Integrated Communications and Control Co-Design for Wireless Vehicular Platoon Systems

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***Abstract*—Vehicle platoons will play an important role in improving on-road safety in tomorrow's smart cities. Vehicles in a platoon can exploit vehicle-to-vehicle (V2V) communications to collect information, such as velocity and acceleration, from surrounding vehicles so as to coordinate their operations and maintain the target velocity and inter-vehicle distance required by the platoon. However, due to the interference and uncertainty of the wireless channel, V2V communications within a platoon will experience a wireless transmission delay which can impair the vehicles' ability to stabilize their speed and distances within their platoon. In this paper, the problem of integrated communication and control is studied for wireless-connected platoons. In particular, a novel approach is proposed for optimizing a platoon's stability while taking into account, jointly, the state of the wireless V2V network and the stability of the platoon's control system. Based on the proposed integrated communication and control strategy, the plant and string stability for the platoon are analyzed. The signal-to-interference-plus-noise-ratio (SINR) threshold, which will prevent the instability of the control system, is also determined. Moreover, the reliability of the wireless system, defined as the probability that the wireless system meets the control system's delay needs, is derived. Simulation results shed light on the benefits of the proposed approach and the synergies between the wireless network and the platoon's control system.**

I. Introduction

Intelligent transportation systems (ITSs) will be one of the major components of smart cities. In essence, ITSs will provide a much safer and more coordinated traffic network by using efficient traffic management approaches [1]. One promising ITS service is *autonomous vehicular platoons*. A platoon system is essentially a group of vehicles that operate together and continuously coordinate their speed and distance. By allowing vehicles to self-organize into a platoon, the road capacity can increase so as to prevent traffic jams [2]. Furthermore, platoons can provide people with a more comfortable driving environment, especially during long travels [3].

To reap the full benefits of platooning, one must ensure that each vehicle in the platoon has enough awareness of its relative distance and velocity with its platoon neighbors. This is needed to enable vehicles in a platoon to coordinate their acceleration and deceleration. In essence, enabling autonomous platooning requires two technologies: adaptive cruise control (ACC) [4] and vehicle-to-vehicle (V2V) communications [5]. ACC is primarily a control system that allows controlling the distances between vehicles. Meanwhile, V2V communications enable vehicles to exchange information, such as velocity and acceleration. Effectively integrating the operation of the ACC system and the V2V communication network is a necessary step for effective platooning in ITSs.

Nevertheless, due to the uncertainty of the wireless channel, the V2V communication links among vehicles will inevitably suffer from time-varying transmission delays. Unfortunately, the delayed information can be detrimental to the stability of the vehicles' control system which can jeopardize the entire operation of the platoon [6]. Therefore, to maintain the stability of a platoon, the control system must be robust to such wireless transmission delays. To this end, one must jointly design the control and wireless systems of a platoon to guarantee low latency and stability.

The prior art on vehicular platooning [7]-[12] can be grouped into two categories. The first category focuses on the inter-vehicle communication network for improving connectivity [7], data dissemination and routing [8], and medium access control (MAC) [9]. The second category designs control strategies that guarantee a platoon's stability. Such strategies include enhanced ACC [10], cooperative adaptive cruise control (CACC) [11], and connected cruise control (CCC) [12]. However, these works are limited in two aspects. The communication-centric works in [7]-[9] completely abstract the control system and do not study the impact of wireless communications on the platoon's stability. Meanwhile, the control-centric works in [10]-[12] focus solely on the stability, while assuming a constant performance from the communication network. Such an assumption is certainly not practical when platoons operate over 5G cellular networks in which interference and wireless dynamics can substantially impact the network performance. Clearly, despite the necessity of integrated communication and control designs, there is a lack in existing works that jointly study the wireless and control system performance of vehicular platoons.

The main contribution of this paper is a novel, integrated control system and V2V wireless communication co-design framework for wireless vehicular platoons. In particular, we first analyze two notions of control system stability for the platoon: string and plant stability, and, then, we determine the maximum time delay that a single platoon can tolerate. Accordingly, we derive the signal-to-interference-plus-noise-ratio (SINR) threshold that ensures both string and plant stability for the platoon. This threshold can, in turn, be used to identify the reliability requirements, in terms of transmission power and bandwidth, for the wireless communication system.

