VARYING MOTOR ASSISTANCE DURING BICEPS CURLS INDUCED VIA FUNCTIONAL ELECTRICAL STIMULATION

Courtney A. Rouse *
Christian A. Cousin
Department of Mechanical
and Aerospace Engineering
University of Florida
Gainesville, Florida 32611
Email:{courtneyarouse,ccousin}@ufl.edu

Victor H. Duenas
Warren E. Dixon
Department of Mechanical
and Aerospace Engineering
University of Florida
Gainesville, Florida 32611
Email:{vhduenas,wdixon}@ufl.edu

ABSTRACT

Robot-assisted therapy has been established as a useful rehabilitation tool for motor recovery in people with various neurological impairments; however, balancing human and robot contribution, such that the target muscle is sufficiently exercised, is necessary to improve the outcome of rehabilitation protocols. Functional Electrical Stimulation (FES) can assist a person to move their limb by contracting the muscle; however, motor assistance is often necessary to accurately follow a desired limb trajectory, especially since stimulation can be limited due to various factors (e.g., subject comfort, stimulation saturation). In this paper, a motor is tasked with intermittently assisting the FES-activated biceps brachii in tracking a desired forearm trajectory whenever the FES input reaches a pre-set comfort threshold. A Lyapunov-like switched systems stability analysis is used to prove exponential stability of the human-robot system. Preliminary experiments demonstrate the feasibility and performance of the controller on two subjects with neurological impairments.

1 INTRODUCTION[1]

Robot-assisted therapy has been shown to be beneficial for rehabilitation and neurological relearning for people with movement impairments. Specifically, in [1] robot-assisted therapy was used to aid in stroke recovery. The review in [2] compares various therapeutic robots that have been developed for the upper extremity. While exercises involving robotic assistance are advantageous for rehabilitation, intensive active involvement from the person (rather than only passive motion) is necessary for motor recovery [3–5]. Balancing human and robot contributions often requires patient-specific adaptations of the controller [6]. Many methods have been used to facilitate maximum human contribution, such as challenge-based robots [7], assist-as-needed controllers [8, 9], and an algorithm for altering allowable error and decaying disturbance rejection [10].

Functional Electrical Stimulation (FES) can be used to elicit muscle contractions when the person cannot volitionally produce sufficient force to follow the desired rehabilitation protocol. FES has been found to increase strength in the biceps brachii [11] and motor function on the post-stroke paretic arm [12]; however, stimulation often must be limited due to subject comfort, resulting in a need for alternative actuator input such as a motor. A combination of robotic assistance and FES resulted in arm mobility improvements in severely impaired stroke survivors [13]. Thus, there is motivation to design an upper arm robot-assisted FES rehabilitiation protocol where the robot only assists when the necessary stimulation input reaches a saturation limit (e.g., due to a predefined comfort level by the participant).

Repetitive movements (e.g., biceps curls) are known to improve muscle strength and movement coordination for people with neurological conditions [2, 14]. In [15], FES induces biceps curls and motor assistance is used intermittently whenever the stimulation reaches the pre-set comfort threshold; and, although the error remains bounded in practice, tracking performance may be improved by extending bouts of motor assistance,

*Address all correspondence to this author.
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during which the muscle is still activated for sufficient contractions. In this paper, FES is used as the primary actuator for inducing biceps curls through multiple stimulation channels placed across the muscle, as in [15] and [16], and the motor contributes when the upper stimulation threshold is reached; however, here the motor will continue to contribute until the stimulation decays to the lower stimulation threshold. This new approach eliminates the potential for motor chatter about a single threshold is prevented without activating the motor throughout the entire experiment duration. Additionally, a strategy is developed to decrease the lower stimulation threshold for which the motor turns off whenever the upper stimulation threshold is reached more than once in a single biceps curl. Thus, the motor is active for longer as the person fatigues and/or performance decreases. Lyapunov methods for switched systems are used to prove exponential stability by establishing a common Lyapunov function for all subsystems. The performance of the controller is demonstrated by experiments on two subjects with neurological conditions that result in hemiparesis, with average RMS position errors of 4.55 degrees and 4.36 degrees for the impaired and unimpaired arms, respectively.

