Split-Crank Cadence Tracking for Switched Motorized FES-Cycling with Volitional Pedaling

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***Abstract*—A wide variation in muscle strength and asymmetry exists in people with movement disorders. Functional electrical stimulation (FES) can be used to induce muscle contractions to assist and a motor can be used to both assist and resist a person's volitional and/or FES-induced pedaling. On a traditional cycle with coupled pedals, people with neuromuscular asymmetries can primarily use their dominant (i.e., stronger) side to successfully pedal at a desired cadence, neglecting the side that would benefit most from rehabilitation. In this paper, a multi-level switched system is applied to a two-sided control objective to maintain a desired range of cadence using FES, an electric motor, and volitional pedaling. The non-dominant leg tracks the cadence range while the dominant leg tracks the position (offset by 180 degrees) and cadence of the first leg. Assistive, uncontrolled, and resistive modes are developed based on cadence and position for the non-dominant and dominant legs, respectively. Lyapunov-based methods for switched systems are used to prove global exponential tracking to the desired cadence range for the combined FES-motor control system.**

I. Introduction

Various neuromuscular conditions that result in movement disorders are known to limit a person's lower limb function, and oftentimes affect one side of the body more than the other, resulting in hemiparesis. FES-cycling has been shown to be a successful rehabilitation tool for improving many rehabilitation outcome measures [1]–[5]. However, people with hemiparesis can mask their asymmetry when pedaling with mechanically coupled pedals on a traditional single-crank cycle. The work in [6] concluded that future efforts in cycling for stroke rehabilitation should promote equal contribution from the dominant and non-dominant legs. While FES-cycling on a single crank has been shown to improve coordination between contralateral legs post-stroke [7], a split-crank cycle is proposed in this paper to enforce symmetrical contribution in people with movement disorders resulting in asymmetric function. Previous rehabilitation studies have used a split-crank cycle [8]–[11]; however, only [11] used closed-loop control. Furthermore, none of the aforementioned studies used FES to control the muscles. The work in [12] focuses on balancing torques generated from each leg on a single-crank FES-cycle, such that the dominant leg is tasked with only producing as much force as that acheivable by the non-dominant leg, rather than maximizing overall torque, which could result in heavy reliance on the stronger, dominant leg. While the control objective in this paper is to track a desired cadence rather than force, there is similar motivation to balance contribution. Here the dominant (i.e., more capable) side will only be tasked with pedaling as fast as the non-dominant side, even if the non-dominant side does not meet the cadence goal. However, the subsequently developed protocol remains applicable to people with no asymmetry.

Among different populations that participate in physical therapy, there is a large variation of strength and abilities, providing motivation for an exercise protocol that accommodates each user by assisting when they do not meet minimum performance metrics and resisting when the person exceeds a desired target range. Moreover, most people with movement disorders have some level of muscle control, and should be encouraged to contribute volitionally. Motivated to develop globally effective rehabilitation tools, [13] established a novel closed-loop state dependent switched system strategy that automatically transitions between assistive, uncontrolled, and resistive modes according to cadence bounds, and also encouraged volitional pedaling. To benefit a broad range of strength, ability, and hemiparesis, in this paper, a similar strategy for volitional contribution during FES-cycling is implemented on a split-crank cycle. As such, the non-dominant side will be resisted by the electric motor when pedaling above the desired cadence region and assisted via a combination of FES and the electric motor when below. Similarly, the dominant leg will be appropriately assisted or resisted when its position is outside a desired range centered at 180 degrees out of phase from the non-dominant leg.

Figure 1 depicts two levels of arbitrary and state-dependent switching that occurs between and within the three modes as the continuous state-dynamics evolve. Switching amongst the three modes is considered the high-level switching and is based on cadence on the non-dominant side and based on position on the dominant side. Low-level position-dependent switching within the assistive mode of each side will occur between the quadriceps, gluteal, and hamstring muscle groups, and the electric motor. As in [14], each muscle group is only activated when it is efficient to produce positive torque (i.e., FES regions) and the electric motor is activated elsewhere (i.e., motor regions). Motivated by [15], a comfort threshold is set on the stimulation input and a motor is activated only when the FES threshold is reached, so that full control authority can be maintained. Similarly, in the

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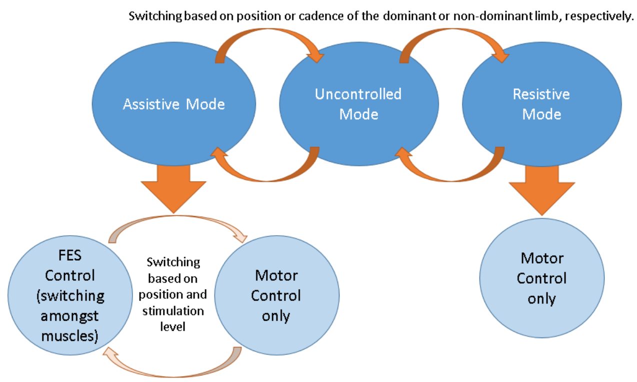

Figure 1. Diagram illustrating the combined switched system.

current work, arbitrary switching will activate the motor when the FES threshold is reached in the FES regions of the assistive mode, in addition to the cadence-based switching that activates the motor in the resistive mode and the motor regions of the assistive mode.