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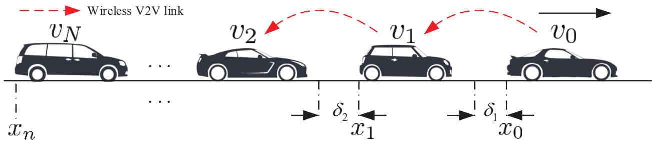


Fig. 1. Leader-follower model: a vehicle platoon with one leader and N followers. The first car is the leader of the platoon, and other cars are followers. x_i denotes the location of car i, $i \in \mathcal{N}$. The spacing error for car i is δ_i.

To the best of our knowledge, this is the first work that jointly considers the design of the control mechanism and the wireless communication strategy for connected platoon systems. Simulation results validate the effectiveness of the proposed integrated communication and control strategy, and shed light on benefits of co-designing the control system and the wireless network for a platoon. In particular, the results show that, in order to maintain platoon stability where connected vehicles suffer from interference, the transmit power of each vehicle should be greater than 1.2 W.

The rest of the paper is organized as follows. Section II presents the system model. In Section III, we perform stability analysis for the platoon and derive expressions for the reliability of the wireless network. Section IV provides the simulation results, and conclusions are drawn in Section V.

II. System Model

Consider a platoon system organized into a leader-follower model where one leader and following vehicles drive in the same lane, as shown in Fig. 1. Consider a set $\mathcal{M}$ of M cars and define $\mathcal{N} \subseteq \mathcal{M}$ as a set of $N+1$ cars that form a platoon. In the platoon, there are N followers and one leader. In essence, the first vehicle is the leader and other vehicles are the followers. The location of each vehicle is captured by the position x_i of its rear bumper, $i \in \mathcal{N}$. For each vehicle, an embedded radar can sense the distance between its rear bumper and the rear bumper of the preceding vehicle. Moreover, every vehicle can communicate with its neighbors via V2V communication links to obtain information, such as velocity and acceleration.

A. Control System Model

The driver or the ACC system in the vehicle will brake or accelerate according to the difference between the actual distance and the target spacing slot to the preceding vehicle. Hence, if the difference is positive, the vehicle must speed up so that the distance to the preceding car meets the platoon's requirement. Otherwise, the vehicle must slow down. This distance difference is defined as the spacing error δ_i:

$$\delta_i(t) = x_{i-1}(t) - x_i(t) - L_t, i \in \mathcal{N}, \tag{1}$$

where L_t is the target spacing for the platoon. The distance difference, $h_{i-1,i}(t) = x_{i-1}(t) - x_i(t)$, is usually known as the *headway*. We also define the velocity error:

$$w_i(t) = v_i(t) - v_t, \tag{2}$$

where $v_i(t)$ represents the velocity of vehicle i at time t, and v_t is the target velocity for the platoon system.

Similar to the optimal velocity model (OVM) introduced in [13], to realize the stability of a platoon system, the

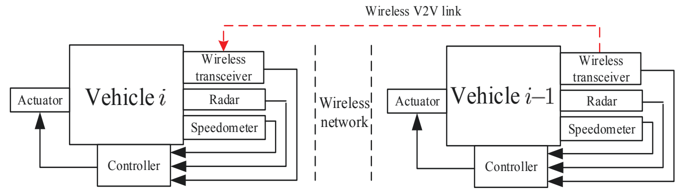


Fig. 2. Basic structure of a platoon system.

acceleration or deceleration of each vehicle must be determined by two components. One is the difference between headway-dependent and actual velocities, and the other is the velocity difference between the given vehicle and the vehicle immediately ahead. As shown in Fig. 2, we can use the following control law to determine the acceleration u_i of vehicle i [13]:

$$u_i(t)=a_i(V(h_{i-1,i}(t))-v_i(t))+b_i(v_{i-1}(t-\triangle\tau_{i-1,i}(t))-v_i(t)), \tag{3}$$

where $\triangle\tau_{i-1,i}(t)$ captures the V2V communication link delay between car i and its preceding car, a_i is the associated gain of car i for the difference of the headway-dependent velocity and the actual speed, and b_i is the associated gain for the velocity difference between cars $i-1$ and i. The headway-dependent velocity $V(h)$ should satisfy following properties: 1) in dense traffic, the vehicle will stop, i.e., $V(h) = 0$ for $h < h_{\text{dense}}$, 2) in sparse traffic, the vehicle can travel with its maximum speed, which is also called free-flow speed, i.e., $V(h) = v_{\max}$ for $h > h_{\text{sparse}}$, and 3) when $h_{\text{dense}} < h < h_{\text{sparse}}$, $V(h)$ is a monotonically increasing function of h. Similar to [14], we define function $V(h)$ as follows:

$$V(h) = \begin{cases} 0, & \text{if } h < h_{\text{dense}}, \\ v_{\max} \times \left(\frac{h - h_{\text{dense}}}{h_{\text{sparse}} - h_{\text{dense}}}\right), & \text{if } h_{\text{dense}} \leq h \leq h_{\text{sparse}}, \\ v_{\max}, & \text{if } h_{\text{sparse}} < h. \end{cases} \tag{4}$$

For this model, our goal is to design a control law that is equivalent to finding the control parameters, a_i and b_i, $i \in \mathcal{N}$ so that all followers can drive with the target velocity v_t, which is the speed of the leader, and maintain the target inter-vehicle distance L_t. Note that $v_t = V(L_t)$.