2 MODEL

The dynamics of the motorized FES system are[2]

$$M\ddot{q} + V\dot{q} + G + \tau_p + \tau_b + \tau_d = \tau_m + \tau_e, \quad (1)$$

where $q : \mathbb{R}_{>0} \to Q$ denotes the angular forearm position about the elbow joint, and $Q \subseteq [0, \pi)$ denotes the set of forearm angles. The states q and $\dot{q}$ are assumed to be measurable and calculable, respectively. Inertial effects are denoted by $M : Q \to \mathbb{R}_{>0}$, $V : Q \times \mathbb{R} \to \mathbb{R}$ denotes centripetal and Coriolis effects, and $G : Q \to \mathbb{R}$ denotes gravitational effects. Torques applied about the elbow joint by passive viscoelastic tissue forces are denoted by $\tau_p : Q \times \mathbb{R} \to \mathbb{R}$; $\tau_b : \mathbb{R} \to \mathbb{R}$ denotes torques applied about the elbow by viscous damping of the testbed's hinge; $\tau_d : \mathbb{R}_{\geq 0} \to \mathbb{R}$ denotes unknown disturbances (e.g., spasticity or changes in load); $\tau_m : Q \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ denotes torques applied about the elbow joint axis by muscle contractions; and $\tau_e : Q \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ denotes torques applied about the elbow joint axis by the electric motor.

2.1 Switched Muscle Subsystem

As in [15] and [16], stimulation is applied to $w \in \mathbb{N}$ distinct electrode channels that are placed along the biceps brachii in predefined regions of Q, where w is a predetermined constant and each combination of channels comprises a subsystem. Let $i \in S$ denote the i^{th} electrode channel and $S \triangleq \{1, 2, ..., w\}$ denote a finite indexed set of all channels. As in [17], the normalized isometric torque produced by the i^{th} channel, $\tau_i \in \mathbb{R}$, was measured a priori every 10 degrees throughout a range of angles defining a biceps curl. The portion of the dynamic biceps curl trajectory over which a particular electrode channel is stimulated is denoted by $Q_i \subset Q$, defined as

$$Q_i \triangleq \{q \in Q \,|\, \max(\tau_i(q), \tau_i(q+10), \tau_i(q-10)) > \varepsilon\},$$

and $\bigcup_{i \varepsilon S} Q_i = Q$. The threshold, $\varepsilon \in [0, 1]$, is a design constant. The torque due to muscle contractions in Eqn. (1) is generated by applying an electric potential field across the biceps brachii muscle and is defined as

$$\tau_m(q, \dot{q}, t) \triangleq \sum_{i \in S} B_i(q, \dot{q})\, u_i(q, \dot{q}, t), \quad (2)$$

$i \in S$, where $B_i : Q \times \mathbb{R} \to \mathbb{R}_{>0}$ denotes an unknown, nonlinear, auxiliary function relating the stimulation intensity applied to the i^{th} stimulation channel to the torque produced by the activated sensory-motor structures. In Eqn. (2), $u_i : Q \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ denotes the control input and the electrical stimulation intensity applied to the i^{th} electrode, defined as

$$u_i(q, \dot{q}, t) \triangleq \sigma_i(q)\, T_i(q)\, u_m(q, \dot{q}, t),\ i \in S, \quad (3)$$

where $\sigma_i(q) \in \{0, 1\}$ is a piecewise switching signal for each channel such that

$$\sigma_i \triangleq \begin{cases} 1, \text{ if } (q \in Q_i) \wedge (\dot{q}_d > 0) \\ 0 \quad \text{otherwise} \end{cases},\ i \in S,$$

where $q_d : \mathbb{R}_{>0} \to \mathbb{R}$ is the desired forearm position, designed so its first and second derivatives exist and are bounded. The switching signals σ_i ensure that FES is only used during regions of desired elbow flexion. Note that the muscle control switches based on desired velocity, rather than the actual velocity, because it cannot be guaranteed that the actual velocity is positive throughout flexion and negative throughout extension. The subsequently designed control input to the biceps muscle is denoted by $u_m : Q \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$, and is distributed amongst all biceps electrodes according to σ_i and the known function of the elbow angle $T_i : Q \to \mathbb{R}$ in Eqn. (3), which is calculated a priori based on the i^{th} channel's effectiveness in producing torque at the given angle, as in [15].

The torque due to the motor is defined as

[2]For notational brevity, functional dependence on system states and time may be hereafter suppressed.

$$\tau_e(q,\dot{q},t) \triangleq B_e u_e(q,\dot{q},t), \tag{4}$$

where the subsequently designed current input applied to the motor is denoted as $u_e : Q \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ and $B_e \in \mathbb{R}_{>0}$ is the electric motor control constant relating input current to output torque. Substituting Eqns. (2), (3), and (4) into Eqn. (1) yields

$$M\ddot{q} + V\dot{q} + G + \tau_p + \tau_b + \tau_d = B_\sigma u_m + B_e u_e, \tag{5}$$

where $B_\sigma : Q \times \mathbb{R} \to \mathbb{R}$ is the combined switched control effectiveness for the entire biceps brachii, defined as

$$B_\sigma = \sum_{i \in S} \sigma_i B_i T_i. \tag{6}$$

A combination of w channels allows for $2^w - 1$ possible subsystems, assuming ε is selected such that stimulation is always sent to at least one channel (i.e., exclude the empty set). The system model in Eqn. (5) has the following properties [18–20].

