

A Survey on MIMO Transmission With Finite Input Signals: Technical Challenges, Advances, and Future Trends

This paper provides a comprehensive overview of multiple-input–multiple-output transmission design with finite input signals and delves into unsolved technical challenges and future trends in this field.

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ABSTRACT | Multiple antennas have played an essential role in spatial multiplexing and diversity transmission for a wide range of communication applications. Most advances in the design of high-speed wireless multiple-input–multiple-output (MIMO) systems have been based on information-theoretic principles that demonstrate how to efficiently transmit signals conforming to Gaussian distribution. However, although the Gaussian signal is capacity-achieving, practical systems transmit signals belonging to finite and discrete constellations. Therefore, capacity-achieving transceiver processing based on a Gaussian input signal can be quite suboptimal for practical MIMO systems with discrete constellation input signals. To address this shortcoming, this paper aims

to provide a comprehensive overview of MIMO transmission design with finite input signals. It first summarizes existing fundamental results for MIMO systems with finite input signals. Next, focusing on basic point-to-point MIMO systems, it examines transmission schemes based on the three most important criteria for communication systems: mutual-information-driven designs, mean-square-error-driven designs, and diversity-driven designs. In particular, a unified framework is developed for the design of low-complexity transmission schemes applicable to massive MIMO systems in forthcoming 5G wireless networks for the first time. Furthermore, adaptive transmission designs are proposed that switch among these criteria based on channel conditions to formulate the best transmission strategy. A survey is then given of transmission designs with finite input signals for multiuser MIMO scenarios, including MIMO uplink transmission, MIMO downlink transmission, MIMO interference channel, and MIMO wiretap channel. Additionally, transmission designs with finite input signals are discussed for other multi-antenna systems. Finally, a number of technical challenges that remain unresolved at the time of writing are highlighted, and future trends in transmission design with finite input signals are discussed.

KEYWORDS | Diversity; finite input signals; MIMO; MSE; multi-user systems; mutual information

I. INTRODUCTION

The explosive growth in the use of smart mobile devices such as smartphones and wireless modems has led to

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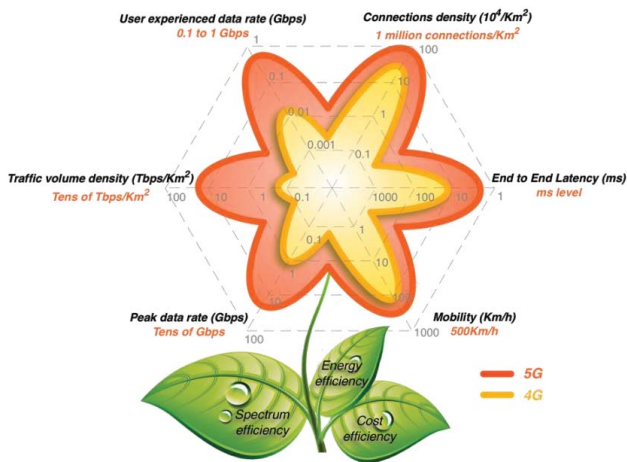


Fig. 1. Fundamental requirements of 5G: the "5G flower" (from [11]).

an exponentially increasing demand for wireless data services. Accordingly, substantial effort has been directed at improving the spectral efficiency and data throughput of wireless communication systems. In particular, multiple-input-multiple-output (MIMO) technologies promise to provide gains in multiplexing and/or diversity. By increasing the number of antennas at base stations and mobile terminals, communication systems can achieve better performance in terms of both system capacity and link reliability [1]–[3]. To date, MIMO technology has been adopted by the Long-Term Evolution (LTE) standard of the third-Generation Partnership Project (3GPP), IEEE 802.11n, and other wireless communication standards.

A. Future Wireless Communication Networks

The unprecedented growth in the number of mobile data and connected machines continues to push the limits of modern wireless network capacity to accommodate the enormous data demand. For example, mobile data traffic is expected to grow to 24.3 exabytes per month by 2019 [4], and the number of items connected by the Internet of Things (IoT) is estimated to reach 50 billion by 2020 [5]. Also, emerging new services such as ultra-high-definition multimedia streaming and cloud computing, storage, and retrieval require high cell capacity/end-user data rates and extremely low latency, respectively. It is becoming evident that current 4G technologies will be soon be stretched to near breaking point and will struggle to cope with these emerging demands. Therefore, the development of fifth-generation (5G) wireless communication technologies is essential for the deployment of future wireless communication networks [6], [7].

Fig. 1 provides a vivid representation of the fundamental requirements for building 5G wireless networks. From the point of view of system performance, 5G wireless networks must support massive capacity (with tens of Gb/s peak data rate) and massive connectivity (with 1 million/ km^2 connection density and tens of Tb/s/ km^2 traffic volume

density), extremely divergent requirements for different user services (with user-experienced data rates ranging from 0.1 to 1 Gb/s), high mobility (with user mobilities up to 500 km/h), and low-latency communication (with a 1ms level of end-to-end latency). From the point of view of transmission efficiency, 5G wireless networks must simultaneously deliver high spectral efficiency, energy efficiency, and cost efficiency. The evolution of 5G wireless communication will be a cornerstone of future human-centric and connected machine-centric wireless networks, allowing near-instantaneous communication for people and connected machines.

