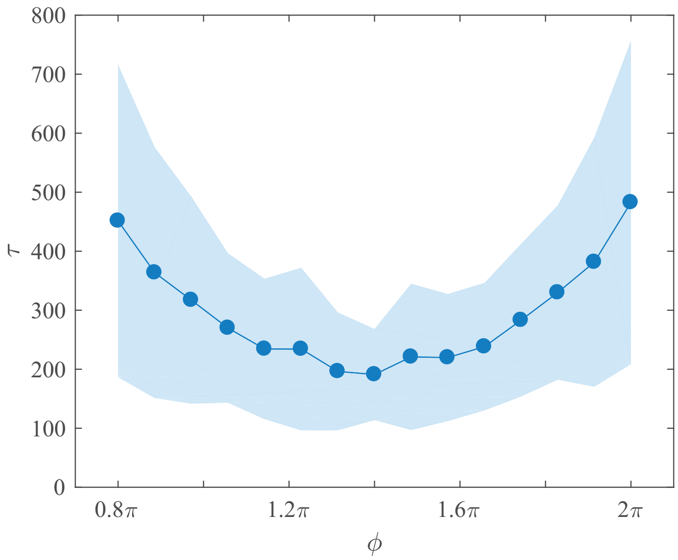

FIGURE 3. LENGTH OF TRANSIENT AS A FUNCTION OF VIEW ANGLE. LINE PLOT SHOWS MEAN VALUE OVER ALL PARTICLES FOR 100 DIFFERENT INITIAL CONDITIONS, WITH SHADED REGION SHOWING ± ONE STANDARD DEVIATION

compare the number of interactions during the transient time for different view angles. To account for the effect of random initial conditions, the simulation was performed for 100 repetitions.

SIMULATION RESULTS

The length of the transient as a function of view angle is plotted in figure 3. The line plot shows the length of transient averaged over 100 simulations with different initial conditions, and the error bars show the standard deviation over these repetitions. In line with the results in [21], there is a minimum in the length of transient τ which occurs at some view angle which is less than the maximum possible angle 2π. We refer to this as an optimal view angle, and we seek to understand the dynamics from which it originates.

Based on the hypothesis that the optimal view angle results from restricting the number of interactions, we compute the total number of interactions over the entire transient, averaged over all particles in the group and varying with view angle, shown in figure 4. This plot shows that, on average, the total number of interactions among particles is not monotonically increasing with view angle. In fact, it decreases as view angle decreases from 2π until it reaches a minimum and then increases. Notably, the angle at which the minimum number of interactions occurs is close to the optimal view angle in figure 3. In line with this analysis, the number of distinct neighbors over the entire transient, averaged over all particles in the group and varying with view angle,

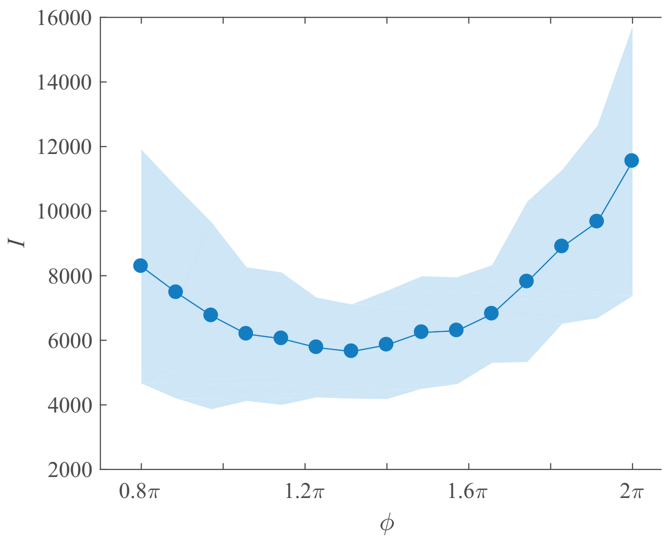

FIGURE 4. TOTAL NUMBER OF INTERACTIONS AVERAGE OVER PARTICLES AS A FUNCTION OF VIEW ANGLE. LINE PLOT SHOWS MEAN VALUE OVER ALL PARTICLES FOR 100 DIFFERENT INITIAL CONDITIONS, WITH SHADED REGION SHOWING ± ONE STANDARD DEVIATION

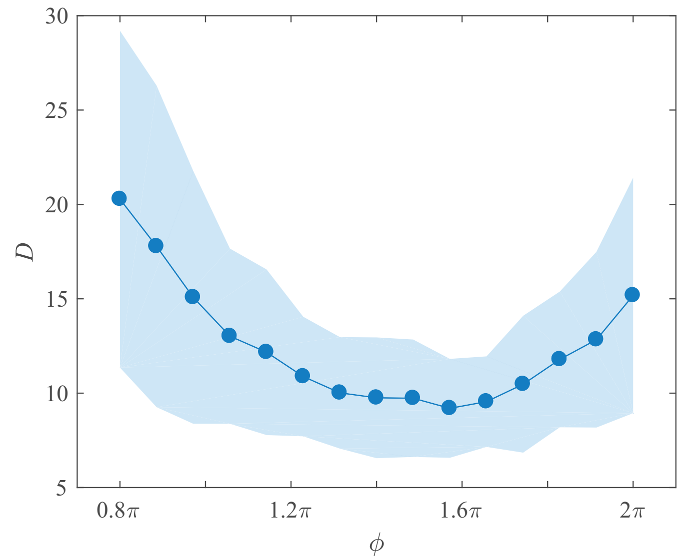

FIGURE 5. NUMBER OF DISTINCT NEIGHBORS AVERAGED OVER PARTICLES AS A FUNCTION OF VIEW ANGLE. LINE PLOT SHOWS MEAN VALUE OVER ALL PARTICLES FOR 100 DIFFERENT INITIAL CONDITIONS, WITH SHADED REGION SHOWING ± ONE STANDARD DEVIATION

is shown in figure 5. This plot shows that the total number of different particles interacting with a specific particle is also not monotonic as view angle is varied, and it shows a minimum close the optimal angle.