Property 1. $c_\sigma \leq B_\sigma(q, \dot{q}) \leq c_\Sigma, \forall i \in S$, where $c_\sigma, c_\Sigma \in \mathbb{R}_{>0}$ are known constants. **Property 2.** $c_m \leq M(q) \leq c_M$, where $c_m, c_M \in \mathbb{R}_{>0}$. **Property 3.** $|V(q, \dot{q})| \leq c_V|\dot{q}|$, where $c_V \in \mathbb{R}_{>0}$ is a known constant. **Property 4.** $|G(q)| \leq c_G$, where $c_G \in \mathbb{R}_{>0}$ is a known constant. **Property 5.** $|\tau_b(\dot{q})| \leq c_b|\dot{q}|$, where $c_b \in \mathbb{R}_{>0}$ is a known constant. **Property 6.** $|\tau_d| \leq c_d$, where $c_d \in \mathbb{R}_{>0}$ is a known constant. **Property 7.** $\frac{1}{2}\dot{M}(q) = V(q, \dot{q})$.

3 CONTROL DEVELOPMENT

3.1 Open-Loop Error System

The control objective is to track a desired forearm trajectory, quantified by the position tracking error, $e_1 : \mathbb{R}_{\geq 0} \to \mathbb{R}$, defined as

$$e_1(t) \triangleq q_d(t) - q(t). \tag{7}$$

To facilitate the subsequent development, an auxiliary tracking error $e_2 : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is defined as

$$e_2(t) \triangleq \dot{e}_1(t) + \alpha e_1(t), \tag{8}$$

where $\alpha \in \mathbb{R}_{>0}$ is a selectable constant gain. Taking the time derivative of Eqn. (8), multiplying by M, adding and subtracting e_1, and using Eqns. (5) and (7) yields

$$M\dot{e}_2 = \chi - Ve_2 - B_\sigma u_m - B_e u_e - e_1, \tag{9}$$

where the auxiliary term $\chi : Q \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ is defined as $\chi \triangleq M\left(\ddot{q}_d + \alpha e_2 - \alpha^2 e_1\right) + V\left(\dot{q}_d + \alpha e_1\right) + G + \tau_p + \tau_b + \tau_d + e_1$. From Properties 1-6, χ can be bounded as

$$|\chi| \leq c_1 + c_2 \| z \| + c_3 \| z \|^2, \tag{10}$$

where $c_1, c_2, c_3 \in \mathbb{R}_{>0}$ are known constants, $\| \cdot \|$ denotes the Euclidean norm, and the error vector $z \in \mathbb{R}^2$ is defined as $z \triangleq \left[e_1 \ e_2 \right]^T$. Based on Eqns. (9), (10), and the subsequent stability analysis, the control input to the muscle is designed as

$$u_m \triangleq \mathrm{sat}_\Gamma \left[c_\sigma^{-1} \left(k_1 e_2 + \left(k_2 + k_3 \| z \| + k_4 \| z \|^2 \operatorname{sgn}(e_2) \right) \right) \right], \tag{11}$$

where $\{k_i\}_{i=1,\ldots,4} \in \mathbb{R}_{>0}$, are selectable constant control gains, $\mathrm{sat}_\Gamma(\cdot)$ is defined as $\mathrm{sat}_\Gamma(\kappa) \triangleq \kappa$ for $|\kappa| \leq \Gamma$ and $\mathrm{sat}_\Gamma(\kappa) \triangleq \operatorname{sgn}(\kappa)\Gamma$ for $|\kappa| > \Gamma$, where $\Gamma \in \mathbb{R}_{>0}$ is a design constant, and $\operatorname{sgn}(\cdot) : \mathbb{R} \to [-1, 1]$ is the signum function.

