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On the Properties of Neutrinos

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Abstract

This article reviews our present understanding of neutrino properties with a particular emphasis on observable differences between Majorana and Dirac neutrinos. We summarize current and future experimental efforts toward measuring neutrino properties and describe consequences of the Majorana versus Dirac nature of neutrinos on neutrino masses, neutrino decays, and neutrino electromagnetic properties.

Contents

1. INTRODUCTION

In the last several decades, neutrino physics has progressed at a breathtaking pace. We now know that only three active flavors couple to the *W* and *Z* bosons. The electroweak eigenstates of these neutrinos are linear combinations of their mass eigenstates. Parts of the worldwide neutrino program have reached precision stage. Short- and long-baseline neutrino oscillation experiments, as well as observations of neutrinos produced by the nuclear fusion reactions in the Sun and those produced by cosmic rays in the upper atmosphere, have determined two nonzero differences between the squares of the masses of the three mass eigenstates, demonstrating that at least two of these eigenstates have nonzero mass. The same experiments have also measured three of the parameters of the mixing transformation, so-called mixing angles, with unprecedented precision. Nevertheless, many unanswered questions remain. We have only limits on the absolute values of the neutrino masses from direct detection experiments and cosmology. The unitarity of the transformation connecting the mass eigenstates to the electroweak eigenstates is not firmly established. There are tantalizing hints, but no firm evidence, of the existence of sterile neutrinos that do not couple to the vector bosons of the Standard Model (SM), but nevertheless mix with the neutrinos that do. We do not know the transformation properties of the neutrinos under particle–antiparticle conjugation (i.e., whether the neutrinos are Majorana or Dirac fermions). We do not know just how small the neutrino magnetic moments are. We have just started exploring the full potential of the neutrinos in astrophysics and cosmology. Knowing the correct answer to these questions could lead to paradigm-shifting developments in physics and astrophysics.

Two of the three leptonic mixing angles are much larger than the quark mixing angles, a fact that needs to be understood by an appropriate extension of the SM. *CP* violation in the leptonic sector may shed light on the origin of the baryon–antibaryon asymmetry in the Universe. The interaction of neutrinos with ordinary matter is rather feeble except when the density is very large. Consequently, neutrinos can easily transfer a significant amount of energy and entropy in astrophysical settings, affecting many cosmic phenomena.

The purpose of this article is to explore our current understanding of the properties of neutrinos. In particular, we cover neutrino masses, the nature of the relation between neutrinos and antineutrinos (Dirac versus Majorana), and the electromagnetic properties of neutrinos, with particular emphasis on the connection between the latter two subjects. We limit the discussion of empirical observations primarily to terrestrial probes, and briefly mention relevant insights from astrophysics and cosmology.

In view of the central importance of the question of whether the neutrinos are Dirac particles or Majorana particles, we conclude this introduction by defining these terms. A Dirac neutrino $\nu^{(D)}$ is one that is distinct from its antiparticle: $\nu^{(D)} \neq \overline{\nu^{D}}$. When the neutrinos are Dirac particles, there is a conserved lepton number L that is $+1$ for leptons, both charged and neutral, and -1 for antileptons, both charged and neutral. The distinction between a Dirac neutrino and its antiparticle, then, is that they carry opposite values of L. A Majorana neutrino $\nu^{(M)}$ is one that is identical to its antiparticle: $\nu^{(M)} = \overline{\nu^{M}}$. When the neutrinos are Majorana particles, there is no conserved lepton number (for a relatively recent review of the physics of Majorana fermions, see Reference 1; for an earlier pedagogical discussion of this physics, see Reference 2).

The free field of a Dirac neutrino is a spinor with four independent components. This field is distinct from its charge conjugate. In contrast, the free field of a Majorana neutrino is identical to its charge conjugate, apart from a possible phase factor. While this field may be written in four-component form (as it is in Section 3.2), only two of its components are independent.

2. PRESENT EXPERIMENTAL STATUS

Neutrino oscillation experiments at different baselines have firmly established that the neutrino flavor states that are produced by the weak interactions are combinations of mass eigenstates; that is,

$$|\nu_f\rangle = \sum_i U_{fi}|\nu_i\rangle, \qquad 1.$$

where f and i are flavor and mass basis indices, respectively. Precise measurement of the invisible decay width of the Z boson restricts the number of flavors that can participate in weak interactions to three so-called active neutrinos: $f = e, \mu$, or τ. Clearly, to have three linearly independent flavor eigenstates, one needs at least three mass eigenstates. If the number of mass eigenstates is also three, imposing the condition that one can use either flavor or mass basis to describe the same physics requires the 3×3 matrix U to be unitary. However, there is no fundamental reason or symmetry principle that would limit the number of mass eigenstates to three. In case of more than three mass eigenstates, the only constraint is that only three "active" combinations of these mass eigenstates couple to the electroweak gauge bosons; the remaining orthogonal "sterile" combinations do not. Here one point is worth clarifying: Sometimes in the literature mass eigenstates are called sterile states if their contributions to the three active flavors are very small. (For example, there may be a fourth mass eigenstate, and the coefficient of that mass eigenstate in the linear combination that defines the electron neutrino is likely to be very small.) Strictly speaking, such a description is misleading, since the word "sterile" refers to the lack of interaction with electroweak gauge bosons and hence should be reserved for flavor states.

A 3×3 unitary matrix with unit determinant has nine independent variables. This matrix can be parameterized using trigonometric functions of three Euler angles and six additional phases. For

Dirac neutrinos, five of these phases can be absorbed in the definitions of neutrino states, and one is left with three angles and one *CP*-violating phase describing mixing of three flavors. However, for Majorana neutrinos, it is not possible to absorb two more of these additional phases, since the Majorana fields must remain self–charge conjugate. The number of parameters quickly increases with increasing number of mass eigenstates. For example, inclusion of a fourth mass eigenstate necessitates a parameterization with six angles and three *CP*-violating phases to describe mixing of three active and one sterile Dirac flavor states.

Several anomalies in various experiments can be interpreted as coming from one or more sterile neutrino admixtures. However, concrete experimental evidence for sterile neutrinos is still lacking. For three flavors, a combination of solar, atmospheric, reactor, and accelerator experiments has measured the three angles in the mixing matrix with a different precision for each angle. The value of the *CP*-violating phase is not yet determined.

2.1. Oscillation Experiments

From the equation describing the evolution of mass eigenstates for noninteracting neutrinos,

$$i\frac{\partial}{\partial t}|\nu_i\rangle = E_i|\nu_i\rangle, \qquad 2.$$

one can write an equation describing evolution in the flavor space:

$$i\frac{\partial}{\partial t}|\nu_f\rangle = \sum_{f'}\left(U\Lambda U^\dagger\right)_{ff'}|\nu_{f'}\rangle; \quad \Lambda_{ij} = E_i\delta_{ij}. \qquad 3.$$

From this equation it is clear that flavor changes will depend on the differences $E_i - E_j \sim (m_i^2 - m_j^2)/(2E)$, since one can write Λ as the sum of a matrix proportional to the identity and a matrix that depends only on those differences. Thus, oscillation experiments in which neutrinos do not interact between production and detection measure only the differences $\delta m_{ij}^2 = m_i^2 - m_j^2$, not the individual masses. In fact, experiments looking only at the disappearance of the original flavor are not even sensitive to the signs of these differences:

$$P(\nu_f \to \nu_f) = 1 - 2\sum_{i\neq j}|U_{fi}|^2|U_{fj}|^2\sin^2\left(\frac{\delta m_{ij}^2}{4E}L\right). \qquad 4.$$

If neutrinos interact between their source and their detection, the situation changes. Except in circumstances where matter densities are exceedingly large, their collisions with background particles can be neglected with reasonably good accuracy since the relevant cross sections are proportional to G_F^2 (where G_F is the Fermi coupling constant). However, neutrinos would coherently scatter in the forward direction from the background particles with an amplitude proportional to G_F. The inclusion of this effect modifies Equation 3 according to

$$\left(U\Lambda U^\dagger\right)_{ff'} \to \left(U\Lambda U^\dagger\right)_{ff'} + a_f\delta_{ff'} \qquad 5.$$

if the background is static and free of large polarizing magnetic fields. In the SM, both charged- and neutral-current interactions contribute to the quantities a_f. At the tree level (but not when one includes the first-order loop corrections), the neutral current contributes the same amount to all active flavors, and in the absence of sterile neutrinos, the resulting proportional-to-identity matrix does not affect the flavor change. The remaining term, a_e, is proportional to the background electron density in the SM for locally charge-neutral backgrounds. The value of the combined δm^2 and a_e terms can vary significantly depending on the sign of δm^2; it can even be zero at the so-called Mikheev–Smirnov–Wolfenstein (MSW) resonance (3, 4).

For three active flavors, the mixing matrix can be parameterized as

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}, \quad 6.$$

where we have used the notation $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ with all angles in the range $0 \leq \theta_{ij} \leq \pi/2$ and designated the *CP*-violating Dirac phase as δ. Majorana phases, which do not appear in the probabilities measured by the oscillation experiments, are not shown. The Euler angles are measured to be $\sin^2\theta_{12} = 0.307 \pm 0.013$, $\sin^2\theta_{23} = 0.51 \pm 0.04$, and $\sin^2\theta_{13} = 0.0210 \pm 0.0011$ (5). For three flavors, one can write two distinct differences of the squares of masses. Combining all the measurements yields the smaller one as $(7.53 \pm 0.18) \times 10^{-5}$ eV^2 and the larger one as $\sim 2 \times 10^{-3}$ eV^2 (5). Solar neutrino physics has determined that, of the two mass eigenstates separated by the smaller difference, δm^2_{21}, the mass eigenstate that is approximately two-thirds the electron flavor is the lighter one. For the larger difference, δm^2_{31}, there remain two possibilities: The possibility of $\delta m^2_{31} > 0$ is referred to as a normal hierarchy, and the possibility $\delta m^2_{31} < 0$ is referred to as an inverted hierarchy. Assuming a normal mass hierarchy yields a value of $(2.45 \pm 0.05) \times 10^{-3}$ eV^2 for the larger δm^2. For the inverted mass hierarchy, one obtains a value of $(2.52 \pm 0.05) \times 10^{-3}$ eV^2.

