Cognitive Relay Networks with Energy and Mutual-Information Accumulation

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***Abstract*—Cognitive radio networks, a.k.a. dynamic spectrum access networks, offer a promising solution to the problems of spectrum scarcity and under-utilization. In this paper, we consider two single-user links: primary and secondary links. To increase secondary user (SU) transmission opportunities and increase primary user (PU) throughput, we consider a cognitive relay network where a SU relays PU packets that are unsuccessfully received at the primary receiver (PR). At the PR side, two protocols are suggested: i) energy accumulation (EA), and ii) mutual-information accumulation (MIA). The average stable throughput of the secondary link is derived under these protocols for a specific throughput selected by the primary link. Results show that EA and MIA can significantly improve the secondary throughput compared with the no accumulation scenario, especially under extreme environment.**

I. Introduction and Motivation

With the radio spectrum being scarce and under-utilized, cognitive radio networks offer a solution to improve end-user data rates. In such a network, unlicensed users or secondary users (SUs) periodically sense the spectrum to detect any licensed or primary user (PU) activity before transmission. The objective of the SU becomes maximizing its opportunistic transmissions while being "transparent" to PU's activities. The broadcast nature of wireless systems allows cooperative communications to bring great benefits to the network, especially that intermediate nodes can act as potential relay nodes. To increase SU transmission opportunities and increase PU throughput, we turn to cognitive relay networks (CRN) where a SU relays PU packets that are unsuccessfully received at the primary receiver (PR). As introduced in [1]–[4], this would help free up PU's queue faster so that the SU can have more channel access. In this paper, we consider two approaches to enhance the performance of PUs: i) energy accumulation (EA) where two transmissions are combined at the PR side using maximal ratio combining (MRC), and ii) mutual-information accumulation (MIA) which can be realized through hybrid techniques such as rateless codes. Rateless codes allow the transmitter to generate an unlimited number of encoded packets, such that the receiver can decode the data after receiving a high enough number of encoded packets, irrespective of which ones it has received [5]. In addition, rateless codes do not require knowledge of the channel state information (CSI) at the transmitter, and offer robustness, reliability and efficiency compared to fixed-rate codes [5].

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Mutual-information accumulation is an enhanced physical layer technique which enables a user to accumulate mutual-information even when the wireless link is too weak to decode the entire packet successfully. The idea of MIA was introduced to cooperative relay networks in [6]. In [7], optimal/near-optimal routing and resource allocation algorithms for ad hoc networks were designed. The diversity backpressure scheduling and routing algorithm was introduced in [8] for MIA-based wireless networks that are capable of routing multiple packet streams. The introduced algorithm is shown to be throughput-optimal and outperform traditional schemes with respect to the achievable throughput. Recently, we applied MIA to underlay cognitive radio networks and it was shown that MIA can reduce 70-80 percent of end-to-end delay [9], [10]. Similar technology has also been applied to massive machine type communications (MTC) showing significant performance gains [11].

Most of the existing studies focus on enhancing the throughput by employing energy accumulation at the receiver using MRC (e.g., [2], [12]), and few papers discuss employing MIA at the receiver such as [13]–[15]. In [13], the authors use rateless codes (RLC) to maximize the throughput of SU under a delay constraint, and a distributed relay selection is employed. In [14], SU uses dirty paper coding (DPC) to transmit both its own data and PU data to the destination. In [15], a secondary transmitter (ST) employs rateless codes to transmit the encoded packets to N relays. Whenever any subset of the relays acquires enough coded packets, they will transmit to the destination to decode the message.

In this work, we consider a time-slotted CRN where only one packet is sent in each time slot. At the beginning of the time slot, a ST senses the primary transmitter's (PT) activity. If detected idle, the ST will transmit a packet, provided it has at least one in its queue. Furthermore, we consider that both PT and ST are equipped with queues of infinite length, with ST having two queues: one holding its own data while the other holding PT's packets that were unsuccessfully received by the PR. Compared with the no accumulation scenario in [1], our goal in this paper is to show that the average throughput of the CRN can be significantly improved under EA and MIA, such that the stability of the system is guaranteed, i.e., the

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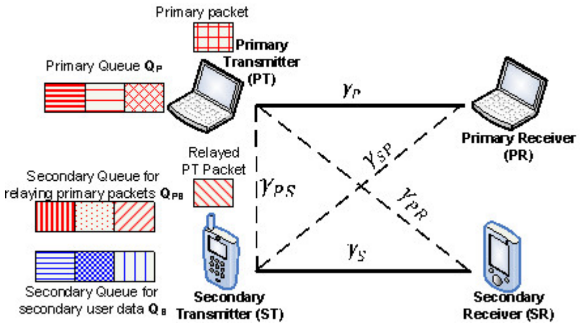

Fig. 1: General system model.

finiteness of all queues in the network.

