

Math Difficulties and Working Memory Growth in ELL children:

Does Bilingual Proficiency Play a Significant Role?

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Abstract

Purpose

The purpose of this study was to determine those components of working memory (WM) that play a significant role in predicting math growth in children who are English language learners ($N=157$) with serious math difficulties (MD).

Method

A battery of tests was administered in English and Spanish that assessed computation, reading, vocabulary, inhibition and components of WM in grade 1 children with follow-up testing in grades 2 and 3.

Results

The results indicated that growth in the executive component of WM was related to growth in math performance. Proficient bilingual children (proficient in both Spanish and English vocabulary) with MD outperformed less proficient bilingual children with MD children on measures of math calculation, fluid intelligence, reading and Spanish WM at grade 3.

Conclusion

Growth in the executive component of WM is significantly related to growth in math computation and increased bilingual proficiency across testing waves yielded positive gains in both math and cognitive performance in children with MD.

Key Words: English Language Learners, Working Memory, Math Difficulty, Longitudinal, Bilingual

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Does Bilingual Proficiency Play a Significant Role?

Children with Spanish as a first language in the United States have been found to yield low mathematics scores when compared to ELL other groups on national assessments across several years (e.g., NAEP, National Assessment of Education Progress, 2004, 2015, 2017).

Although closing achievement gaps has been a goal in national and state education policies, the average mathematics scores for non-ELL students in grades 4 and 8 have been higher than the scores of then ELL student whose first language is Spanish since 1996 (e.g., August & Hakuta, 1997; Bumgarner, Martin, Brooks-Gunn, 2013). No doubt, there are long-term implications related to this achievement gap. Serious difficulties in math in the elementary grades have been shown to have detrimental effects on high school performance (e.g., drop rates) as well as later employment (e.g., Grégoire & Desoete, 2009; Polk, 2016).

Although some of the difficulties in math experienced by ELL children with Spanish as a first language have been partially attributed to cross language transfer, oral language, linguistic complexity, and reading skill (e.g., Farnia & Geva, 2011; Han, 2012; Macizo, Herra, roman, & Martin, 2011; Martiniello, 2008; 2009; Ockey, 2007; Vukovic & Lesaux, 2013), other processes besides language may play a critical role in such children's math difficulties (MD). Therefore, it is important to determine some of the cognitive measures that predict success on computation measures for Spanish speakers so intervention programs can be developed and tested. There is recent evidence to suggest that one domain-general cognitive process, working memory (WM), plays a significant role in math for monolingual children who suffer from serious math difficulties (e.g., Swanson 2011; Swanson & Beebe-Frankenberger, 2004). Working memory is

defined as consisting of a limited capacity system of temporary stores, functions related to the preservation of information while simultaneously processing other information and attention control related to these functions (e.g., Baddeley, 2012). Previous studies have found that growth in WM is significantly related to growth in math for monolingual children with math disabilities (e.g., Swanson, Jerman, & Zheng, 2008), and therefore it was of interest in determining whether WM growth may also play a significant role in computation development in ELL children suffering from math difficulties.

One framework to capture diverse memory processes as they apply to second-language math performance is Baddeley's multicomponent WM model (Baddeley & Logie, 1999). Clearly, there are other models (Cowan, 2005, 2014; Engle et al., 1999), but the present study focuses on Baddeley's model since it is commonly used to capture academic difficulties in children (see Swanson & Alloway, 2012, for review). This multicomponent model characterizes WM as comprising a central executive controlling system that interacts with a set of two subsidiary storage systems: the speech-based phonological loop and the visual-spatial sketchpad. According to Baddeley (Baddeley, 2012; Baddeley & Logie, 1999), the central executive coordinates the two systems, focusing and switching attention, and activating representations within long-term memory (LTM). This model has been revised to include an episodic buffer (Baddeley, 2012), but support for the tripartite model has been found across various age groups of children (Gathercole Pickering, Ambridge & Wearing., 2004).

One component of WM that has been associated with math difficulties is executive processing (e.g., Blair Ursache, Greenberg, & Vernon-Feagan, 2015; Bull & Scerif, 2001; Cai, Georgiou, Wen, & Das, 2016; Martin et al., 2013; Menon, 2016; Swanson & Beebe-Frankenberger, 2004). Bilingual children are viewed as experiencing some advantages in

executive processing when compared to monolingual children (e.g., Bialystok, 2007; Blom, Kuntay, Messer, Verhagen & Leseman, 2014). That is, several studies suggest that bilingual children's continual practice in inhibiting the irrelevant language in order to communicate effectively in the relevant language enhances cognitive performance (e.g., Bialystok, 2007; Morales, Calvo, & Bialystok, 2013). Because such inhibition practices have been attributed to executive processing (e.g., Alloway, Gathercole, Willis, & Adams, 2004; Baddeley, 1996; Friedman et al., 2007), one would expect that variations in the executive component of WM among bilingual children would play an important role in predictions of math proficiency. However, although some studies demonstrate a bilingual benefit in WM (e.g., Morales et al., 2013), not all studies find such an effect (e.g., de Bruin, Barbara & Della Sala, 2015; Namazi & Thordardotir, 2010). Thus, the role of the executive component of WM on growth in math computation for bilingual children is unclear.

It is important to note that the majority of these studies on executive processing and bilingualism have focused children who learned L1 and L2 simultaneously. However, ELL children in U.S. public schools frequently represent children who learn L1 first and L2 later. Thus, few studies have focused on sequential bilinguals (who learn their L1 first, then L2 later) with different levels of language proficiency on executive processing and math. If bilingualism influences cognitive processes, it is possible that a positive cognitive impact on math performance appears as bilinguals gain higher degrees of bilingual proficiency. However, the relationship between sequential bilinguals' cognitive, language and math proficiency is unclear. Cummins's threshold hypothesis (1979) suggests that a certain threshold level of language proficiency in L1 and L2 should be attained to demonstrate the cognitive advantages of

bilingualism. Therefore, it is important to examine WM and math performance in relation to the degree of bilingualism acquired.

In summary, the purpose of this study was to determine if growth in math performance is related to growth in WM in ELL children. Because executive processing is associated with bilingualism and WM, we tested the hypothesis that growth math is related to the executive component of WM. This hypothesis is tested through a series of mixed regression models. Current studies suggest that when phonological STM is partialled out from the effects of WM on achievement (math) measures, the remaining residual variance in regression modeling reflects the executive or controlled attention component of WM (e.g., Engle, Tuholski, Laughlin, & Conway, 1999). Thus, consistent with the literature, the importance of the executive component of WM in predictions of math performance was assessed by entering STM (phonological storage) into the regression analyses. In general, we expect that significant increases in math proficiency will be related to growth in the executive component of WM. Further, regardless of bilingual proficiency, because WM is viewed as an important component of both language acquisition and math processes (e.g., Abreu & Gathercole, 2012), children with higher and lower WM would be expected to yield observable differences in math performance. Likewise, children with lower WM and math performance but with higher levels of bilingual proficiency would be expected to show greater improvement in math as well as WM than children with MD but lower in bilingual proficiency. This prediction is based on studies (e.g., Morales et al., 2013) showing that when comparing monolingual and bilingual children, executive processing plays an important role in accounting for the cognitive advantages of bilingual children.

Two questions direct this study:

1. Does growth in the executive component of WM predict growth in math performance in children who vary in bilingual proficiency?