B. Wireless Communication System

For V2V communications, we consider an orthogonal frequency-division multiple access (OFDMA) scheme to facilitate simultaneous transmissions among vehicles within the platoon. That is, different subcarriers will be allocated to different V2V links within the platoon. Also, the number of subcarriers is equal to the number of followers in the platoon. In this case, the V2V links can coexist simultaneously without experiencing interference from other links in the *same* platoon. However, interference from other vehicles or platoons that use the same frequency will be accounted for. Similar to [15], we consider a Rician fading channel for the V2V links within the platoon. In other words, the channel gain follows a Rician distribution with shape parameter K_1. The received power at any car $i \in \mathcal{N}$ will be $P^r_{i-1,i}(t) = P^t_{i-1} g_{i-1,i}(h_{i-1,i}(t))^{-\alpha}$, where P^t_{i-1} is the transmission power of vehicle $i-1$, $g_{i-1,i}$ is the channel gain, and α is the path loss exponent. Also, we can express the interference at car i as $I_i(t) =$

$\sum_{j_1,j_2\in\mathcal{M}\setminus\mathcal{N}} \mathbb{1}_{j_1,j_2} P^t_{j_1} g_{j_1,i}(h_{j_1,i}(t))^{-\alpha}$, where $g_{j_1,i}$ refers to the channel gain from vehicle j_1 to i, which follows a Rayleigh distribution, and $\mathbb{1}_{j_1,j_2}$ is a binary variable where $\mathbb{1}_{j_1,j_2} = 1$ if the V2V link between vehicle j_1 and j_2 uses the same subcarrier as the link between vehicle $i-1$ and i; otherwise, $\mathbb{1}_{j_1,j_2} = 0$.

The SINR of the V2V link from car $i-1$ to i will be $\gamma_{i-1,i}(t) = \frac{P^r_{i-1,i}(t)}{I_i(t)+\sigma^2}$, where σ^2 is the variance of the Gaussian noise. Then, the data rate will be: $R_{i-1,i}(t) = w\log_2(1+\gamma_{i-1,i}(t))$, where $w=\frac{W}{N}$ is the bandwidth of each subcarrier. Whenever all packets are of equal size S bits, the communication delay of the V2V link from cars $i-1$ to i can be derived as:

$$\triangle\tau_{i-1,i}(t) = \frac{S}{w\log_2(1+\gamma_{i-1,i}(t))}. \tag{5}$$

Next, we take into account the time-varying wireless communication delay in (5) and analyze its effect on the stability of the platoon's control system.

III. Stability Analysis for the Platoon System

For the car-following model, the inevitable V2V communication delay in (5) can negatively impact the stability of the platoon system. Here, we first perform stability analysis for the control system in presence of a wireless communication time delay. In particular, we analyze two types of stability: plant stability and string stability. In essence, plant stability focuses on the convergence of error terms related to the inter-vehicle distance and velocity, while string stability pertains to the error transition along with the platoon. Based on the stability analysis, we obtain design guidelines for the allocation of transmit power and spectral resources to vehicles in the platoon's communication system. Moreover, based on the communication and control co-design requirements, we characterize the *reliability* of the wireless system, defined as the probability of the wireless system meeting the control system's delay needs.