We summarize the contributions of this paper as follows:

- Unlike existing works, we use queueing analysis to provide an analytical framework of the average stable throughput of the secondary link. And three different scenarios (with no accumulation, with EA, with MIA) are considered in the derivation.
- By considering measurement errors at PT and random packet arrivals, we show that both EA and MIA can create more transmission opportunities for ST, by increasing the probability of successful primary packets reception at PR.

The paper is organized as follows. The system model is presented in Section II. The problem formulations of cognitive relay protocol with no accumulation, with EA, and with MIA are presented in Section III-A, Section III-B, and Section III-C, respectively. Results are analyzed in Section IV. Finally, this paper concludes in Section V.

II. System Model

The system model is depicted in Fig. 1. The model consists of a CRN where a pair of SUs coexist with a pair of PUs, and the ST is assumed to stay between the PT and the PR. This basic model has been investigated in many papers such as [1] and [16]. Denote the following queues notations: Q_P holding PT packets, Q_S holding ST own packets and Q_{PS} holding PT packets to be relayed by ST. Independent and stationary traffic arrival processes are considered for primary and secondary queues, with arrival rates λ_i, and departure rates μ_i packets/sec, where i reads "P" for Q_P, and "S" for Q_S, and "PS", "SP" for Q_{PS} arrival and departure rates, respectively. Similar to [1], we consider independent stationary Rayleigh flat-fading channels, $h_i(t)$, between users, where $E[|h_i(t)|^2] = 1$. The average channel power gain due to pathloss and shadowing is denoted by γ_i, where i reads "P" for primary network, "S" for secondary network, "PS" for PT-ST link, "SP" for ST-PR link, and "PR" for PT-SR link; the noise power spectral density is normalized to 1 at all receivers. Note that block fading is assumed where the channel remains constant in each slot. PT transmits with normalized power $P_P = 1$, while ST transmits with power $P_S \leq 1$. It is obvious that a small P_S leads to an extreme transmission environment and further incurs lower successful transmission probability. On the other hand, if P_S approaches P_P, the SU has a high probability to interfere PUs. Thus, the P_S should be carefully selected such that its stable throughput is maximized while its activity is being "transparent" to the primary link [1]. We consider that a packet is successfully received at the intended destination if the instantaneous received signal-to-noise ratio (SNR) is above a threshold β_i, where i reads "P" for primary, and "S" for secondary. The probability of an outage event is

$$P_{\text{out},i} = P[\gamma_i |h_i(t)|^2 P_i < \beta_i] = 1 - e^{-\frac{\beta_i}{\gamma_i P_i}}. \quad (1)$$

Due to fading and pathloss, ST may misdetect PT's transmission with probability p_m, causing undesirable interference to the primary link. We assume that ST fails to detect PT activity if its instantaneous SNR falls below threshold α, then

$$p_m = P\left[\gamma_{PS} |h_{PS}(t)|^2 P_S < \alpha\right] = 1 - e^{\frac{-\alpha}{P_S \gamma_{PS}}}. \quad (2)$$

It is also assumed that when PT is not transmitting, ST might detect the idle slot as busy with a false alarm probability p_f.

In case of successful primary packet reception at the primary receiver, PR transmits an ACK signal to PT, and the latter drops the packet from its queue. Otherwise if PR fails to decode the packet, it transmits a NACK, which will be heard by the ST. If ST successfully decodes PT packet, it assumes the role of relaying PT packet to PR, by sending an ACK to PT, then the latter drops the packet from its queue. ST will attempt to retransmit once since if it fails, it is unlikely that the packet will be successfully transmitted in subsequent rounds. If PR fails to decode the packet relayed by ST, the packet is dropped and never transmitted. Note that in the case when both PR and ST successfully decode the packet, ST will not put it in its queue. It is assumed that ACK/NACK feedback are perfectly known and received as in [1]. When the slot is sensed idle, ST transmits PT packet with a probability ϵ while it transmits its own packet with probability $1-\epsilon$. The secondary user operates in full-duplex mode, i.e., it can simultaneously transmit and listen for primary transmission without being affected by self-interference [17]. In what follows, we derive the stable cognitive throughput for three different scenarios: i) under no accumulation, ii) under EA, and iii) under MIA.