Because the literature suggests there are cognitive advantages to increasing L2 proficiency, i.e., bilingualism, one would expect a strong relationship between WM and math as ELL children become increasingly proficient in both languages. This relationship can be inferred from a number of studies that have shown that proficiency in L1 and L2 (i.e., bilingualism) positively affects executive functioning, flexibility, and intentional control (e.g., Bialystok, 2007; Bialystok & Martin, 2004). These studies suggest that navigating between two languages, as well as frequent opportunities to inhibit one language when using the other, to shift between languages and hold linguistic information in mind while manipulating another is related to the development of executive processes (e.g., Bialystok, 2007, 2011; Bialystok & Martin, 2004). Because these processes have been attributed to WM (Engle, 2002; Friedman et al., 2007), individual differences related to bilingualism are assumed to play an important role in WM performance.

2. Do children with MD who become increasingly proficient in both languages excel developmentally in math performance when compared less proficient bilinguals with MD?

For this study, children with MD who varied in bilingual proficiency were identified in grade 1. After controlling for reading and aptitude measures at grade 1, we determined whether differences in math skills emerged between the two groups in the later grades. Although the ELL children in this study were bilingual to some degree, it is assumed that an increased transition to L2 will facilitate academic progress. Therefore, we test the hypothesis that after controlling for measures of reading, fluid intelligence and related processes, that children with MD who are weak bilinguals fall further behind their MD counterparts who are becoming

increasingly proficient bilinguals in the later grades on measures of math and the executive component of WM.

Methods

Participants

Data for this study were derived from two larger federally-funded studies investigating the role of cognitive processes on academics achievement in children at risk (e.g., Swanson, Orosco, & Lussier, 2015). A total of 157 ELL students in Grades 1 were selected from four large school districts in the U. S. Southwest. All children participating in the study were designated as English language learners (ELLs) based on school administration of the California English Language Development Test (CELDT). Ninety-seven percent of the children participating in this program participated in a Federal lunch program. All children were Hispanic and exposed to both English and Spanish languages. Appendix A reports the mean standard scores on math computation, English and Spanish vocabulary and reading measures. Also reported are the normed-referenced scores on a measure of fluid intelligence (Raven Colored Progressive Matrices Test; Raven, 1976). As noted, fluid intelligence and reading scores were in the normal range for this sample, whereas overall vocabulary scores were in the low average range.

Performance of children at grade 1 was assessed in the spring and these children were retested in the spring of grade 2 and grade 3. Performance patterns across the three grades are reported in Appendix A. Parent interviews indicated that in 90% of households the children's primary current home spoken language was Spanish.

Math difficulty status. In terms of common cut-off score designations for children at risk for math difficulties (MD), the 25th percentile on a normed referenced math measures is

commonly used to designate children at risk for MD, and therefore, it is useful to use cut-off scores as practiced in the schools (Fuchs et al., 2006; Geary, 2011; Geary, Hoard, Nugent, 2012). Consistent with other studies (e.g., Martin et al., 2013), math computation from the Wide Range Achievement Test (Wilkinson, 2003; with English and Spanish instructions) was used to identify children at risk for MD. Children who performed in the lower 25th percentile in math calculation at Grade 1 were considered at risk for MD. Thus, our criteria for identifying participants at risk for MD was performance in the average range (85 to 120) on measures of fluid intelligence (Progressive Matrices Test; Raven, 1976) and below the 25th percentile on a norm-referenced measure of math computation.

Bilingual status. As with the definition of MD, the definition of ELL in terms of emerging bilingual proficiency is also controversial (i.e., whether to use expressive vs. receptive language, the frequency of English spoken at home, etc.). The first language for all children participating in this study was Spanish and performance on the California English Language Development Test (CELDT) was at a low level of proficiency at grade 1. However, our sampling of ELL children yielded variations in English and Spanish vocabulary. Thus, two math subgroups (children at risk and not at risk for MD) were further divided into proficient bilinguals (those children with relatively higher overall vocabulary scores) and less proficient bilinguals (those children with lower overall vocabulary scores). Children whose average vocabulary scores in English and Spanish was above a norm-referenced standard score of 85 were considered proficient bilinguals and those with average scores at or below a standard score of 85 were considered less proficient bilinguals.¹

Measures

The study included group and individual administrations of a battery of tests. The series

of tests were counterbalanced into one of six presentation orders. No Spanish and English versions of the same test were presented simultaneously. Several measures that required Spanish-translated versions were developed in previous studies (e.g., Swanson, Sáez, Gerber, & Leafstedt, 2004; Swanson, Orosco, Lussier, Gerber & Guzman-Orth, 2011). All participants were administered both English and Spanish versions of each measure. The description below indicates whether tests were administered individually or in small groups (four to six children). Instructions were given in Spanish for all tasks requiring Spanish responses unless noted otherwise. There were some tasks (e.g., memory tasks) that required calibration for task difficulty.² Three native Spanish speakers made judgments on the difficulty of the items in relation to the task presented in English. Interrater agreements exceeded 90%. Appendix B reports the sample reliability for each task.

Classification Measures

Math calculation. The arithmetic subtest from the Wide Range Achievement Test (WRAT-III; Wilkinson, 2003) was administered to measure basic calculation ability at grades 1, 2, and 3 of the study. The WRAT-3 subtest required the child to perform written computation on number problems that increased in difficulty. The items vary from single-digit addition ($2 + 2 = ?$) to more advanced skills such as algebra. The WRAT-3 math calculation subtest allows up to 15 minutes for students to complete math calculations. The dependent measure was the number of problems correct (raw score range was 15-55), which yielded a standard score ($M = 100$, $SD = 15$). The test was administered in both English and Spanish. Norms for the WRAT-3 assessment were originally derived from a sample of 4,433 English speaking people, age 5-75, across four regions of the US. For the WRAT-3, internal consistency was adequately measured by coefficient alphas and was reported for the math subtest (Arithmetic - Blue Form) as .81 to

.92 for the age ranges involved in the present study. Test-retest for the WRAT-3 coefficients ranged from .91 to .98 on the subtest.

Vocabulary-English. The Peabody Picture Vocabulary Test (PPVT-III; Dunn & Dunn, 1981) was administered to assess English receptive vocabulary knowledge. Children were presented with four pictures and were asked, after hearing a word spoken in isolation, to select the picture that matched the meaning of the word. The technical manual states a parallel form reliability of .91.

Vocabulary-Spanish. The Test de Vocabulario en Imagenes (TVIP) is similar to the PPVT-III in the presentation and administration (Dunn, Lugo, Padilla, & Dunn, 1986). Children were presented with four pictures and asked to identify the picture for a word read aloud in Spanish. The split-half reliability presented in the manual was .91 to .94.

Reading-English and Spanish. The Woodcock-Muñoz Language Survey-Revised (WMLS-R) Spanish and English word identification tests were administered to establish a normed-referenced reading level in English and Spanish (Woodcock, 1998; Woodcock, Muñoz-Sandoval, & Alverado, 2005). The test reliabilities range from the mid-.70s to high-.90s for the word identification tests in the various age clusters.

Fluid (nonverbal) intelligence. It was necessary to ensure that the sample did not have low general intellectual performance. Thus, the Raven Colored Progressive Matrices (CPM; Raven, 1976) was used as an indicator of nonverbal or fluid intelligence. Children were given a booklet with patterns displayed on each page, which revealed a missing piece. For each pattern, six possible replacement pieces were displayed. The dependent measure was the number of matrices solved correctly. The technical manual reports internal consistency reliability ratings

ranging from .80 to .90.

