A Capacity-based Sensing Matrix Design Method for Block Compressive Imaging Applications

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***Abstract*—In many compressive imaging applications, it can be difficult to design sensing matrices with suitable reconstruction capabilities. In this paper, we presents a novel method, based upon capacity maximization, for designing sensing matrices with enhanced block-sparse signal reconstruction capabilities. Numerical results, which demonstrate the design method's capabilities in a practical imaging application, are presented.**

I. Introduction

A classical problem in science and engineering is reconstructing an unknown vector $\mathbf{x} \in \mathbb{C}^N$ from a set of linear measurements $\mathbf{y} = \mathbf{A}\mathbf{x} \in \mathbb{C}^M$. When $M < N$, there exist an infinite number of solutions satisfying $\mathbf{y} = \mathbf{A}\mathbf{x}$; and, therefore, regularization techniques need to be employed in order to induce a unique solution. In practice, the regularization term is selected from prior knowledge of the unknown vector. When the vector is known to be sparse, then Compressive Sensing (CS) theory [1]–[3] states that it can be recovered exactly as the solution to a convex and computationally tractable ℓ_1-norm minimization problem, provided that the sensing matrix is "well-behaved" according to a performance metric such as the mutual coherence [4] or the Restricted Isometry Property (RIP) [5].

CS theory also considers the case where the unknown vector is block sparse. When a signal is block sparse, the non-zero values are distributed over $K = N/L$ disjoint blocks of size L. Although block sparse signals can be reconstructed using the standard techniques, such as ℓ_1-norm minimization, applied to general sparse signals, specialized techniques [6]–[11] based on joint ℓ_2/ℓ_1 minimization have been shown to provide better reconstruction performance. In practice, it is difficult to deterministically generate sensing matrices with enhanced reconstruction capabilities; and, therefore, many researchers resort to using randomized sensing matrices. Unfortunately, this approach does not achieve the desired results in many applications, such as electromagnetic imaging, where the elements of the sensing matrix are constrained by practical limitations. In this paper, we introduce a method based upon maximizing the sensing capacity [12], [13] for designing sensing matrices with enhanced block sparse signal reconstruction capabilities.

II. Capacity-based Design Method

Suppose that the sensing matrix $\mathbf{A} \in \mathbb{C}^{M \times N}$ is a function of $\mathbf{p} \in \mathbb{C}^P$ design variables according to the nonlinear and differentiable relationship $\mathbf{A} = \mathbf{F}(\mathbf{p})$. Without loss of generality, we will assume that this function outputs the sensing matrix with normalized columns. We define the projection matrices $\mathbf{\Phi}_r \in \{0,1\}^{N \times M_r},\ r = 1, \ldots, R$ for the R blocks on which the capacity will be evaluated. The design algorithm then seeks the minimizer to the following non-convex optimization program:

$$\underset{\mathbf{p}}{\text{minimize}} \quad \max_{r=1,\ldots,R} -\log\det\left(\mathbf{\Phi}_r^T \mathbf{F}^H(\mathbf{p})\mathbf{F}(\mathbf{p})\mathbf{\Phi}_r + \beta \mathbf{I}_{M_r,M_r}\right) \tag{1}$$

$$\text{subject to} \quad \mathbf{p} \in Q_p$$

where β is a small positive constant that ensures that the arguments to $\det$ are positive-definite, and Q_p is the feasible set for the design variables. In other words, this optimization program seeks the design vector $\mathbf{p}$ that maximizes the smallest capacity of the sub-matrices $\mathbf{F}(\mathbf{p})\mathbf{\Phi}_r,\ r = 1, \ldots, R$.

III. Numerical Results

The design algorithm was applied to an electromagnetic imaging application, in which a single transmitting and receiving antenna is used to excite a region of interest with a single frequency. The discretized measurement process for this system can be modeled as follows:

$$y_m = \sum_{n=1}^{N} x_n e^{-j2k\|\mathbf{r}_m - \mathbf{r}_n\|_{\ell_2}} = \sum_{n=1}^{N} A_{mn} x_n \tag{2}$$

where y_m is the $m-$th scattered field measurement, $\mathbf{r}_m$ is the position of the $m-$th antenna, $\mathbf{r}_n$ is the $n-$th position in the imaging region, k is the wavenumber, and x_n is the reflectivity at the $n-$th position in the imaging region. Keeping the wavenumber fixed, the objective is to select the antenna positions $\mathbf{r}_m$ such that the minimum capacity over the set of blocks is optimized.