To this end, a 5G air interface is proposed, incorporating three basic technologies: massive MIMO, millimeter-wave, and small-cell [8]. For millimeter-wave transmission, beamforming achieved with the use of large antenna arrays, including both analog and digital beamforming, can reduce propagation losses [9]. Cell shrinking also requires the deployment of large conformal antenna arrays, the cost of which depends inversely on infrastructure density [10]. Therefore, MIMO technology will continue to play an important role in upcoming 5G and other future wireless networks.

B. MIMO Transmission Design

While the benefits of MIMO can be seen even when only the receiver has communication channel knowledge, performance gains can be even greater if the transmitter also exploits channel knowledge and performs certain processing operations on the signal before transmission. For example, a MIMO transmission design can double its capacity at -5 dB signal-to-noise ratio (SNR) and achieve $1.5 \text{ bs}^{-1}\text{Hz}^{-1}$ additional capacity at 5 dB SNR by using four transmit (Tx) and two receive (Rx) antennas over independent identically distributed (i.i.d.) Rayleigh flat-fading channels [12]. These are normal SNR ranges for practical applications such as LTE, WiFi, and WiMax. MIMO transmission designs can be improved even further with the use of correlated fading channels [13], [14]. For the massive MIMO technology needed in future wireless communication networks, it has been shown in [15] that, with perfect channel knowledge, linear precoding at the transmitters, and simple signal detection at the receivers, the performance approaches its optimum level as the number of antennas at the base station tends to infinity. A two-step joint spatial division and multiplexing transmission scheme has also been proposed to achieve massive MIMO gains with reduced channel state information at the transmitter (CSIT) feedback [16]. Therefore, MIMO transmission designs based on some form of CSIT are of great practical interest for wireless communication systems.

Typically, MIMO transmission designs can be classified into essentially three categories [17]: 1) designs based on mutual information; 2) designs based on the mean-squared error (MSE); and 3) diversity-driven designs. The

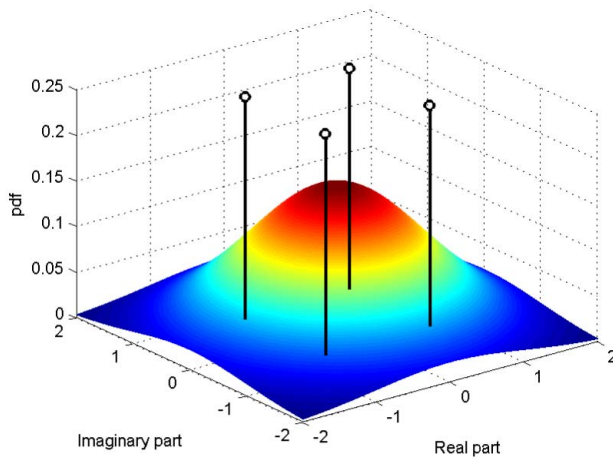


Fig. 2. The pdf of a Gaussian signal and the pmf of a QPSK signal.

first category adopts mutual information such as ergodic or outage capacity for transmission optimization. The second category performs the transmission design by optimizing MSE-related objective functions subject to various system constraints. The third category maximizes the diversity of the communication system based on pairwise error probability (PEP) analysis.

C. MIMO Transmission Design With Finite Input Signals

From the information-theoretic point of view, a Gaussian-distributed source signal achieves the fundamental limit of MIMO communication [1]. However, Gaussian transmit signals are rarely used in practical communication systems, mainly for two reasons: 1) the amplitude of a Gaussian transmit signal is unbounded, which may result in substantial distortion for power-limited transmitters; and 2) the probability distribution function (pdf) of a Gaussian transmit signal is continuous, which will significantly complicate the task of signal detection at the receiver. Therefore, in practice, transmit signals are non-Gaussian input signals taken from finite discrete constellations, such as pulse amplitude modulation (PAM) and/or quadrature amplitude modulation (QAM). Fig. 2 compares the pdf of a standard Gaussian-distributed signal and the probability mass function (pmf) of a quadrature phase-shift keying (QPSK) signal. We can see that the QPSK signal is significantly different from the Gaussian signal. Therefore, transceivers designed for finite input signals¹ will normally be quite different from those for Gaussian signals. One cannot simply use a design optimized for Gaussian input signals to replace the optimization that needs to be performed for finite input signals.

There are three main areas of difficulty arising in MIMO transmission design with finite input signals.

¹By “finite input signals,” we mean the signals used in practice for transmission, which are finite and discrete.

- 1) The mutual information and the minimum mean squared error (MMSE) of MIMO channels with finite discrete constellation signals usually do not have closed forms. As a result, the elegant water-filling type solutions available for MIMO channels with Gaussian input signals no longer hold. Therefore, sophisticated numerical algorithms are required to search for both the power allocation matrix and the eigenvector matrix.
- 2) The complexity of MIMO transmission designs with finite discrete constellation signals normally grows exponentially with both the number of antennas and the number of users, which often makes the design optimization process computationally prohibitive.
- 3) Depending on the actual channel conditions, adaptive transmission is needed in practical systems in order to determine the best transmission policy according to various criteria such as mutual information, MSE, and diversity. This procedure is challenging since it involves the selection of a large number of parameters.