A. Plant Stability

Plant stability requires all followers in a platoon to drive with the same speed as the leader and keep a target distance to the vehicle immediately ahead. In other words, plant stability requires both the spacing and speed errors of each vehicle to converge to zero. To this end, we take the first-order derivatives of (1) and (2) as:

$$\begin{cases} \dot{\delta}_i(t) = w_{i-1}(t) - w_i(t), \\ \dot{w}_i(t) = A\delta_i(t) + Bw_{i-1}(t-\triangle\tau_{i-1,i}(t)) - Cw_i(t), \end{cases} \tag{6}$$

where $A = \frac{a_i v_{\max}}{h_{\text{sparse}}-h_{\text{dense}}}$, $B = b_i$, and $C = a_i+b_i$. Since the leading vehicle, car 0, always drives with the target velocity and has no car ahead of it, its velocity (spacing) error is $w_0(t)=0$ ($\delta_0(t)=0$). Also, as the channel gains of different V2V links follow the same distribution and two adjacent vehicles in a platoon are always close to each other, we assume that the time delay $\triangle\tau_{i-1,i}(t) = \triangle\tau(t), \forall i \in \mathcal{N}$. Therefore, after collecting spacing and velocity errors for all of the followers, we can find the augmented error state vector $\boldsymbol{e}(t)=[\delta_1(t),\delta_2(t),...,\delta_N(t),w_1(t),w_2(t),...,w_N(t)]^T$ and obtain

$$\dot{\boldsymbol{e}}(t) = \begin{bmatrix} \mathbf{0}_{N\times N} & \boldsymbol{\Omega}_1 \\ \boldsymbol{\Omega}_2 & \boldsymbol{\Omega}_3 \end{bmatrix} \boldsymbol{e}(t) + \begin{bmatrix} \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} \\ \mathbf{0}_{N\times N} & \boldsymbol{\Omega}_4 \end{bmatrix} \boldsymbol{e}(t-\triangle\tau(t)), \tag{7}$$

where

$$\boldsymbol{\Omega}_1 = \begin{bmatrix} -1 & 0 & 0 & \dots & 0 & 0 \\ 1 & -1 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & -1 \end{bmatrix}_{N\times N}, \tag{8}$$

$$\boldsymbol{\Omega}_2 = \text{diag}\left\{\frac{a_1 v_{\max}}{h_{\text{sparse}}-h_{\text{dense}}}, \dots, \frac{a_N v_{\max}}{h_{\text{sparse}}-h_{\text{dense}}}\right\}_{N\times N}, \tag{9}$$

$$\boldsymbol{\Omega}_3 = \text{diag}\{-(a_1+b_1),...,-(a_N+b_N)\}_{N\times N}, \tag{10}$$

$$\boldsymbol{\Omega}_4 = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ b_2 & 0 & 0 & \dots & 0 & 0 \\ 0 & b_3 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & b_N & 0 \end{bmatrix}_{N\times N}, \tag{11}$$

and

$$\boldsymbol{e}(t-\triangle\tau(t)) = [\delta_1(t-\triangle\tau(t)),...,\delta_N(t-\triangle\tau(t)), w_1(t-\triangle\tau(t)),...,w_N(t-\triangle\tau(t))]^T. \tag{12}$$

For ease of presentation, we rewrite $\boldsymbol{M}_1=\begin{bmatrix} \mathbf{0}_{N\times N} & \boldsymbol{\Omega}_1 \\ \boldsymbol{\Omega}_2 & \boldsymbol{\Omega}_3 \end{bmatrix}$ and $\boldsymbol{M}_2=\begin{bmatrix} \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} \\ \mathbf{0}_{N\times N} & \boldsymbol{\Omega}_4 \end{bmatrix}$, and replace $\boldsymbol{e}(t)$ with $\boldsymbol{e}$ hereinafter. Since plant stability requires the spacing and velocity errors to approach zero, the error vector $\boldsymbol{e}(t) = \mathbf{0}_{2N\times 2N}$ should be asymptotically stable.

Guaranteeing plant stability for a wireless-connected platoon will hence require an SINR that is high enough to support a small V2V transmission time delay. Without loss of generality, we assume that all vehicles in the platoon are identical with equal control parameters a and b. Therefore, in the following theorem, we characterize the minimum SINR threshold needed to guarantee plant stability.

Theorem 1. *The plant stability of the system in (3) can be guaranteed if the received SINR γ of any V2V link in the platoon satisfies:*

$$\gamma > \gamma_1 = 2^{\frac{S}{w\triangle\tau^{(1)}_{\max}}} - 1, \tag{13}$$

where $\triangle\tau^{(1)}_{\max} = \lambda_{\min}(-\boldsymbol{M}_1 - \boldsymbol{M}_2 - (\boldsymbol{M}_1 + \boldsymbol{M}_2)^T)/\lambda_{\max}(\boldsymbol{M}_2\boldsymbol{M}_1\boldsymbol{M}_1^T\boldsymbol{M}_2^T + \boldsymbol{M}_2\boldsymbol{M}_2\boldsymbol{M}_2^T\boldsymbol{M}_2^T + 2k\boldsymbol{I}_{2n\times 2n})$ *with* $k > 1$, *and* $\lambda_{\max}(\boldsymbol{M})$ *and* $\lambda_{\min}(\boldsymbol{M})$ *represent the maximum and minimum eigenvalues of matrix* $\boldsymbol{M}$, *respectively.*