Working Memory Measures

Short-term memory (STM) measures (phonological loop). Three measures of STM were administered in Spanish and English: Forward Digit Span, Word Span, and Pseudoword Span. The Forward Digit Span task (taken from the WISC-III; Wechsler, 1991) and a Spanish translated version were administered. The Forward Digit Span task required children to recall sequentially ordered sets of digits that increased in set size, which were spoken by the examiner. The technical manual reported a test-retest reliability of .91. The dependent measure was the largest set of items recalled in order (range = 0 to 8). For the translated Spanish version of the Digit Span subtest, identical numbers were presented in the same order as the English version. There were no deviations in the procedure, except for language use.

The Word Span and Pseudoword Span tasks were presented in the same manner as the Forward Digit Span task. In the Word Span task examiners read lists of one- or two-syllable, high frequency words that included unrelated nouns and then asked the children to recall the words. Word lists gradually increased in set size, from a minimum of two words to a maximum of eight. The Pseudoword Span task (Phonetic Memory Span task) uses strings of one-syllable nonsense words, which are presented one at a time in sets of 2 to 6 nonwords (e.g., DES, SEEG, SEG, GEEZ, DEEZ, DEZ). A parallel version was developed in Spanish for the Word Span and Pseudoword Span tests. The dependent measure for all STM measures was the highest set of items retrieved in the correct serial order (range = 0 to 7).

Executive component of WM. Conceptual Span, Listening Sentence Span, and Updating task were administered in English and Spanish to capture the executive component of WM. Previous studies have shown that these measures load on the executive component of WM

(see Swanson, 2008). The WM tasks required children to hold increasingly complex information in memory while simultaneously responding to a question about the task. For example, after children listened to a list of words they were asked, “Which word from the list did I say, X or Y?” They were then asked to recall words from the list. This balance of simultaneous storage and processing is consistent with a number of studies of WM processing, including Daneman and Carpenter's (1980) seminal WM measure. A previous study (Swanson, 1996) with a different sample, established the reliability and the construct validity of the WM measures with the Daneman and Carpenter measure.

The Conceptual Span task (Swanson, 2008) was used as an indicator of WM processing that involves the ability to organize sequences of words into abstract categories. Children listened to a set of words that, when re-organized, could be grouped into meaningful categories. For example, they were told a word set, such as, “shirt, saw, pants, hammer, shoes, nails.” After answering the distracter question, they were asked to recall the words that “go together” (i.e., shirt, pants, and shoes; saw, hammer, and nails). The range of set difficulty was two categories containing two words each to four categories with four words each. A Spanish-translated version was also administered. Care was taken in the development of the measure to keep the abstract categories the same in both languages (e.g., clothes and tools); however, WM-level appropriate words were used in cases where direct translation resulted in significantly harder words to recall. The dependent measure for both versions was the number of sets recalled correctly (range = 0 to 6).

The children's adaptation of Daneman and Carpenter's (1980) Listening Sentence Span task was administered. This task required the presentation of groups of sentences, read aloud, for which children tried to simultaneously understand the sentence contents and to remember the last

word of each sentence. The number of sentences in the group gradually increased from two to six. After each group of sentences was presented, the child answered a question about a sentence and then was asked to recall the last word of each sentence. The dependent measure was the total number of correctly recalled word items in order up to the largest set of items (e.g., set 1 contained 2 items, set 2 contained 3 items, set 3 contained 4 items, etc.), in which the process question was also answered correctly.

Because WM tasks were assumed to tap a measure of controlled attention referred to as updating (e.g., Miyake, Friedman, Emerson, Witzki, & Howerter, 2000), an experimental Updating task, adapted from Swanson et al. (2004), was also administered. A series of one-digit numbers was presented that varied in set length. No digit appeared twice in the same set. The examiner told the child that the length of each list of numbers might be 3, 5, 7, or 9 digits. Children were then told that they should only recall the last three numbers presented. Each digit was presented at approximately one-second intervals. After the last digit was presented the child was asked to name the last three digits in order. The dependent measure was the total number of sets correctly repeated (range = 0 to 16).

Visual-spatial WM (sketchpad). Two measures were administered to assess visual-spatial WM: Visual Matrix and Mapping & Directions tasks (Swanson, 2008). The Visual Matrix task assessed the ability of participants to remember visual sequences within a matrix. Participants were presented a series of dots in a matrix and were allowed 5 seconds to study the matrix. The matrix was then removed and participants were asked, in both English and Spanish, "Are there any dots in the first column?" To ensure the understanding of columns prior to the test, participants were shown the first column location and then practiced finding it on blank matrices. In addition, for each test item the experimenter pointed to the first column on a blank

matrix (a grid with no dots) as a reminder of first column location. After answering the discrimination question, students were asked to draw the dots they remembered seeing in the corresponding boxes of their blank matrix response booklet. The task difficulty ranged from a matrix of 4 squares and 2 dots to a matrix of 45 squares and 12 dots. The dependent measure was the number of matrices recalled correctly (range = 0 to 11).

The Mapping and Directions task required children to remember a sequence of directions on a map. The experimenter presented a street map with dots connected by lines; arrows illustrated the direction a bicycle would go to follow this route through the city. The dots represented stoplights, while lines and arrows mapped the route through the city. The child was allowed 10 seconds to study the map. After the map was removed, the child was asked a process question (i.e., "Were there any stop lights on the first street (column)?"). The child was then presented a blank matrix on which to draw the street directions (lines and arrows) and stop lights (dots). Difficulty ranged on this subtest from 4 dots to 19 dots. The dependent measure was the highest set of correctly drawn maps (range = 0 to 9), in which the distracter process question was also answered correctly.

Inhibition measure. A random generation task was used to assess inhibition in this study. The use of Random Generation tasks has been well articulated in the literature as a measure of inhibition (e.g., Baddeley, 1996; Cooper, 2016; Towse & Cheshire, 2007). The task is considered to tap inhibition because participants are required to actively monitor candidate responses and suppress responses that would lead to well-learned sequences, such as 1-2-3-4 or a-b-c-d (Baddeley, 1996). The Random Number and Random Letter Generation Tasks were administered to assess inhibition (Swanson & Beebe-Frankenberger, 2004). Depending on whether the English or Spanish version was administered, children were first asked to write, as

quickly as possible, numbers (or letters) in a non-random sequential order to establish a baseline. They were then asked to write numbers as quickly as possible, out of order, in a 30-second period. Scoring included an index for randomness, information redundancy, and percentage of paired responses to assess the tendency of participants to suppress response repetitions. The measure of inhibition was calculated as the number of sequential letters or numbers, minus the number of correctly unordered numbers or letters, divided by the number of sequential letters or numbers, plus the number of unordered letters or numbers.

Procedures

Children were tested individually after informed consent was obtained for participation. Two sessions of individual testing were conducted, each lasting thirty minutes (for limited English or Spanish speakers) to one hour. The presentation order of tasks was counterbalanced into one of six presentation orders. Children were randomly assigned to each participation order and randomly assigned to an examiner. No Spanish and English versions of the same test were presented consecutively.