It should be noted that conceptually there are two approaches to transmission design for finite input signals. One is to perform precoding under finite and fixed transmission constellation constraints. The other is to optimize the input distributions with finite signal support. For the transmission designs in Sections III–V, we focus mainly on the former, in which precoders are designed to maximize the mutual information, minimize the MSE, or maximize the diversity of a MIMO system under practical finite discrete constellation signals. For the fundamental results in Section II, in order to maintain a consistent description and also provide readers with a more comprehensive introduction to the fundamental research on finite input signals, we will also mention some important work on input distribution optimization.

D. Contributions of This Paper

In this paper, we present, for the first time, a comprehensive review of MIMO transmission design with finite input signals with respect to various design criteria. We first introduce fundamental research results on finite input signals, including the mutual information and MMSE relationships [18]–[20], the properties of the MMSE function [21], [22], and the definition of MMSE dimension [23]. Then, focusing on basic point-to-point MIMO systems, we provide a detailed description of transmission designs with finite input signals based on the criteria of mutual information (e.g., [17], [24]–[28]), MSE (e.g., [29]–[31]), and diversity (e.g., [32]–[37]), respectively. In particular, for the first time, we provide a unified framework for low-complexity designs that can be implemented in large-scale MIMO antenna arrays for 5G wireless networks. We further present some practical adaptive transmission schemes that switch among these criteria, corresponding to channel variations for both uncoded systems (e.g., [38]–[41])

and coded systems (e.g., [42]–[45]). We introduce specific adaptive transmission schemes defined in the 3GPP LTE, IEEE 802.16e, and IEEE 802.11n standards. Moving toward multiuser MIMO systems, we outline designs based on the above-mentioned criteria for multiuser MIMO uplink transmission (e.g., [46]–[49]), multiuser MIMO downlink transmission (e.g., [50]–[53]), MIMO interference channels (e.g., [54]–[57]), and MIMO wiretap channels (e.g., [58]–[60]). Finally, we give an overview of MIMO transmission designs with respect to finite input signals for other systems, such as MIMO cognitive radio systems (e.g., [61]–[63]), green MIMO communication systems (e.g., [64], [65]), and MIMO relay systems (e.g., [66]–[69]). Fig. 3 shows the overall outline of this paper.

To provide a comprehensive introduction to research on MIMO transmission with finite input signals, we also summarize the state of the art with respect to multiuser MIMO systems with finite input signals as well as MIMO wiretap channels and MIMO interference channels. However, we employ a different approach when introducing point-to-point MIMO systems and other scenarios to give readers a clearer focus. For point-to-point MIMO systems, we provide an explicit explanation of the differences between transmission design with finite input signals and that with Gaussian input signals, and give a detailed introduction to every key research point. For multiuser MIMO and other scenarios, we give a brief general introduction to related research work.

II. FUNDAMENTAL RESULTS FOR FINITE INPUT SIGNALS

In this section, we briefly review existing fundamental results for finite input signals. The design of transmission schemes for MIMO systems with finite input signals must rely on these fundamental results.

A. Fundamental Link Between Information Theory and Estimation Theory

The relationship between information theory and estimation theory is a classic topic, dating back 50 years to the relationship obtained between the derivative of the q -entropy and generalized Fisher information [70]. This relationship is called De Bruijn's identity. For a scalar Gaussian noise channel, there is another identity between the MMSE function and the Fisher information called Brown's identity [71]. The representation of mutual information as a function of causal filtering error was given in [72] as Duncan's theorem. Other early connections between fundamental quantities in information theory and estimation theory can be found in [73]–[77] and references therein.

1) *Gaussian Channel Model:* More recently, Guo *et al.* [18] established a fundamental relationship between mutual information and the MMSE, which applies for both discrete-time and continuous-time

vector channels² with additive Gaussian noise. In particular, for a real-valued linear vector Gaussian channel $\mathbf{y} = \sqrt{\text{snr}} \mathbf{H} \mathbf{x} + \mathbf{n}$, where $\mathbf{y}$, $\mathbf{H}$, $\mathbf{x}$, and $\mathbf{n}$ denote the received signal, the channel matrix, the transmit signal, and the Gaussian noise vector, respectively, Guo *et al.* found the following relationship regardless of the input distribution $\mathbf{x}$:

$$\frac{d}{d\text{snr}} I(\mathbf{x}; \sqrt{\text{snr}} \mathbf{H} \mathbf{x} + \mathbf{n}) = \frac{1}{2} \text{mmse}(\text{snr}) \quad (1)$$

where

$$I(\mathbf{x}; \sqrt{\text{snr}} \mathbf{H} \mathbf{x} + \mathbf{n}) = \sum_{\mathbf{x}, \mathbf{y}} p(\mathbf{x}, \mathbf{y}) \ln \frac{p(\mathbf{x}, \mathbf{y})}{p(\mathbf{x}) p(\mathbf{y})} \quad (2)$$

$$\text{mmse}(\text{snr}) = E \left[\left\| \mathbf{H} \mathbf{x} - E[\mathbf{H} \mathbf{x} | \sqrt{\text{snr}} \mathbf{H} \mathbf{x} + \mathbf{n}] \right\|^2 \right]. \quad (3)$$

$I(\mathbf{x}; \sqrt{\text{snr}} \mathbf{H} \mathbf{x} + \mathbf{n})$ denotes the mutual information in nats. The right-hand side of (3) denotes the expectation of the squared Euclidean norm of the error corresponding to the best estimation of $\mathbf{H} \mathbf{x}$ upon the observation $\mathbf{y}$ for a given SNR.