Experiments measuring the appearance of a flavor not present at the neutrino source are sensitive to the *CP*-violating phase and to the sign of δm^2. Note that oscillation experiments do not determine the overall neutrino mass scale, that is, the value of the smallest neutrino mass.

2.2. Direct Neutrino Mass Measurements

It is possible to measure the neutrino masses using nuclear β decays. Near the endpoint of the β spectrum, corresponding to the highest values of the measured electron energies, at least two of the mass eigenstates are nonrelativistic, implying a linear dependence of the decay probability on the masses. The maximum kinetic energy of the electron is $Q = E_0 - m_e$, where E_0 is the total decay energy. In a β decay experiment, the spectrum is measured up to an electron energy E near E_0. The fraction of decays in the interval $E_0 - E$ is given by $(E_0 - E)^3/Q^3$. Therefore, one needs a nucleus with a small value of Q. A relatively short decay lifetime is also helpful to reduce the amount of line broadening. The tritium nucleus satisfies both of these constraints.

Direct mass measurements are very robust since they depend only on conservation of energy. Since neutrinos mix, these measurements probe the quantity (6)

$$m^2_\beta = \sum_i |U_{ei}|^2 m^2_i. \quad 7.$$

So far, two experiments have carefully measured the endpoint of the tritium β decay spectrum. The Troitsk experiment reported $m^2_\beta = -0.67 \pm 2.53$ eV^2, corresponding to a limit of $m_\beta < 2.2$ eV (7). The Mainz experiment reported $m^2_\beta = -0.6 \pm 2.2(\text{stat.}) \pm 2.1(\text{syst.})$ eV^2, corresponding to a limit of $m_\beta < 2.3$ eV (8). A third experiment, KATRIN, is expected to start taking data in 2018 and to eventually reach a sensitivity of 0.2 eV (9). KATRIN is a high-resolution spectrometer based on magnetic adiabatic collimation combined with an electrostatic filter, using a well-characterized gaseous tritium source. To increase the sensitivity of a KATRIN-like experiment, one needs a larger spectrometer. However, given the size of the existing KATRIN spectrometer, scaling up this approach does not seem to be realistic. In order to circumvent this problem, another measurement based on cyclotron radiation emission spectroscopy has been proposed. This approach uses the principle that the energy of the emitted electron can be determined very accurately by detecting the radiation it emits when moving in a magnetic field. The planned experiment, Project 8, has the potential to reach sensitivities down to $m_\beta \sim 40$ meV by use of an atomic tritium source (10).

2.3. Cosmological Considerations

A quantity that is sometimes misstated as the number of neutrino species is the quantity called N_{eff} in Big Bang cosmology. This quantity is defined in terms of the radiation energy density deduced from the observations of cosmic microwave background radiation at photon decoupling as

$$\rho_{\text{rad}} \equiv \frac{\pi^2}{15} T_\gamma^4 \left[1 + \frac{7}{8} N_{\text{eff}} \left(\frac{4}{11} \right)^{4/3} \right], \qquad 8.$$

where T_γ is the photon temperature. The radiation density on the left side of this equation receives contributions from photons, plus three active flavors of neutrinos as well as antineutrinos, and all other particles that may be present. In the limit that all masses and lepton asymmetries are set to zero, all interactions and plasma effects are ignored, and no particles other than photons and active neutrinos are assumed to be present, N_{eff} has a value of 3. Including the SM interactions and masses slightly changes the thermal blackbody spectra of neutrinos and increases this value by a small amount. The Planck Collaboration (11) reports a value of $N_{\text{eff}} = 3.15 \pm 0.23$ (a careful evaluation of the assumptions that need to be taken into account to assess what N_{eff} represents is provided in Reference 12). A related analysis by the Planck Collaboration results in an upper limit of 0.23 eV for the sum of the neutrino masses, $\sum_i m_i$.

From oscillation experiments we know that at least two neutrinos have a nonzero mass. The lightest neutrino may also be massive, but it could have zero mass. This leaves open the possibility that one of the cosmic background neutrinos is relativistic but the other two are nonrelativistic. In Section 3.5, we discuss an interesting consequence of such a possibility.

3. DIRAC AND MAJORANA MASSES AND THEIR CONSEQUENCES

3.1. Neutrino Mass

The discovery and study of the Higgs boson at CERN's Large Hadron Collider have provided strong evidence that the quarks and charged leptons derive their masses from an interaction with the SM Higgs field. Conceivably, the neutrinos derive their masses in the same way. However, because the neutrinos are electrically neutral, the origin of their masses could involve a component—a "Majorana mass"—that is forbidden to the quarks and charged leptons.

Figure 1 depicts the essence of a Majorana mass, and shows how this mass differs from a "Dirac mass," which is the kind of mass a quark has. For simplicity, this figure treats a world with only one flavor and, correspondingly, only one neutrino mass eigenstate. The neutrinos ν and $\bar{\nu}$ in the figure are not the mass eigenstate and its antiparticle, but rather underlying neutrino states in terms of which we construct the picture of neutrino mass. The underlying states ν and $\bar{\nu}$ are distinct from each other.

As shown in **Figure 1**, when a Dirac mass term in the Lagrangian acts on an incoming ν, it leaves this particle a ν, and when it acts on a $\bar{\nu}$, it leaves this particle a $\bar{\nu}$. By contrast, when a Majorana mass term acts on a ν, it turns it into a $\bar{\nu}$, and when it acts on a $\bar{\nu}$, it turns it into a ν. Thus, Majorana neutrino masses do not conserve the lepton number L that is defined as $+1$ for a lepton, neutral or charged, and -1 for an antilepton.

Beyond neutrinos, a Majorana mass acting on any fermion turns it into its antiparticle. If the fermion is electrically charged, this transition reverses its electric charge, violating electric-charge conservation. That is why the quarks and charged leptons cannot have Majorana masses.

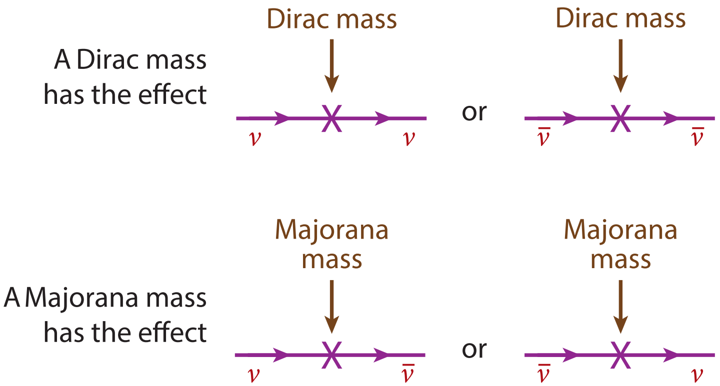

Figure 1

The effects of Dirac and Majorana mass terms in the Lagrangian.

When the neutrino mass term is a Majorana mass term, its mass eigenstate is $\nu + \bar{\nu}$, since this is clearly the state that the mass term sends back into itself (**Figure 2**). Noting that $\nu + \bar{\nu}$ is self-conjugate under particle–antiparticle interchange, we see that when the neutrino mass term is a Majorana mass term, the neutrino mass eigenstate will be a Majorana neutrino. It is easily shown that when there are several flavors, and correspondingly several neutrino mass eigenstates, if the neutrino mass term—now a matrix in flavor space—is a Majorana mass term, all the neutrino mass eigenstates will be Majorana neutrinos. Correspondingly, if the neutrino mass term is a Dirac mass term, all the mass eigenstates will be Dirac neutrinos.

What if, in the real world of multiple, mixed flavors, there are both Dirac and Majorana neutrino mass terms? So long as there are Majorana mass terms, the lepton number L that would distinguish Dirac neutrinos from their antiparticles is no longer conserved. Thus, one would expect that the neutrino mass eigenstates will be Majorana particles. This expectation is indeed correct (13). If there are n flavors, and one starts with Dirac mass terms alone, there will be n Dirac mass eigenstates. Of course, each mass eigenstate will be a collection of four states: the two helicity states of a neutrino, plus the two of an antineutrino. If one then adds Majorana mass terms, the n Dirac neutrinos will be replaced by $2n$ Majorana neutrinos, each comprising just the two helicity states of any spin-1/2 particle.

Let us now turn to the possible origins of Dirac and Majorana neutrino masses. For simplicity, we neglect mixing. Any mass term couples two neutrino fields of opposite chirality. Charge-conjugating a field of definite chirality reverses its chirality, and in a Majorana mass term, one of the two coupled neutrino fields is simply the charge conjugate of the other. In a Dirac mass term, the chirally left-handed neutrino field ν_L belongs to a SM weak-isospin doublet, while the chirally

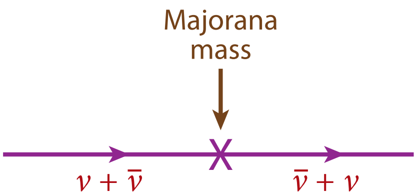

Figure 2

Effect of a Majorana mass term on its mass eigenstate, $\nu + \bar{\nu}$.

right-handed field ν_R must be added to the SM, which contains no right-handed neutrino fields, before a Dirac mass term is possible. Once ν_R has been introduced, the Lagrangian may contain the Yukawa interaction

$$\mathcal{L}_Y = -y\, H^0\, \overline{\nu_R}\, \nu_L + \text{h.c.}, \qquad 9.$$

where H^0 is the SM neutral Higgs boson field and y is a real Yukawa coupling constant. When H^0 develops its vacuum expectation value $\langle H^0 \rangle_0 = 174$ GeV $\equiv v$, this Yukawa interaction leads to the Dirac mass term

$$\mathcal{L}_D = -m_D\, \overline{\nu_R}\, \nu_L + \text{h.c.}, \qquad 10.$$

where $m_D = yv$ is the Dirac neutrino mass. If this mass term is the sole source of mass for a neutrino ν, then the field of this neutrino is given by $\nu = \nu_L + \nu_R$. In terms of this field, the mass term can be written as

$$\mathcal{L}_D = -m_D\, \bar{\nu}\, \nu, \qquad 11.$$

a form from which it is obvious that m_D is the mass of ν. If ν is one of the three established neutrino mass eigenstates, whose masses are $\lesssim$0.1 eV, then $y = m_D/v \lesssim 10^{-12}$. A coupling constant this much smaller than unity leaves many theorists skeptical of the notion that Dirac mass terms are the sole source of neutrino masses.