Once the motor begins assisting the muscle, it is activated until u_m decreases to the lower threshold denoted by $\gamma_j : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, which is initialized at $\gamma_1 \in \mathbb{R}_{>0}$, such that $\gamma_1 \leq \Gamma$. The threshold γ_j resets to γ_1 at the beginning of each biceps curl and updates every time it is reached, according to $\gamma_{j+1} = \rho\gamma_j$, where $j \in \mathbb{N}$ denotes the j^{th} time during the n^{th} biceps curl for which u_m decreases to γ_j after the FES control input u_m saturates at Γ. The selected constant $\rho \in (0, 1)$ denotes the amount that γ_j should decrease after each saturation. At the beginning of each biceps curl, the motor is not activated until u_m reaches Γ, and is again deactivated the next time that $u_m = \gamma_j$ or when a new biceps curl starts (i.e., when $\dot{q}_d > 0$). Let $T_{ext,n}, T_{flex,n} \in \mathbb{R}_{>0}$ denote the initial times during the n^{th} biceps curl for which $\dot{q}_d \leq 0$ and $\dot{q}_d > 0$. The switched control input to the motor is designed as

$$u_e \triangleq \delta B_e^{-1} \left(k_{5,\beta} e_2 + \left(k_{6,\beta} + k_{7,\beta} \| z \| + k_{8,\beta} \| z \|^2 \right) \operatorname{sgn}(e_2) \right), \tag{12}$$

where $\left\{k_{i,\beta}\right\}_{i=5,\ldots 8} \in \mathbb{R}_{>0}$, are constant control gains and β indicates which of two sets of control gains are implemented. Movements involving both FES and motor (i.e., desired flexion, $\dot{q}_d > 0$) are indicated by $\beta = 1$ and movements for which only the motor is activated (i.e., desired extension, $\dot{q}_d \leq 0$) are indicated by $\beta = 0$. The motor's switched signal, $\delta : \mathbb{R}_{\geq 0} \to \{0, 1\}$, is defined as

$$\delta \triangleq \begin{cases} 1, & \dot{q}_d \leq 0 \\ 1, & \min(u_m(t)) > \gamma_j, \forall t \in \left[T_{n,j}^u, \ T_{n,j}^l \right) \\ 0, & \text{otherwise} \end{cases},$$

where the superscripts $\{u, l\}$ refer to the upper and lower saturation thresholds, such that $T_{n,j}^{u} \in \mathbb{R}_{>0}$ is the j^{th} time during the n^{th} biceps curl for which u_m saturates at the upper threshold Γ, after either beginning the biceps curl (i.e., when $j = 1$) or after u_m rises from γ_j, and $T_{n,j}^{l} \in \mathbb{R}_{>0}$ is the j^{th} time during the n^{th} biceps curl for which u_m saturates at the lower threshold γ_j after u_m falls from Γ. Note that u_m may reach Γ multiple times before falling to γ_j, and vice versa; however, $T_{n,j}^{u}$ and $T_{n,j}^{l}$ only refer to the first occurrence. Thus, within the n^{th} biceps curl, the times occur in succession such that $\left\{T_{n,j}^{u},\, T_{n,j}^{l},\, T_{n,j+1}^{u},\, T_{n,j+1}^{l}, T_{n,j+2}^{u}, \ldots\right\}$. Substituting Eqns. (11) and (12) into Eqn. (9) yields

$$\begin{aligned} M\dot{e}_2 = \chi - Ve_2 - e_1 - B_\sigma \Big[\text{sat}_\Gamma \Big(c_\sigma^{-1} \Big(k_1 e_2 \\ + \left(k_2 + k_3 \parallel z \parallel + k_4 \parallel z \parallel^2\right) \text{sgn}\left(e_2\right) \Big)\Big)\Big] \\ - B_e \Big[\delta B_e^{-1} \Big(k_{5,\beta} e_2 + \big(k_{6,\beta} + k_{7,\beta} \parallel z \parallel \\ + k_{8,\beta} \parallel z \parallel^2 \big) \text{sgn}\left(e_2\right)\Big)\Big]. \end{aligned} \tag{13}$$

4 STABILITY ANALYSIS

Let $V_L : \mathbb{R}^2 \to \mathbb{R}$ be a continuously differentiable, positive definite, common Lyapunov function candidate defined as

$$V_L(t) \triangleq \frac{1}{2}e_1^2 + \frac{1}{2}Me_2^2, \tag{14}$$

which satisfies the following inequalities:

$$\lambda_A ||z||^2 \leq V_L \leq \lambda_B ||z||^2, \tag{15}$$

where $\lambda_A, \lambda_B \in \mathbb{R}_{>0}$ are known positive constants defined as $\lambda_A \triangleq \min\left(\frac{1}{2}, \frac{c_m}{2}\right)$, $\lambda_B \triangleq \max\left(\frac{1}{2}, \frac{c_M}{2}\right)$.