For a continuous-time real-valued scalar Gaussian channel $R_t = \sqrt{\text{snr}} X_t + N_t$, $t \in [0, T]$, where X_t is the input process and N_t is a white Gaussian noise with a flat double-sided power spectral density of unit height. Guo *et al.* established another fundamental relationship between the causal MMSE and the noncausal MMSE with any input process

$$\text{cmmse}(\text{snr}) = \frac{1}{\text{snr}} \int_0^{\text{snr}} \text{mmse}(\gamma) d\gamma \quad (4)$$

where

$$\text{cmmse}(\text{snr}) = \frac{1}{T} \int_0^T \text{cmmse}(t, \text{snr}) dt \quad (5)$$

$$\text{cmmse}(t, \text{snr}) = E[(X_t - E[X_t | Y_0^t; \text{snr}])^2] \quad (6)$$

$$\text{mmse}(\text{snr}) = \frac{1}{T} \int_0^T \text{mmse}(t, \text{snr}) dt \quad (7)$$

$$\text{mmse}(t, \text{snr}) = E[(X_t - E[X_t | Y_0^T; \text{snr}])^2]. \quad (8)$$

Equation (4) was proved in [18] by comparing (1) with Duncan's theorem. The fundamental relationships in (1) and (4), and their generalizations in [18], are referred to as the I-MMSE relationships.

The I-MMSE relationships in [18] can be generalized.

²It should be noted that for a scalar Gaussian noise channel, Brown's identity [71] indicates that the I-MMSE relationship is equivalent to De Bruijn's identity. Therefore, the main contribution of the result of Guo *et al.* is the extension of the I-MMSE relationship to the vector case.

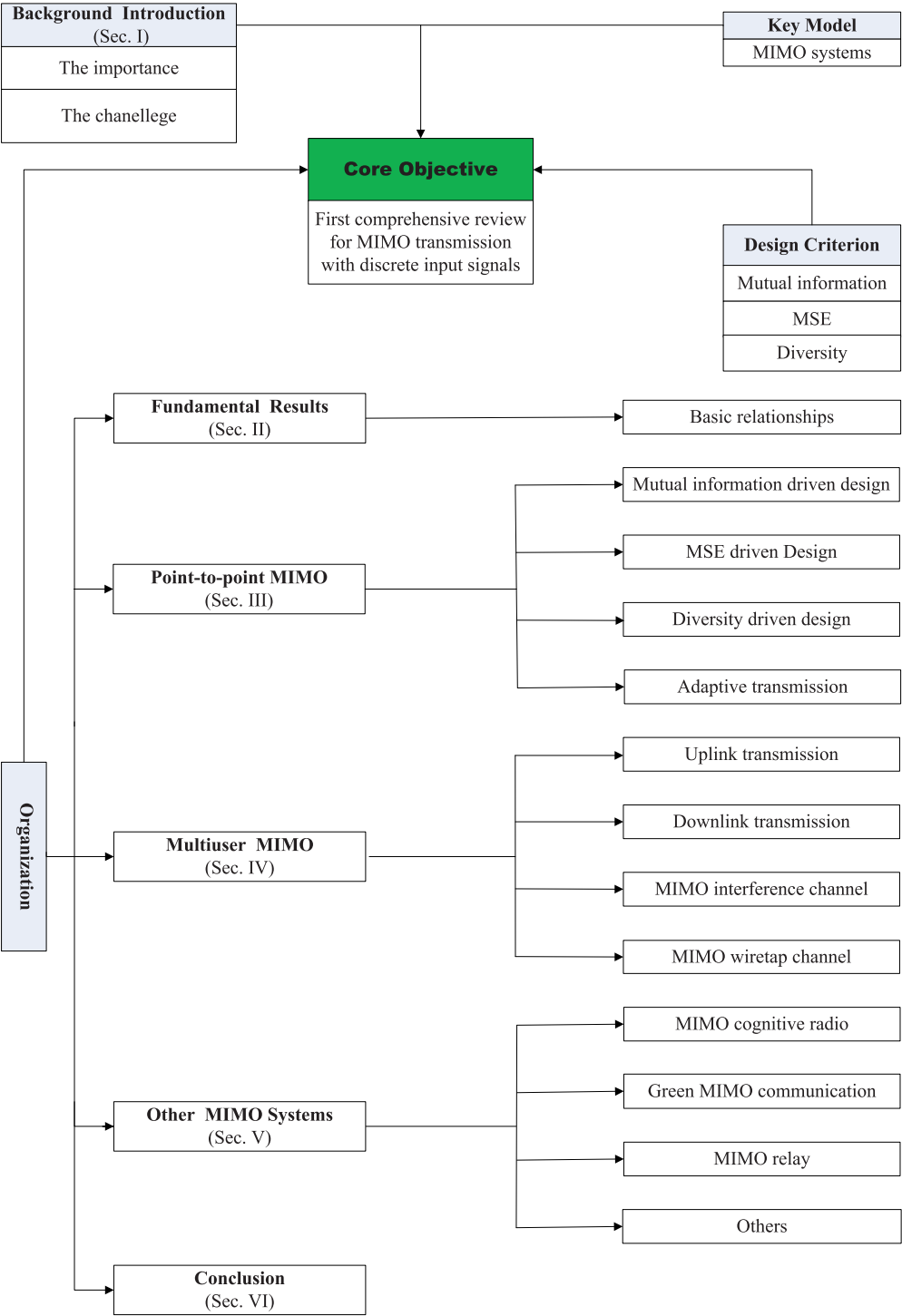


Fig. 3. Overall outline of this paper.