Statistical Analyses

The statistical analysis involved three steps. The first analysis determined those memory variables related to growth in math performance. In this model, to account for the influence of children nested within classrooms, a multilevel regression model included the random effects for children's assignment to the various math classroom/teachers at Grades 1, 2, and 3. Because children changed classroom membership for Grade 1 to Grade 2 and 3, a series of cross-classified random effects models were tested (see Hox, 2010, Chapter 9, for a review).

After establishing a baseline model that included memory measures (Model 1), five models were tested. Model 2 determined if entering measures of fluid intelligence, vocabulary,

and reading provided a better fit to the data than the entry of WM measures. Thus, Model 2 considered only the covariates in the analysis. Model 3 assessed the role of L1 (Spanish) in predictions of math performance. In contrast, Model 4 considered only the English measures (L2) and the covariates in the regression model. Model 5 (Full model) entered both L1 and L2 measures as well as the covariates into the regression model. Of interest is whether both L1 and L2 memory processes contributed independent variance to math performance. Model 6 entered into the regression model those variables found only significant in the Full model (Model 5). All models were fit to the data using the SAS PROC Mixed software (SAS, 2010). Maximum likelihood (ML) procedures were used to determine the parameter estimates because the ML estimation procedure has several advantages over other missing data-techniques (see Peugh & Enders, 2004, for discussion). To compare competing or alternative models, the Bayesian Information Criterion (BIC), Akaike Information Criterion (AIC) and deviance scores were used. Models with smaller values are preferred to models with higher values when determining the best model fit. The AIC is primarily focused on comparing competing nonhierarchical models, and the BIC is recommended when the sample size is large and the number of parameters is small. Further, the BIC is more likely to penalize for additional model parameters than the AIC. For all HLM analyses computed, we centered the data at grade 3. This grade level of performance resulted in an intercept value that assessed the level of performance (z-score in this case) after three years. (It is important to note that slope remains the same whether the data are centered at grade 1, 2, or 3. However, the correlation between the intercept and slope does vary as a function of different centering.)

The previous aforementioned mixed regression analyses focused on the relationship between growth in WM and growth in math in the total sample. The second analysis focused on

subgroups at risk for math difficulties. Of interest was whether subgroup comparisons between children at risk and not at risk for MD on measures of WM varied as a function of bilingual proficiency. A mixed regression model was again implemented. The intercept and slopes of the bilingually proficient children without MD served as the comparison group. The remaining three subgroups were statistically compared to the intercept and slope of the proficient bilingual children without MD.

Finally, because of our sample size and because subgroup analyses reduces statistical power, we computed effect sizes comparing each subgroup. Effect sizes (ESs) considered the magnitude of differences on the various cognitive and achievement measures between each subgroup at Grades 1, 2 and 3. Of interest was whether the magnitude of the ESs between less proficient and proficient bilinguals with MD varied across the testing waves.

Results

The means and standard deviations of the measures used in this study across the three testing waves are reported in Appendix A. The reliability scores based on the sample for all measures are reported in Appendix B.

We used the beta weightings from the model in Grade 1 to compute latent (factor) scores for the memory measures across the testing waves.

Total Sample

Correlations. The correlation between the latent measures with the classification measures at each testing wave are shown in Table 1. As shown, none of the measures showed high collinearity and therefore both English and Spanish measures were entered into the regression analysis. The results in Table 1 showed that the magnitude of the correlations related

to computation increased across the testing waves. For example, math computation correlated with reading, fluid intelligence, and vocabulary and yielded coefficients at .18, .33, and .17 in grade 1 and .45, .46, .45 at grade 3, respectively. For the cognitive measures, few measures of moderate magnitude correlated with math computation. The largest coefficients for cognitive performance with math computation occurred in the third grade for the executive component of English and Spanish WM (r 's = .46, .38, respectively).

Because classroom instruction was in English (L 2) starting at grade 1, we determined the number of children with higher norm referenced Spanish vocabulary scores than norm-referenced English vocabulary scores. The percentage of children with higher Spanish than English scores in grade 1 was 58.71 %, and this percentage decreased to 42.11% in grade 2 and 31.30% percent in grade 3. Thus, there was a gradual increase in English vocabulary proficiency across the grades.

Growth modeling. Of interest was whether growth in WM played an important role in the growth of math computation performance. For this analysis, the criterion measure for math computations was the mean z-scores computed at grade 3, based on grade 1 raw score means and standard deviations.

To determine the best fitting model, six models are shown in Table 3. As shown in Table 3 for the Model 1, the average math performance for the total sample showed over a standard deviation of improvement from grade 1. The fourth model entered only English measures. When compared with the Spanish (Model 3) versus English presentation, the fit indices were lower for the English presentation, suggesting a better fit to the data. Growth in math was significantly related to growth in the executive component of WM.

The Full model (Model 5) entered both English and Spanish measures along with the covariates into the analysis. Growth in the executive component of WM was related to computation growth, even when STM and inhibition were entered into the analysis.

In general, the results support the notion that growth in the executive component of WM was related to growth in math computation. The results also suggested the models that included English WM performance provided a better fit to the data than Spanish WM performance. The results also suggested that English STM, Visual-Spatial WM, Fluid intelligence and English inhibition measures played a significant role in predicting math outcomes.

Comparison of Risk Groups

Whether the advantages related to children with higher math skills were related to the executive component of WM and/or to bilingual (i.e., vocabulary) proficiency were of interest. To address this issue, the sample was divided into children with MD and children without MD (NMD) at grade 1. The 25th percentile (standard score of 90) was used as the cut-off criterion. Because of our interest in the role of bilingualism in the present sample, we also separated the math groups in terms of their proficiency in English and Spanish vocabulary. As previously mentioned, children with English (PPVT) and Spanish (TVIP) vocabulary scores above an 85 standard score were considered proficient bilinguals (PB) and those with scores below 85 were considered less proficient bilinguals (LB). Table 3 provides the means and standard deviations across the three grades for each of the four subgroups.

Also shown in Table 3 was the “split” or absolute difference score between English and Spanish vocabulary scores. As noted, the split between Spanish and English vocabulary was larger for children considered proficient bilinguals in the earlier grades, but this difference in

score between the two vocabulary systems narrowed substantially by grade 3. Proficient bilingual (PB) children were substantially higher in normative scores for Spanish vocabulary than for English vocabulary.

Subgroup comparisons. The next analysis determined whether significant performance differences emerged on WM measures as a function of the four subgroups. To address this issue, a mixed regression analysis was computed using English, Spanish STM, the executive component of WM and visual-spatial WM as the criterion measures. The covariates in the analysis were reading, English and Spanish STM and English and Spanish inhibition measures. The criterion (memory) measures were z-scores centered at grade 3. As shown for the predictions of the English WM performance in Table 4, the average z-score for children without MD at grade 3 was .55. Since the z-score was based on the means and SD at grade 1, the intercept indicated that proficient bilinguals without MD increased by approximately 1/2 standard deviation in English WM from grade 1. The values for the slope (.23) indicated that growth was approximately .23 units at each testing wave.

As shown in Table 4, two regression models were tested. Model 1 addressed the question as to whether individual differences in WM performance were related to math abilities and/or bilingual (less proficient vs. proficient) status. Model 2 determined whether the significant differences that emerged in Model between the ability groups were merely an artifact of performance on reading, fluid intelligence, STM and/or inhibition measures. A comparison of these two models yielded four important findings.