Majorana mass terms can have several different origins, all of which entail physics that is far outside the SM. Once a right-handed field ν_R has been introduced, we can have a "right-handed Majorana mass term,"

$$\mathcal{L}_R = -\frac{m_R}{2}\, \overline{(\nu_R)^c}\, \nu_R + \text{h.c.}, \qquad 12.$$

where the superscript c denotes charge conjugation and m_R is a positive, real constant. Note that $\overline{(\nu_R)^c}\, \nu_R$ absorbs a neutrino and emits an antineutrino, while its Hermitian conjugate, $\overline{\nu_R}\,(\nu_R)^c$, absorbs an antineutrino and emits a neutrino, in conformity with the effects attributed to Majorana mass terms in the above discussion. Since a right-handed neutrino does not carry any nonzero quantum number that is conserved in the SM, or as far as we know in nature, the right-handed Majorana mass term of Equation 12 does not violate any known conservation law, despite the fact that it transforms neutrinos and antineutrinos into one another. This Majorana mass term does not arise from a neutrino coupling to the SM Higgs field, so its origin is quite different from the Brout–Englert–Higgs mechanism that leads to masses in the SM. If this mass term is the only source of mass for a neutrino ν, then the field of this neutrino is given by $\nu = \nu_R + (\nu_R)^c$. As we see, ν is a self-conjugate (i.e., Majorana) neutrino, in agreement with the above conclusion that when the mass term is a Majorana mass term, its mass eigenstate will be a Majorana neutrino. In terms of the field ν, the right-handed Majorana mass term can be written as

$$\mathcal{L}_R = -\frac{m_R}{2}\, \bar{\nu}\, \nu + \text{h.c.} \qquad 13.$$

This has the expected form for the mass term of a mass eigenstate ν of mass m_R, except for the factor of one-half. To understand that factor, recall that the mass term for a fermion mass eigenstate must absorb and then reemit that fermion with an amplitude that is the fermion's mass. As we have noted, in the mass term of Equation 13, the field is a "Majorana" (i.e., self-conjugate) field. As a result, in contrast to the mass term for a Dirac neutrino, where only the field ν can absorb the incoming neutrino and only the field $\bar{\nu}$ can emit the outgoing one, now there is an additional term in which the field $\bar{\nu}$ absorbs the incoming neutrino and the field ν emits the outgoing one. This additional term is equal in size to the sole term in the Dirac case, so the amplitude produced by $(m_R/2)\bar{\nu}\nu$ is m_R, not $(m_R/2)$. Since by definition this amplitude must be the mass of ν, our notation has been chosen such that the parameter m_R is the mass of ν.

Even if there is no right-handed neutrino field, if there is a non-SM weak-isospin triplet of scalar fields Δ, there can be an interaction of the form $\Delta^0 \overline{(\nu_L)^c}\, \nu_L$, where Δ^0 is the neutral member of the triplet. If Δ^0 has a nonzero vacuum expectation value, this interaction leads to a "left-handed Majorana mass term"

$$\mathcal{L}_L = -\frac{m_L}{2} \overline{(\nu_L)^c}\, \nu_L + \text{h.c.}, \qquad 14.$$

where m_L is a positive, real constant. As in the case of the right-handed Majorana mass term, the eigenstate of this left-handed one is a Majorana particle. Its field is the self-conjugate $\nu = \nu_L + (\nu_L)^c$.

Non-SM physics at a high mass scale can lead at present-day energies to an effective interaction of the form $\overline{(\nu_L)^c}\, H^0 H^0 \nu_L/\Lambda$, where Λ is the high mass scale from whose physics this effective interaction comes (14). Once the SM Higgs field H^0 develops its vacuum expectation value v, this interaction leads in turn to the effective left-handed Majorana mass term,

$$\mathcal{L}_L^{\text{effective}} = -\frac{m'_L}{2} \overline{(\nu_L)^c}\, \nu_L + \text{h.c.}, \qquad 15.$$

where $m'_L/2 = v^2/\Lambda$.

Perhaps the leading candidate for an explanation of how the neutrino masses, although nonzero, can be so small is the seesaw mechanism (15). To explain this mechanism (16), let us again treat a world with just one flavor. The most straightforward (so-called type-I) seesaw model adds to the SM of this world a right-handed neutrino ν_R and both Dirac and right-handed Majorana neutrino mass terms. The neutrino mass part of the Lagrangian is then

$$\mathcal{L}_m = -m_D\, \overline{\nu_R} \nu_L - \frac{m_R}{2} \overline{(\nu_R)^c}\, \nu_R + \text{h.c.}, \qquad 16.$$

where ν_L is the neutrino in a SM left-handed lepton doublet and m_D and m_R are of course constants. Using the identity $\overline{(\nu_L)^c}\, m_D\, (\nu_R)^c = \overline{\nu_R}\, m_D \nu_L$, one can rewrite this $\mathcal{L}_m$ as

$$\mathcal{L}_m = -\frac{1}{2} \left[\overline{(\nu_L)^c}, \overline{\nu_R} \right] \begin{bmatrix} 0 & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} \nu_L \\ (\nu_R)^c \end{bmatrix} + \text{h.c.} \qquad 17.$$

As discussed above, the right-handed Majorana mass term in Equations 16 and 17 does not violate any known conservation law. Thus, m_R could be extremely large, and the seesaw model assumes that this is the case. By contrast, the Dirac neutrino mass m_D is presumably of the same order as the masses of the quarks and charged leptons, all of which are Dirac masses. Thus, $m_D \ll m_R$. The neutrino mass matrix

$$M_\nu = \begin{bmatrix} 0 & m_D \\ m_D & m_R \end{bmatrix} \qquad 18.$$

in Equation 17 can be diagonalized by the transformation

$$Z^T M_\nu Z = D_\nu, \qquad 19.$$

where, with the assumption that $m_D/m_R \equiv \rho \ll 1$,

$$Z \cong \begin{bmatrix} 1 & \rho \\ -\rho & 1 \end{bmatrix} \begin{bmatrix} i & 0 \\ 0 & 1 \end{bmatrix}, \qquad 20.$$

and

$$D_\nu \cong \begin{bmatrix} m_D^2/m_R & 0 \\ 0 & m_R \end{bmatrix}. \qquad 21.$$

Here, the second matrix in Equation 20 is included so that both diagonal elements in D_ν will be positive. Defining

$$\begin{bmatrix} \nu'_{\rm L} \\ N'_{\rm L} \end{bmatrix} \equiv Z^{-1} \begin{bmatrix} \nu_{\rm L} \\ (\nu_{\rm R})^c \end{bmatrix} \qquad 22.$$

and

$$\begin{bmatrix} \nu \\ N \end{bmatrix} \equiv \begin{bmatrix} \nu'_{\rm L} + (\nu'_{\rm L})^c \\ N'_{\rm L} + (N'_{\rm L})^c \end{bmatrix}, \qquad 23.$$

we may rewrite Equation 16 as

$$\mathcal{L}_m = -\frac{1}{2}\frac{m_{\rm D}^2}{m_{\rm R}}\bar{\nu}\nu - \frac{1}{2}m_{\rm R}\bar{N}N. \qquad 24.$$

From this relation, we see that ν and N are mass eigenstates, and from the definition of Equation 23, we see that they are Majorana (self-conjugate) particles. Equation 24 shows that the mass of ν is $m_{\rm D}^2/m_{\rm R}$, while that of N is $m_{\rm R}$. Thus,

$$(\text{mass of } \nu) \times (\text{mass of } N) = m_{\rm D}^2 \sim (\text{mass of quark or charged lepton})^2. \qquad 25.$$

This is the famous seesaw relation, which states that the heavier N is, the lighter ν will be.

It is no surprise that the seesaw model predicts that the neutrino mass eigenstates will be Majorana particles. As mentioned above, any model that includes Majorana masses will do that. Nor is it a surprise that, even though we were treating a world with just one flavor, we ended up with two Majorana neutrinos. As noted above, when a Majorana mass term is added to a Dirac one in a world with n flavors, the n Dirac neutrinos of that world are replaced by $2n$ Majorana neutrinos.

How large is the mass m_N of the heavy neutrino N predicted to be? While there is no sharp prediction because of the unknown parameter $m_{\rm D}$, if we assume that $m_{\rm D}$ is of the order of the mass m_μ of the muon, the charged lepton in the "middle" generation of leptons, and if we take the mass m_ν of the light neutrino ν to be $\sim$0.1 eV, as suggested by neutrino oscillation experiments, then the seesaw relation predicts that

$$m_N = \frac{m_{\rm D}^2}{m_\nu} \sim \frac{m_\mu^2}{0.1\ \text{eV}} = 10^8\ \text{GeV}. \qquad 26.$$

Thus, according to the seesaw model, the known light neutrinos are a window into physics at a very high mass scale (unless $m_{\rm D}$ is much smaller than we have guessed). By contrast, since 10^8 GeV is obviously far out of reach of current or near-future particle accelerators, it may be a while before the seesaw picture can be tested.

3.2. Why Determining Whether Neutrinos Are Dirac or Majorana Particles Is Very Challenging

Why is it that we do not know whether neutrinos are Dirac or Majorana particles? To answer this question, we note first that all the neutrinos that we have so far been able to study directly have been ultrarelativistic. As explained below, when a neutrino is ultrarelativistic, its behavior is almost completely insensitive, under almost all circumstances, to whether it is a Dirac particle or a Majorana one.

There may or may not be a conserved lepton number L that distinguishes antileptons from leptons. However, regardless of whether such a conserved quantum number exists, the SM weak interactions are chirally left-handed. As a result, when the particle we call the "neutrino," ν, is created in, for example, the decay $W^+ \to e^+ + \nu$, this "ν" will be of left-handed (i.e., negative) helicity extremely close to 100% of the time. In contrast, when the particle we call the "antineutrino," $\bar{\nu}$, is created in the decay $W^- \to e^- + \bar{\nu}$, this "$\bar{\nu}$" will be of right-handed (i.e., positive) helicity extremely close to 100% of the time. In the Majorana case, helicity will be the sole difference between this "ν" and this "$\bar{\nu}$."