For a complex-valued linear vector Gaussian channel with arbitrary signaling, Palomar *et al.* [19] derived the gradient of the mutual information with respect to the channel matrix and other arbitrary parameters of the system through the chain rule. They showed that the partial

derivative of the mutual information with respect to the channel matrix is equal to the product of the channel matrix and the error covariance matrix of the optimal estimation of the input given the output. The Hessian of the mutual information and the entropy with respect to various

parameters of the system were derived in [20]. In a special case where the left singular vectors of the precoder matrix coincide with the eigenvectors of the channel correlation matrix, it was proved in [20] that the mutual information is a concave function of the squared singular values of the precoder matrix. Other generalizations of the I-MMSE relationships include their extensions to an abstract Wiener space for a scalar Gaussian channel in [78] and for a vector Gaussian channel in [79].

Guo *et al.* [21] investigated the properties of the MMSE as a function of the SNR and the input distribution for a real-valued scalar Gaussian channel. They showed that the MMSE is concave in the input distribution for a given SNR and infinitely differentiable for all SNRs for a given input distribution. An important conclusion in [21] is that the MMSE curve of a Gaussian input and the MMSE curve of a non-Gaussian input intersect at most once throughout the entire SNR regime. The MMSE properties were extended to a real-valued diagonal MIMO Gaussian channel in [80], where it was proved that each eigenvalue of the MMSE matrix with a Gaussian input has at most a single crossing point with that of the MMSE matrix with a non-Gaussian input for all SNR values.

The properties of the mutual information and the MMSE for a scalar Gaussian channel have been analyzed further. Based on the Lipschitz continuity of the MMSE and the I-MMSE relationships, Wu *et al.* [81] proved that the mutual information with input cardinality constraints converges to the usual Gaussian channel capacity as the constellation cardinality tends to infinity. Wu *et al.* [82] further provided a family of input constellations, generated from the roots of Hermite polynomials, that achieve this convergence exponentially. Merhav *et al.* [83] introduced techniques from statistical physics to evaluate the mutual information and the MMSE, and they established several important relationships between the mutual information, the MMSE, the differential entropy, and the statistical-mechanical variables. They also presented some examples of application examples showing how the statistical physics tools could be used to provide a useful analysis of the MMSE. Venkat *et al.* [84] investigated the statistical distribution of the difference between the input-output information density and half the causal estimation error. Han *et al.* [85] presented a new proof of the I-MMSE relationship by choosing a probability space independent of the SNR to evaluate the mutual information. Based on this approach, they extended the I-MMSE relationship to both discrete and continuous-time Gaussian channels with feedback.

Further extensions of the I-MMSE relationship were studied. There is a random variable with distribution P at the receiver. The receiver performs MSE estimation of this random variable under the assumption that its distribution is Q . This is referred to as “mismatched estimation.” Verdú [86] established a new connection between the relative entropy and mismatched estimation for a real-valued scalar Gaussian channel, with the key relationship

being as follows:

$$D(P \parallel Q) = \frac{1}{2} \int_0^\infty \text{mse}_Q(\gamma) - \text{mse}_P(\gamma) d\gamma \quad (9)$$

where $D(P \parallel Q)$ is the relative entropy, and $\text{mse}_Q(\gamma)$ and $\text{mse}_P(\gamma)$ are MMSEs obtained by estimators that assume the input signal is distributed according to Q and P , respectively, for a real-valued scalar Gaussian channel. Equation (9) is a generalization of (1) for a scalar Gaussian channel with a mismatched estimation. Similarly, using the mutual information and the relative entropy as a bridge, Weissman [87] generalized (4) to the case of mismatched estimation. The statistical properties of the difference between the mismatched and matched estimation losses were studied in [84]. The relationship between relative entropy and mismatched estimation was extended to a real-valued vector Gaussian channel in [88]. For a more complicated case of mismatched estimation where the channel distribution is mismatched at the estimator instead of the input signal distribution, Huleihel *et al.* [89] derived the mismatched MSE for a vector Gaussian channel using tools from statistical physics. A brief summary of recent research results on the I-MMSE relationships for a Gaussian channel is given in Table 1.³

2) Other Channel Models: The I-MMSE relationships have also been investigated for other channel models. For a scalar Poisson channel, Guo *et al.* [90] proved that the derivative of the input-output mutual information with respect to the intensity of the additive dark current is equal to the expected error between the logarithm of the actual input and the logarithm of its conditional mean estimate (noncausal in the continuous-time case). A similar relationship holds for the derivative of the mutual information with respect to input scaling, by replacing the logarithm function by $x \log x$ [90]. Moreover, Atar *et al.* [91] proved that the I-MMSE relationships that hold for Gaussian channels [18], [86], [87], including both discrete- and continuous-time channels, and matched and mismatched filters, also hold for a scalar Poisson channel on replacing the squared error loss by the corresponding loss function. Wang *et al.* [92] derived the gradient of mutual information with respect to the channel matrix for the vector Poisson channel. In addition, they defined a Bregman matrix based on the Bregman divergence to formulate a unified framework for the gradient of mutual information for both vector Gaussian and Poisson channels. The Bregman divergence has also been used to characterize the derivative of the relative entropy and the mutual information for scalar binomial and negative binomial channels [93]. The I-MMSE relationships for a scalar channel with expo-

³For every important research point in this paper, we provide a table summarizing each key research result currently available, including the detailed system model, the important assumptions made, and the main achievement. Based on these tables, researchers interested in a specific research point can easily find the results that have already been obtained and the open problems that remain to be solved.