III. Stable Throughput Analysis

A. Cognitive Relay Protocol

The departure rate $X_P(t)$ of the PT, can be defined as the number of primary packets that are successfully received at either PR or ST at time t. It can be expressed as [1][1]

$$X_P(t) = \mathbb{1}\left\{O_P(t) \cup O_P^I(t)\right\}, \quad (3)$$

where $\mathbb{1}\{.\}$ is the indicator function; $O_P(t)$ and $O_P^I(t)$ denote the events that PT transmits successfully (i.e. the packet is successfully received by either PR or ST) without interference

[1]The calculations in this section are based on [1]. The reader is encouraged to refer to them for an in-depth understanding of the formulations.

from ST and under ST interference. The average primary departure becomes [1]:

$$\mathbf{E}[X_P(t)] = \mu_P^r = (1-p_m)\left(e^{-\frac{\beta_P}{\gamma_P}} + e^{-\frac{\beta_P}{\gamma_{PS}}} - e^{(\frac{-\beta_P}{\gamma_P} - \frac{\beta_P}{\gamma_{PS}})}\right) + p_m\left(\frac{\gamma_P e^{-\frac{\beta_P}{\gamma_P}}}{\gamma_P + \beta_P\gamma_{SP}P_S}\right).$$

The departure rate $X_S(t)$ of the secondary queue Q_S is defined as the number of secondary packets that are successfully transmitted to the intended SR at time t [1]

$$X_S(t) = \mathbb{1}\left\{\left(A_S(t)\cap O_S(t)\right)\cup\left(A_S^I(t)\cap O_S^I(t)\right)\right\}, \quad (4)$$

where $A_S(t)$ and $A_S^I(t)$ denote the events that slot t is available for transmission by ST when correctly sensed as idle and when misdetected as idle, respectively; $O_S(t)$ and $O_S^I(t)$ denote the events of successful transmission by ST under no interference from PT and under PT interference, respectively. And

$$P[A_S(t)] = \left(1-p_f\right)P[Q_P(t)=0] = \left(1-p_f\right)\left(1-\frac{\lambda_P}{\mu_P^r}\right).$$

The average number of packets successfully departing Q_S is:

$$\mathbf{E}[X_S(t)] = \mu_S(P_S,\epsilon) = (1-p_f)\left(1-\frac{\lambda_P}{\mu_P^r}\right)e^{-\frac{\beta_S}{P_S\gamma_S}}(1-\epsilon) + p_m\left(\frac{\lambda_P}{\mu_P^r}\right)(1-\epsilon)\frac{P_S\gamma_S e^{\frac{-\beta_S}{P_S\gamma_S}}}{P_S\gamma_S + \beta_S\gamma_{PR}}$$

The departure process $X_{SP}(t)$ is defined as the number of successfully transmitted packets from ST to PR at time t, and

$$X_{SP}(t) = \mathbb{1}\Big\{\left(A_S(t)\cap O_{SP}(t)\right)\cup\left(A_S^I(t)\cap O_{SP}^I(t)\right)\Big\}, \quad (5)$$

where $O_{SP}(t)$ and $O_{SP}^I(t)$ denote the events of successful reception by PR from ST without interference from PT and under PT interference, respectively. The average departure rate of Q_{PS} can be expressed as [1]:

$$\mathbf{E}[X_{SP}(t)] = \mu_{SP}(P_S,\epsilon) = (1-p_f)\left(1-\frac{\lambda_P}{\mu_P^r}\right)e^{-\frac{\beta_P}{\gamma_{SP}P_S}}\cdot\epsilon + p_m\left(\frac{\lambda_P}{\mu_P^r}\right)\frac{P_S\gamma_{SP}e^{-\frac{\beta_P}{\gamma_{SP}P_S}}}{P_S\gamma_{SP}+\beta_P\gamma_P}\cdot\epsilon$$