Proof: Similar to the consensus problem considered in [16], we use the following candidate Lyapunov function: $V(\boldsymbol{e}) =$

$\boldsymbol{e}^T\boldsymbol{P}\boldsymbol{e}$, where $\boldsymbol{P}=\boldsymbol{I}_{2N\times 2N}$ is a positive definitive matrix. We also assume that there is a continuous nondecreasing function $\psi(x)\geq x$, $x>0$. Then, the time derivative for $V(\boldsymbol{e})$ will be:

$$\begin{aligned}\dot{V}(\boldsymbol{e}) &= \boldsymbol{e}^T\big((\boldsymbol{M}_1+\boldsymbol{M}_2)+(\boldsymbol{M}_1+\boldsymbol{M}_2)^T\big)\boldsymbol{e}\\ &- 2\boldsymbol{e}^T\int_{-\triangle\tau(t)}^{0}\boldsymbol{M}_2\boldsymbol{M}_1\boldsymbol{e}(t+x)dx \\ &- 2\boldsymbol{e}^T\int_{-\triangle\tau(t)}^{0}\boldsymbol{M}_2\boldsymbol{M}_2\boldsymbol{e}(t+x-\triangle\tau(t+x))dx.\end{aligned} \tag{14}$$

Note that for a positive definite matrix Φ, we have $2\boldsymbol{v}_1^T\boldsymbol{v}_2\leq \boldsymbol{v}_1^T\Phi\boldsymbol{v}_1+\boldsymbol{v}_2^T\Phi^{-1}\boldsymbol{v}_2$. Thus, let $\boldsymbol{v}_1=-2\boldsymbol{e}^T\boldsymbol{M}_1\boldsymbol{M}_2$, $\Phi=\boldsymbol{P}$, and $\boldsymbol{v}_2=\boldsymbol{e}(t+x)$. Then, the inequality for the second term of the right-hand side in (14) can be expressed as

$$\begin{aligned}&-2\boldsymbol{e}^T\int_{-\triangle\tau(t)}^{0}\boldsymbol{M}_2\boldsymbol{M}_1\boldsymbol{e}(t+x)dx\leq\int_{-\triangle\tau(t)}^{0}\boldsymbol{e}(t+x)^T\boldsymbol{e}(t+x)dx\\ &+\triangle\tau(t)\boldsymbol{e}^T\boldsymbol{M}_2\boldsymbol{M}_1\boldsymbol{M}_1^T\boldsymbol{M}_2^T\boldsymbol{e}.\end{aligned} \tag{15}$$

When $V(\boldsymbol{e}(t+x))\leq\psi(V(\boldsymbol{e}(t)))=kV(\boldsymbol{e}(t))$ with $k>1, x\in(-\triangle\tau(t),0)$, (15) can be further simplified as:

$$\begin{aligned}&-2\boldsymbol{e}^T\int_{-\triangle\tau(t)}^{0}\boldsymbol{M}_2\boldsymbol{M}_1\boldsymbol{e}(t+x)dx\leq\\ &\triangle\tau(t)\boldsymbol{e}^T(\boldsymbol{M}_2\boldsymbol{M}_1\boldsymbol{M}_1^T\boldsymbol{M}_2^T+k\boldsymbol{I}_{2N\times 2N})\boldsymbol{e}.\end{aligned} \tag{16}$$

Similarly, we can perform the same steps for the third term of the right-hand side in (14). Finally, we can obtain

$$\begin{aligned}\dot{V}(e)\leq&\boldsymbol{e}^T\big(\boldsymbol{M}_1+\boldsymbol{M}_2+(\boldsymbol{M}_1+\boldsymbol{M}_2)^T+\triangle\tau(t)\boldsymbol{M}_2\boldsymbol{M}_1\boldsymbol{M}_1^T\boldsymbol{M}_2^T\\ &+\triangle\tau(t)\boldsymbol{M}_2\boldsymbol{M}_2\boldsymbol{M}_2^T\boldsymbol{M}_2^T+2\triangle\tau(t)k\boldsymbol{I}_{2N\times 2N}\big)\boldsymbol{e}.\end{aligned} \tag{17}$$