Table 1 Recent Research Results on the I-MMSE Relationships for a Gaussian Channel

Paper	Model	Main contribution
Guo <i>et al.</i> [18]	Real-valued MIMO	Reveals a fundamental relationship between mutual information and MMSE
	Discrete-time, Continuous-time	Reveals a fundamental relationship between causal and noncausal MMSEs
Palomar <i>et al.</i> [19]	Complex-value MIMO	Derives the gradient of the mutual information
	Discrete-time	with arbitrary parameters of the systems
Payaró <i>et al.</i> [20]	Complex-value MIMO	Derives the Hessian of the mutual information and entropy
	Discrete-time	with arbitrary parameters of the systems
Zakai [78]	Real-valued scalar	Extends the I-MMSE relationships in [18] to an abstract Wiener space (scalar version)
Wolf <i>et al.</i> [79]	Real-valued MIMO	Extends the I-MMSE relationships in [18] to an abstract Wiener space (vector version)
Guo <i>et al.</i> [21]	Real-valued MIMO	Obtains various properties of the MMSE
	Discrete-time	with respect to SNR and input distribution
Bustin <i>et al.</i> [80]	Real-valued diagonal MIMO	Obtains properties of the MMSE matrix
	Discrete-time	
Wu <i>et al.</i> [81]	Real-valued scalar	Proves the mutual information with input cardinality constraints converges to
	Discrete-time	the Gaussian channel capacity as the constellation cardinality tends to infinity
Wu <i>et al.</i> [82]	Real-valued scalar	Proves the convergence speed in [81] is exponential
	Discrete-time	
Merhav <i>et al.</i> [83]	Real-valued scalar	Introduces techniques of statistical physics to
	Discrete-time	evaluate the mutual information and the MMSE
Venkat <i>et al.</i> [84]	Real-valued scalar, mismatched case	Derives the statistical properties of the difference between the
	with/without feedback	input-output information density and half the causal estimation error
Han <i>et al.</i> [85]	Real-valued scalar, feedback	Extends the I-MMSE relationship to channels with feedback
	Discrete-time, Continuous-time	
Verdú <i>et al.</i> [86]	Real-valued scalar	Establishes a new connection between the relative entropy
	Discrete-time, mismatched case	and the mismatched estimation
Weissman [87]	Real-valued scalar	Establishes a relationship between causal and noncausal
	Continuous-time, mismatched case	MMSEs in the mismatched case
Chen <i>et al.</i> [88]	Real-valued MIMO	Extends the relationship in [86] to MIMO
	Discrete-time, mismatched case	
Huleihel <i>et al.</i> [89]	Real-valued MIMO	Derives the MSE when the mismatched estimation
	Discrete-time, mismatched case	occurs for the channel distribution

nentially distributed additive noise have been established in [94]. For a continuous-time scalar channel with additive Gaussian/Poisson noise in the presence of feedback, the relationships between the directed information and the causal estimation error have been established in [95].

More general channel models have also been considered. A scalar channel with arbitrary additive noise was studied in [96], where it was found that the increase in mutual information due to improvement in channel quality is equal to the correlation of two conditional mean estimates associated with the input and the noise, respectively.

Palomar *et al.* [97] represented the derivative of mutual information in terms of the conditional input estimation given the output for general channels (not necessarily additive noise). The general results in [97] embrace most of the popular channel models, including binary symmetric channels, binary erasure channels, discrete memoryless channels, scalar/vector Gaussian channels, arbitrary additive-noise channels, and Poisson channels. For a scalar channel with arbitrary additive noise, Guo [98] obtained the derivative of the relative entropy with respect to the energy of perturbation, which can be expressed as

Table 2 Recent Research Results on the I-MMSE Relationships for Non-Gaussian Channels

Paper	Model	Main contribution
Guo <i>et al.</i> [90]	Scalar Poisson	Obtains the derivative of the mutual information with respect to the additive dark current
	Discrete-time, Continuous-time	Obtains the derivative of the mutual information with respect to the input scaling
Atar <i>et al.</i> [91]	Scalar Poisson, mismatched case	Obtains I-MMSE relationships
	Discrete-time, Continuous-time	
Wang <i>et al.</i> [92]	MIMO Poisson	Obtains the gradient of the mutual information with respect to the channel matrix
	Discrete-time	
Taborda <i>et al.</i> [93]	Scalar binomial and negative binomial	Obtains the derivative of the relative entropy and the mutual information with respect to the scaling factor
	Discrete-time	
Raginsky <i>et al.</i> [94]	Scalar, exponential noise	Obtains the I-MMSE relationships
	Discrete-time	
Weissman <i>et al.</i> [95]	Scalar Gaussian and Poisson	Obtains relationships between the directed information and the causal estimation error channels
	Continuous-time with feedback	
Guo <i>et al.</i> [96]	Scalar, arbitrary additive noise	Obtains the I-MMSE relationships
	Discrete-time	
Palomar <i>et al.</i> [97]	General channel	Obtains the derivative of the mutual information in terms of the conditional input estimation given the output
	Discrete-time	
Guo <i>et al.</i> [98]	Scalar, arbitrary additive noise	Obtains the derivative of the relative entropy with respect to the energy of perturbation of the output
	Discrete-time	
Wu <i>et al.</i> [22]	Scalar, arbitrary additive noise	Analyzes the properties of the MMSE as a function of the input–output joint distribution
	Discrete-time	
Merhav <i>et al.</i> [99]	MIMO, general channel	Analyzes the MMSE
	Discrete-time, mismatched case	
Merhav <i>et al.</i> [100]	Scalar, general channel	Establishes relationships between the rate-distortion function and the MMSE
	Discrete-time	