In this paper, a switched controller is developed for the FES and the motor on both sides of the cycle. A separate analysis is presented for each side of the cycle-rider system. Since arbitrary switching between stable subsystems can result in overall system instability [16], a Lyapunov-based analysis for switched systems involving a common Lyapunov function candidate with a set valued generalized derivative is used to prove global exponential stability of the controllers operating in the assistive and resistive modes. The trajectories in the uncontrolled mode are bounded above and below by the controlled subsystems.

II. Model

Switched dynamics are considered separately for both sides of the cycle-rider system as[1]

$$\tau_{e_l}(t) \triangleq \tau_{c_l}(\dot{q}_l, \ddot{q}_l, t) + \tau_{r_l}(q_l, \dot{q}_l, \ddot{q}_l, t), \quad (1)$$

$\forall l \in S \triangleq \{1, 2\}$, which indicates the non-dominant ($l = 1$) and dominant ($l = 2$) side, respectively. The measureable crank angle is denoted by $q_l : \mathbb{R}_{\geq 0} \to \mathcal{Q}$, where $\mathcal{Q} \subseteq \mathbb{R}$ is the set of all possible crank angles. The torques applied about the crank axis by the cycle and the rider are denoted by $\tau_{c_l} : \mathbb{R}^2 \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ and $\tau_{r_l} : \mathcal{Q} \times \mathbb{R}^2 \times \mathbb{R}_{\geq 0} \to \mathbb{R}$, respectively. The torque applied about the crank axis by the electric motor is denoted by $\tau_{e_l} : \mathbb{R}_{\geq 0} \to \mathbb{R}$ and can be expressed as

$$\tau_{e_l}(t) \triangleq B_{e_l} u_{e_l}(t), \quad (2)$$

$\forall l \in S$, where the motor control constant, $B_{e_l} \in \mathbb{R}_{>0}$, relates the motor's input current to output torque, and $u_{e_l} : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is the subsequently designed motor control current input. The cycle and rider torques, τ_{c_l} and τ_{r_l}, are defined as

$$\tau_{c_l}(q_l, \dot{q}_l, \ddot{q}_l, t) \triangleq J_{c_l}(q_l)\ddot{q}_l + b_{c_l}\dot{q}_l + d_{c_l}(t), \quad (3)$$

$$\begin{aligned}\tau_{r_l}(q_l, \dot{q}_l, \ddot{q}_l, t) \triangleq\; & \tau_{p_l}(q_l, \dot{q}_l, \ddot{q}_l) \\ & -\tau_{M_l}(q_l, \dot{q}_l, t) + d_{r_l}(t),\end{aligned} \quad (4)$$

$\forall l \in S$, respectively, where $J_{c_l} : \mathcal{Q} \to \mathbb{R}$, $b_{c_l} \in \mathbb{R}_{>0}$, and $d_{c_l} : \mathbb{R}_{\geq 0} \to \mathbb{R}$, denote inertial effects, viscous damping effects, and disturbances applied by the cycle, respectively. The sum of torques produced by muscle, by both FES and volitional contribution, is denoted by $\tau_{M_l} : \mathcal{Q} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ and the disturbances from the rider (e.g., spasticity or changes in load) are denoted by $d_{r_l} : \mathbb{R}_{\geq 0} \to \mathbb{R}$. Passive torques applied by the rider's legs are denoted by $\tau_{p_l} : \mathcal{Q} \times \mathbb{R}^2 \to \mathbb{R}$ and can be further expanded as

$$\begin{aligned}\tau_{p_l}(q_l, \dot{q}_l, \ddot{q}_l) \triangleq\; & M_{p_l}(q_l)\ddot{q}_l + V_l(q_l, \dot{q}_l)\dot{q}_l \\ & + G_l(q_l) + P_l(q_l, \dot{q}_l),\end{aligned} \quad (5)$$

$\forall l \in S$, where $M_{p_l} : \mathcal{Q} \to \mathbb{R}_{>0}$, $V_l : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$, $G_l : \mathcal{Q} \to \mathbb{R}$, and $P_l : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$ denote the inertial, centripetal-Coriolis, gravitational, and passive viscoelastic tissue forces, respectively. The cumulative torque applied by the muscles can be separated into volitional contribution and the sum of each muscle's individual torque production due to FES-induced contractions, written as

$$\tau_{M_l}(q_l, \dot{q}_l, t) \triangleq \sum_{m \in \mathcal{M}} B_m(q_l, \dot{q}_l)\, u_{m_l}(t) + \tau_{vol_l}(t), \quad (6)$$

$\forall l \in S, \forall m \in \mathcal{M}$, where $u_{m_l} : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is the subsequently designed FES control input (i.e., stimulation intensity), and the subscript $m \in \mathcal{M} = \{Q, G, H\}$ indicates the quadriceps femoris (Q), gluteal (G), and hamstring (H) muscle groups, respectively. The rider's volitional torque is denoted by $\tau_{vol_l} : \mathbb{R}_{\geq 0} \to \mathbb{R}$. The uncertain muscle control effectiveness is denoted by $B_m : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}_{>0}$, as in [14], [17].