Theorem 1. *When the motor is inactivated,* $\delta = 0$ *and* $B_\sigma > 0$, *the FES controller in Eqn. (11) ensures exponential tracking such that*

$$\parallel z(t) \parallel \leq \sqrt{\frac{\lambda_B}{\lambda_A}} \parallel z(t_{n,1}) \parallel \exp\left[-\frac{1}{2}\lambda_2 \left(t - t_{n,1}\right)\right], \tag{16}$$

$\forall t \in [t_{n,1}, t_{n,2})$, *where* $t_{n,1}, t_{n,2} \in \mathbb{R}_{>0}$ *are defined as* $t_{n,1} \triangleq \max\left(T_{flex,n},\, T_{n,j}^{l}\right)$ *and* $t_{n,2} \triangleq \min\left(T_{ext,n},\, T_{n,j+1}^{u}\right)$, *respectively, and* $\lambda_2 \in \mathbb{R}_{>0}$ *is defined as*

$$\lambda_2 \triangleq \frac{1}{\lambda_B} \min\left(\alpha, k_1\right), \tag{17}$$

provided the following gain conditions are satisfied:

$$k_2 \geq c_1,\; k_3 \geq c_2,\; k_4 \geq c_3. \tag{18}$$

where c_1, c_2, c_3 *are introduced in Eqn. (10).*

Proof. The motor is inactivated when $\dot{q}_d > 0$ and the FES control input has not yet reached the selected comfort threshold Γ since either starting the current biceps curl or decreasing to the lower threshold. Because of the signum function in the closed-loop error system in Eqn. (13), the time derivative of Eqn. (14) exists almost everywhere (a.e.), and $\dot{V}_L \overset{a.e.}{\in} \dot{\tilde{V}}_L$ [21] such that

$$\begin{aligned} \dot{\tilde{V}}_L \overset{a.e.}{=} e_1\left(e_2 - \alpha e_1\right) + \left(\frac{1}{2}\dot{M}e_2^2 - V\right)e_2^2 + e_2\chi - e_2 e_1 \\ - K\left[B_\sigma c_\sigma^{-1}\left(k_1 e_2^2 + \left(k_2 + k_3 \parallel z \parallel + k_4 \parallel z \parallel^2\right)|e_2|\right)\right], \end{aligned} \tag{19}$$

where $K[\cdot]$ is defined in [22]. Upper bounding Eqn. (19) using Prop. 7 and Eqn. (10) results in

$$\begin{aligned} \dot{\tilde{V}}_L \overset{a.e.}{\leq} -\alpha e_1^2 - k_1 e_2^2 - \left(k_2 - c_1\right)|e_2| \\ - \left(k_3 - c_2\right)|e_2| \parallel z \parallel - \left(k_4 - c_3\right)|e_2| \parallel z \parallel^2, \end{aligned}$$

where $K[\text{sgn}(\cdot)] = \text{SGN}(\cdot)$, such that $\text{SGN}(\cdot) = \{1\}$ if $(\cdot) > 0$, $[-1, 1]$ if $(\cdot) = 0$, and $\{-1\}$if $(\cdot) < 0$. Since $\dot{V}_L \overset{a.e.}{\in} \dot{\tilde{V}}_L$, further upper bounding of the Lyapunov derivative, provided the gain conditions in Eqn. (18) are satisfied, results in

$$\dot{V}_L \leq -\lambda_2 V_L(t), \tag{20}$$

where λ_2 is defined in Eqn. (17). Using Eqn. (15), the result in Eqn. (16) can be obtained. □

Theorem 2. *When the desired trajectory indicates flexion (i.e.,* $\dot{q}_d > 0$*), but the FES control input in Eqn. (11) is saturated, the motor controller in Eqn. (12) ensures exponential tracking such that*

$$\parallel z(t) \parallel \leq \sqrt{\frac{\lambda_B}{\lambda_A}} \parallel z\left(T_{n,j}^{u}\right) \parallel \exp\left[-\frac{1}{2}\lambda_3 \left(t - T_{n,j}^{u}\right)\right], \tag{21}$$

$\forall t \in [T_{n,j}^{u}, \min\left(T_{ext,n},\, T_{n,j}^{l}\right))$, *where* $T_{ext,n}$ and $T_{n,j}^{l}$ *were previously defined, and* $\lambda_3 \in \mathbb{R}_{>0}$ *is defined as*

$$\lambda_3 \triangleq \frac{1}{\lambda_B} \min(\alpha, k_{5,1}), \quad (22)$$

provided the following gain conditions are satisfied:

$$k_{6,1} \geq c_1 + c_\Sigma \Gamma,\ k_{7,1} \geq c_2,\ k_{8,1} \geq c_3. \quad (23)$$

where c_1, c_2, c_3 are introduced in Eqn. (10), c_Σ in Prop. 1, and Γ in Eqn. (11).