Now, suppose the "ν" or the "$\bar{\nu}$" that has been created in W boson decay interacts with some target via the SM charged-current weak interaction, creating a charged lepton in the process. Neglecting mixing, the leptonic part of the first-generation weak interaction Lagrangian is

$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}} \left[\bar{e}\gamma^\lambda \frac{(1-\gamma_5)}{2} \nu W^-_\lambda + \bar{\nu}\gamma^\lambda \frac{(1-\gamma_5)}{2} e W^+_\lambda \right], \quad 27.$$

where g is the semiweak coupling constant and $(1-\gamma_5)/2$ is a left-handed chirality projection operator. If neutrinos are Dirac particles, the lepton number L is conserved, so the ν created in W^+ decay, with $L = +1$, can create only an e^-, not an e^+, and it will do so via the first term on the right-hand side of Equation 27. Similarly, the $\bar{\nu}$ created in W^- decay, with $L = -1$, can create only an e^+, not an e^-, and it will do so via the second term in Equation 27.

Now suppose that neutrinos are Majorana particles. Then there is no longer a conserved lepton number, and the neutrino field ν is now a Majorana field that, with a suitable choice of phase convention, takes the form

$$\nu = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_h \left(f_{\mathbf{p},h} u_{\mathbf{p},h} e^{-ipx} + f^\dagger_{\mathbf{p},h} v_{\mathbf{p},h} e^{ipx} \right). \quad 28.$$

Here, $\mathbf{p}$ is the momentum of the neutrino, E_p is its energy, and h is its helicity. The operator $f_{\mathbf{p},h}$ absorbs a neutrino of momentum $\mathbf{p}$ and helicity h, and $f^\dagger_{\mathbf{p},h}$ creates such a neutrino. The functions $u_{\mathbf{p},h}$ and $v_{\mathbf{p},h}$ are the usual Dirac u and v wave functions for a particle of momentum $\mathbf{p}$ and helicity h. We note that the field operator ν can both absorb and create a neutrino, and the same is true of the conjugate field operator:

$$\bar{\nu} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_h \left(f_{\mathbf{p},h} \bar{v}_{\mathbf{p},h} e^{-ipx} + f^\dagger_{\mathbf{p},h} \bar{u}_{\mathbf{p},h} e^{ipx} \right). \quad 29.$$

In particular, an incoming neutrino can be absorbed by either the field ν or the field $\bar{\nu}$. Thus, a Majorana neutrino created in, say, W^+ decay can be absorbed, in principle, by either the first or the second term in the charged-current Lagrangian $\mathcal{L}_{cc}$ of Equation 27. However, the Majorana neutrino from decay of a W^+ will be essentially 100% polarized with left-handed helicity. Thus, because of the left-handed chirality projection operator $(1-\gamma_5)/2$ in $\mathcal{L}_{cc}$, and the ultrarelativistic energy of the neutrino, in practice it can be absorbed only by the first term. The action of $(1-\gamma_5)/2$ on the $\bar{\nu}$ field in the second term almost totally suppresses this term for a left-handed ultrarelativistic neutrino. Since the action of the first term in $\mathcal{L}_{cc}$ always creates an e^-, never an e^+, our Majorana neutrino form W^+ decay will always create an e^- rather than an e^+, just as a Dirac neutrino would. Similarly, a Majorana neutrino born in W^- decay will essentially always be of right-handed helicity, and consequently can in practice be absorbed only by the second term in $\mathcal{L}_{cc}$, whose action always produces an e^+, never an e^-. This is exactly how a Dirac antineutrino would behave.

As the neutrinos from W decay illustrate, for ultrarelativistic neutrinos, helicity is a substitute for lepton number. Even if there is no conserved lepton number, ultrarelativistic neutrinos will behave as if there is such a quantum number. That is, Majorana neutrinos will behave as if they are Dirac neutrinos. To address experimentally the question of whether neutrinos are actually Dirac or actually Majorana particles, we have to find exceptions to this rule, or find an effective way of working with nonrelativistic neutrinos, or find some process that addresses the question even though it does not involve neutrinos at all.

3.3. Double β Decay

Numerous experiments are seeking to answer the Dirac versus Majorana question through the study of special nuclear decays. In the nuclear shell model, each nucleon is assumed to interact with an average mean field, filling well-defined nuclear shells. Since a heavy nucleus typically has more neutrons than protons, the last filled shell orbital can be very different for protons and neutrons. In a heavy nucleus with an even number of protons and an even number of neutrons, which we denote as X, like nucleons tend to pair up. If one of the neutrons in X is replaced by a proton, that proton and the second neutron from the broken pair cannot pair up, since they would sit in different shells. Without the benefit of pairing energy, this could raise the ground-state energy of the new nucleus, which we call Y, to a value higher than the ground-state energy of the original nucleus, X. Thus, the ground state–to–ground state β decay of X into Y is energetically impossible. Replacing the odd neutron in Y with a second proton could create a third nucleus, Z, the ground-state energy of which is lower than that of the other two nuclei since those two protons again pair up. The exact conditions under which such a scenario takes place depend on the details of the shell structure, but there are a handful of triplets of nuclei for which this scenario is realized. For such triplets, the first-order β decay of X into Y is energetically prohibited, but the second-order β decay of X into Z is possible (17):

$$X \rightarrow Z + 2e^- + 2\bar{\nu}_e. \qquad 30.$$

Such double β decays with two neutrinos in the final state ($2\nu\beta\beta$) have been observed in a number of nuclei.

Already in 1937 it was pointed out (18) that, if the neutrinos are Majorana fermions, the neutrino emitted by one of the nucleons can be absorbed by another one. The resulting process,

$$X \rightarrow Z + 2e^-, \qquad 31.$$

is called neutrinoless double β decay ($0\nu\beta\beta$). This process has not yet been experimentally observed. However, if one starts with a very large number of parent nuclei and waits patiently for a handful of them to undergo $0\nu\beta\beta$ decay, this process may well prove to be an exception to the rule that, when relativistic, Majorana neutrinos behave just like Dirac ones (19).

For the Majorana neutrino exchange, the leptonic part of the amplitude comes from the operator

$$L_{\mu\nu} = \sum_i \bar{e}(x)\gamma_\mu(1-\gamma_5)U_{ei}\nu_i(x)\overline{\nu_i^c}(y)U_{ei}\gamma_\nu(1+\gamma_5)e^c(y), \qquad 32.$$

where the sum is over the neutrino mass eigenstates. Contraction of the two neutrino fields in the above tensor yields the neutrino propagator

$$\frac{\not{q} - m_i}{q^2 - m_i^2}. \qquad 33.$$

The $\not{q}$ term does not contribute to the traces, leaving the leptonic tensor proportional to the quantity

$$m_{\beta\beta} \equiv \sum_i m_i U_{ei}^2. \qquad 34.$$

Calculation of the hadronic part of the amplitude, a nuclear matrix element, is significantly more complicated. Both the $2\nu\beta\beta$ and $0\nu\beta\beta$ decay modes involve virtual intermediate states of the nucleus Y. When two real neutrinos are emitted, the virtual momentum transfers are relatively small. The ground states of the even–even nuclei X and Z have spin parity 0^+. Thus, to calculate the $2\nu\beta\beta$ rate it is sufficient to include transitions through only a few low-lying 1^+ states in the nucleus Y. In contrast, when the neutrinos remain virtual, as in the $0\nu\beta\beta$ decay, virtual momentum transfers can reach values of up to a few hundred MeV, necessitating inclusion of transfers through many intermediate states, including most of the particle-hole excitations in the nucleus Y. For this reason, the nuclear matrix elements in the $2\nu\beta\beta$ and $0\nu\beta\beta$ decays would be significantly different. For a comprehensive review of the nuclear matrix elements in double β decay, we refer the reader to Reference 20.

We can then write the half-life for the $0\nu\beta\beta$ decay where a Majorana neutrino is exchanged as

$$\left[T_{1/2}^{0\nu\beta\beta}\left(0_X^+ \to 0_Z^+\right)\right]^{-1} = G_{0\nu}\,|M_{0\nu}|^2\,|m_{\beta\beta}|^2, \qquad 35.$$

where $M_{0\nu}$ is the appropriate nuclear matrix element and $G_{0\nu}$ contains all the other easily calculable contributions, including phase-space factors. The $2\nu\beta\beta$ decay rate can be written in a similar form:

$$\left[T_{1/2}^{2\nu\beta\beta}\left(0_X^+ \to 0_Z^+\right)\right]^{-1} = G_{2\nu}\,|M_{2\nu}|^2. \qquad 36.$$

Note that there are considerable uncertainties coming from both nuclear physics and neutrino physics in the calculated $0\nu\beta\beta$ decay rates. The factor $m_{\beta\beta}$ depends on the neutrino masses themselves, not their squares. This makes the $0\nu\beta\beta$ decay exquisitely sensitive to the neutrino mass hierarchy and the possible presence of additional mass eigenstates that mix into the electron neutrino. Also, because of the Majorana character of the neutrinos, $m_{\beta\beta}$ depends on the square of the mixing matrix element, not on the square of its absolute value. As a result, the $0\nu\beta\beta$ rate also depends on the phases in the mixing matrix, including two (or more if there are sterile states) additional phases that do not come into the neutrino oscillations. Those phases could interfere either constructively or destructively.

Since a $0\nu\beta\beta$ decay experiment can observe only the daughter nucleus and two electrons, the exchanged particle does not have to be a light neutrino. Lepton number–violating interactions taking place at a scale Λ above the electroweak scale could lead to exchange of heavier particles instead of the light neutrino. The contribution of such a heavy particle to the $0\nu\beta\beta$ decay amplitude is roughly $\sim G_F^2 M_W^4/\Lambda^5$, whereas the light neutrino exchange contributes a factor of $\sim G_F^2 m_{\beta\beta}/k^2$ (where k is the exchanged virtual momentum) of the order of a few 100 MeV, as mentioned above. These two contributions become comparable at approximately $\Lambda = 1$ TeV. Calculations using effective field theory also come up with a similar scale (21).