Figure 6 demonstrates in detail the combined effect of distinct neighbors and number of interactions. In this figure, the horizontal axis shows the rank of the neighbor with the most number of interactions for each particle and the vertical axis shows the frequency of the visit. These ranked frequencies are computed for four values of the view angle, and are averaged over all particles in the group and over all repetitions of the initial conditions (error bars are omitted for clarity). As a rule, all curves are monotonically decreasing since they order the most to least frequently visited neighbors. In addition, steeper curves show more interactions with a smaller cohort of neighbors, while flatter curves show more mixing or *shuffling* among the neighbor set of all particles.

DISCUSSION

The optimal view angle associated with minimum convergence time correlates well with parameters associated with low numbers of interactions and few distinct neighbors. From figures 3, 4, and 5, we can see that the minimum of all plots occurs in a similar range of the view angle ϕ. This suggests that fewer interactions are associated with faster convergence when view angle is in a certain range. Although this result seems counterintuitive initially, it may indicate that the restricted view angle imposes a kind of sorting that allows particles to preferentially select neighbors whose motion is well-aligned with theirs. This possibility is supported by results from figure 6.

Another way to view these results is to notice that mixing among particles' neighbor sets is sensitive to the view angle. Considering figure 6, we see that ranked interaction frequency curves become increasingly more peaked as ϕ increases from 0.8π to 1.4π. This shows that neighbor sets become more mixed as view angle is restricted. However, between $\phi = 1.4\pi$ and $\phi = 2\pi$, the curve becomes flatter, indicating that more mixing occurs when view angle increases beyond a critical value. This suggests that increasing view angle above critical may cause each particle to be *disturbed* by surrounding particles leading to a longer transient.

Based on these result, one possible explanation about existence of view angle could be based on a trade off between two different interaction mechanisms. When the view angle is small, it is less likely for a typical particle to observe the same set of particles for several consecutive time steps. In other words, the neighbors of that particle are changing too fast for local alignment to happen. On the other hand, when the view angle is larger, each particle needs to average its heading direction with more number of neighbors at each time step and that increases the *inertia* of the particle. Therefore, both number of total interaction and distinct neighbors are high. In the optimal case, however, the

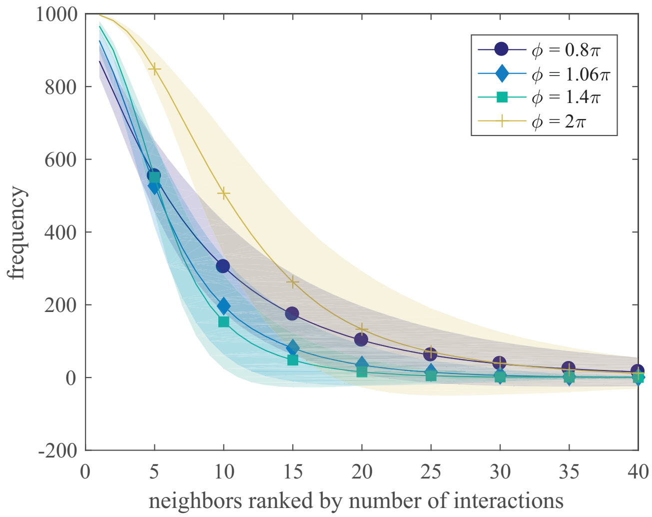

FIGURE 6. RANKED FREQUENCY OF NUMBER OF INTERACTIONS WITH DISTINCT NEIGHBORS, AVERAGED OVER PARTICLES AND 100 DIFFERENT INITIAL CONDITIONS. SHADED REGION SHOWS $\pm$ ONE STANDARD DEVIATION OVER INITIAL CONDITIONS. LINE PLOTS ARE FOR FOUR DIFFERENT VALUES OF VIEW ANGLE.

view angle is large enough to provide enough time for neighboring particles to locally align, yet not be disturbed by too many neighbors. This may result in a more cohesive growing network for the cases where view angle is close to the optimal value in comparison to cases with smaller or larger view angles.

Another observation from figures 3-5 is the larger error bars in small and large view angles comparing to the optimal angle. Since these error bars show the standard deviation of the corresponding parameter for different initial conditions, the large error bars show that the length of transient for small and large view angles depends more on initial condition. This dependency to initial condition can also be a sign that the group struggles in building a cohesive growing network if the initial condition is not well-aligned to start with.

CONCLUSION AND FUTURE WORK

In this paper, we studied the two-dimensional Vicsek model with restricted view angle to explore fa possible cause of the seemingly counterintuitive optimal view angle which leads to faster convergence for the algorithm. This optimal view angle is reported in several studies in the literature, and we also showed its existence here. We studied the statistics of the interaction between particles and found that there is a correlation between the length of transient part of the system and the averaged number of interactions in the group and number of distinct neighbors each individual has in average. The results of this study show that the optimal view angle corresponds to an interaction network with fewer interactions and fewer independent neighbors, yet both interactions and neighbors seem to be selected in a more targeted way comparing to smaller and larger view angles.

More investigation is needed to get a better understanding on the origin of the optimal angle in the two-dimensional Vicsek model. For example comparing the results of the same analysis for different values of group density (number of particles per unit area of the domain), and particle speed can be enlightening. Also, it is possible to get more local information about the interaction of particles using local order parameters.

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