The arrival rate of Q_{PS} is defined as the number of primary packets that are to be relayed by ST, and

$$Y_{PS}(t) = \mathbb{1}\left\{\{Q_P(t)\neq 0\}\cap P_{\text{out},P}\cap P_{\text{out},PS}^c\right\}, \quad (6)$$

where $P_{\text{out},PS}^c = 1-P_{\text{out},PS}$; and $P_{\text{out},P}$, $P_{\text{out},PS}$ are given in (1). Then, the average arrival rate of Q_{PS} can be written as:

$$\lambda_{PS} = \frac{\lambda_P}{\mu_P^r}\left(1-e^{\frac{-\beta_P}{\gamma_P}}\right)e^{\frac{-\beta_P}{\gamma_{PS}}}. \quad (7)$$

To guarantee the stability of $Q_{PS}(t)$, $\lambda_{PS} < \mu_{SP}(P_S,\epsilon)$ should be guaranteed by Loynes' theorem [1]. Thus,

$$\epsilon \geq \frac{\frac{\lambda_P}{\mu_P^r}\left(1-e^{-\frac{\beta_P}{\gamma_P}}\right)e^{-\frac{\beta_P}{\gamma_{PS}}}}{(1-p_f)\left(1-\frac{\lambda_P}{\mu_P^r}\right)e^{-\frac{\beta_P}{\gamma_{SP}P_S}} + p_m\left(\frac{\lambda_P}{\mu_P^r}\right)\frac{P_S\gamma_{SP}e^{-\frac{\beta_P}{\gamma_{SP}P_S}}}{P_S\gamma_{SP}+\beta_P\gamma_P}}.$$

B. Cooperative Cognitive Protocol under EA

For EA and MIA, all calculation in Section III-A remain valid except for $O_{SP}(t)$. With EA, the two transmissions from PT and ST at time slots t_1 and t_2 are combined using MRC. The probability that ST transmits in t_2 conditioned on the primary transmission at t_1 is given by:

$$p_t = \left(1-\lambda_p/\mu_p^r\right)\left(1-p_f\right) + p_m\left(\lambda_p/\mu_p^r\right). \quad (8)$$

Denote M_1 and M_2 as the SNRs of PT without and with ST interference, respectively. Let N_1 and N_2 be the SNRs of ST without and with PT interference, respectively, then we have

$$\begin{aligned} M_1 &= \gamma_P|h_P(t_1)|^2, \\ M_2 &= \frac{\gamma_P|h_P(t_1)|^2}{1+P_S\gamma_{SP}|h_{SP}(t_1)|^2}, \\ N_1 &= P_S\gamma_{SP}|h_{SP}(t_2)|^2, \\ N_2 &= \frac{P_S\gamma_{SP}|h_{SP}(t_2)|^2}{1+\gamma_P|h_P(t_2)|^2}. \end{aligned} \quad (9)$$

Proposition 1. *Recall that $O_{SP}(t_2)$ denotes the event of successful reception by PR from ST, then:*

$$\begin{aligned} P[O_{SP}(t_2)] =& w_1P\left[M_1+N_1>\beta_P\right] + w_2P\left[M_2+N_2>\beta_P\right] \\ &+ w_3P\left[M_1+N_2>\beta_P\right] + w_4P\left[M_2+N_1>\beta_P\right]. \end{aligned} \quad (10)$$

$$w_1 = \frac{(1-p_m)\left(1-\frac{\lambda_p}{\mu_p^r}\right)\left(1-p_f\right)}{p_t}, \qquad w_2 = \frac{p_m^2\left(\frac{\lambda_p}{\mu_p^r}\right)}{p_t},$$

$$w_3 = \frac{(1-p_m)\left(\frac{\lambda_p}{\mu_p^r}\right)p_m}{p_t}, \qquad w_4 = \frac{p_m\left(1-\frac{\lambda_p}{\mu_p^r}\right)\left(1-p_f\right)}{p_t}.$$

Eq. (10) shows that the sum of individual SNRs needs to exceed a threshold β_P for EA. The calculation takes into account all possible scenarios with/without PT/ST interference. Note that we normalize the weight factors w_i by dividing them by p_t. To guarantee the stability of the queue $Q_{PS}(t)$, $\epsilon \geq \lambda_{PS}/f_s(P_S)$, with $P[O_{SP}(t_2)] \triangleq f_s(P_S)$ should be satisfied. The calculation of $P[O_{SP}(t_2)]$ is listed in Appendix A.

C. Cooperative Cognitive Protocol under MIA

In this section, we consider MIA at PR side. Remember that with MIA, nodes accumulate information as $C_{i,j}t$ bits/Hz, where $C_{i,j}$ (bits/sec/Hz) is the channel capacity between nodes i and j. Let $g(x) = \log_2(1+x)$.