Experimental observation of the $0\nu\beta\beta$ decay, no matter what the underlying mechanism is, would imply that nature contains a Majorana neutrino mass and that, therefore, neutrinos are Majorana fermions. This is because the decay implies that the lepton number–violating amplitude converting two d quarks into two u quarks plus two electrons is nonzero. If this amplitude is not zero, then each initial d quark and final u quark pair can be contracted to a W boson. The two W bosons can then combine with the two electrons in the final state. The resulting diagram is nothing but a contribution to the Majorana neutrino mass, as pointed out long ago (22).

3.4. An Exotic Exception

A different kind of exception to the rule that ultrarelativistic Majorana and Dirac neutrinos behave indistinguishably can occur if there exists a heavy neutrino mass eigenstate N satisfying $m_e \ll m_N \ll m_K$, where m_e, m_N, and m_K are the e^-, N, and kaon masses, respectively (23).

Since leptons mix, we would expect this N to be a (small) component of the ν_e. Then the N can be produced by the decay $K^+ \to e^+ + N$, driven by the SM weak interaction. In this decay, because the kaon is spinless, the kaon rest-frame helicities of the e^+ and N must be of the same sign. Since $m_e \ll m_N \ll m_K$, the left-handed chiral projection operator in the SM weak interaction will almost always give the e^+ the usual right-handed helicity of a relativistic antilepton, forcing the N to have right-handed helicity as well. If, for example, $m_N = 50$ MeV, the N will have right-handed helicity 99.99% of the time. Now suppose this N undergoes a charged-current or neutral-current SM weak interaction with some target. If neutrinos are Dirac particles, then lepton number is conserved, so the N from K^+ decay will be a neutrino, not an antineutrino. Given its right-handed helicity, its interaction will then be extremely suppressed by the left-handed chiral projection operator in the SM weak interaction. However, if neutrinos are Majorana particles, it can interact just like a right-handed Dirac antineutrino would, and there is no suppression.

3.5. The Special Role Nonrelativistic Neutrinos Could Play

As shown above, in almost all situations where a neutrino is relativistic, its helicity is a substitute for lepton number, so that its behavior will not reveal whether it is a Dirac or a Majorana particle. However, if the neutrino is nonrelativistic instead, its behavior can depend quite a lot on whether it is a Dirac or a Majorana particle. To illustrate, let us consider the capture of the relic neutrinos from the Big Bang on tritium. At their current temperature, these neutrinos have $kT = 1.7 \times 10^{-4}$ eV. Given the known values of Δm^2_{32} and Δm^2_{21}, if the mass ordering is normal, then mass(ν_3) $\geq 5.0 \times 10^{-2}$ eV and mass(ν_2) $\geq 8.6 \times 10^{-3}$ eV. If the ordering is inverted, then mass(ν_2 and ν_1) $\geq 5.0 \times 10^{-2}$ eV. Thus, for either ordering, two of the three known mass eigenstates are nonrelativistic, and if the lightest member of the spectrum is not too light, all three of them are. Neglecting the small kinetic energy of one of the nonrelativistic mass eigenstates, ν_i, the capture of this mass eigenstate on a tritium nucleus via the reaction $\nu_i + {}^3\text{H} \to {}^3\text{He} + e^-$ will yield a monoenergetic electron with energy $E_e \cong (m_\text{H} - m_\text{He}) + m_{\nu_i}$, where m_H, m_He, and m_{ν_i} are the masses of the two participating nuclei and the neutrino, respectively. In contrast, the β decay of a tritium nucleus yielding the lightest neutrino mass eigenstate ν_Li, ${}^3\text{H} \to {}^3\text{He} + e^- + \overline{\nu_\text{Li}}$, will yield an electron with energy $E_e \leq (m_\text{H} - m_\text{He}) - m_{\nu_\text{Li}}$. To prove that relic neutrinos are being captured, an experiment must have sufficient energy resolution to establish that some of the electrons it observes have energies very slightly beyond the endpoint of the electron energy spectrum from β decay.

The relic neutrinos were highly relativistic when they were produced in the hot early Universe, and the SM interactions that produced them yielded the same number of particles with negative helicity as with positive helicity. The number of particles produced with each helicity did not depend on whether neutrinos are Dirac or Majorana particles, because, as described above, Dirac and Majorana neutrinos behave identically when they are relativistic. Of course, if neutrinos are Dirac particles, then the relics created with negative helicity were (and still are) *neutrinos*, whereas those created with positive helicity were *antineutrinos*. After decoupling, the neutrinos free-streamed, and as the Universe expanded and cooled, many (and possibly all) of them became nonrelativistic. Equality between the number of negative-helicity particles and positive-helicity ones was preserved during this evolution.

For either Dirac neutrinos (not antineutrinos) or Majorana neutrinos, the amplitude for the capture of relic mass eigenstate ν_i on tritium obeys

$$\text{amplitude}(\nu_i + {}^3\text{H} \to {}^3\text{He} + e^-) \propto \overline{u_e}\gamma^\lambda(1-\gamma_5)u_{\nu_i}J^\text{nucl}_\lambda. \qquad 37.$$

Here, u_e and u_{ν_i} are Dirac wave functions for the electron and the neutrino, respectively, and J_λ^{nucl} is a current describing the nuclear part of the process. The product $(1-\gamma_5)u_{\nu_i}$ leads to a factor

$$1 - 2h_{\nu_i}\sqrt{(E_{\nu_i} - m_{\nu_i})/(E_{\nu_i} + m_{\nu_i})} \equiv F(h_{\nu_i}, E_{\nu_i}) \tag{38.}$$

in the amplitude, where E_{ν_i} is the energy of the neutrino and h_{ν_i} is its helicity. Now, suppose the relic neutrinos were still highly relativistic, with $E_{\nu_i} \gg m_{\nu_i}$, in the rest frame of the tritium today. Then, if neutrinos are Majorana particles, capture of the half of the relic population with $h_{\nu_i} = +1/2$ would be extremely suppressed by $F(h_{\nu_i}, E_{\nu_i})$. [It can be shown that the details of J_λ^{nucl} do not affect this argument. However, if we view the Lorentz-invariant amplitude of Equation 37 from the rest frame of the neutrino, in which $F(h_{\nu_i}, E_{\nu_i}) = 1$, but in which the target nucleus is moving at high speed, the details of J_λ^{nucl} are all important, and lead to the same suppression of capture that we find when viewing the amplitude from the rest frame of the target (16).] If neutrinos are Dirac particles, the half of the relic population with $h_{\nu_i} = +1/2$ cannot be captured by tritium to make an electron, because the positive-helicity relics are antineutrinos, and lepton number is conserved. Capture of the half of the relic population with $h_{\nu_i} = -1/2$ would be described by the amplitude of Equation 37 in either the Dirac or Majorana case, and would not be suppressed. Thus, the event rate would have no visible dependence on whether neutrinos are Dirac or Majorana particles.

In reality, as we have discussed, many, and perhaps all, of the relic neutrinos have become nonrelativistic in the tritium/laboratory rest frame. For the nonrelativistic relics, $F(h_{\nu_i}, E_{\nu_i}) \cong 1$, causing little suppression and depending very little on the neutrino helicity. If neutrinos are Majorana particles, then the amplitude for capture of a neutrino with either positive or negative helicity is given by Equation 37, and with $F(h_{\nu_i}, E_{\nu_i}) \cong 1$ independent of the helicity, the relic populations with positive and negative helicity will contribute equally to the capture rate. If neutrinos are Dirac particles, then the amplitude for capture of a neutrino with negative helicity is again given by Equation 37 and is the same as in the Majorana case, but because of lepton number conservation, the amplitude for capture of a neutrino with positive helicity, which is an antineutrino, is zero. Thus, the total capture rate is twice as large in the Majorana case as in the Dirac case (24).

While this dependence of the capture rate on whether neutrinos are Dirac or Majorana particles is substantial, it must be acknowledged that actually using relic capture on tritium to determine whether neutrinos are of Dirac or Majorana character faces major challenges. First, the observation of this capture is very difficult, and has not yet been accomplished (for a description of the PTOLEMY experiment, which is being developed in an effort to detect capture of the relics on tritium, see Reference 25). Second, the capture rate obviously depends not only on the cross section for the process but also on the local density of relic neutrinos. Owing to gravitational clustering, this local density could be very different from the average density in the Universe as a whole, and is much less precisely predicted than the latter (26). Third, if the lightest neutrino mass eigenstate is light enough to be relativistic today, finite experimental energy resolution could well make it impossible to determine that an electron from its capture is not one from tritium β decay. Thus, its capture would not be counted. Now, $|U_{e1}|^2 \cong 2/3$, so if the mass ordering is normal such that the lightest mass eigenstate is ν_1, two-thirds of the captures would not be counted.