First, Model 1 showed that subgroup differences emerged on English and Spanish WM measures, but not on visual-spatial WM measures. When compared to the less proficient

bilingual children with MD on English and Spanish WM measures found in Model 1, Model 2 showed that the advantages for the proficient bilingual children without MD were sustained when the covariates were entered into the regression analysis. Second, significant growth differences in performance emerged only among subgroups on the English WM measures. Children without MD who were proficient bilinguals yielded significantly higher growth rates in English WM than less proficient bilingual children with MD. Third, measures of reading were significant covariates in predictions of English and Spanish WM performance. Fluid intelligence was a significant predictor for all criterion measures. Finally, language specific effects emerged on measures of STM and inhibition. English STM and English inhibition were significant predictors of English WM. In contrast, both English and Spanish STM and Spanish Inhibition were significant predictors of Spanish WM.

Taken together, the important findings were that significant intercept and slope (growth) advantages in WM performance occurred for proficient bilingual children with MD when compared to less proficient children with MD. However, proficient bilingual children without MD did not supersede the performance of less proficient children without MD on the WM measures. Thus, the results suggest that subgroup differences in WM between children with and without MD may be merely an artifact of math status and not necessarily related to bilingual proficiency status. Because these findings may be related to power in the analysis, further analysis was necessary.

Effect sizes. Because the previous effects may be related to sample size, effect sizes were computed. According to Cohen's (1988) criteria, effect sizes $> .50$ and $.80$ are considered moderate and high, respectively. However, for discussion purposes, ESs values $> .40$ were considered important in describing the results of this study. The effect sizes are shown

in Table 5. The left side of Table 5 shows comparisons of children with and without MD and the right side shows comparisons of children with low bilingual proficiency with children labeled as proficient bilinguals.

As shown on the left side of Table 5, comparisons between children with and without MD were separated within language proficient subgroups. For the low proficient group, Table 5 shows an advantage (moderate to large effect sizes) at grade 3 for children without MD when compared to children with MD on measures of calculation, reading, fluid intelligence, English STM, Spanish WM, visual-spatial WM and English inhibition. As expected, the largest ESs emerged for measures of calculation and fluid intelligence. For the language proficient group, the magnitude of the ESs between children with MD and without MD was larger on measures of English STM than on measures of the executive component of WM.

As shown on the right side of Table 5, the comparison between language proficient groups occurred within math subgroups. For children with MD, proficient bilingual children with MD outperformed less proficient bilingual children with MD children on measures of calculation, reading, vocabulary, fluid intelligence and Spanish WM at grade 3. These findings suggest that the gap in math performance for less proficient bilingual children with MD increased substantially when compared to proficient bilingual children with MD at wave 3 ($ES = -.99$). Although the difference in overall vocabulary was expected, the important finding here was that math differences increased in favor of the proficient bilinguals on measures of calculation. For children without MD, bilingually proficient children outperformed less proficient bilingual children at grade 3 on measures of vocabulary.

In summary, three important findings occurred. First, regardless of bilingual proficiency, children without MD outperformed children with MD on measures of English STM. Second, regardless of math status, proficient bilingual children outperformed less proficient bilinguals on the executive components of WM. These two findings suggest that executive processes are tied to variations in bilingual proficiency whereas STM measures are tied to variations in MD status. Finally, proficient bilingual children with MD clearly outperformed less proficient bilingual children with MD on math computation measures at grade 3. This finding contrasted with grade 1 performance where the differences between the two groups in math computation were minimal.

Discussion

This study determined whether growth in the executive component of WM was related to growth in computation among ELL children. The results clearly showed that computation growth was related to the development of the executive component (controlled attention) of WM and this component was also shown to operate independent of vocabulary, reading, language specific measures of STM and inhibition in its predictions of math performance. Thus, the pattern of findings suggests that growth in the executive component of WM is inherently linked to growth in computation. The results also show that growth in the executive component of WM in proficient bilingual children with MD was significantly larger than growth in less bilingually proficient children with MD. Taken together, the results indicate that the executive component of WM plays a critical role in math development and that increases in bilingualism for children with MD yields greater improvements in computation when compared to their less proficient bilingual counterpart. No doubt, there are competing interpretations of this finding. Three are considered.

First, the phonological loop (in this case STM) plays a more important role in computation growth than the executive components of WM. The full regression model showed that the intercept for English STM (as well as the visual component of WM) was significantly related to math computation. This interpretation also finds support by considering the performance of bilingually proficient children with and without MD at grade 3 on the STM measures. As shown in Table 5, the ESs at grade 3 comparing children with and without MD who were proficient bilinguals were $-.56$ and $-.41$ at grade 3 on English and Spanish STM, respectively. In contrast, the ESs comparing children with and without MD who were proficient bilinguals was $-.35$ and $-.05$ at grade 3 and $-.28$ and $.16$ at grade 1 on English and Spanish WM measures, respectively. A key finding of this study, however, was that growth in math computation was significantly related to growth in WM even when measures of STM were entered into the analysis. In addition, the results showed that the slopes for STM were not significantly related to the slopes of math performance. Thus, although the storage component of WM was related to math performance, especially among children with MD, it is the controlled attention component of WM that played a major role in math computation growth.

A second interpretation of the findings suggests that the relationship between WM and math performance was primarily due to increases in bilingualism. As noted earlier, the absolute differences between Spanish and English vocabulary standard scores decreased across grades suggesting the sample was becoming increasingly proficient in both languages. In addition, the sample with the highest scores (proficient bilinguals without MD) had clear advantages on the English and Spanish WM measures, suggesting that increases in bilingualism (i.e., the standard score differences diminished between Spanish and English vocabulary as grade level increased) were related to increases WM and math performance. Further, the literature has established that

increased familiarity with a language influences WM (i.e., first language superiority leads to higher recall in that language; Thorn & Gathercole, 2001). However, we found that the effects of English WM growth on computation were not partialled out when measures of vocabulary were entered into the regression model (see Table 2). Thus, attributing the results related to English WM growth in predicting math growth as merely a function of increased bilingual proficiency, in this case, does not provide an adequate account of the findings.

A final interpretation of the findings was that children had difficulty switching between the language codes. However, entering English and Spanish inhibition measures into the analysis did not eliminate the significant contribution of WM growth to computation. Thus, we did not find that L1 and L2 language inhibition played a major role in mitigating the effects of WM on math computation performance. Although the English inhibition measure was found to play an important role in predicting math computation, it does not necessarily directly underlie the relationship between the executive component of WM and math. Growth in WM, as far as it relates to growth in math computation, appears to operate independent of the distractions related to inhibition of L1 and L2.

Thus, the question emerges as to whether poor math computation in ELL children is further compounded by weaknesses in language proficiency and/or the executive component of WM. Before answering this question, it is important to note the sample reflected sequential bilingualism (L2 follows L1 development) and therefore may not reflect bilingualism when two languages are learned simultaneously. Given these qualifications, the findings in Table 2 suggest that when the effects of vocabulary, reading, aptitude, STM, and inhibition were partialled out, growth in the executive component of WM was related to growth in computation ability. However, the subgroup analysis suggested a clear advantage emerged for proficient bilingual

children with MD when compared to less proficient bilingual children with MD in the final testing wave. The results also showed that proficient bilingual children with MD outperformed less proficient bilingual children with MD on both the intercept and growth parameters related to the English WM measures. Taken together, the results suggest that bilingually proficient children with MD have performance advantages over less bilingually proficient children with MD not only in measures of math, but also on the executive component of WM.