High-level switching occurs between assistive, uncontrolled, and resistive modes according to subsequently designed switching signals. On the non-dominant side, the high-level switching laws are based on velocity, such that

$$\sigma_{a_1} \triangleq \begin{cases} 1 & \text{if} \quad \dot{q}_1 \leq \dot{q}_{\underline{d}1} \\ 0 & \text{if} \quad \dot{q}_1 > \dot{q}_{\underline{d}1} \end{cases}, \quad \sigma_{r_1} \triangleq \begin{cases} 1 & \text{if} \quad \dot{q}_1 \geq \dot{q}_{\overline{d}1} \\ 0 & \text{if} \quad \dot{q}_1 < \dot{q}_{\overline{d}1} \end{cases}, \quad (7)$$

where the switching signals $\sigma_{a_1} : \mathbb{R} \to \{0, 1\}$ and $\sigma_{r_1} : \mathbb{R} \to \{0, 1\}$ determine when the non-dominant side is in the assistive and resistive modes, respectively, and $\dot{q}_{\underline{d}1} : \mathbb{R}_{>0} \to \mathbb{R}$ and $\dot{q}_{\overline{d}1} : \mathbb{R}_{>0} \to \mathbb{R}$ are the selectable minimum and maximum desired cadence values that bound the non-dominant side's uncontrolled mode. High-level switching between the three modes on the dominant side is based on position, such that

$$\sigma_{a_2} \triangleq \begin{cases} 1 & \text{if} \quad q_2 \leq q_{\underline{d}2} \\ 0 & \text{if} \quad q_2 > q_{\underline{d}2} \end{cases}, \quad \sigma_{r_2} \triangleq \begin{cases} 1 & \text{if} \quad q_2 \geq q_{\overline{d}2} \\ 0 & \text{if} \quad q_2 < q_{\overline{d}2} \end{cases}, \quad (8)$$

where the switching signals $\sigma_{a_2} : Q \to \{0, 1\}$ and $\sigma_{r_2} : Q \to \{0, 1\}$ determine when the dominant side is in the assistive and resistive modes, respectively, and $q_{\underline{d}2} : \mathbb{R}_{>0} \to \mathbb{R}$ and $q_{\overline{d}2} : \mathbb{R}_{>0} \to \mathbb{R}$ are the selectable minimum and

[1] For notational brevity, all explicit dependence on time, t, within the terms $q(t)$, $\dot{q}(t)$, $\ddot{q}(t)$ is suppressed.

maximum desired position values that bound the dominant side's uncontrolled mode, and are designed to be centered around $q_1 - \pi$ to maintain a 180 degree offset. For both the non-dominant and dominant sides, the subsystem is in the uncontrolled mode when $\sigma_{a_l} = \sigma_{r_l} = 0$, $\forall l \in S$. Within the assistive mode for both the non-dominant and dominant subsystems, low-level switching amongst the muscle groups and motor is based on definitions for the subsequent FES regions for each muscle group $\mathcal{Q}_m \subset \mathcal{Q}$, $\forall m \in \mathcal{M}$, as in [13]. The stimulation intensity applied to each muscle group, $u_{m_l} : Q \times \mathbb{R}_{>0} \to \mathbb{R}$, is defined as[2]

$$u_{m_l} \triangleq \sigma_{a_l}\sigma_{m_l}k_{m_l}u_{s_l}, \tag{9}$$

$\forall l \in S$, $\forall m \in \mathcal{M}$, where σ_{a_l} was defined in (7) and (8), the subsequently designed FES control input is denoted by $u_{s_l} : \mathbb{R}_{>0} \to \mathbb{R}$, and $k_{m_l} \in \mathbb{R}_{>0}$ is a selectable constant control gain. The low-level switching signal $\sigma_{m_l}(q_l) \in \{0, 1\}$ is designed for each muscle group such that $\sigma_{m_l}(q_l) = 1$ when $q_l \in \mathcal{Q}_m$ and $\sigma_{m_l}(q_l) = 0$ when $q_l(t) \notin \mathcal{Q}_m$, $\forall l \in S$, $\forall m \in \mathcal{M}$. The overall FES region, $\mathcal{Q}_{FES}$, is the union of individual muscle regions on the particular side, defined as $\mathcal{Q}_{FES} \triangleq \bigcup_{m\in\mathcal{M}} \{\mathcal{Q}_m\}$, $\forall m \in \mathcal{M}$.

