

Angle Estimation Using Wideband Frequency Modulation and an Active Distributed Array

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Abstract—A new method of target angle estimation that leverages wideband waveforms and a distributed array receiver is presented. The distributed array receiver generates a grating lobe pattern, the spatially-distributed phase of which changes as a function of frequency. By implementing a wideband linear frequency modulated (LFM) waveform, a phase response manifests in the distributed array receiver output whose instantaneous frequency is proportional to the angle of the object. The receiver frequency response is derived, and the angle estimation approach is experimentally demonstrated. A 4.8 GHz distributed array radar was implemented, with a receiver baseline of 19.36λ using a waveform modulated over 2 GHz of bandwidth. Measurements with a single point target were conducted, and the measured frequency responses show the expected correspondence to the angle of the target with an average error of 2.8° (0.049 rad) and a standard deviation of 1.8° (0.031 rad).

Index Terms—Distributed arrays, distributed radar, angle estimation, correlation interferometry

I. INTRODUCTION

Estimation of the angle of a target is a basic and crucial measurement in remote sensing and radar applications spanning aerial object detection [1], automotive radar [2], and other applications. Along with the estimation of range, range-rate (Doppler), and more recently angle-rate [3], angle estimation represents one of the basic functions of a radar system. Traditionally, angle estimation was performed by scanning a narrow antenna beam, either electronically or mechanically, and determining the target angle based on the pointing angle where detections occur [4]. While yielding high directivity, and thus the ability to detect objects at long ranges, such raster scanning systems inherently involve delays due to the scanning speed of the beam, and are limited in resolution by the beamwidth of the antenna. Digital array MIMO techniques have also been employed [5]–[7], and subspace techniques including MUSIC [8] and ESPRIT [9], where the relative phases of the received signals are compared to determine the most likely angle of origin. These approaches leverage *a priori* information including the number of independent signals being received to achieve angle estimation below traditional Rayleigh-limited resolution, however they necessitate more signal processing than competing techniques, requiring the formation and subsequent inversion of a covariance matrix,

which can lead to computational inefficiencies. The use of distributed array systems for phased interferometry has been studied for a number of years as well (e.g. [10]), however these systems tend to be passive, limiting their sensitivity and thus operational range, and require multiple antenna elements to ensure unambiguous estimation of the target angle.

In this letter, a new radar angle estimation technique that uses a wideband transmit signal combined with a two-element distributed array receiver is presented. Distributed array apertures generate a large number of grating lobes, whose specific locations and relative phases are determined by the electrical separation of the antenna baseline. By implementing a wideband LFM transmitted signal, the complex interference pattern changes over the duration of the waveform, creating a deterministic sinusoidal temporal phase pattern at each angle in space. By cross-correlating the received signal outputs, the sinusoidal response manifests as a frequency shift on the received signal that uniquely corresponds to a specific angle in space. The proposed technique requires no beam scanning, and unlike other distributed array techniques, only two antenna elements are required on the receiving array. Additionally, since the response is a simple frequency-modulated signal, the baseband processing is simple and computationally efficient, necessitating only a frequency estimation to determine the target angle in real time. Furthermore, since the system actively transmits a signal, the sensitivity can be increased compared to passive phased interferometric techniques.

In the following sections, the active distributed array angle estimation theory is given, and the correspondence between baseband signal frequency and angle is derived. An experimental system operating at 4.8 GHz with a 2 GHz bandwidth is then presented. Measurements showing the frequency output as a function of the angle of a corner reflector are shown, and the error between the measured and expected frequency is calculated.

II. ACTIVE DISTRIBUTED ARRAY ANGLE ESTIMATION THEORY

The angle estimation technique utilizes a wide-baseline distributed array receiver combined with a wideband transmitted signal. The transmitted signal can be either emitted by a separate antenna, as shown in Fig. 1, or it can be emitted by one of the two receiving antennas provided that sufficient isolation between the transmit and receive hardware is implemented. The received signals are cross-correlated using a multiplier, the output of which is low-pass filtered; the correlation processing

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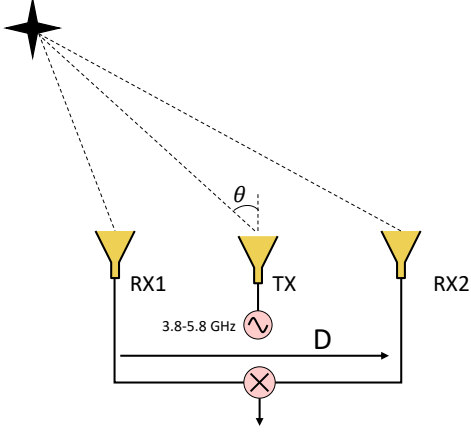


Fig. 1. Active distributed array observing a target at angle θ . The transmitter generates a LFM waveform over the 3.8-5.8 GHz band, which is received by the two receiver antennas separated by the baseline D . The signals are complex correlated, yielding a frequency response proportional to the angle of the object.

may be realized in either analog hardware or digital signal processing.