Based on the Lyapunov-Razumikhin theorem introduced in [17], if $\triangle\tau(t)<\lambda_{\min}(-\boldsymbol{M}_1-\boldsymbol{M}_2-(\boldsymbol{M}_1+\boldsymbol{M}_2)^T)/\lambda_{\max}(\boldsymbol{M}_2\boldsymbol{M}_1\boldsymbol{M}_1^T\boldsymbol{M}_2^T+\boldsymbol{M}_2\boldsymbol{M}_2\boldsymbol{M}_2^T\boldsymbol{M}_2^T+2k\boldsymbol{I}_{2N\times 2N})$, the system in (3) is asymptotically stable and the augmented error state vector will converge to a zero vector. We can then obtain the SINR threshold (13) using (5). □

B. String Stability

Beyond plant stability, we must ensure that the platoon is *string stable*. In particular, if the disturbances, in terms of velocity or distance, of preceding vehicles do not amplify along with the platoon, the system will be string stable [2].

To analyze string stability, we consider the worst-case scenario in which all V2V links in the platoon experience the maximum time delay $\triangle\tau_{\max}^{(2)}$ due to the wireless channel. Hence, we can obtain the transfer function between two adjacent vehicles using the Laplace transform on (6), as follows:

$$T(s)=\frac{w_i(s)}{w_{i-1}(s)}=\frac{A+sBe^{-s\triangle\tau(t)}}{s^2+Cs+A}. \tag{18}$$

By using the Padé approximation, $e^x\approx\frac{1+0.5x}{1-0.5x}$ [18], we further simplify (18) and derive the minimum SINR threshold needed to maintain the string stability for the platoon system.

Theorem 2. *The string stability of the system in (3) can be guaranteed if the received SINR γ of any V2V link in the platoon satisfies:*

$$\gamma>\gamma_2=2^{\frac{S}{w\triangle\tau_{\max}^{(2)}}}-1. \tag{19}$$

where $\triangle\tau_{\max}^{(2)}=\frac{C^2-2A-B^2}{2AB}$.

Proof: To ensure string stability, the magnitude of the transfer function must satisfy $|T(j\omega)|\leq 1$, for $\omega\in\mathbb{R}^+$, where ω represents the frequency of sinusoidal excitation generated by the leader [19]. The magnitude inequality is equivalent to

$$\mathit{\Gamma}(\omega)=D\omega^4+E\omega^2+F>0, \tag{20}$$

where $D=\frac{1}{4}(\triangle\tau(t))^2>0$, $E=(\frac{1}{4}C^2-\frac{1}{2}A-\frac{1}{4}B^2)(\triangle\tau(t))^2+1$, and $F=C^2-2A-B^2-2AB(\triangle\tau(t))$. To solve (20), we need $\mathit{\Gamma}(\bar{\omega})>0$, where $\frac{d\mathit{\Gamma}(\omega)}{d\omega}|_{\omega=\bar{\omega}}=0$. We can easily find that $\triangle\tau(t)<\triangle\tau_{\max}^{(2)}=\frac{C^2-2A-B^2}{2AB}$, and, using (5), we obtain the SINR threshold (19). □

Hence, to guarantee plant and string stability for a platoon, we must ensure that the SINR threshold $\gamma>\max(\gamma_1,\gamma_2)$.

C. Reliability Analysis of the Wireless System

For a system with fixed control parameters, a and b, we can meet the SINR requirements of Theorems 1 and 2 by improving the wireless system performance, such as by managing interference or increasing the transmission power. However, when the control parameters are not fixed, we can choose proper values for parameters a and b in the control law to reduce the minimum SINR threshold without jeopardizing the system stability. Thus, we are able to relax the constraints on the design of the wireless system by properly designing the platoon's control mechanism. Moreover, control and communication synergies can be used to introduce a notion of *reliability* for the wireless system, defined as the probability of the wireless system meeting the control system's delay needs.

Nevertheless, in presence of interference, analytically finding the reliability for the wireless system is challenging due to the difficulty of finding a general interference model for a V2V system. Instead, we consider the system model without suffering interference from other transmitters, and derive its reliability in the following theorem. The case with interference is then analyzed via simulations in Section IV.

Theorem 3. *If receivers in the platoon system do not experience any interference, the probability $F(\gamma_1,\gamma_2)$ for meeting both plant and string stability can be expressed as*

$$F(\gamma_1,\gamma_2)=Q\left(\sqrt{2K},\frac{\sigma\max(\gamma_1,\gamma_2)}{PL_t^{-\alpha}}\right), \tag{21}$$

where K represents the Ricean K factor, and $Q(\cdot,\cdot)$ denotes the Marcum Q-function.