The applied motor current in (2) is denoted by $u_{e_l} : Q \times \mathbb{R} \times \mathbb{R}_{>0} \to \mathbb{R}$ and defined as

$$u_{e_l} \triangleq (\sigma_{r_l} + \sigma_{a_l}\sigma_{e_l})\, u_{r_l}, \tag{10}$$

$\forall l \in S$, where $u_{r_l} : \mathbb{R}_{>0} \to \mathbb{R}$ denotes the subsequently designed motor control input, and $\sigma_{e_l} : Q \times \mathbb{R} \to \{0, 1\}$ is an auxiliary low-level switching signal for activation of the electric motor within the assistive mode, defined as

$$\sigma_{e_l} \triangleq \begin{cases} 1 & \text{if} \quad q_l \notin \mathcal{Q}_{FES} \\ 1 & \text{if} \quad q_l \in \mathcal{Q}_{FES},\, u_{m_l} = \beta_{m_l} \\ 0 & \quad\;\; q_l \in \mathcal{Q}_{FES},\, u_{m_l} \neq \beta_{m_l} \end{cases}, \tag{11}$$

$\forall l \in S$, $\forall m \in \mathcal{M}$, where $\beta_{m_l} \in \mathbb{R}_{>0}$ is the stimulation comfort threshold and the motor is activated in FES regions when the stimulation input reaches β_{m_l}. Thus, the switching laws autonomously activate subsets of muscle groups and the motor based on position, velocity, and stimulation level.

Substituting (2)-(6), (9), and (10) into (1) yields

$$\begin{aligned} B_{M_l}u_{s_l} + B_{E_l}u_{r_l} + \tau_{vol_l} \;=\; & M_l\ddot{q}_l + b_{c_l}\dot{q}_l + d_{c_l} \\ & + V_l\dot{q}_l + G_l + P_l + d_{r_l}, \end{aligned} \tag{12}$$

$\forall l \in S$, where $M_l : \mathcal{Q} \to \mathbb{R}$ is defined as the summation $M_l \triangleq J_{c_l} + M_{p_l}$. The combined switched FES control effectiveness $B_{M_l} : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$ is defined as

$$B_{M_l} \triangleq \sum_{m\in\mathcal{M}} B_m\sigma_{a_l}\sigma_{m_l}k_{m_l}, \tag{13}$$

and $B_{E_l} : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$ is the switched motor control effectiveness, defined as

$$B_{E_l} \triangleq B_e\,(\sigma_{r_l} + \sigma_{a_l}\sigma_{e_l}). \tag{14}$$

[2]For notational brevity, all functional dependencies are hereafter suppressed unless required for clarity of exposition.

The switched system in (12) has the following properties and assumptions $\forall l \in S$ [18]:

Property: 1 $c_m \le M_l \le c_M$, where $c_m, c_M \in \mathbb{R}_{>0}$ are known constants. **Property: 2** $|V_l| \le c_V|\dot{q}_l|$, where $c_V \in \mathbb{R}_{>0}$ is a known constant. **Property: 3** $|G_l| \le c_G$, where $c_G \in \mathbb{R}_{>0}$ is a known constant. **Property: 4** $|P_l| \le c_{P1} + c_{P2}|\dot{q}_l|$, where $c_{P1}, c_{P2} \in \mathbb{R}_{>0}$ are known constants. **Property: 5** $b_{c_l} \le c_b$, where $c_b \in \mathbb{R}_{>0}$ is a known constant. **Property: 6** $|d_{r_l} + d_{c_l}| \le c_d$, where $c_d \in \mathbb{R}_{>0}$ is a known constant. **Property: 7** $\frac{1}{2}\dot{M}_l = V_l$. **Property: 8** B_{m_l} is lower bounded $\forall m \in \mathcal{M}$, and thus, when $\sum_{m\in\mathcal{M}} \sigma_{m_l} > 0$, $c_{b_m} \le B_{M_l} \le c_{b_M}$, where $c_{b_m}, c_{b_M} \in \mathbb{R}_{>0}$ are known constants. **Property: 9** $c_{b_e} \le B_{e_l}$, where $c_{b_e} \in \mathbb{R}_{>0}$ is a known constant. **Assumption: 1** The volitional torque produced by each leg of the rider is bounded due to human physical limitations as $|\tau_{vol_l}| \le c_{vol}$, where $c_{vol} \in \mathbb{R}_{>0}$ is a known constant.

III. Control Development

The following control development for the two sides of the cycle-rider system is applicable to any combination of tracking desired ranges of position or cadence for each subsystem. Without loss of generality, in this paper the control objective is for the non-dominant subsystem to track a desired cadence range and for the dominant subsystem to track a desired position range and single cadence value such that a phase shift within a desired range of 180 degrees from the non-dominant leg is maintained.

A. Non-dominant Side

The cadence tracking objective for the non-dominant leg is quantified by the velocity error $e_1 : \mathbb{R}_{\ge 0} \to \mathbb{R}$ and auxiliary error $r_1 : \mathbb{R}_{\ge 0} \to \mathbb{R}$, defined as

$$e_1 \triangleq \dot{q}_{\underline{d}1} - \dot{q}_1, \tag{15}$$

$$r_1 \;\triangleq\; e_1 + (1 - \sigma_{a_l})\,\Delta_{d1}. \tag{16}$$

The uncontrolled cadence thresholds, $\dot{q}_{\underline{d}1}$ and $\dot{q}_{\overline{d}1}$, were defined previously and can be related as $\dot{q}_{\overline{d}1} \triangleq \dot{q}_{\underline{d}1} + \Delta_{d1}$, where $\Delta_{d1} \in \mathbb{R}_{>0}$ is the range of the uncontrolled mode. Note that $e_1 = r_1$ when $\sigma_{a_l} = 1$. Taking the time derivative of (15), multiplying by M_1, and using (12) with $l = 1$ yields