a mean squared difference of the score functions of the two distributions. Furthermore, Wu *et al.* [22] proved that the MMSE is a concave function of the input–output joint distribution. Merhav [99] analyzed the MMSE for the cases of both matched and mismatched estimations using statistical-mechanical techniques. The main advantage of the results in [99] is that they apply to very general vector channels with arbitrary joint input–output probability distributions. By exploiting statistical-mechanical techniques, Merhav [100] also established relationships between the rate-distortion function and the MMSE for a general scalar channel. Simple lower and upper bounds on the rate-distortion function can then be obtained based on the bounds on the MMSE. A brief summary of recent research results on the I-MMSE relationships for non-Gaussian channels is given in Table 2.

It should be noted that the I-MMSE relationships are an important tool for studying finite input signals. A typical example is the work in [101], where the I-MMSE relationships are used to show that finite inputs are optimal for an amplitude-constrained channel in a small-amplitude regime. More importantly, to design an optimal precoder

for maximization of the mutual information of MIMO systems with finite input signals, the first step is to calculate the derivative of the mutual information with respect to the precoder. The corresponding result is obtained based on the I-MMSE relationships [17], [24]–[28]. Thus, the I-MMSE relationships play a fundamental role in MIMO transmission design with finite input signals.

B. Some Analytical Results With Finite Input Signals

1) *Asymptotic Results in the Low- and High-SNR Regimes:* Shannon's pioneering work [102] showed that binary antipodal inputs are as good as the Gaussian input in the low-SNR regime. More recently, Verdú [103] investigated the spectral efficiency in the low-SNR regime for a general class of signal inputs and channels and provided two important criteria to characterize the spectral efficiency in the low-SNR regime: the minimum energy per information bit required for reliable communication and the wideband slope of the spectral efficiency. Verdú defined any input signals that achieve the same minimum energy per information bit achieved by the Gaussian input as

Table 3 Recent Analytic Results for Finite Input Signals in the Asymptotic-SNR Regime

Paper	Model	Main results in the asymptotic SNR regime
Verdú [103]	MIMO, low-SNR A general class of channels	Analyzes the minimum energy per information bit Analyzes the wideband slope
Prelov <i>et al.</i> [104]	MIMO, low-SNR A general class of channels	Obtains the second-order expansion of the mutual information
Lozano <i>et al.</i> [105]	Parallel MIMO Gaussian channels high-SNR	Provides lower and upper bounds on the MMSE
Cruz <i>et al.</i> [106]	MIMO Gaussian channels high-SNR	Provides lower and upper bounds on the MMSE and the mutual information
Wu <i>et al.</i> [23]	Scalar, arbitrary additive noise high-SNR	Defines the MMSE dimension
Duyck <i>et al.</i> [107]	MIMO Gaussian channels block Rayleigh fading, high-SNR	Proves the mutual information is a monotonically increasing function of the minimum Euclidean distance
Alvarado <i>et al.</i> [108]	Scalar Gaussian channels high-SNR	Obtains the exact high-SNR expansion of the mutual information
Ramos <i>et al.</i> [109]	Scalar Gaussian channels, Rician fading Nakagami fading, low-SNR, high-SNR	Obtains asymptotic expansions of the ergodic mutual information and the average MMSE
Rodrigues [110]	MIMO Gaussian channels, high-SNR Kronecker fading with LOS	Obtains asymptotic expansions of ergodic mutual information and the average MMSE

being first-order optimal. If perfect channel knowledge is available at the receiver, then both BPSK and QPSK are first-order optimal for a general MIMO channel. Verdú further defined any first-order optimal input signals that achieves the wideband slope achieved by a Gaussian input as being second-order optimal. For a MIMO Gaussian channel with perfect channel knowledge at the receiver, Verdú proved that equal-power QPSK is second-order optimal. Moreover, Prelov *et al.* [104] were able to obtain a second-order expansion of the mutual information in the low-SNR regime for very general classes of inputs and channel distributions.