Given a transmitted LFM signal of the form

$$s(t) = A \cos \left[2\pi \left(f_0 t + \frac{K}{2} t^2 \right) \right] \quad (1)$$

where A is the signal amplitude (which will be normalized in the following equations), f_0 is the carrier frequency, and K (Hz/s) is the chirp rate. The normalized signals received by two antennas separated by a baseline D after reflecting off of an ideal point source at angle θ can be given by

$$v_1(t) = \cos \left[2\pi \left(f_0 t + \frac{K}{2} t^2 \right) \right] \quad (2)$$

$$v_2(t - \tau_g) = \cos \left\{ 2\pi \left[f_0 (t - \tau_g) + \frac{K}{2} (t - \tau_g)^2 \right] \right\} \quad (3)$$

where $\tau_g = \frac{D}{c} \sin \theta$ represents the geometrical time delay of the wavefront face to the second element. The two received signals are then complex correlated by separately multiplying and integrating the in-phase and quadrature components of the received signals. The output of the in-phase multiplier is

$$\begin{aligned} r^I(\tau_g) &= \langle v_1(t) v_2(t - \tau_g) \rangle \\ &= \left\langle \cos \left[2\pi \left(f_0 t + \frac{K}{2} t^2 \right) \right] \times \right. \\ &\quad \left. \cos \left\{ 2\pi \left[f_0 (t - \tau_g) + \frac{K}{2} (t - \tau_g)^2 \right] \right\} \right\rangle \\ &= \cos \left[2\pi \left(f_0 + Kt - \frac{K}{2} \tau_g \right) \tau_g \right]. \end{aligned} \quad (4)$$

where the angled brackets $\langle \cdot \rangle$ indicate averaging, which can be accomplished with low-pass filtering, removing the high frequency signal components. Similarly, the quadrature component is given by

$$r^Q(\tau_g) = \sin \left[2\pi \left(f_0 + Kt - \frac{K}{2} \tau_g \right) \tau_g \right]. \quad (5)$$

Combining the in-phase and quadrature correlator outputs results in the complex signal response

$$r(\tau_g) = e^{j2\pi \left(f_0 + Kt - \frac{K}{2} \tau_g \right) \tau_g}. \quad (6)$$

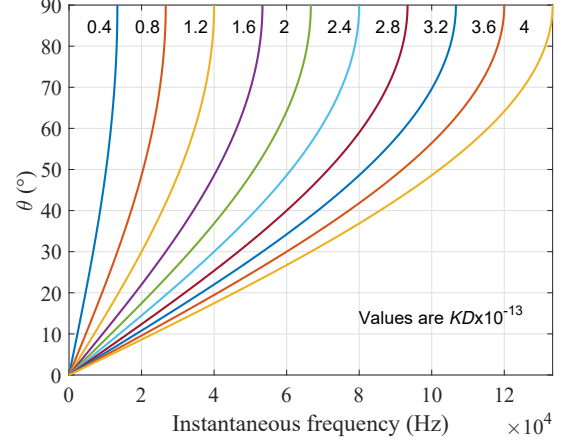


Fig. 2. Theoretical curves of the relationship between frequency and angle for various values of $KD \times 10^{-13}$ Hz-m/s. Using $K = 2 \cdot 10^{13}$ Hz/s, the values of D extend from 0.2 m to 2 m.

The instantaneous frequency of the receiver circuit output can be found from the derivative of the phase through

$$f_i = \frac{1}{2\pi} \frac{d\phi}{dt} = K\tau_g = K \frac{D}{c} \sin \theta. \quad (7)$$

Thus the instantaneous frequency is proportional to the chirp rate K , the baseline D and the angle θ at which the reflecting target resides. By choosing a large enough chirp rate and baseline, the angle of a reflecting target can be measured directly with very low complexity hardware and signal processing via a simple frequency estimation.

To demonstrate the angle estimation approach, the response was simulated using practical system parameters. Fig. 2 shows the relationship between target angle and output signal frequency for a transmitted LFM waveform with chirp rate $K = 2 \cdot 10^{13}$ Hz/s and values of D extending from 0.2 m to 2 m. The curves show that up to an angle of 90° , the frequency demonstrates a one-to-one correspondence with the angle of the target.

III. EXPERIMENTAL SYSTEM DESIGN AND MEASUREMENT RESULTS

To validate the theory of operation, an experimental system operating at 4.8 GHz was developed. Measurements were conducted in a 7.3 m semi-enclosed arch range with two receive elements separated by a baseline $D = 1.21$ m (19.36 λ) with a transmitting antenna placed between them (see Fig. 3). Wideband horn antennas with a gain of 10 dBi were used as the receivers and a standard gain horn antenna with a gain of 20 dBi was used for the transmitter. The LFM transmit signal was generated with a Keysight M8190 Arbitrary Waveform Generator (AWG) with a chirp rate of $2 \cdot 10^{13}$ Hz/s and a pulse duration of 100 μ s resulting in a linear increase between 3.8 and 5.8 GHz. The transmitted signal was amplified by a 9 dB power amplifier, while the received signals were amplified by low-noise amplifiers with gains of 20 dB and noise figures of 3 dB. The two receiver responses were captured using a