3.6. Angular Distributions in Decays

The search for nonrelativistic neutrinos whose behavior might be revealing leads us to consider neutrino decays, since of course a neutrino undergoing decay is totally nonrelativistic in its rest

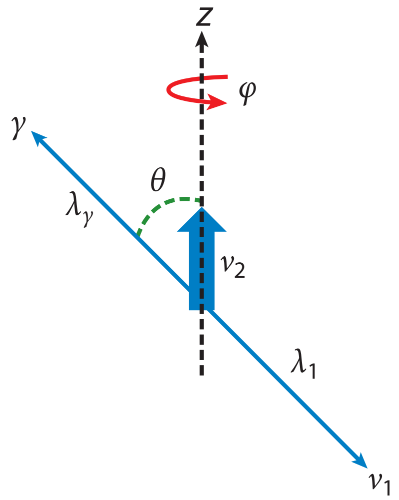

Figure 3

Decay configuration of the Majorana fermion.

frame. Let us first consider the decay of a Majorana neutrino $\nu_2^{(M)}$ into another Majorana neutrino $\nu_1^{(M)}$ and a photon:

$$\nu_2^{(M)} \rightarrow \nu_1^{(M)} + \gamma. \tag{39}$$

Angular momentum conservation implies that the amplitude of such a process in the helicity formalism is given by

$$D^{j*}_{m,\lambda}(\phi, \theta, -\phi) A_{\lambda_1,\lambda_\gamma}. \tag{40}$$

Here, D is the Wigner rotation function, j and m are the spin and third component of the spin along the z axis for the decaying particle $\nu_2^{(M)}$, λ_1 and λ_γ are the helicities of the decay products, and $\lambda = \lambda_\gamma - \lambda_1$. With no loss of generality, we assume that $\nu_2^{(M)}$ is polarized in the $+z$ direction and evaluate the decay amplitude in its rest frame (**Figure 3**). In this configuration, from angular momentum conservation it follows that

$$|\lambda_\gamma - \lambda_1| \leq j = 1/2. \tag{41}$$

There are two possible helicities for the photon, both of which contribute for Majorana fermions. For the case of $\lambda_\gamma = +1$, Equation 41 implies $\lambda_1 = +1/2$, and hence $\lambda = +1/2$. For $\lambda_\gamma = -1$, one has $\lambda_1 = -1/2$, and hence $\lambda = -1/2$. The tree-level amplitudes are then

$$\langle \gamma(\mathbf{p}, +1)\ \nu_1(-\mathbf{p}, +1/2)|H_{\mathrm{EM}}|\nu_2(0, +1/2)\rangle = d^{1/2}_{+1/2,+1/2} A_{+1,+1/2} \tag{42}$$

and

$$\langle \gamma(\mathbf{p}, -1)\ \nu_1(-\mathbf{p}, -1/2)|H_{\mathrm{EM}}|\nu_2(0, +1/2)\rangle = d^{1/2}_{+1/2,-1/2} A_{-1,-1/2} \tag{43}$$

up to phases not explicitly shown. Here, H_{EM} is the effective Hamiltonian for the decay and $d^j_{m\lambda}$ is the reduced Wigner rotation function. Imposing the condition of invariance under *CPT* transformation (27), described by the operator ζ, one can write

$$\begin{aligned} \langle \gamma(\mathbf{p}, \lambda_\gamma)\ \nu_1(-\mathbf{p}, \lambda_1)|H_{\mathrm{EM}}|\nu_2(0, +1/2)\rangle &= \langle \zeta H_{\mathrm{EM}} \zeta^{-1} \zeta[\nu_2(0, +1/2)]|\zeta[\gamma(\mathbf{p}, \lambda_\gamma)\ \nu_1(-\mathbf{p}, \lambda_1)]\rangle \\ &= \langle \gamma(\mathbf{p}, -\lambda_\gamma)\ \nu_1(-\mathbf{p}, -\lambda_1)|H_{\mathrm{EM}}|\nu_2(0, -1/2)\rangle^* \end{aligned} \tag{44}$$

up to phases not explicitly shown. Substituting the values $\lambda_\gamma = +1$ and $\lambda_1 = +1$ in the first and the third entries in Equation 44 and using Equation 40, we obtain

$$d^{1/2}_{+1/2,+1/2} A_{+1,+1/2} = d^{1/2}_{-1/2,-1/2} A^*_{-1,-1/2} \Rightarrow A_{+1,+1/2} = A^*_{-1,-1/2} \equiv A \quad 45.$$

up to phases not shown. The first-order decay rate into photons with helicity $\lambda_\gamma = +1$ can be read from Equation 42 to be

$$\frac{\mathrm{d}\Gamma_+}{\mathrm{d}\cos\theta} = \left(d^{1/2}_{+1/2,+1/2}\right)^2 |A_{+1,+1/2}|^2 = \cos^2\frac{\theta}{2}|A_{+1,+1/2}|^2. \quad 46.$$

Similarly the decay rate into photons with helicity $\lambda_\gamma = -1$ from Equation 43 is

$$\frac{\mathrm{d}\Gamma_-}{\mathrm{d}\cos\theta} = \left(d^{1/2}_{+1/2,-1/2}\right)^2 |A_{-1,-1/2}|^2 = \sin^2\frac{\theta}{2}|A_{-1,-1/2}|^2. \quad 47.$$

Summing over final helicities, we find the total decay rate for a spin-up Majorana fermion to be

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\cos\theta} = \cos^2\frac{\theta}{2}|A_{+1,+1/2}|^2 + \sin^2\frac{\theta}{2}|A_{-1,-1/2}|^2, \quad 48.$$

which, using Equation 45, takes the form

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\cos\theta} = |A|^2. \quad 49.$$

In other words, the distribution of photons is isotropic. Note that this isotropy is a consequence of angular momentum conservation and *CPT* invariance alone.[1] It does not depend on any further details of the interactions involved in the decay.

In contrast to the Majorana case, if neutrinos are Dirac particles one finds by explicit calculation that the radiative decay $\nu_2^{(D)} \rightarrow \nu_1^{(D)} + \gamma$ of polarized neutrinos $\nu_2^{(D)}$ will in general yield a nonisotropic distribution of photons if the decay is driven by both magnetic and electric transition dipole moments. Thus, the angular distribution of photons from radiative neutrino decay can in principle be used to determine whether neutrinos are Dirac or Majorana particles. (This possibility was noted in Reference 28. We emphasize that it depends not on the details of the authors' calculations but rather on nothing more than rotational and *CPT* invariance.) Should it be less challenging to measure the polarization of these photons than their angular distribution, the polarization can also be used for this purpose. If neutrinos are Majorana particles, their helicity will be $\cos\theta$, which, given their isotropic angular distribution, will result in an angle-integrated helicity of zero (28). If they are Dirac particles, their angle-integrated helicity will in general not be zero.

What if there exists an as-yet-undiscovered neutrino N that is much heavier than the three known ones? (The CMS experiment at CERN is hunting for such a heavy neutrino, as reported in Reference 29. For discussion of other ways to search for such a neutrino, and of its possible physical consequences, see, e.g., Reference 30.) In this case there would be new and potentially quite revealing decay modes. Among these are decays of the form $N \rightarrow \nu + X$, where ν is one of the three known light neutrino mass eigenstates, and $X = \overline{X}$ is a particle that is identical to its antiparticle. Depending on the mass of N, X can be, for example, a γ, π^0, ρ^0, Z^0, or H^0. What these modes could teach us has been analyzed (A.B. Balantekin, A. de Gouvêa & B. Kayser, manuscript in preparation). For each of them, if the neutrinos, including N, are Majorana particles, the decay rate is twice as large as it is if the neutrinos are Dirac particles (31). However, the decay rate for any given mode also depends on unknown mixing angles, so a measurement of the rate may not

[1]We thank S. Petcov for long-ago discussions of this point.

Table 1 **The coefficient α in the angular distribution $(1+\alpha\cos\theta)$ of the particle X from the decay $N^{(D)} \rightarrow \nu^{(D)} + X$ of a heavy Dirac neutrino $N^{(D)}$ fully polarized with its spin in the +z direction**[a]

X	γ	π^0	ρ^0	Z^0	H^0
α	$[2\Im m(\mu d^*)]/(\lvert\mu\rvert^2+\lvert d\rvert^2)$	1	$(m_N^2-2m_\rho^2)/(m_N^2+2m_\rho^2)$	$(m_N^2-2m_Z^2)/(m_N^2+2m_Z^2)$	1

[a]The quantities μ and d are, respectively, the magnetic and electric transition dipole moments that drive $N^{(D)} \rightarrow \nu^{(D)} + \gamma$, and m_N, m_ρ, and m_Z are, respectively, the masses of $N^{(D)}$, ρ^0, and Z^0.

reveal whether neutrinos are of Dirac or Majorana character. Therefore, it is quite interesting that the angular distribution of the outgoing particles in these decays is another feature that depends on whether neutrinos are Dirac or Majorana particles. Furthermore, except when $X = \gamma$, this dependence does not involve elusive parameters.

Let us assume that the reaction producing the heavy neutrino N in some experiment leaves it fully polarized with its spin in the $+z$ direction in its rest frame. Let us also assume, first, that N is a Majorana particle $N^{(M)}$ and ν is a Majorana particle $\nu^{(M)}$ (and that $X = \overline{X}$ is a self-conjugate particle as well). Then, through a generalization of the analysis given above for $\nu_2^{(M)} \rightarrow \nu_1^{(M)} + \gamma$, we can show that, purely as a result of angular momentum conservation and *CPT* invariance, the angular distribution of X particles from the decay $N^{(M)} \rightarrow \nu^{(M)} + X$ will be isotropic in the $N^{(M)}$ rest frame. Next, let us assume instead that N is a Dirac particle $N^{(D)}$, ν is a Dirac particle $\nu^{(D)}$, and X is the same self-conjugate particle as before. Then the angular distribution of X particles from $N^{(D)} \rightarrow \nu^{(D)} + X$ in the $N^{(D)}$ rest frame will be

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}(\cos\theta)} = \Gamma_0(1+\alpha\cos\theta), \qquad 50.$$

where Γ_0 is a normalization constant, and α, the asymmetry parameter, is given for each X considered (A.B. Balantekin, A. de Gouvêa & B. Kayser, manuscript in preparation) in **Table 1**. In the calculations summarized by this table, it has been assumed that when $X = \pi^0$ or ρ^0, the decay is dominated by a virtual Z^0 that is emitted by the lepton line and becomes the X, and that the coupling of the H^0 to the lepton line is a Yukawa coupling. In the corresponding antineutrino decays, $\overline{N^{(D)}} \rightarrow \overline{\nu^{(D)}} + X$, the angular distribution is the same except for a reversal of the sign of α.

As **Table 1** shows, except under very special circumstances, such as $m_N^2 = 2m_\rho^2$, the angular distribution in the Dirac case is not isotropic. When X is a ρ^0 or a Z^0, the only presently unknown parameter in this distribution is the mass of N, which would likely be measured once N is discovered. When X is a π^0 or an H^0, the angular distribution does not depend on any parameters at all. Thus, the study of angular distributions in the decays of a heavy neutrino could tell us whether neutrinos are Dirac or Majorana particles.

Imagine, finally, that a heavy neutrino N is created together with an e^+ in the decay of some particle that is not a lepton. If it is found that this same N can undergo the decay $N \rightarrow e^+ + \pi^-$, then lepton number conservation is obviously violated and this N must be a Majorana particle. Needless to say, such chains of events are well worth searching for.