Proof: To ensure plant and string stability, the wireless system must provide enough bandwidth or transmission power so that

TABLE I
SIMULATION PARAMETERS.

Parameter	Meaning	Value
N	Number of followers	5
h_{sparse}	Sparse spacing distance	35 m [11]
h_{Dense}	Dense spacing distance	5 m [11]
$v_{\max}$	Maximum velocity	30 m/s [11]
a	Associated gain for headway	4
b	Associated gain for velocity	4
k	Coefficient of nondecreasing function	1.01
K_1	Ricean shape parameter	3 [15]
α	Path loss exponent	3.5
σ^2	Power of noise	-174 dBm/Hz
S	Packet size	3200 bits [20]
W	Bandwidth	20 MHz

the actual received SINR exceeds the threshold $\max(\gamma_1, \gamma_2)$. In the case with no interference, we can derive the reliability

$$\begin{aligned} F(\gamma_1, \gamma_2) &= \mathbb{P}(\gamma > \max(\gamma_1, \gamma_2)) \\ &\overset{(a)}{=} \mathbb{P}\left(g > \frac{\sigma^2 \max(\gamma_1, \gamma_2)}{P L_t^{-\alpha}}\right) \\ &\overset{(b)}{=} Q\left(\sqrt{2K}, \frac{\sigma \max(\gamma_1, \gamma_2)}{P L_t^{-\alpha}}\right), \end{aligned} \tag{22}$$

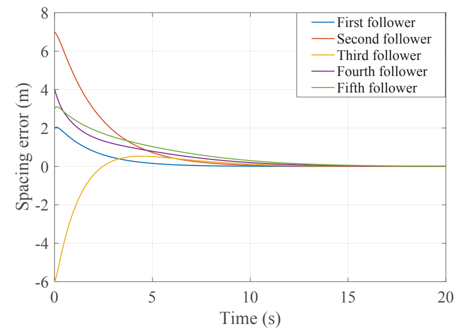
where (a) follows the fact receiving vehicle will not encounter interference from other transmitting vehicles. Also, in (b), we use the cumulative density function of Rice distribution [21]. □

Using Theorem 3, as long as we know the SINR requirements for plant and string stability, the transmission power, and the distance from the transmitter to the receiver, we can obtain the reliability of each V2V link. Since the vehicles in the platoon are identical, we can consider the platoon to be reliable when the reliability of the wireless link between two adjacent vehicles is higher than a target reliability threshold.

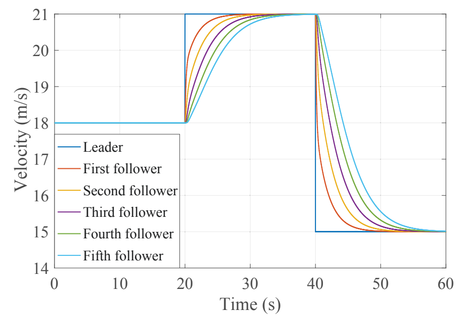
IV. SIMULATION RESULTS AND ANALYSIS

For our simulation, we first validate Theorems 1 and 2. Then, we study the impacts of the wireless parameters on the stability of the platoon's control system. All simulation parameters are summarized in Table. I. Using the parameters of Table I, based on Theorems 1 and 2, we can find that the maximum time delay for the plant and the string stability are approximately 14.90 ms and 1.25 s, respectively. To guarantee both types of stability, we assume that the maximum delay for the platoon is 14.90 ms.

We first corroborate our analytical results on both types of stability under the derived 14.90 ms delay. In this simulation, we consider a platoon with one leader and five followers. For this first result, we focus on the control system and, hence, we model the uncertainty of the wireless channel pertaining to the V2V links between two adjacent vehicles in the platoon system as a time-varying delay in the range (0, 14.90 ms). The vehicles in the platoon are initially assigned different velocities and different inter-vehicle distances. Here, the target velocity is v_t=15 m/s, and the target inter-vehicle distance is $L_t = 20$ m. Fig. 3(a) shows the time evolution of the spacing errors. We can observe that the spacing error will converge to 0 (a similar result is observed for the velocity error but is omitted due to space limitations). Thus, by choosing the maximum time delay derived from Theorems 1 and 2, we can ensure the plant stability of the platoon. Next, to verify the string stability, we add disturbances to the leader, leading to the velocity increase from 18 to 21 m/s at $t = 20s$ and the decrease from 21 to 15 m/s at $t = 40s$. The disturbance might come from bad driving habits of drivers or malfunctions of ACC systems. As shown in Fig. 3(b), the velocity error is not amplified when propagating along with the platoon, guaranteeing string stability. In particular, when the velocity of the leader jumps from 18 to 21 m/s, the velocity curve of the fifth follower is more smooth compared with the counterpart of the first follower. Clearly, the SINR thresholds, found by Theorems 1 and 2, can guarantee the platoon's plant and string stability.