Proposition 2. *In the case of MIA,*

$$\begin{aligned} P[O_{SP}(t_2)] =& w_1P\Big[g(M_1)+g(N_1)>g(\beta_P)\Big] \\ &+ w_2P\Big[g(M_2)+\underbrace{g(N_2)}_{Y_1}>g(\beta_P)\Big] + w_3P\Big[g(M_1)+g(N_2) \\ &> g(\beta_P)\Big] + w_4P\Big[g(M_2)+\underbrace{g(N_1)}_{Y_2}>g(\beta_P)\Big] \\ =& w_1\left(e^{-\frac{\beta_P}{\gamma_{SP}P_S}} + \frac{e^{\frac{1}{\gamma_{SP}P_S}+\frac{1}{\gamma_P}}}{\gamma_{SP}P_S}\int_1^{1+\beta_P} e^{-\frac{g(M_1)}{\gamma_{SP}P_S}-\frac{1+\beta_P}{g(M_1)\gamma_P}}dg(M_1)\right) \\ &+ w_2\left(\int_0^{g(\beta_P)}\frac{\gamma_P e^{-\frac{2^{g(\beta_P)-y_1}-1}{\gamma_P}}}{\gamma_P+P_S\gamma_{SP}\left(2^{g(\beta_P)-y_1}-1\right)}f_{Y_1}(y_1)dy_1\right. \end{aligned}$$

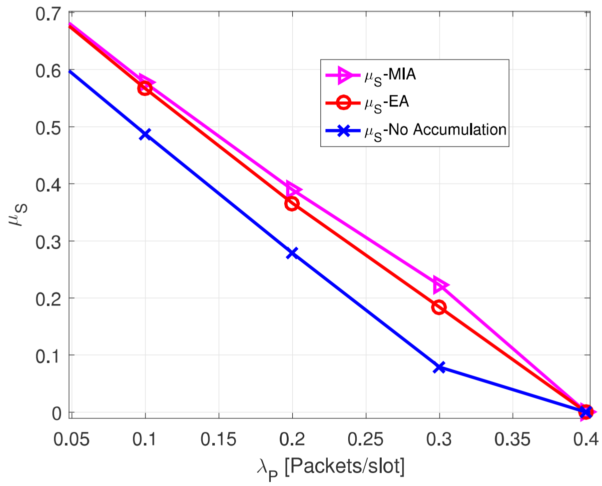

Fig. 2: μ_S versus λ_P.

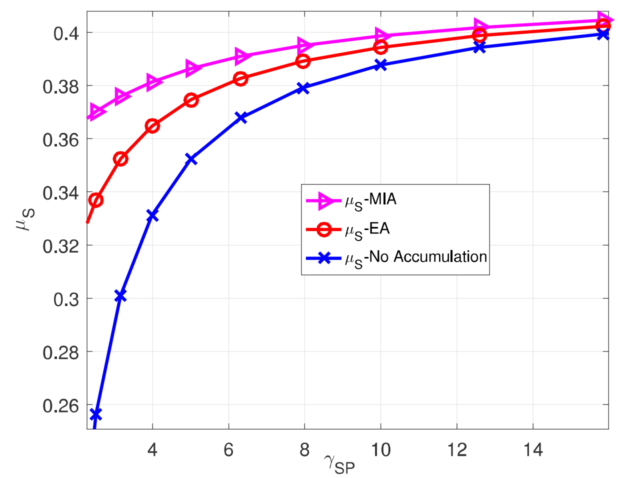

Fig. 3: μ_S versus γ_{SP} ($\lambda_P \approx \exp(-\beta_P/\gamma_P)$ is fixed to be 0.37).

$$+ \frac{P_S\gamma_{SP} e^{-\frac{2^{g(\beta_P)}-1}{P_S\gamma_{SP}}}}{P_S\gamma_{SP} + \gamma_P \left(2^{g(\beta_P)} - 1\right)}\Bigg) + w_3\Bigg(\frac{P_S\gamma_{SP} e^{-\frac{2^{g(\beta_P)}-1}{P_S\gamma_{SP}}}}{P_S\gamma_{SP} + \gamma_P \left(2^{g(\beta_P)} - 1\right)}$$

$$+ \int_0^{g(\beta_P)} e^{-\frac{2^{g(\beta_P)-1}-1}{\gamma_P}} f_{Y_1}(y_1)dy_1\Bigg) + w_4\Bigg(e^{-\frac{2^{g(\beta_P)}-1}{P_S\gamma_{SP}}} +$$