A question emerges as to “what cognitive processes are critically related to the development of math in ELL children whose first language is Spanish?” The regression analyses clearly show that the executive component of WM uniquely predicted growth effects in math computations. However, the results also strongly suggest that a number of other processes were related to the math computation. Specifically, vocabulary, reading, fluid intelligence and cognitive processes, such as STM and inhibition, were important correlates of math computation. Each of these processes contributed unique variance, suggesting that a number of processes play an important role in math performance. However, when considering various components of WM that may play a role in math performance, the executive component of WM appears to play a more important role in computation growth than the storage component of WM (i.e., STM).

Summary

Taken together, we interpret our findings as suggesting that growth in math computation is directly tied to the development of the executive WM system. However, this system does not appear to act independent of cross-language skills across grades. The present study also has identified key cognitive variables in ELL children that predict math proficiency. Future research must focus on the interaction between executive components of WM within and across language systems during math computation to disentangle alternative interpretations of the results.

Footnote

1. Although the PPVT and TVIP are commonly used to assess vocabulary in ELL children, they are not without limitations (see Peña, 2007, for review). In addition, it is important to note that using average performance in our designation of bilingual proficiency yielded a large range between Spanish and English vocabulary scores in the language proficient group (see Table 4). This range was reduced substantially in the later grades suggesting that the alignment L1 and L2 for sequential bilinguals may not occurred until the later grades, even though the threshold for 12 language comprehension may be age appropriate at the younger grades. Thus, the scores for proficient children met a minimum threshold in L2 (> 85 standard score), whereas the less proficient children did not meet the 85 standard score threshold in their first language (L1).
2. Clearly, as indicated by one reviewer, the translation could not control for the number of syllables, frequency of use, imagery, and meaning across measures. Although we relied on expert judgment, the factor structure of the measures in a previous study suggested measures were tapping a similar construct (Swanson et al., 2012). In addition, it is important to note the difficulty of the tasks were scaled (z-scores) individually for the total sample.

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Table 1

Correlations between Calculation, Classification, Covariates and Memory Latent Measures at each Grade

Variables

Grade 1		1	2	3	4	5	6	7	8	9	10	11
1	Math	1										
2	Reading	0.18	1									
3	FIQ	0.33	0.23	1								
4	Vocab.	0.17	0.32	0.12	1							
5	E-STM	0.19	0.29	0.21	0.14	1						
6	S-STM	0.21	0.45	0.15	0.21	0.53	1					
7	E-WM	0.29	0.17	0.26	0.08	0.30	0.27	1				
8	S-WM	0.27	0.42	0.16	0.39	0.32	0.54	0.46	1			
9	Vis-WM	0.28	0.24	0.15	0.09	0.23	0.19	0.28	0.21	1		
10	E-Inhib	0.11	0.09	0.27	0.13	0.17	0.26	0.17	0.29	0.28	1	
11	S-Inhib	0.03	0.07	0.07	0.20	0.23	0.17	0.12	0.24	0.11	0.43	1
Grade 2												
1	Math	1										
2	Reading	0.37	1									
3	FIQ	0.34	0.39	1								
4	Vocab.	0.24	0.43	0.31	1							
5	E-STM	0.21	0.33	0.12	0.19	1						
6	S-STM	0.15	0.35	0.16	0.31	0.50	1					
7	E-WM	0.22	0.30	0.32	0.17	0.31	0.24	1				
8	S-WM	0.12	0.36	0.16	0.27	0.25	0.41	0.44	1			
9	Vis-WM	0.21	0.13	0.16	0.02	0.08	0.13	0.21	0.17	1		

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10	E-Inhib	0.18	0.08	0.19	0.16	0.20	0.24	0.15	0.15	-0.05	1	
11	S-Inhib	0.13	0.14	0.25	0.18	0.03	0.29	0.10	0.26	0.06	0.28	1
Grade 3												
1	Math	1										
2	Reading	0.45	1									
3	FIQ	0.46	0.24	1								
4	Vocab.	0.45	0.54	0.29	1							
5	E-STM	0.27	0.34	0.26	0.21	1						
6	S-STM	0.17	0.43	0.03	0.41	0.53	1					
7	E-WM	0.46	0.36	0.29	0.32	0.41	0.31	1				
8	S-WM	0.38	0.45	0.28	0.44	0.52	0.52	0.47	1			
9	Vis-WM	0.23	0.15	0.30	0.13	0.18	0.03	0.15	0.16	1		
10	E-Inhib	0.33	0.25	0.11	0.09	0.18	0.04	0.28	0.19	0.19	1	
11	S-Inhib	0.12	0.28	0.05	0.16	0.19	0.10	0.10	0.31	0.24	0.19	1

Note. E=English, S=Spanish, Math=calculation from Wide Range Achievement Test, Reading= word identification from English and Spanish test, Vocab=vocabulary from English and Spanish measures, STM=short-term memory or phonological loop, FIQ=fluid intelligence or Raven Colored Progressive Test, WM=working memory task that included process and storage, executive component. Vis=Visual-spatial, Inhib=inhibition or random generation tasks

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Table 2

Hierarchical Growth Models Predicting Math Calculation

	Model 1		Model 2		Model 3		Model 4		Model 5		Model 6	
Effect	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
Intercept	1.09***	0.26	1.21***	0.19	1.27***	0.28	1.42***	0.25	1.44***	0.25	1.28***	0.20
E-STM	0.77	0.42	.	.	.	.	0.72*	0.34	.79**	0.40	.45*	0.25
S-STM	0.19	0.39	.	.	0.42	0.33	.	.	-0.01	0.38	.	.
E-Exec	0.27	0.38	.	.	.	.	-0.06	0.34	0.01	0.37	.	.
S-Exec	0.24	0.54	.	.	-0.02	0.5	.	.	-0.07	0.52	.	.
VS-WM	0.66*	0.33	.	.	0.57*	0.32	0.49	0.32	0.48	0.32	.39**	0.14
Reading	.	.	0.01*	0.005	0.01	0.01	0.005	0.005	0.004	0.005	.	.
Fintell.	.	.	0.59***	0.08	0.45***	0.08	0.41***	0.08	.42***	0.08	.45***	0.08
Vocab.	.	.	0.01	0.009	0.01	0.01	0.01	0.008	0.01	0.008	.	.
E-Inhib	.	.	.	.	.	.	0.43**	0.19	0.48***	0.19	.45*	0.19
S-inhib	.	.	.	.	-0.11	0.21	.	.	-0.23	0.21	.	.
Growth	0.08***	0.01	0.09***	0.008	0.07***	0.01	0.07***	0.01	0.07***	0.01	.05***	0.008
E-STM	-0.02	0.03	.	.	.	.	-0.03	0.02	-0.04	0.03	.	.
S-STM	-0.02	0.02	.	.	-0.03	0.02	.	.	-0.01	0.02	.	.
E-Exec	0.05*	0.02	.	.	.	.	0.05**	0.02	0.04*	0.02	.05***	0.01
S-Exec	0.03	0.03	.	.	0.06**	0.03	.	.	0.04	0.03	0.03	0.015
VS-WM	-0.01	0.02	.	.	0.01	0.01	0.01	0.01	-0.003	0.01	.	.
Random Effects												
Grade1	0.09	0.08	0.02	0.06	0.03	0.06	0.004	0.05	0.02	0.05	0.02	0.06
Grade2	0.16*	0.09	0.09	0.08	0.08	0.07	0.07	0.06	0.06	0.06	0.10	0.07
Grade3	0.003	0.05	0.06	0.07	0.02	0.06	0.02	0.05	0.01	0.05	0.008	0.05
Residual	1.865***	0.15	2.007***	0.16	1.82***	0.14	1.75***	0.14	1.74***	0.14	1.76***	0.14
Fit Statistics												
Deviance	1273.1		1290.7		1254.3		1239.2		1234.1		1242.5	
AIC	1305.1		1309.2		1286.3		1271.2		1276.1		1266.2	

BIC	1273.1	1290.7	1254.3	1239.2	1234.1	1242.2
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- $P < .05$, ** $p < .01$, *** $p < .001$. Model 1=Working Memory component-only, Model 2=reading, fluid intelligence, language and covariates-only, Model 3=Spanish Memory and covariates, Model 4=English Memory and covariates, Model 5=Full Model, Model 6= reduced model.