$$M_1\dot{e}_1 = -B_{E_1}u_{r_1} - B_{M_1}u_{s_1} - \tau_{vol_1} - V_1r_1 + \chi_1, \tag{17}$$

where the auxiliary term $\chi_1 : Q \times \mathbb{R} \times \mathbb{R}_{\ge 0} \to \mathbb{R}$ is defined as $\chi_1 \triangleq b_{c_1}\dot{q}_1 + d_{c_1} + G_1 + P_1 + d_{r_1} + V_1\dot{q}_{\underline{d}1} + V_1(1 - \sigma_{a_1})\,\Delta_{d1} + M_1\ddot{q}_{\underline{d}1}$. From Properties 1-6, χ_1 can be bounded as

$$\chi_1 \le c_1 + c_2|e_1|, \tag{18}$$

where $c_1, c_2 \in \mathbb{R}_{>0}$ are known constants, and $|\cdot|$ denotes the absolute value. Based on (17), (18), and the subsequent stability analysis, the FES control input to the muscle groups on the non-dominant side is designed as

$$u_{s_1} = \text{sat}_{\beta_1}\left[k_{1s} + k_{2s} r_1\right], \tag{19}$$

where $k_{1s}, k_{2s} \in \mathbb{R}_{>0}$ are constant selectable control gains, $\text{sat}_{\beta_1}(\cdot)$ is defined as $\text{sat}_{\beta_1}(\kappa) \triangleq \kappa$ for $|\kappa| \le \beta_1$ and $\text{sat}_{\beta_1}(\kappa) \triangleq \text{sgn}(\kappa)\beta_1$ for $|\kappa| > \beta_1$, and $\beta_1 \in \mathbb{R}_{>0}$ is the rider-selected comfort threshold for the non-dominant leg. The switched control input to the motor is designed as

$$u_{r_1} = k_{1e}\text{sgn}(r_1) + k_{2e} r_1, \tag{20}$$

where $k_{1e}, k_{2e} \in \mathbb{R}_{>0}$ are constant selectable control gains. Substituting (19) and (20) into (17) yields

$$\begin{aligned} M_1 \dot{e}_1 \quad = \quad & -B_{E_1}\left(k_{1e}\text{sgn}(r_1) + k_{2e} r_1\right) \\ & -B_{M_1}\left(\text{sat}_{\beta_1}\left[k_{1s} + k_{2s} r_1\right]\right) \\ & -\tau_{vol_1} - V_1 r_1 + \chi_1. \end{aligned} \tag{21}$$

B. Dominant Side

The position tracking objective for the dominant leg is quantified by the error $e_2 : \mathbb{R}_{\ge 0} \to \mathbb{R}$ and auxiliary errors $r_2 : \mathbb{R}_{\ge 0} \to \mathbb{R}$ and $r_3 : \mathbb{R}_{\ge 0} \to \mathbb{R}$, defined as

$$e_2 \quad \triangleq \quad q_{\underline{d}2} - q_2, \tag{22}$$

$$r_2 \quad \triangleq \quad e_2 + (1 - \sigma_{a2})\,\Delta_{d2}, \tag{23}$$

$$r_3 \quad \triangleq \quad \dot{e}_2 + \alpha e_2 \tag{24}$$

where $\alpha \in \mathbb{R}_{>0}$ is a constant selectable control gain. The position thresholds for the uncontrolled mode, $q_{\underline{d}2}$ and $q_{\overline{d}2}$, were defined previously and are now related as $q_{\overline{d}2} \triangleq q_{\underline{d}2} + \Delta_{d2}$, where $\Delta_{d2} \in \mathbb{R}_{>0}$ is the range of desired position values for the dominant leg, and $q_{\underline{d}2} \triangleq q_1 - \pi - \frac{\Delta_{d2}}{2}$ so that $q_{\underline{d}2}$ and $q_{\overline{d}2}$ are centered around $q_1 - \pi$. Note that $e_2 = r_2$ when $\sigma_{a_2} = 1$. Taking the time derivative of (24), multiplying by M_2, and using (12) with $l = 2$ and (22) yields

$$M_2 \dot{r}_3 = -B_{E_2} u_{r_2} - B_{M_2} u_{s_2} - \tau_{vol_2} - V_2 r_3 - r_2 + \chi_2, \tag{25}$$

where the auxiliary term $\chi_2 : Q \times \mathbb{R} \times \mathbb{R}_{\ge 0} \to \mathbb{R}$ is defined as $\chi_2 \triangleq b_{c_2}\dot{q}_2 + d_{c_2} + G_2 + P_2 + d_{r_2} + V_2 \dot{q}_{\underline{d}2} + V_2 \alpha e_2 + M_2 \ddot{q}_{\underline{d}2} + M_2 \alpha r_3 - M_2 \alpha^2 e_2 + r_2$. From Properties 1-6, χ_2 can be bounded as