$$\int_0^{g(\beta_P)} \frac{\gamma_P e^{-\frac{2^{g(\beta_P)-y_2}-1}{\gamma_P}}}{\gamma_P + P_S\gamma_{SP}\left(2^{g(\beta_P)-y_2} - 1\right)} f_{Y_2}(y_2)dy_2\Bigg) \triangleq k_s(P_S),$$

where

$$f_{Y_1}(y_1) = \frac{2^{y_1} e^{-\frac{2^{y_1}-1}{P_S\gamma_{SP}}} \ln 2}{P_S\gamma_{SP} - \gamma_P + 2^{y_1}\gamma_P} + \frac{2^{y_1} P_S\gamma_P\gamma_{SP} e^{-\frac{2^{y_1}-1}{P_S\gamma_{SP}}} \ln 2}{(P_S\gamma_{SP} - \gamma_P + 2^{y_1}\gamma_P)^2},$$

$$f_{Y_2}(y_2) = \left(2^{y_2} e^{-\frac{2^{y_2}-1}{P_S\gamma_{SP}}} \ln 2\right) P_S^{-1}\gamma_{SP}^{-1}.$$

To guarantee the stability of the queue $Q_{PS}(t)$, $\epsilon \geq \lambda_{PS}/k_s(P_S)$ should be met.

IV. Results and Analysis

In this section, we evaluate the performance gain of the introduced cooperative cognitive protocols (MIA and EA) over the no accumulation case. We assume the following system parameters in our performance evaluation: $\gamma_P = 4$ dB, $\gamma_S = \gamma_{PR} = \gamma_{PS} = 10$ dB, $\gamma_{SP} = 5$ dB, and $\beta_P = \beta_S = 4$ dB.

Fig. 2 shows the maximum stable throughput μ_S versus the throughput selected by the primary node λ_P. We can see from the figure that over any $\lambda_P < \exp(-\beta_P/\gamma_P) = 0.37$, EA and MIA achieve higher stable throughput of the cognitive link than that with no accumulation. Furthermore, the MIA protocol achieves higher performance gain over the EA protocol. This gain is mainly due to the increase of the probability of successful primary packets reception at the PR, thereby creating more transmission opportunities for ST.

To further elaborate the performance improvement achieved by EA and MIA, we also plot the maximum stable throughput μ_S of the secondary link for different values of γ_{SP}, as shown in Fig. 3, where we set $\lambda_P \approx \exp(-\beta_P/\gamma_P) = 0.37$. As γ_{SP} increases, the relaying gain increases for all scenarios under study, since most of the traffic is redirected to the secondary link due to the better channel conditions on the ST-PR link. Furthermore, we do see that the maximum stable throughput μ_S for EA and MIA protocols are much higher than that of no accumulation scenario. In addition, the performance of MIA is slightly better than that of EA when γ_{SP} is low, and the two become similar in performance as γ_{SP} becomes high. This is because EA and MIA will approach each other in the low SNR regime. Finally, Fig. 4 shows the maximum stable throughput μ_S by changing values of P_S. It can be seen from the figure that the performance of all the three scenarios increases as P_S increases. Furthermore, the MIA protocol always performs the best among the three strategies. Also, the gap between the no accumulation protocol and the others is large when P_S is small, and reduces with the increase of P_S. This suggests that cooperation, especially cooperation through MIA, is extremely useful under extreme environments.

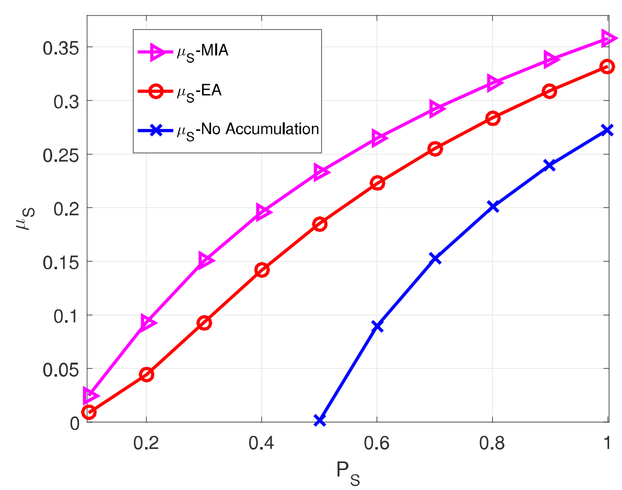

Fig. 4: μ_S versus P_S.