Proof. When the FES is activated, but has saturated at the upper threshold at least once since $T_{flex,n}$ or $T^l_{n,j-1}$, the motor is also activated so $\delta = 1$, $\beta = 1$, and $B_\sigma > 0$. Because of the signum function in the closed-loop error system in Eqn. (13), the time derivative of Eqn. (14) exists a.e., and $\dot{V}_L \overset{a.e.}{\in} \dot{\tilde{V}}_L$ [21] such that

$$\begin{aligned}\dot{\tilde{V}}_L \overset{a.e.}{=} & -\alpha e_1^2 + e_2\chi - K\left[B_\sigma e_2 \left(\mathrm{sat}_\Gamma\left(c_\sigma^{-1}(k_1 e_2\right.\right.\right. \\ & \left.\left.\left. + \left(k_2 + k_3 \| z \| + k_4 \| z \|^2\right)\mathrm{sgn}(e_2)\right)\right)\right)\Big] \\ & -K\left[k_{5,1}e_2^2 - \left(k_{6,1}\right.\right. \\ & \left.\left. + k_{7,1} \| z \| + k_{8,1} \| z \|^2\right)|e_2|\right] \end{aligned} \quad (24)$$

Noting the definitions of $K[\cdot]$ and $\mathrm{sat}_\Gamma(\cdot)$, Eqn. (24) can be expressed as

$$\begin{aligned}\dot{\tilde{V}}_L \overset{a.e.}{=} & -\alpha e_1^2 + \chi e_2 - B_\sigma e_2 \Gamma - k_{5,1} e_2^2 \\ & - \left(k_{6,1} + k_{7,1} \| z \| + k_{8,1} \| z \|^2\right)|e_2|. \end{aligned} \quad (25)$$

After using Eqn. (10) and Prop. 1, Eqn. (25) can be upper bounded as

$$\dot{\tilde{V}}_L \overset{a.e.}{\leq} -\alpha e_1^2 - k_{5,1} e_2^2, \quad (26)$$

assuming the gain conditions in Eqn. (23) are satisfied, the first of which is formed noting that $\gamma_j \leq \Gamma$, $\forall n$. Using Eqns. (15) and (22), Eqn. (21) can be obtained. □

Theorem 3. *When the desired trajectory indicates extension (i.e., $\dot{q}_d \leq 0$), only the motor is activated (i.e., $\delta = 1$, $\beta = 0$, $B_\sigma = 0$), and the motor controller in Eqn. (12) results in global exponential tracking in the sense that*

$$\| z(t) \| \leq \sqrt{\frac{\lambda_B}{\lambda_A}} \| z(T_{ext,n}) \| \exp\left[-\frac{1}{2}\lambda_1 (t - T_{ext,n})\right], \quad (27)$$

$\forall t \in [T_{ext,n}, T_{flex,n+1})$, and $\lambda_1 \in \mathbb{R}_{>0}$ is defined as

$$\lambda_1 \triangleq \frac{1}{\lambda_B} \min(\alpha, k_{5,0}), \quad (28)$$

provided the following gain conditions are satisfied:

$$k_{6,0} \geq c_1,\ k_{7,0} \geq c_2,\ k_{8,0} \geq c_3. \quad (29)$$

where c_1, c_2, c_3 are introduced in (10).

Proof. Because of the signum function in the closed-loop error system in Eqn. (13), the time derivative of Eqn. (14) exists a.e., and $\dot{V}_L \overset{a.e.}{\in} \dot{\tilde{V}}_L$ [21] such that

$$\begin{aligned}\dot{\tilde{V}}_L \overset{a.e.}{=} & e_1(e_2 - \alpha e_1) + \left(\frac{1}{2}\dot{M} - V\right) e_2^2 + e_2\chi - e_2 e_1 \\ & - K\left[k_{5,0}e_2^2 + \left(k_{6,0} + k_{7,0} \| z \| + k_{8,0} \| z \|^2\right)|e_2|\right], \end{aligned} \quad (30)$$

Cancelling common terms and using Prop. 7 and Eqn. (10) allows Eqn. (30) to be upper bounded as

$$\begin{aligned}\dot{\tilde{V}}_L \overset{a.e.}{\leq} & -\alpha e_1^2 - k_{5,0} e_2^2 - (k_{6,0} - c_1)|e_2| \\ & - (k_{7,0} - c_2)|e_2| \| z \| - (k_{8,0} - c_3)|e_2| \| z \|^2. \end{aligned}$$