$$\chi_2 \le c_3 + c_4\|z\| + c_5\|z\|^2, \tag{26}$$

where $z \triangleq [r_2\ r_3]^T$, $\|\cdot\|$ is the Euclidean norm, and $c_3, c_4, c_5 \in \mathbb{R}_{>0}$ are known constants. Based on (25), (26), and the subsequent stability analysis, the FES control input to the muscle groups on the dominant side is designed as

$$\begin{aligned} u_{s_2} \quad = \quad & \text{sat}_{\beta_2}\Big[k_{3s} r_3 + \\ & \left(k_{4s} + k_{5s}\|z\| + k_{6s}\|z\|^2\right)\text{sgn}(r_3)\Big], \end{aligned} \tag{27}$$

where $k_{3s}, k_{4s}, k_{5s}, k_{6s} \in \mathbb{R}_{>0}$ are constant selectable control gains and $\beta_2 \in \mathbb{R}_{>0}$ is the rider-selected comfort threshold for the dominant leg. The switched control input to the motor on the dominant side is designed as

$$u_{e_2} = k_{3e} r_3 + \left(k_{4e} + k_{5e}\|z\| + k_{6e}\|z\|^2\right)\text{sgn}(r_3), \tag{28}$$

where $k_{3e}, k_{4e}, k_{5e}, k_{6e} \in \mathbb{R}_{>0}$ are constant selectable control gains. Substituting (27) and (28) into (25) yields

$$\begin{aligned} M_2 \dot{r}_3 \quad = \quad & -B_{E_2}\text{sat}_{\beta_2}\Big[k_{3s} r_3 + \\ & \left(k_{4e} + k_{5e}\|z\| + k_{6e}\|z\|^2\right)\text{sgn}(r_3)\Big] \\ & -B_{M_2}\Big[k_{3s} r_3 + \\ & \left(k_{4s} + k_{5s}\|z\| + k_{6s}\|z\|^2\right)\text{sgn}(r_3)\Big] \\ & -\tau_{vol_2} - V_2 r_3 - r_2 + \chi_2. \end{aligned} \tag{29}$$

IV. Stability Analysis

The stability analysis is divided into non-dominant (Section IV, A) and dominant (Section IV, B) subsystems. To facilitate the analysis of switching signals, switching times are denoted by $\left\{t_{n,l}^i\right\}$, $i \in \{a, r, p\}$, $n \in \{0, 1, 2, ...\}$, $\forall l \in S$, representing the times when each side's subsystem switches into the assistive ($i = a$), resistive ($i = r$), or uncontrolled ($i = p$) modes.

A. Stability of the Non-Dominant Subsystem

Let $V_{L1} : \mathbb{R} \to \mathbb{R}$ be a continuously differentiable, positive definite, common Lyapunov function candidate defined as

$$V_{L1} \triangleq \frac{1}{2} M_1 r_1^2, \tag{30}$$

which satisfies the following inequalities:

$$\frac{c_m}{2} r_1^2 \le V_{L1} \le \frac{c_M}{2} r_1^2, \tag{31}$$

where c_m and c_M are introduced in Property 1. Theorems 1 and 2 apply, provided some gain conditions are satisfied.

Theorem 1. *Throughout the assistive mode, when $\dot{q}_1 \le \dot{q}_{\underline{d}1}$, the closed-loop error system in (21) results in exponential convergence of the cadence on the non-dominant side to $\dot{q}_{\underline{d}1}$, provided some gain conditions are satisfied.*

Proof: When $\dot{q}_1 \le \dot{q}_{\underline{d}1}$; $e_1 = r_1 \ge 0$, $\sigma_{a_1} = 1$, and $\sigma_{r_1} = 0$ (i.e., the non-dominant side subsystem is controlled in the assistive mode by either FES, the motor, or both). It can be demonstrated that, due to B_{M_1} and B_{E_1} discontinuously varying over time, the time derivative of (30) exists almost everywhere (a.e.) within $t \in \left[t_{n,1}^a, t_{n+1,1}^p\right)$, $\forall n$, and $\dot{V}_{L1} \overset{a.e.}{\in} \dot{\tilde{V}}_{L1}$ [19]. After substituting (13), (14), and (21), the derivative of (30) can be solved using Filippov's differential inclusion [20], and then upper bounded using Properties 7 and 8, Assumption 1, and (18) as

$$\dot{\tilde{V}}_{L1} \overset{\text{a.e.}}{\le} -\left(A - c_{vol} - c_1\right) r_1 - \left(B - c_2\right) r_1^2, \tag{32}$$

which is negative definite in all cases since $r_1 \ge 0$, provided some gain conditions are satisfied. The values of $A \in \mathbb{R}_{>0}$

and $B \in \mathbb{R}_{>0}$ depend on the switching signals, and are defined as

$$A \triangleq \begin{cases} c_{b_e} k_{1e} & \text{if } \sigma_{e_1} = 1,\ q_1 \notin Q_{FES} \\ c_{b_m} k_{1s} & \text{if } \sigma_{e_1} = 0,\ q_1 \in Q_{FES} \\ c_{b_e} k_{1e} + c_{b_m}\beta_1 & \text{if } \sigma_{e_1} = 1,\ q_1 \in Q_{FES} \end{cases},$$

$$B \triangleq \begin{cases} c_{b_e} k_{2e} & \text{if } \sigma_{e_1} = 1 \\ c_{b_m} k_{2s} & \text{if } \sigma_{e_1} = 0 \end{cases}.$$