V. Conclusion

In this paper we showed that the throughput of the secondary link in a CRN can be improved if it has the capability to relay the transmitted packets from the PU. To be specific, we investigated the throughput of the SU where no accumulation, EA, and MIA are adopted for the cooperative relay communication respectively. The queueing analysis has been conducted and measurement errors are considered in the corresponding queueing characterization. Simulation results show that the EA-based and the MIA-based cooperation schemes can improve SU's throughput significantly compared with the no accumulation case, especially under extreme environments.

VI. Acknowledgment of Support and Disclaimer

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Appendix A

Let $U = |h_P(t_1)|^2$, $V = |h_{SP}(t_2)|^2$, $a = (P_S\gamma_{SP})/\gamma_P$, and $b = \beta_P/\gamma_P$. Since the channel gains are exponentially distributed, we have the following for the first term in Eq. (10):

$$P[U + aV > b] = 1 - \int_0^{\frac{b}{a}} \int_0^{b-av} e^{-u-v} du dv = \frac{a}{a-1} e^{\frac{-b}{a}} - \frac{1}{a-1} e^{-b}.$$

Substituting back the values of a and b, we obtain the result for the 1st term in (10). The 2nd term in (10) can be written as:

$$P\left[M_2 > \beta_P - N_2\right] = \int_0^{\beta_P} \int_{\beta_P - n_2}^{\infty} f_{M_2}(m_2) f_{N_2}(n_2) dm_2 dn_2 + \int_{\beta_P}^{\infty} \int_0^{\infty} f_{M_2}(m_2) f_{N_2}(n_2) dm_2 dn_2 = \int_0^{\beta_P} \left(1 - F_{M_2}(\beta_P - n_2)\right) f_{N_2}(n_2) dn_2 + 1 - F_{N_2}(\beta_P),$$

where

$$F_{M_2}(m_2) = P\left[M_2 \le m_2\right] = 1 - \frac{\gamma_P e^{-\frac{m_2}{\gamma_P}}}{\gamma_P + m_2 P_S \gamma_{SP}},$$

$$F_{N_2}(n_2) = P\left[N_2 \le n_2\right] = 1 - \frac{P_S\gamma_{SP} e^{-\frac{n_2}{P_S\gamma_{SP}}}}{P_S\gamma_{SP} + n_2\gamma_P},$$

and $f_{N_2}(n_2)$ is given at the end of the proof. This gives the expression for the 2nd term. Using similar calculations, we can get the expressions for the other terms. Finally, we have

$$P[O_{SP}(t_2)] = w_1 \left(\frac{P_S\gamma_{SP} e^{-\frac{\beta_P}{P_S\gamma_{SP}}} - \gamma_P e^{-\frac{\beta_P}{\gamma_P}}}{P_S\gamma_{SP} - \gamma_P} \right) + w_2 \left(\frac{P_S\gamma_{SP} e^{-\frac{\beta_P}{P_S\gamma_{SP}}}}{P_S\gamma_{SP} + \beta_P\gamma_P} + \int_0^{\beta_P} \frac{\gamma_P e^{-\frac{\beta_P - n_2}{\gamma_P}}}{\gamma_P + (\beta_P - n_2) P_S\gamma_{SP}} f_{N_2}(n_2) dn_2 \right) + w_3 \left(\int_0^{\beta_P} e^{-\frac{\beta_P - n_2}{\gamma_P}} f_{N_2}(n_2) dn_2 + \frac{P_S\gamma_{SP} e^{-\frac{\beta_P}{P_S\gamma_{SP}}}}{P_S\gamma_{SP} + \beta_P\gamma_P} \right) + w_4 \left(\int_0^{\beta_P} \frac{\gamma_P e^{-\frac{\beta_P - n_1}{\gamma_P}}}{\gamma_P + (\beta_P - n_1) P_S\gamma_{SP}} f_{N_1}(n_1) dn_1 + e^{-\frac{\beta_P}{P_S\gamma_{SP}}} \right),$$

where

$$f_{N_1}(n_1) = \frac{e^{-\frac{n_1}{P_S\gamma_{SP}}}}{P_S\gamma_{SP}},$$

$$f_{N_2}(n_2) = \frac{e^{-\frac{n_2}{P_S\gamma_{SP}}}}{P_S\gamma_{SP} + \gamma_P n_2} + \frac{P_S\gamma_P\gamma_{SP} e^{-\frac{n_2}{P_S\gamma_{SP}}}}{(P_S\gamma_{SP} + \gamma_P n_2)^2}.$$