Table I displays the design parameters and constraints. Figure 1 displays the positions of the baseline random antenna configuration, which was used as the starting point to the optimization procedure, and the positions of the optimized antenna configuration. The shaded blocks in the background of Figure

1 represent the nine blocks on which the unknown signal is block-sparse. The optimization procedure was configured so that the minimum capacity of block pairs (36 in total) was maximized. The minimum capacity was increased from -12.6 in the baseline design to -3.3 in the optimized design. This directly led to an improvement in CS reconstruction accuracy, as can be seen in Figure 2. The optimized antenna positions were able to reconstruct $> 90\%$ of block-sparse vectors up to a block sparsity $S_B = 2$ (total sparsity $S = 32$), whereas the baseline random positions reconstructed $< 20\%$. The fact that the optimized design performs so well up to the theoretical maximum sparsity level, $M/2 = 32$, truly demonstrates the capabilities of the design method.

Design Parameters and Constraints		
Parameter	Baseline Value	Constraint
M	64	–
N	144	–
K	9	–
L	16	–
$\mathbf{r}_n$	5λ by 5λ grid centered at origin	–
$\mathbf{r}_m$	Uniformly spaced over 5λ by 5λ grid at $z = 5\lambda$	$\lvert x_m\rvert \leq 2.5\lambda$ $\lvert y_m\rvert \leq 2.5\lambda$ $z_m = 5\lambda$

TABLE I
SUMMARY OF DESIGN PARAMETERS AND CONSTRAINTS IN THE OPTIMIZATION PROBLEM.

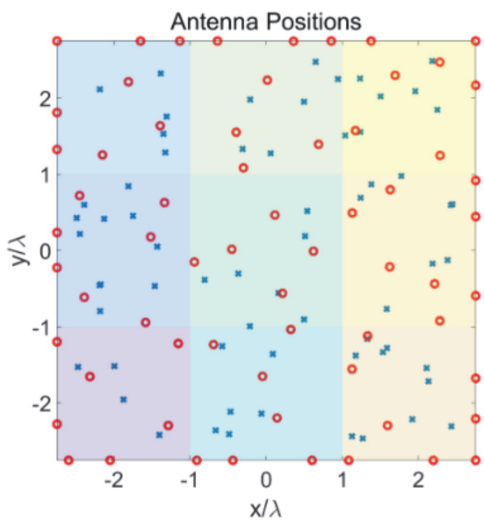

Fig. 1. Antenna positions of the baseline (blue) and optimized (red) designs. The shaded boxes in the background represent the squares on which the capacity was evaluated.

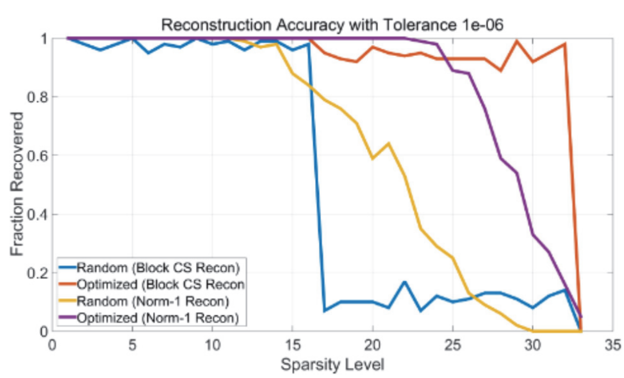

Fig. 2. Numerical comparison of the reconstruction accuracies of joint ℓ_2/ℓ_1 reconstruction and standard ℓ_1 reconstruction using the baseline random (blue) and optimized (red) designs.

IV. Conclusion

This paper describes a novel method for designing sensing matrices with enhanced block-sparse signal recovery capabilities. The numerical results presented for a monostatic imaging application demonstrate the design method's capabilities. Although this paper only considered a simple monostatic imaging application, the design method can be applied to any compressive sensing application where the unknown signal is known to be block sparse.

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