Further upper bounding of the Lyapunov derivative results in

$$\dot{V}_L \leq -\lambda_1 V_L(t), \quad (31)$$

where λ_1 is defined in Eqn. (28). Using Eqn. (15), the result in Eqn. (27) can be obtained. □

Remark 4. Using Eqns. (20), (26), (31) and Thms. 1-3, a common bound is created for the Lyapunov derivative, $\dot{V}_L$, as $\dot{V}_L \overset{a.e.}{\leq} -\lambda_s V_L$, and hence, the controllers in Eqn. (11) and Eqn. (12) yield global exponential tracking $\forall t \in [t_0, \infty)$, such that

$$\|z(t)\| \leq \sqrt{\frac{\lambda_B}{\lambda_A}} \|z(t_0)\| \exp\left[-\frac{1}{2}\lambda_s (t - t_0)\right], \quad (32)$$

where $\lambda_s \in \mathbb{R}_{>0}$ is defined as $\lambda_s \triangleq \min(\lambda_1, \lambda_2, \lambda_3)$. From [23, Th. 2.1, Remark 2.1], since all subsystems share the radially unbounded common Lyapunov function in Eqn. (14), global exponential convergence to the desired trajectory holds true in all cases, according to Eqn. (32).

5 EXPERIMENTS

The performance of the controllers in Eqn. (11) and Eqn. (12) was demonstrated on two participants with neurological conditions that impaired their right arm. The first subject had post-polio syndrome and the second subject had both a spinal cord injury (SCI) and an elbow that had been surgically removed and autografted with shoulder tissue, preventing any supination. Average position and velocity errors for the impaired and unimpaired arms of each subject are compared in Tab. 1.

5.1 Arm Testbed

The testbed used for the experiments in this study was composed of two aluminum plates, one of which the upper arm rested on and the other of which was strapped to the forearm and rotated about a hinge aligned with the elbow. The designed motor controller was applied to a 27 Watt, brushed, parallel-shaft 12 VDC gearmotor and the FES controller regulated the pulsewidth sent to the biceps brachii via a Hasomed stimulator and six 0.6" x 2.75" PALS® electrodes. The controllers were implemented using real-time control software (QUARC, MATLAB 2015b/Simulink, Windows 8). For consistent biceps coverage and evenly spaced electrode placement, the first electrode was placed at 21% of the distance from the elbow crease to the acromion, the sixth electrode at 50% of this distance, and the other four biceps electrodes spaced evenly between the first and last. A seventh electrode (3" x 5") was placed on the shoulder as the reference electrode for all six biceps electrode channels. Based on comfort and necessary torque values, stimulation amplitude was fixed at a current of 30 mA with a frequency of 35 Hz for each channel, while the closed-loop FES controller modulated the pulse-width.

5.2 Protocol

After all seven electrodes were placed on the subject's upper arm, the subject was seated such that the upper arm and forearm could comfortably rest on their respective parts of the testbed. The desired angular position, q_d, of the forearm was selected as $q_d(t) = \begin{cases} \frac{7\pi}{18}\left(1-\cos\left(\frac{\pi}{2}\frac{t-5}{T}\right)\right)+\frac{\pi}{9}, & t \geq 5 \\ 4t, & t < 5 \end{cases}$, where the period, T, or amount of time for the forearm to move from 20 to 90 degrees, was 5 seconds. The motor first brought the arm to 20 degrees, which was found to be the beginning of the region where the muscle could always produce sufficient torque, and from there 10 biceps curls were completed.

The control gains, $\{k_i\}_{i=1,..,4}$, $\{k_{i,\beta}\}_{i=5,...8}$, introduced in Eqn. (11) and Eqn. (12), were adjusted to yield acceptable tracking performance with values for both the right and left arms as follows: $k_1 = 25$, $k_2 = k_3 = k_4 = 1$, $k_{5,0} = 15$, $k_{5,1} = 35$, $k_{6,\beta} = k_{7,\beta} = k_{8,\beta} = 1$. A saturation limit for the muscle control input was established based on comfort. The decay constant for γ_j was selected as $\rho = 0.8$. When the muscle control input was below saturation, electrical stimulation was used to control the forearm from 20 to 90 degrees, whereas both muscle stimulation and the DC motor were used at any point that the muscle controller reached the saturation limit. Only the DC motor brought the forearm from the highest forearm angle (90 degrees) to the starting position (20 degrees). The set of channels used to stimulate within the muscle control region (i.e., during flexion) varies with angular position as in [15], where $\varepsilon = 0.22$ was selected as the normalized torque threshold for all but the impaired right arm of the subject 1, which was set to 0.10 due to no electrode locations producing sufficient isometric torque.