References

[1] O. Simeone, Y. Bar-Ness, and U. Spagnolini, "Stable throughput of cognitive radios with and without relaying capability," *IEEE Trans. Commun.*, vol. 55, pp. 2351–2360, Dec. 2007.
[2] X. Zhang *et al.*, "On the study of outage performance for cognitive relay networks (CRN) with the nth best-relay selection in rayleigh-fading channels," *IEEE Wireless Commun. Lett.*, vol. 2, pp. 110–113, Feb. 2013.
[3] H. Chen, L. Liu, J. D. Matyjas, and M. J. Medley, "Optimal resource allocation for sensing-based spectrum sharing D2D networks," *Comput. & Electr. Eng.*, vol. 44, pp. 107 – 121, May 2015.
[4] W. Su, J. D. Matyjas, and S. Batalama, "Active cooperation between primary users and cognitive radio users in heterogeneous ad-hoc networks," *IEEE Trans. Signal Process.*, vol. 60, pp. 1796–1805, Apr. 2012.
[5] J. Perry, P. A. Iannucci, K. E. Fleming, H. Balakrishnan, and D. Shah, "Spinal codes," in *ACM SIGCOMM*, pp. 49–60, Aug. 2012.
[6] A. F. Molisch, N. B. Mehta, J. S. Yedidia, and J. Zhang, "Performance of fountain codes in collaborative relay networks," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 4108–4119, Nov. 2007.
[7] S. C. Draper, L. Liu, A. F. Molisch, and J. S. Yedidia, "Cooperative transmission for wireless networks using mutual-information accumulation," *IEEE Trans. Inf. Theory*, vol. 57, pp. 5151–5162, Aug. 2011.
[8] H. Feng and A. F. Molisch, "Diversity backpressure scheduling and routing with mutual information accumulation in wireless ad-hoc networks," *IEEE Trans. Inf. Theory*, vol. 62, pp. 7299–7323, Dec. 2016.
[9] L. Liu, H. Chen, J. Matyjas, and M. Medley, "Cooperative transmission for cognitive radio networks using mutual-information accumulation," in *IEEE WOCC*, pp. 1–5, May 2014.
[10] H. Chen, L. Liu, J. Matyjas, and M. Medley, "Cooperative routing for underlay cognitive radio networks using mutual information accumulation," *IEEE Trans. Wireless Commun.*, vol. 14, pp. 7110–7122, Dec. 2015.
[11] H. Chen, L. Liu, N. Mastronarde, L. Ma, and Y. Yi, "Cooperative retransmission for massive mtc under spatiotemporally correlated interference," in *IEEE GLOBECOM*, pp. 1–6, Dec. 2016.
[12] A. A. Sharifi, F. Ashtiani, H. Keshavarz, and M. Nasiri-Kenari, "Impact of cognition and cooperation on MAC layer performance metrics, part i: Maximum stable throughput," *IEEE Trans. Wireless Commun.*, vol. 11, pp. 4252–4263, Dec. 2012.
[13] B. Shahrasbi and N. Rahnavard, "Rateless-coding-based cooperative cognitive radio networks: Design and analysis," in *IEEE SECON*, pp. 224–232, Jun. 2011.
[14] Y. Chen, H. Huang, Z. Zhang, P. Qiu, and V. K. N. Lau, "Cooperative spectrum access for cognitive radio network employing rateless code," in *IEEE ICC Workshop*, pp. 326–331, May 2008.
[15] H. Kushwaha, Y. Xing, R. Chandramouli, and H. Heffes, "Reliable multimedia transmission over cognitive radio networks using fountain codes," *Proc. IEEE*, vol. 96, pp. 155–165, Jan. 2008.
[16] N. Devroye, P. Mitran, and V. Tarokh, "Achievable rates in cognitive radio channels," *IEEE Trans. Inf. Theory*, vol. 52, pp. 1813–1827, May 2006.
[17] S. ElAzzouni, O. Ercetin, A. El-Keyi, T. ElBatt, and M. Nafie, "Full-duplex cooperative cognitive radio networks," in *IEEE WiOpt*, pp. 475–482, May 2015.