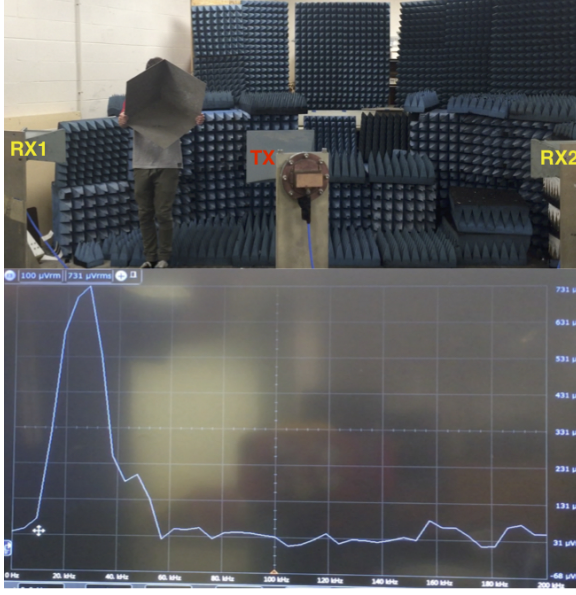


Fig. 3. (Top) Experimental configuration with trihedral reflector inside the semiarch range, two receivers and a single transmitter. (Bottom) Real-time frequency response in the oscilloscope after multiplying the two elements' responses and taking the fast Fourier transform of their product. The horizontal axis goes from 0 Hz to 200 kHz in 20 kHz increments.

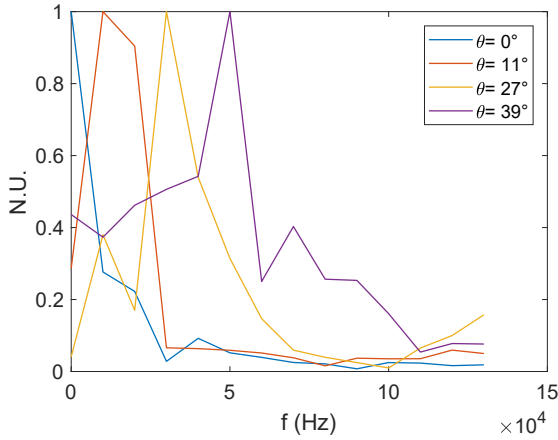


Fig. 4. The frequency response of the cross-correlated output for four different target angles, showing the increasing response frequency as the angle increases away from broadside to the receiving antennas.

Keysight MSOX92004A mixed signal oscilloscope. A trihedral corner reflector was used as the target.

The signal processing consisted of digitally bandpass filtering the two signal responses in the 3.8-5.8 GHz region and then directly multiplying them. Fig. 4 shows the frequency response for different angle locations. It is evident that the instantaneous frequency is increasing with the change in angle. In Fig. 5 the instantaneous frequency from the experimental measurements are plotted against the theoretical values, showing a strong agreement between them. The estimated angles had an average error of 2.8° (0.049 rad) with a standard deviation of 1.8° (0.031 rad), which can be furthermore improved by increasing the duration of the received waveforms and using

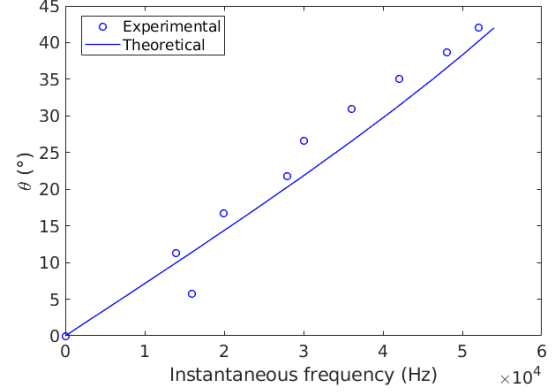


Fig. 5. The value of the maximum instantaneous frequency from the experimental measurements and the theoretical values from Eq. 7. The error of the corresponding angle estimation from the measured frequencies had an average of 2.8° (0.049 rad) and a standard deviation of 1.8° (0.031 rad).

analog filters to remove additional noise. The measurements results above were conducted in real-time, with the frequency response of the received signal plotted on the oscilloscope as shown in Fig. 3. Moving a trihedral reflector angularly as shown in the top of Fig. 3 changed accordingly the location of the peak of the fast Fourier transform of the output as shown in the bottom of Fig. 3. This demonstrated that real-time monitoring is possible via the proposed technique using only a multiplication and Fourier transform operation.

IV. CONCLUSIONS

A new active, direct angle estimating technique using an LFM waveform and a wide-baseline distributed array receiver has been demonstrated and verified through measurement. Theoretical and experimental results have shown that the frequency response of the receiver directly corresponds to the angle of the target, and even with minimal signal filtering in the system, the angle errors are quite low. The simple relationship between the angle and the output frequency, along with the minimal signal processing required, may lead to implementations in various applications like automotive radar, maritime radar and air traffic scanning, enabling direct angle estimation with low system cost.

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