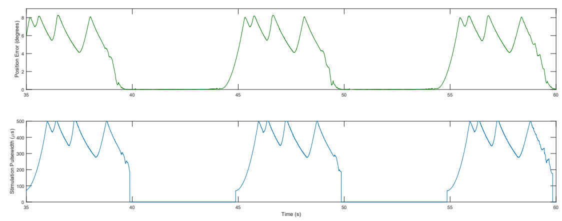

FIGURE 1: Position error and stimulation pulsewidth (i.e., FES input) for the right arm of subject 2 during trials where the lower stimulation threshold iteratively decreased according to the constant $\rho = 0.8$. The zoomed view of biceps curls 4-6 is provided to easily compare the change in FES control input to the position error.

5.3 Results

Results from all four experiments (right and left arms of two subjects) are included in Tab. 1, which presents the position and velocity RMS errors, as well as the FES and motor control inputs, averaged over times of desired flexion. Fig. 1 shows both the position error and FES control input (stimulation pulsewidth) for the right (impaired) arm of Subject 2.

6 DISCUSSION

As seen in Tab. 1, the position and velocity errors of the impaired and unimpaired arms for both subjects are similar, despite each having movement disorders that significantly limit their impaired arm in daily activities. Thus, the motor and FES controllers developed in this paper enable a subject with muscular asymmetries to perform similar tasks. Moreover, the motor only contributes as needed and the FES activates the biceps throughout flexion.

In [15], exponential tracking is achieved and the motor assists as needed when the stimulation comfort threshold Γ is reached; however, since it only assists for an instant before the error drops and the stimulation falls below the single threshold Γ, the motor is activated and deactivated frequently, to the point of chattering, in addition to the chattering due to sliding mode cotnrol. In the current development, the motor continues to assist the muscle until the lower threshold γ_j is reached by u_m, and

TABLE 1: Average position and velocity errors, FES control input, and motor control input for both arms (one impaired, one unimpaired) for both subjects. S1 and S2 denote subjects 1 and 2; R and L denote the right and left arms.

	RMS Position Error (deg)	RMS Velocity Error (deg/s)	Average FES Control Input (μs)	Average Motor Control Input (Amps)
S1, Impaired/R Arm	4.26	3.70	286.7	2.08
S1, Unimpaired/L Arm	3.75	4.33	317.6	1.61
S2, Impaired/R Arm	4.83	5.56	354.0	1.79
S2, Unimpaired/L Arm	4.96	5.04	346.0	1.67

motor assistance is deactivated. The constant ρ was used to decrease the lower threshold after every time the comfort threshold was reached in a single biceps curl. Lowering the lower threshold was motivated by the expectation that as the muscle fatigues, the FES control input would rise quicker to the comfort threshold after each successive bout of motor assistance. Thus, to prevent the motor from turning on and off more quickly towards the end of a biceps curl, the motor remains activated over a longer range of biceps curl angles. However, if desired, $\rho = 1$ would cause the lower threshold γ_j to remain constant throughout the protocol.

As seen in Fig. 1, as an example of a typical portion of an experiment, changes in the stimulation pulsewidth mirrors changes in the position error. The relation is dependent on control gains; however, with a high dependence on the position error due to $\alpha = 40$ being selected (i.e., e_2 is 40 times more dependent on the position than the velocity error), the control input nearly mirrors the position error, which decreases during the bouts of continuous motor assistance.

The control technique in this paper may depend on muscle delay even more so than other FES protocols [24, 25]. Because the motor instantaneously switches off after the γ_j condition is met, the muscle must react to the rapid increase in stimulation back to Γ that often occured, as seen in Fig. 1, which is likely a combination of fatigue, an insufficiently high comfort threshold, and/or muscle delay.

7 CONCLUSION

The muscle and motor track a desired forearm trajectory resembling a typical biceps curl. FES is the primary actuator for controlling the arm movement since it is desired to work the muscle as much as possible; however, the motor assists in tracking when the stimulation input reaches the subject's comfort threshold. To avoid chattering and to allow the error and stimulation to decay, even briefly, the motor continues to assist until the calculated stimulation input decreases to a lower threshold that discretely changes depending on controller performance. Switched sliding mode controllers are designed for both the FES and motor control input and exponential tracking is proved via Lyapunov methods. Experimental data is obtained from two subjects with neurological disorders that cause asymmetrical impairments.

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