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Table 3

Means and Standard Deviations as a Function of MD and Bilingual Status

Variable	MD/LP		NMD/LP		MD/PB		NMD/PB	
	1		2		3		4	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Grade 1	N=56		N=29		N=33		N=40	
Calculation ^a	88.07	9.92	104.36	8.02	90.76	9.32	102.9	6.60
Reading	93.25	13.94	98.38	13.3	102.34	18.07	105.09	15.99
Vocabulary	77.49	7.29	78.31	8.15	92.91	5.21	92.36	7.01
Split ^b	10.88	8.66	11.03	10.53	17.29	15.19	18.73	13.57
Fluid Intelligence	92.89	13.98	107.84	13.94	99.00	14.13	105.33	13.46
E-STM ^c	-0.06	0.36	-0.03	0.33	-0.03	0.33	0.09	0.40
S-STM	-0.07	0.36	-0.06	0.35	0.01	0.41	0.03	0.38
E-Exec	-0.09	0.35	0.02	0.37	-0.03	0.29	0.02	0.30
S-Exec	-0.11	0.19	-0.07	0.32	0.10	0.28	0.06	0.19
Visual WM	-0.07	0.3	0.07	0.37	-0.1	0.36	0.09	0.25
E-Inhibition	-0.12	0.38	-0.05	0.40	-0.04	0.37	0.13	0.38
S-Inhibition	-0.10	0.34	-0.01	0.40	0.07	0.34	0.03	0.37
Grade 2	N=45		N=25		N=30		N=35	
Calculation	92.58	10.00	109.12	6.7	94.53	10.73	108.56	7.67
Reading	91.00	16.04	104.88	12.78	101.28	18.91	108.47	20.86
Vocabulary	79.33	7.99	81.54	7.9	88.58	7.8	90.41	11.81
Split	13.33	9.74	14.04	8.96	13.17	8.32	15.69	11.34
Fluid Intelligence	95.28	14.85	103.27	13.58	99.38	13.08	104.43	16.35
E-STM	0.17	0.37	0.30	0.27	0.23	0.30	0.26	0.37
S-STM	0.11	0.35	0.14	0.33	0.19	0.42	0.19	0.36
E-Exec	0.13	0.27	0.08	0.31	0.25	0.4	0.24	0.39
S-Exec	0.05	0.22	-0.01	0.2	0.17	0.28	0.17	0.27
Visual WM	0.2	0.48	0.29	0.49	0.14	0.36	0.31	0.41
E-Inhibition	0.08	0.37	0.06	0.32	0.15	0.5	0.14	0.46
S-Inhibition	0.11	0.44	0.15	0.38	0.21	0.34	0.06	0.4
Grade 3	N=40		N=21		N=28		N=30	
Calculation	91.00	10.13	107.33	7.45	100.75	10.63	107.43	8.34
Reading	82.29	19.59	94.39	18.21	96.59	21.65	98.55	19.38
Vocabulary	77.68	10.61	80.82	7.41	90.56	9.11	89.22	10.68
Split	15.40	11.17	15.82	11.05	13.63	10.19	12.90	9.81
Fluid Intelligence	93.71	13.36	106.2	12.39	102.6	12.66	100.2	18.22
E-STM	0.32	0.31	0.45	0.31	0.33	0.29	0.50	0.30
S-STM	0.20	0.38	0.24	0.3	0.27	0.35	0.45	0.45
E-Exec	0.27	0.40	0.27	0.35	0.41	0.47	0.52	0.44

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S-Exec	0.15	0.28	0.27	0.21	0.38	0.4	0.43	0.39
Visual WM	0.54	0.72	0.83	0.62	0.64	0.73	0.72	0.76
E-Inhibition	0.21	0.43	0.44	0.37	0.31	0.42	0.42	0.49
S-Inhibition	0.19	0.4	0.24	0.41	0.28	0.36	0.21	0.32

^a Standard Scores (M=100, SD=15), ^b split refers to the difference score between Spanish and English vocabulary, ^c z-scores (M=0, SD=1). E=English, S=Spanish, Exec=executive component of working memory, LP=less proficient bilingual, PB=proficient bilingual, MD=math disabilities, NMD=non math disabilities

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Table 4

Fixed Effects from Subgroup comparisons on Working Memory Measures

	English WM		English WM		Spanish WM		Spanish WM		Visual-spatial WM		Visual-Spatial WM	
Solution for Fixed Effects												
Effect	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
Intercept ^a	.55**	0.06	0.45***	0.06	.39***	0.05	0.33***	0.0404	.69***	0.08	0.60***	0.08
MD/PB	-0.24**	0.08	-0.10	0.08	-0.24***	0.06	-0.12*	0.05	-0.18	0.11	-0.06	0.11
NMD/LP	-0.07	0.09	-0.02	0.08	-0.03	0.06	-0.0006	0.05	-0.09	0.12	-0.07	0.12
MD/LP	-0.26**	0.09	-0.31**	0.09	-0.17*	0.07	-0.14*	0.06	0.07	0.13	0.04	0.13
Read			0.003**	0.001			0.003**	0.0008			0.01	0.002
Fluid			0.07***	0.02			0.03*	0.01			0.08*	0.03
Split ^b			0.003	0.002			-0.002	0.002			-0.003	0.003
E-STM			0.25***	0.07			0.13**	0.05			0.10	0.10
S-STM			0.05	0.06			0.22**	0.04			-0.04	0.09
E-Inhib			0.10*	0.05			0.03	0.03			0.10	0.07
S-Inhib			-0.006	0.05			0.09**	0.04			0.13	0.07
Growth	.23***	0.04	0.14**	0.05	.18***	0.03	0.09***	0.03	.29***	0.06	0.22**	0.07
MD/PB	-0.07	0.05	-0.10b	0.06	-0.06	0.04	-0.05	0.04	-0.02	0.08	-0.02	0.08
NMD/LP	-0.008	0.06	-0.04	0.06	-0.04	0.04	-0.04	0.04	0.07	0.09	0.04	0.09
MD/LP	-0.13*	0.06	-0.17**	0.06	-0.02	0.05	-0.02	0.05	0.05	0.09	0.03	0.10

Note. ^aIntercept and Slope are average values for children without MD who are proficient bilinguals. ^b split refers to the absolute difference score between Spanish and English vocabulary

MD=children with math difficulties, NMD=children without math difficulties, PB=Proficient Bilinguals, LP=less proficient bilinguals.