Furthermore, since $\dot{V}_{L1} \overset{a.e.}{\in} \dot{\tilde{V}}_{L1}$, (31) can be used to upper bound (32) as

$$\dot{V}_{L1} \leq -\lambda_{a1} V_{L1}, \tag{33}$$

$t \in \left[t^{a}_{n,1},\, t^{p}_{n+1,1}\right)$, $\forall n$, where λ_{a1} is a positive constant. Solving the inequality in (33), using (31), and performing some algebraic manipulation yields exponential convergence of r_1 and e_1 to zero. Since exponential convergence is guaranteed for all combinations of σ_{e_1} and σ_{m_1} while $\sigma_{a_1} = 1$, V_{L1} is a common Lyapunov function for switching during the assistive mode of the non-dominant side. ■

Theorem 2. *When $\dot{q}_1 \geq \dot{q}_{\overline{d}1}$, the closed-loop error system in (21) is exponentially stable, provided some gain conditions are satisfied.*

Proof: When $\dot{q}_1 \geq \dot{q}_{\overline{d}1}$; $\sigma_{a_1} = 0$, $\sigma_{r_1} = 1$, and $e_1 + \Delta_{d1} = r_1 \leq 0$ (i.e., the non-dominant side subsystem is in the resistive mode and controlled by the motor). Due to the signum function in (21), the time derivative of (30) exists a.e. within $t \in \left[t^{r}_{n,1},\, t^{p}_{n+1,1}\right)$, $\forall n$, and $\dot{V}_{L1} \overset{a.e.}{\in} \dot{\tilde{V}}_{L1}$. After substituting (16) and (21), the derivative can be upper bounded using Properties 7 and 9, Assumption 1, and (18) as

$$\begin{aligned} \dot{\tilde{V}}_{L1} \overset{\text{a.e.}}{\leq} & -\left(c_{b_e} k_{1e} - c_{vol} - c_1 - c_2 \Delta_{d1}\right)|r_1| \\ & -\left(c_{b_e} k_{2e} - c_2\right) r_1^2, \end{aligned} \tag{34}$$

$\forall t \in \left[t^{r}_{n,1},\, t^{p}_{n+1,1}\right)$, $\forall n$, which is negative definite provided sufficient gain conditions are satisfied. Furthermore, since $\dot{V}_{L1} \overset{a.e.}{\in} \dot{\tilde{V}}_{L1}$, (34) can be upper bounded as

$$\dot{V}_{L1} \leq -\lambda_{r1} V_{L1}, \tag{35}$$

$\forall t \in \left[t^{r}_{n,1},\, t^{p}_{n+1,1}\right)$, $\forall n$, where λ_{r1} was defined previously. Solving (35), rewriting using (31), and performing algebraic manipulation yields exponential convergence. ■

Remark 1. Since the non-dominant side is in the uncontrolled mode when $-\Delta_{d1} < e_1 < 0$, the error is always bounded in the uncontrolled mode. As described in Theorems 1 and 2, $|r_1|$ (which, by (16), is equivalent to e_1 in the assistive mode) decays at an exponential rate in both the assistive and resistive modes to zero. By extension, $|e_1|$ also decays exponentially in the assistive and resistive modes, to values of 0 and Δ_{d1}, respectively. When the subsystem of the non-dominant side enters the resistive mode, the cadence will instantly exponentially decay towards $\dot{q}_{\overline{d}1}$ back into the uncontrolled mode, and when entering the assistive mode, the FES and motor controllers on the non-dominant side will ensure the cadence exponentially increases towards $\dot{q}_{\underline{d}}$ back into the uncontrolled mode. For this particular control objective, where there is a desired cadence range, rather than a single value for the desired trajectory, error convergence to a range (i.e., $[0,\, \Delta_{d1}]$) is desirable, rather than exponential error convergence to zero.

B. Stability of the Dominant Side

Let $V_{L2} : \mathbb{R}^2 \to \mathbb{R}$ be a continuously differentiable, positive definite, common Lyapunov function candidate defined as

$$V_{L2} \triangleq \frac{1}{2} r_2^2 + \frac{1}{2} M_2 r_3^2, \tag{36}$$

which satisfies the following inequalities:

$$\frac{\min\left(c_m,\, 1\right)}{2}\left\|z\right\|^2 \leq V_{L2} \leq \frac{\max\left(c_M,\, 1\right)}{2}\left\|z\right\|^2, \tag{37}$$

where c_m and c_M are introduced in Property 1. Theorems 3 and 4 apply, provided some gain conditions are satisfied.