(a) Spacing error.



(b) Performance of the followers under disturbance of the leader.

Fig. 3. Plant and string stability.

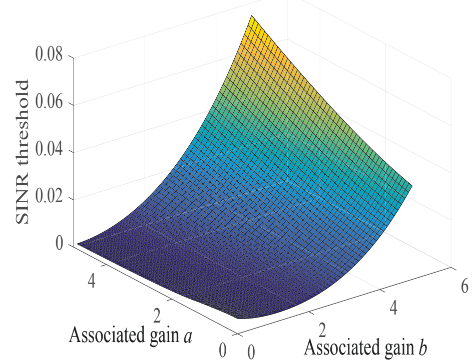


Fig. 4. SINR threshold derived from Theorem 1 for different gains a and b.

Fig. 4 shows the SINR threshold derived by Theorem 1 for different values of the control system gains a and b. As observed from Fig. 4, by properly choosing the control parameters, we can find a small SINR threshold for the wireless system to maintain platoon's plant stability. In other words, we can relax the wireless design constraints by choosing

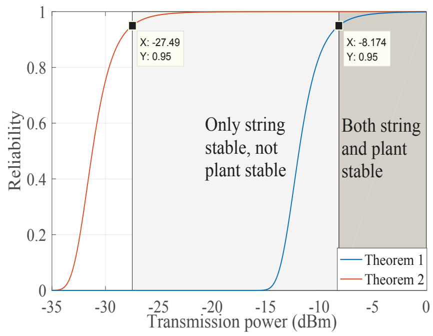


Fig. 5. Reliability analysis for V2V communication without interference.

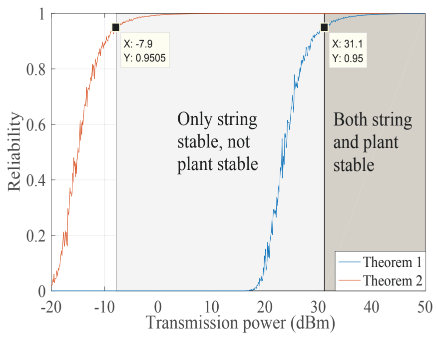


Fig. 6. Reliability analysis for V2V communication with interference.

appropriate control parameters for the control mechanism. However, because of physical limitations, such as the limited power provided by the torque and the maximum speed of a car, the control system might not be able to operate at all values of the control gains a and b in (0,5). Nonetheless, Fig. 4 provides us with key guidelines on how to choose the SINR threshold jointly with the control parameters to optimize the platoon's overall operation.

Fig. 5 shows the reliability for different SINR thresholds obtained from Theorems 1 and 2 when the transmission power ranges from -40 dB to 0 dB for the case without interference. As illustrated in Fig. 5, to be plant stable, the transmission power of each vehicle should exceed 0.15 mW, which is the practical case for most real-world systems. However, to be only string stable, a power of 1.62×10^{-3} mW is sufficient. Moreover, the platoon system cannot achieve neither string stability nor plant stability if the transmission power is below 1.62×10^{-3} mW.

Fig. 6 shows the reliability as function of the V2V transmission power for the scenario with interference generated by other vehicles outside of the platoon. From this figure, we can see that the power needed to guarantee stability is much higher than the counterpart in Fig. 5. In particular, to guarantee string and plant stability, a transmit power of over 1.2 W is needed. This figure also shows that for the range of transmit powers between 0.16 mW and 1.29 W, the network can only guarantee string stability. Clearly, for integrating wireless communications in the platoon system, one must properly manage interference so as to provide stability for the control systems of platoons. In particular, the results of Fig. 5 and Fig. 6 provide a first step towards a more in-depth understanding on the impact of a real-world wireless network environment on the control system of platoons.

V. Conclusion

In this paper, we have proposed a joint design of the wireless V2V network and control mechanism for platoon systems. Based on the proposed integrated communication and control strategy, we have analyzed the plant and string stability for the platoon, and have derived the SINR threshold, which will prevent the instability of the control system. We have also derived the reliability of the wireless system, defined as the probability of meeting the control system's delay requirements. Simulation results have shown how the synergies between control and wireless systems must be leveraged to properly design a stable platooning system.

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