Table 5

Comparison of Effect Sizes Across Grades between Children with and without MD as a Function of Bilingual Proficiency

	Variable	MD/LP	vs. NMD/LP ^a		MD/LP vs. MD/PB ^a		
		Grade 1	Grade 2	Grade 3	Grade 1	Grade 2	Grade 3
1	Calculation	-1.70	-1.93	-1.86^a	0.02	-0.18	-0.99
2	Reading	-0.44	-0.97	-0.64	-0.60	-0.58	-0.75
3	Vocabulary	0.11	-0.18	-0.34	-2.41	-1.33	-1.46
4	Split ^a	-0.21	-0.07	-0.07	-0.50	0.25	0.23
5	Fluid Intell	-1.05	-0.70	-0.96	-0.26	-0.25	-0.60
6	E-STM	-0.20	-0.46	-0.40	-0.20	-0.18	-0.05
7	S-STM	0.04	-0.17	-0.04	-0.20	-0.51	-0.25
8	E-Exec	-0.25	0.05	-0.04	-0.05	-0.41	-0.29
9	S-Exec	-0.19	0.38	-0.49	-0.95	-0.69	-0.81
10	Vis-WM	-0.52	-0.25	-0.44	0.20	0.01	-0.14
11	E-Inhib	-0.22	0.05	-0.56	-0.23	-0.22	-0.25
12	S-Inhib	-0.39	-0.04	-0.06	-0.68	-0.26	-0.31
		MD/PB vs. NMD/PB ^a			NMD/LP vs. NMD/PB ^a		
1	Calculation	-1.71	-1.46	-0.65	0.29	0.08	-0.01
2	Reading	-0.03	-0.22	-0.05	-0.28	-0.02	-0.22
3	Vocabulary	-0.03	-0.12	0.22	-2.54	-0.99	-0.96
4	Split ^a	-0.20	-0.67	-0.01	-0.46	-0.30	0.30
5	Fluid Intell	-0.77	-0.43	0.09	0.21	0.03	0.39
6	E-STM	-0.35	-0.05	-0.56	-0.35	0.22	-0.20
7	S-STM	-0.19	0.18	-0.41	-0.51	-0.18	-0.63
8	E-Exec	-0.28	-0.06	-0.35	-0.02	-0.50	-0.65
9	S-Exec	0.16	0.15	-0.05	-0.49	-0.84	-0.53
10	Vis-WM	-0.68	-0.42	-0.09	0.08	-0.10	0.18
11	E-Inhib	-0.64	0.07	-0.26	-0.57	-0.21	-0.02
12	S-Inhib	0.14	0.46	0.31	-0.09	0.21	0.04

Note. MD=children at risk for math difficulties, NMD=children not at risk, LP=low language proficiency, PB=proficient bilingual. Intell=intelligence, Inhib=inhibition, STM=short-term storage or phonological loop. Exec=Executive component, Vis=visual-spatial. Split=absolute difference in norm scores between Spanish and English vocabulary, Bold=effect sizes in the moderate to high range at grade 3. Negative effect sizes in favor of second comparison group (e.g., -1.86 in favor of NMD/LP group)

^a split refers to the difference score between Spanish and English vocabulary

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Appendix A

Descriptive Statistics for Measures as a Function of Grade

Variable	Label	N	Grade 1 Mean	SD	N	Grade 2 Mean	SD	N	Grade 3 Mean	SD
Math										
Computation	Accuracy	157	95.32	11.28	135	95.16	11.86	119	98.27	12.38
Reading										
E-Word ID		157	105.31	13.08	135	104.61	16.27	119	97.38	16.14
S-Word ID		154	92.32	23.63	134	95.84	24.22	119	86.36	28.08
Average Reading		157	99.07	15.9	135	100.39	18.77	119	91.87	20.74
Vocabulary										
E-Vocab		157	83.17	8.85	135	86.07	10.96	119	87.39	11.78
S-Vocab		156	85.94	17.39	135	83.27	15.19	119	80.79	15.84
Average Vocab		157	84.57	10.17	135	84.67	10.21	119	84.09	11.2
Fluid Intelligence										
Raven		157	100.01	15.01	133	100.25	12.88	119	102.66	11.89
Working Memory										
E-digits		157	3.18	0.83	135	3.53	0.83	119	3.77	0.84
E-words		157	2.12	0.77	135	2.44	0.73	119	2.68	0.68
E-nonwords		157	1.04	0.63	135	1.28	0.68	119	1.49	0.8
E-Concept span		157	2.15	1.79	136	2.69	1.79	119	3.5	2.27
E-Listen Span		157	1.44	0.27	135	1.62	0.35	119	1.69	0.39
E-Update		157	1.49	1.61	135	2.15	1.66	119	2.61	1.59
Visual matrix		157	8.24	5.5	135	10.95	5.8	119	14.39	7.31
Mapping		157	1.79	1.25	135	3.03	2.65	119	5.73	5.2
S-digits		157	3.16	0.64	135	3.25	0.74	119	3.48	0.8
S-words		157	1.58	0.78	134	1.9	0.84	119	1.99	0.88
S-Nonwords		157	1.11	0.61	134	1.37	0.63	119	1.51	0.71
S-Concep Span		157	1.42	1.56	135	2.11	1.54	119	2.55	1.66
S-Listen Span		157	0.48	0.67	135	0.7	0.91	119	1.27	1.32

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S-Update	157	0.98	1.64	135	1.3	1.53	119	1.84	2.03
Inhibition									
E-inhibition	157	-0.02	0.39	135	0.11	0.42	119	0.33	0.44
S-inhibition	157	-0.01	0.36	134	0.13	0.4	119	0.22	0.37

Note. E-=English, S-=Spanish, Average=mean average of English and Spanish, Computation=Arithmetic subtest from the Wide Range Achievement Test, Word ID=word identification, Vocab=vocabulary, digits=digit span, nonwords=pseudoword span, words=word span, Concep=Conceptual span, Listen=Listening Span

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Appendix B

Brief Task Description and Reliability (KR₂₀)

Construct	Task	Brief Description	Sample Reliability	
			English	Spanish
Math	WRAT- math	Arithmetic calculation	0.90	. ^a
Literacy	WMLS-R word ID	Single word reading	0.92	0.92
Receptive vocabulary	PPVT or TVIP	Match vocabulary to picture	0.90	0.96
Fluid intelligence	Raven	Find missing piece of complex design	0.88	.
STM-Phonological loop	Forward digit span	Recall sequentially ordered sets of digits	0.61	0.76
	Word span	Recall sequentially ordered words	0.89	0.85
	Pseudoword span	Recall sequentially ordered sets of nonwords	0.90	0.87
Executive Component of Working Memory	Conceptual span	Answer process ques. & recall categories	0.90	0.85
	Listening sentence span	Answer process ques. & recall words end of sentence	0.92	0.96
	Updating span	Recall three number sequence from varying sequence	0.94	0.92
Visual-spatial Sketchpad	Visual Matrix	Remember visual sequences within a matrix	0.92	.
	Mapping and direction	Remember a sequence of directions on a map	0.93	.
Inhibition	Random number	Write numbers in random order	0.80	0.89
	Random letter	Write letters in random order	0.87	0.86

Note. ^a For Tasks presented simultaneously in both languages (calculation, fluid intelligence, visual-spatial WM), reliability (Cronbach alpha) is the same across both language systems .