4. ELECTROMAGNETIC STRUCTURE OF NEUTRINOS

Having considered whether neutrinos are of Majorana or Dirac character, we now turn to their electromagnetic structure. As discussed below, this structure is not independent of their Majorana versus Dirac nature.

The most general matrix element of the electromagnetic current $J_\mu^{\rm EM}$ between neutrino mass eigenstates ν_i and ν_j is given by

$$\left\langle \nu_j(p_j)|J_\mu^{\rm EM}|\nu_i(p_i)\right\rangle = \bar{u}_j(p_j) \times \left\{\left(\gamma_\mu - q_\mu \frac{\not{q}}{q^2}\right)[f_Q^{ji}(q^2) + f_A^{ji}(q^2)q^2\gamma_5] - i\sigma_{\mu\nu}q^\nu[f_M^{ji}(q^2) + i f_E^{ji}(q^2)\gamma_5]\right\} u_i(p_i). \quad 51.$$

Here, $q = p_i - p_j$ is the momentum transfer and the various factors f are Hermitian matrices of form factors. In particular, the matrices f_Q, f_M, f_E, and f_A contain the charge, magnetic dipole, electric dipole, and anapole form factors, respectively. For the coupling to a real photon ($q^2 = 0$), f_M and f_E reduce to transition (if $j \neq i$) or intrinsic (if $j = i$) magnetic and electric dipole moments, respectively (32, 33).

Excellent comprehensive reviews of neutrino electromagnetic structure are available in the literature (33, 34). Therefore, in this section we limit our discussion to those aspects pertinent to neutrino magnetic moments.

In the minimally extended SM, which includes neutrino masses and mixing, the neutrino magnetic moment is very small (35, 36). For Dirac neutrinos, the magnetic moment matrix in the mass basis is given by

$$\mu_{ij} = -\frac{1}{\sqrt{2}}\frac{eG_{\rm F}}{8\pi^2}\left(m_i + m_j\right)\sum_\ell U_{\ell i}U_{\ell j}^* f(r_\ell), \quad 52.$$

where

$$f(r_\ell) \sim -\frac{3}{2} + \frac{3}{4}r_\ell + \cdots, \qquad r_\ell = \left(\frac{m_\ell}{M_W}\right)^2, \quad 53.$$

and m_ℓ is the mass of the charged lepton ℓ. Since three neutrino mixing angles and the upper bounds on the neutrino mass are known, one can calculate this SM prediction as a function of the unknown neutrino masses m_i and m_j and demonstrate that it is well below experimental reach (37; triangle inequalities that relate the squares of the effective dipole moments of ν_e ν_μ, and ν_τ when neutrinos are Majorana particles and there are no sterile neutrinos have been pointed out in Reference 38).

4.1. Neutrino–Electron Scattering

The differential scattering cross section for ν_e and $\overline{\nu}_e$ on electrons is given by (e.g., 39)

$$\frac{{\rm d}\sigma}{{\rm d}T_e} = \frac{G_{\rm F}^2 m_e}{2\pi}\left[(g_V + g_A)^2 + (g_V - g_A)^2\left(1 - \frac{T_e}{E_\nu}\right)^2 + (g_A^2 - g_V^2)\frac{m_e T_e}{E_\nu^2}\right] + \frac{\pi\alpha^2\mu_\nu^2}{m_e^2}\left[\frac{1}{T_e} - \frac{1}{E_\nu}\right], \quad 54.$$

where T_e is the electron recoil kinetic energy, $g_V = 2\sin^2\theta_W + 1/2$, $g_A = +1/2$ $(-1/2)$ for ν_e ($\overline{\nu}_e$), and the neutrino magnetic moment is expressed in units of Bohr magneton, $\mu_{\rm B}$. The first line in Equation 54 is the weak and the second line is the magnetic moment contribution.[2] Experiments searching for neutrino dipole moments utilize the fact that the magnetic moment cross section

[2]The contribution of the interference of the weak and magnetic amplitudes to the cross section is proportional to the neutrino mass and can be ignored for ultrarelativistic neutrinos.

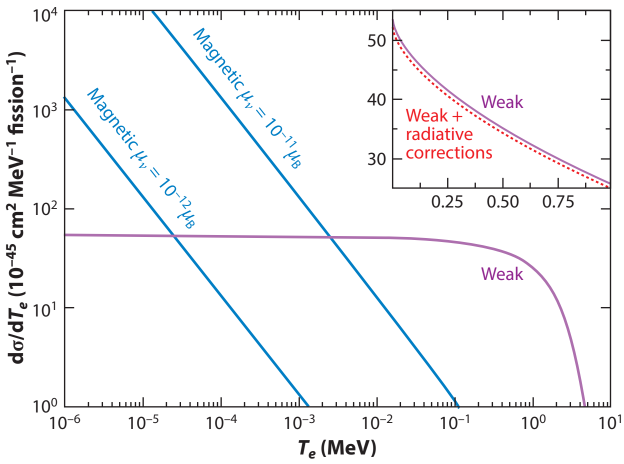

Figure 4

A comparison of the tree-level weak, magnetic, and radiative corrections to the tree-level weak contribution to the differential cross section for electron antineutrino–electron scattering. The axis labels of the inset are the same as the axis labels of the larger figure. Adapted from Reference 37.

exceeds the weak cross section for recoil energies

$$\frac{T_e}{m_e} < \frac{\pi^2\alpha^2}{(G_F m_e^2)^2}\mu_\nu^2. \quad 55.$$

That is, the lower the smallest measurable recoil energy is, the smaller the values of the magnetic moment that can be probed. A reactor experiment measuring the antineutrino magnetic moment by detecting the electron recoil is an inclusive one; in other words, it sums over all the neutrino final states. Note that, because neutrinos oscillate between their source and the detector, the value of the μ_ν of Equation 54 measured at a distance L from the neutrino source is an effective value:

$$\mu_{\text{eff}}^2 = \sum_i \left| \sum_i U_{ej}\mu_{ij} \exp(-iE_j L) \right|^2, \quad 56.$$

where i and j are mass indices, μ_{ij} is the dipole moment matrix in the mass basis, and U_{ej} are elements of the neutrino mixing matrix. Currently the best reactor neutrino limit is given by the GEMMA spectrometer at Kalinin Nuclear Power Plant to be $\mu_\nu < 2.9 \times 10^{-11}\mu_B$ (40).

There are small radiative loop corrections to the tree diagrams calculated in Equation 54 (41). **Figure 4** compares the tree-level weak, magnetic, and radiative corrections with the tree-level weak contribution to the cross section (37).

4.2. Effects of Magnetic Moments in Neutrino Propagation

The precession of neutrino spins in magnetic fields that is induced by magnetic moments has been studied (36). The rotation

$$\nu_{eL} \rightarrow \nu_{eR} \quad 57.$$

produces a right-handed neutrino when magnetic fields transverse to the direction of neutrino propagation are present (42, 43). It was subsequently realized that matter effects would break the vacuum degeneracy of the ν_{eL} and ν_{eR} states, suppressing the spin precession shown above.

However, it was pointed out (44, 45) that this difficulty was naturally circumvented for the process

$$\nu_{e\mathrm{L}} \rightarrow \nu_{\mu\mathrm{R}} \tag{58.}$$

as different matter interactions of the ν_e and ν_μ can compensate for the vacuum $\nu_e - \nu_\mu$ mass difference, producing a crossing similar to the usual MSW mechanism. Such spin-flavor precession can then occur at full strength due to off-diagonal magnetic moments with flavor indices. Note that only the product of the transverse magnetic field and neutrino magnetic moments appears in the equations describing spin-flavor precession. Spin-flavor precession in the Sun has been studied in detail (46), motivated by the Homestake solar neutrino data that suggested an anticorrelation between solar neutrino capture rate and the number of sunspots, a proxy of the magnetic activity in the Sun. Although this correlation was weakened by further observations, it was realized that spin-flavor precession would produce solar antineutrinos if the neutrinos are of Majorana type (47). Solar neutrino experiments searching for such antineutrinos report null results (48, 49). The physics of the Sun does not seem to be affected by neutrino magnetic moments (50), but can be used to place limits on the effective neutrino magnetic moments, yielding $\mu_{\mathrm{eff}} < 2.8 \times 10^{-11} \mu_{\mathrm{B}}$ (49, 51). In contrast, neutrino magnetic moments may play a role in the evolution of massive stars (52).

A good fraction of the heavier nuclei were formed in the rapid neutron capture (r-process) nucleosynthesis scenario. One expects the astrophysical sites of the r-process to be associated with explosive phenomena, since a large number of interactions are required to take place during a rather short time interval. Leading candidates include neutron star mergers and core-collapse supernovae. r-Process element abundances observed in an ultrafaint dwarf (i.e., very old) galaxy were several orders of magnitude greater than has been seen in other such galaxies, implying that a single rare event produced the r-process material (53), an argument in favor of neutron star mergers. A signature of nucleosynthesis in neutron star mergers would be the electromagnetic transients from the decay of radioactive isotopes they would produce (54). The LIGO and Virgo Collaborations (55) reported observation of gravitational waves from a binary neutron star merger. Multimessenger observations of this binary neutron star merger established the presence of an electromagnetic counterpart (56), further supporting neutron star mergers as a site, but not ruling out core-collapse supernovae as another possible secondary site. The salient point for our subject is that both of these sites contain copious quantities of neutrinos. A key quantity for determining the r-process yields is the neutron-to-seed nucleus ratio, which, in turn, is determined by the neutron-to-proton ratio. The neutrino-induced processes such as $\nu_e + n \rightarrow p + e^-$ could significantly alter the neutron-to-proton ratio. During the epoch of α particle formation, almost all the protons and an equal number of neutrons combine into α particles, which have a large binding energy. This "α effect" reduces the number of free neutrons participating in the r-process (57, 58). Naïvely, one may assume that the presence of a neutrino magnetic moment would reduce the electron neutrino flux, resulting in a possible mechanism to suppress α particle formation. This expectation is not realized, since a nonzero magnetic moment suppresses the electron neutrino and antineutrino fluxes at the same time (59). Large values of neutrino magnetic moments would increase the electron fraction and thus amplify the α effect.