Theorem 3. *When $q_2 \leq q_{\underline{d}2}$, the closed-loop error system in (29) results in exponential convergence of the position and cadence on the dominant side to $q_{\underline{d}2}$ and $\dot{q}_1$, provided some gain conditions are satisfied.*

Proof: When $q_2 \leq q_{\underline{d}2}$; $\sigma_{a_2} = 1$, $\sigma_{r_2} = 0$, and $r_2 = e_2 \geq 0$ (i.e., the dominant side subsystem is controlled in the assistive mode by FES and/or the motor). Similar to the proof of Theorem 1, it can be demonstrated that, due to B_{M_2} and B_{E_2} discontinuously varying over time, the time derivative of (36) exists a.e. within $t \in \left[t^{a}_{n,2},\, t^{p}_{n+1,2}\right)$, $\forall n$, and $\dot{V}_{L2} \overset{a.e.}{\in} \dot{\tilde{V}}_{L2}$, and after substituting (29), the derivative of (36) can be upper bounded using Properties 7 and 8, Assumption 1, and (26) as

$$\dot{\tilde{V}}_{L2} \overset{\text{a.e.}}{\leq} -\min\left(c_{b_e} k_{3e},\, c_{b_m} k_{3s}\right) r_3^2 - \alpha r_2^2, \tag{38}$$

$\forall t \in \left[t^{a}_{n,2},\, t^{p}_{n+1,2}\right)$, $\forall n$, provided some gain conditions are satisfied. Furthermore, since $\dot{V}_{L2} \overset{a.e.}{\in} \dot{\tilde{V}}_{L2}$, (37) can be used to upper bound (38) as

$$\dot{V}_{L2} \leq -\lambda_{a2} V_{L2}, \tag{39}$$

$\forall t \in \left[t^{a}_{n,2},\, t^{p}_{n+1,2}\right)$, $\forall n$, where λ_{a2} is a positive constant. The inequality in (39) can be solved, rewritten using (37), and performing some algebraic manipulation yields exponential convergence. Since exponential convergence holds for all combinations of σ_{e_2} and σ_{m_2} while $\sigma_{a_2} = 1$, V_{L2} is a common Lyapunov function for switching during the assistive mode of the dominant side. ■

Theorem 4. *When $q_2 \geq q_{\overline{d}2}$, the closed-loop error system in (29) is exponentially stable, provided some gain conditions are satisfied.*

Proof: When $q_2 \leq q_{\bar{d}2}$; $r_2 \leq 0$, $e_2 \leq 0$, and $\sigma_{r_2} = 1$ (i.e., the cycle-rider system is in the motor-resistance control mode). It can be demonstrated that, due to B_{E_2} discontinuously varying over time, the time derivative of (36) exists a.e. within $t \in \left[t^r_{n,2}, t^p_{n+1,2}\right)$, $\forall n$, and $\dot{V}_{L2} \overset{a.e.}{\in} \dot{\tilde{V}}_{L2}$. After substituting (23) and (29), the derivative can be upper bounded using Properties 7 and 9, Assumption 1, (26), and noting that $r_2 \leq 0$, as

$$\dot{\tilde{V}}_{L2} \overset{\text{a.e.}}{\leq} -c_{b_e} k_{3e} r_3^2 - \alpha r_2^2, \tag{40}$$

which is negative definite provided gain conditions are satisfied. Furthermore, since $\dot{V}_{L2} \overset{a.e.}{\in} \dot{\tilde{V}}_{L2}$, (40) can be upper bounded as

$$\dot{V}_{L2} \leq -\lambda_{r2} V_{L2}, \tag{41}$$

where λ_{r2} is a positive constant, and (41) can be solved and rewritten using (37), and then performing algebraic manipulation yields exponential convergence. Since $\|z\| \to 0$, $\{r_2, r_3\} \to 0$. ■

Remark 2. Since r_2 exponentially decays to zero in both the assistive and resistive modes, by (23), e_2 exponentially decays to 0 in the assistive mode and to Δ_{d2} in the resistive mode. As designed, the position of the dominant leg q_2 exponentially approaches a bound $\left[q_{\underline{d}2}, q_{\bar{d}2}\right]$ surrounding a 180 degree offset from the actual position of the non-dominant leg q_1, and the cadence of the dominant leg exponentially approaches the cadence of the non-dominant leg.

V. Conclusion

The combined motor and FES control system developed in this paper is designed to enable a volitionally contributing rider of a split-crank cycle to maintain a cadence within a desired range, as well as a phase shift within a desired region centered around 180 degrees. A Lyapunov-like analysis proved stability of the controllers for the cycle-rider system, despite unknown disturbances and arbitrary switching, showing exponential convergence to the desired cadence range (i.e., $e_1 \in [0, \Delta_{d1}]$) on the non-dominant side and position range (i.e., $e_2 \in [0, \Delta_{d2}]$) on the dominant side.

With assistive, uncontrolled, and resistive modes, the developed control system has the potential to advance motorized FES-cycling as a rehabilitation exercise for people with movement disorders. The split crank of the cycle in this paper presents a way of addressing the asymmetries associated with some movement disorders. A wide range of volitional abilities could be accomodated, such that any rider could pedal within desired cadence and position ranges. FES and a motor on each side assists those with minimal leg strength or at the onset of fatigue, and motor resistance is provided to someone who can easily pedal above the desired ranges. The authors plan to perform experiments on the split-crank cycle, including on individuals with neurological conditions that result in lower body asymmetric function.

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