Resonant neutrino spin-flavor precession in supernovae and its impact on nucleosynthesis have also been studied (60). Perhaps a more dominant effect in both neutron star mergers and core-collapse supernovae is collective oscillations of neutrinos. These are emergent nonlinear flavor-evolution phenomena instigated by neutrino–neutrino interactions in astrophysical environments with sufficiently high neutrino densities. There may be nonnegligible effects of transition magnetic moments on three-flavor collective oscillations of Majorana neutrinos in core-collapse supernovae (61, 62). Furthermore, the effects of neutrino dipole moments in collective oscillations are intertwined with the CP-violating phases of the neutrino mixing matrix (63).

4.3. Other Astrophysical and Cosmological Consequences

Studies of the red giant cooling process of plasmon decay into neutrinos,

$$\gamma^* \rightarrow \nu_i \bar{\nu}_j, \quad 59.$$

impose limits on the neutrino dipole moments. A large enough neutrino magnetic moment would imply an enhanced plasmon decay rate. Since neutrinos freely escape the star, large neutrino dipole moments cool the red giant star faster, delaying helium ignition. The most recent such limit is $|\mu_\nu| < 4.5 \times 10^{-12} \mu_{\rm B}$ (64).

Limits on the magnetic moments of Dirac neutrinos were obtained in the 1980s by use of cosmological arguments. Since magnetic moments contribute to neutrino electron scattering and electron–positron annihilation into neutrino pairs, high values of the magnetic moments would populate wrong-helicity counterparts, leading to an increase in $N_{\rm eff}$. These arguments limit the dipole moments of Dirac neutrinos to be $\mu_\nu < 10^{-10} \mu_{\rm B}$ (65, 66). However, such a consideration is restricted to Dirac neutrinos since Majorana neutrinos do not have additional neutrino states that can get populated by dipole moment–induced transitions.

It is still possible to explore the impact of Majorana neutrino magnetic moments in the early Universe. Since the energy dependences of the weak and magnetic components of electron–neutrino scattering (see Equation 54) are very different, they can have significantly different contributions to the reaction rates in the early Universe. Thus, a sufficiently large magnetic moment can keep the Majorana neutrinos in thermal and chemical equilibrium below the standard (∼1-MeV) weak decoupling temperature regime and into the Big Bang nucleosynthesis (BBN) epoch. The production of light elements in the BBN epoch is very sensitive to the weak decoupling temperature, since the neutron-to-proton ratio is exponentially dependent on it. This high sensitivity can be exploited to obtain a limit on the effective neutrino magnetic moment through constraints on the observed primordial abundances, such as those of helium and deuterium. It follows that light-element abundances and other cosmological parameters are sensitive to magnetic couplings of Majorana neutrinos on the order of $10^{-10}\ \mu_{\rm B}$ (67).

4.4. Neutrino Decay in Astrophysics

So far, no evidence of neutrino decay has been observed in terrestrial experiments. However, an unidentified emission line was seen in the X-ray spectrum of galaxy clusters: a monochromatic, 3.5-keV line in the X-ray spectrum that could be interpreted as a signal emerging from a decaying 7-keV sterile neutrino that mixes with active ones (68, 69). Such a neutrino can be resonantly produced in the early Universe and constitute dark matter (70). If the sterile neutrino interpretation is indeed correct, the observed X-ray line would imply the presence of new entries in the neutrino dipole moment and mixing matrices. Sterile neutrino dark matter is expected to be in the range suggested by these observations (71) and may have been produced in the early Universe (72). Several planned missions dedicated to the search for X-ray lines from dark matter should elucidate the sterile neutrino decay interpretation of the 3.5-keV line.

4.5. Magnetic Moments of Dirac and Majorana Neutrinos

The neutrino mass and neutrino magnetic moment are not completely independent of one another. For example, one can write a generic expression for the magnetic moment as

$$\mu_\nu \sim \frac{e\mathcal{G}}{\Lambda}, \quad 60.$$

where Λ is the energy scale of the physics beyond the SM generating the magnetic moment at low energies, e is the charge on the electron, and $\mathcal{G}$ represents calculations of the appropriate diagrams connected to a photon. If this external photon is removed, the same set of diagrams contribute a neutrino mass of order

$$\delta m_\nu \sim \mathcal{G}\Lambda, \tag{61.}$$

implying the relationship

$$\delta m_\nu \sim \left(\frac{\mu_\nu}{\mu_{\rm B}}\right)\frac{\Lambda^2}{2m_e}. \tag{62.}$$

However, such a relationship can be circumvented using symmetry arguments, for example, by imposing a new symmetry that would force the neutrino mass to vanish (73).

A more robust connection can be obtained using effective field theory techniques. At lower energies, by integrating out the physics above the scale Λ one can write an effective Lagrangian consisting of local operators written in terms of the SM fields:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum_{n=4}^{\mathcal{N}} \frac{1}{\Lambda^{n-4}} \sum_{j_n} C_{j_n}^{(n)}(\upsilon)\mathcal{O}_{j_n}^{(n)}(\upsilon), \tag{63.}$$

where n is the operator dimension, $\mathcal{N}$ specifies the number of the terms kept, j_n labels all the independent operators of dimension n, and υ is the renormalization scale used. As described above, to obtain a mass term for Dirac neutrinos one introduces a SM singlet field $\nu_{\rm R}$ and writes a mass term of dimension four in a similar way to charged leptons. Using this additional SM field, one can write three independent dimension-six operators:

$$\begin{aligned}
\mathcal{O}_1^{(6)} &= g\bar{L}\tau^a\bar{\epsilon}H^*\sigma^{\mu\nu}\nu_{\rm R}\left(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g\epsilon_{abc}W_\mu^b W_\nu^c\right),\\
\mathcal{O}_2^{(6)} &= g'\bar{L}\bar{\epsilon}H^*\sigma^{\mu\nu}\nu_{\rm R}\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right),\\
\mathcal{O}_3^{(6)} &= \bar{L}\bar{\epsilon}H^*\nu_{\rm R}\left(H^\dagger H\right),
\end{aligned} \tag{64.}$$

where L is the SM left-handed lepton isodoublet, W and B are the $SU(2)_L$ and $U(1)_Y$ gauge fields of the SM, and $\bar{\epsilon} = -i\tau_2$. Noting that $g = e/\sin\theta_W$ and $g' = e/\cos\theta_W$, one observes that, after the spontaneous symmetry breaking, the operators $\mathcal{O}_1^{(6)}$ and $\mathcal{O}_2^{(6)}$ would generate a contribution to the magnetic dipole moment and the operator $\mathcal{O}_3^{(6)}$ would generate a contribution to the neutrino mass. The appropriate renormalization group analysis has been performed (74). Neglecting possible fine-tunings of the coefficients $C_j^{(6)}$, this analysis found that a magnetic moment will rather generically induce a radiative correction to the Dirac neutrino mass of the order of

$$\delta m_\nu \sim \left(\frac{\mu_\nu}{10^{-15}\mu_{\rm B}}\right)[\Lambda\,({\rm TeV})]^2\ {\rm eV}. \tag{65.}$$

This bound was derived for a single flavor. For a hierarchical neutrino mass spectrum, it would be even more stringent.

For Majorana neutrinos the analysis needs to be quite different. One does not introduce a new SM field $\nu_{\rm R}$; instead, the Majorana mass term is given as a unique dimension-five operator. Neutrino magnetic moments and corrections to the neutrino mass then come from dimension-seven operators. A magnetic moment is generated by the operators

$$\begin{aligned}
\mathcal{O}_1^{(7)} &= g(\overline{L^c}\bar{\epsilon}H)\sigma^{\mu\nu}(H^T\bar{\epsilon}\tau_a L)\left(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g\epsilon_{abc}W_\mu^b W_\nu^c\right),\\
\mathcal{O}_2^{(7)} &= g'(\overline{L^c}\bar{\epsilon}H)\sigma^{\mu\nu}(H^T\bar{\epsilon}L)\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right),
\end{aligned} \tag{66.}$$

and a correction to the neutrino mass would be generated by the operator

$$\mathcal{O}_3^{(7)} = (\overline{L^c}\bar{\epsilon}H)(H^T\bar{\epsilon}L)(H^\dagger H). \tag{67.}$$

However, for Majorana neutrinos there is an even more stringent constraint imposed by the flavor symmetries of such neutrinos. Specifically, Majorana neutrinos cannot have diagonal magnetic moments; only transition moments, in either the flavor or the mass basis, are possible. Therefore, the magnetic moment matrix in the flavor space is required to be antisymmetric in flavor indices, even though the mass matrix is symmetric. This feature significantly weakens the constraints on the Majorana neutrino magnetic moments in comparison to the Dirac case. One-loop mixing of the mass and magnetic moment operators leads to rather weak constraints on the Majorana magnetic moment due to the suppression by charged-lepton masses (75). Two-loop matching of the mass and magnetic field operators further reinforces this result (76). The most general bound given in Reference 76 is

$$|\mu_{\alpha\beta}| \leq 4 \times 10^{-9} \mu_{\mathrm{B}} \left(\frac{[m_\nu]_{\alpha\beta}}{1\ \mathrm{eV}} \right) \left(\frac{1\ \mathrm{TeV}}{\Lambda} \right)^2 \frac{m_\tau^2}{|m_\alpha^2 - m_\beta^2|}. \qquad 68.$$

This argument suggests that if the value of the neutrino magnetic moment is measured to be just below the present experimental and observational limits, then neutrinos are very likely Majorana fermions.

5. CONCLUSIONS

Neutrinos are unique among all the elementary fermions of the SM, in that they carry no electric charge. This feature makes it possible for them to possess Majorana masses. There are very interesting consequences of this possibility. In this article, we have reviewed the current status of our knowledge of neutrino properties, then explored theoretical motivations of experiments that can identify whether neutrinos are Dirac or Majorana fermions. This experimental task is not easy, since all the neutrinos that are directly observed are ultrarelativistic. When they are ultrarelativistic, Dirac and Majorana neutrinos behave in exactly the same way. Nevertheless, there are a handful of possibilities, which we have described in some detail, ranging from $0\nu\beta\beta$ decay to the angular distribution of the decay products of heavy neutrinos. Numerous experiments exploring the neutrino properties are in progress or at the planning stage. The much-anticipated answer to the question of Dirac versus Majorana nature may not be too far away.

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