For parallel Gaussian channels with m -ary constellation inputs, Lozano *et al.* [105] provided both lower and upper bounds on the MMSE in the high-SNR regime. For an arbitrary MIMO Gaussian channel, Pérez-Cruz *et al.* [106] further derived high-SNR lower and upper bounds on the MMSE and the mutual information. For a scalar channel with arbitrary inputs and additive noise, Wu *et al.* [23] defined the MMSE dimension in the case when SNR tends to infinity. The MMSE dimension represents the gain of the nonlinear estimation error over the linear estimation error in the high-SNR regime. For MIMO Gaussian block Rayleigh fading channels with m -ary constellation inputs,

it has been proved that a constellation with a higher minimum Euclidean distance achieves a higher mutual information in the high-SNR regime [107, Appendix E]. For a scalar Gaussian channel with arbitrary input distributions independent of SNR, Alvarado *et al.* [108] analyzed how fast the mutual information converges to the entropy of the constellation when the SNR tends to infinity by characterizing the high-SNR behavior of the difference between them. It was shown that the asymptotic mutual information approaches the entropy of the constellation with a speed proportional to the Gaussian Q -function. An asymptotic achievable rate expression for bit-interleaved coded modulation (BICM) was also derived. Based on the expression obtained, it was proved that the Gray code is asymptotically optimal in the high-SNR regime for BICM. By using the Mellin transform, Ramos *et al.* [109] derived low- and high-SNR asymptotic expansions of the ergodic mutual information and the average MMSE for scalar Gaussian channels with arbitrary finite inputs over Rician and Nakagami fading. High-SNR expansions have been derived for a MIMO Gaussian channel over Kronecker fading with the line-of-sight (LOS) [110]. A brief summary of recent analytic results for finite input signals in the asymptotic-SNR regime is given in Table 3.

Table 4 Analytic Results (Transmitter Side) for Finite Input Signals in the Large-System Limit

Paper	Channel	Criterion
Müller [115]	Point-to-point MIMO, spatially correlated Rayleigh	Asymptotic mutual information (BPSK)
Wen <i>et al.</i> [116]	Point-to-point MIMO, Kronecker fading	Asymptotic mutual information
Wen <i>et al.</i> [117]	Distributed MIMO MAC, Kronecker fading	Asymptotic sum-rate
Wen <i>et al.</i> [118]	Centralized MIMO MAC, Kronecker fading	Asymptotic sum-rate
Wen <i>et al.</i> [119]	Centralized MIMO MAC, Kronecker fading with LOS	Asymptotic sum-rate
Wen <i>et al.</i> [120]	Centralized MIMO MAC with AF relay, Kronecker fading	Asymptotic sum-rate
Girnyk <i>et al.</i> [121]	K-hop AF relay MIMO, i.i.d. Rayleigh fading	Asymptotic mutual information
Müller <i>et al.</i> [122]	Point-to-point MIMO, i.i.d. Rayleigh fading, vector precoding	Asymptotic average minimum transmit power
Zaidel <i>et al.</i> [123]	MIMO BC, i.i.d. Rayleigh fading, vector precoding (IRSB ansatz)	Asymptotic average minimum transmit power
Vehkaperä <i>et al.</i> [124]	Point-to-point MIMO, i.i.d. Rayleigh fading, i.i.d. input signals Transmit-side noise and mismatched decoding	General mutual information

2) *Asymptotic Results in the Large-System Limit:* Some analytic results have been obtained for finite input signals in the large-system limit. A technique that has been widely used in statistical physics, referred to as the replica method, is initially exploited to analyze the asymptotic performance of various detection schemes of code-division multiple-access (CDMA) systems in large dimension [111]–[114]. Using this replica method, Müller [115] was the first to derive an asymptotic mutual information expression for a spatially correlated MIMO channel with BPSK inputs. This expression was extended to arbitrary input at the transmitter in [116]. Wen *et al.* obtained an asymptotic sum-rate expression for Kronecker fading channels for the centralized MIMO multiple-access channel (MAC) and the distributed MIMO MAC in [117] and [118], respectively. An asymptotic sum-rate expression including the LOS effect for the centralized MIMO MAC was derived in [119]. Wen *et al.* [120] further obtained an asymptotic sum-rate expression for Kronecker fading channels for the centralized MIMO MAC with amplify-and-forward (AF) relay. Also, Girnyk *et al.* [121] derived an asymptotic mutual information expression for i.i.d. Rayleigh fading for K -hop AF relay MIMO channels. For other performance criteria, Müller *et al.* [122] obtained an asymptotic expression for the average minimum transmit power for i.i.d. Rayleigh fading MIMO channels with nonlinear vector precoding. A similar problem was also considered for the MIMO broadcast channel (BC) in [123]. A more general analysis, referred to as “one-step replica-symmetry-breaking analysis,” was used in [123] to reduce the asymptotic approximation errors in [122] for nonconvex alphabet sets.

The effect of transmit-side noise caused by hardware impairments was investigated in [124] for point-to-point MIMO systems with i.i.d. finite input signals and i.i.d. MIMO Rayleigh fading channels. It was assumed that the

receiver has perfect channel state information (CSI) but the transmitter does not know the CSI. Also, the transmit-side noise is generally unknown at both transmitter and receiver. As a result, the receiver employs mismatched decoding without taking account of the transmit-side noise. In this case, the general mutual information defined in [125] is adopted as the metric to evaluate the performance of mismatched decoding. An asymptotic expression for the general mutual information can be derived when the numbers of transmit and receive antennas both tend to infinity with a fixed ratio between them. Numerical results indicate that the derived asymptotic expression provides an accurate approximation for the exact general mutual information and that the effects of transmit-side noise and mismatched decoding become significant only at high modulation orders. A brief summary of recent analytic results (transmitter side) for finite input signals in the large-system limit is given in Table 4.

3) *Numerical Results:* There are some other numerical algorithms for finite input signals. For a scalar Gaussian channel with a finite and fixed input signal constellation, Varnica *et al.* [126] proposed a modified Arimoto–Blahut algorithm to search for the optimal input distribution that maximizes the information rate. The corresponding maximum information rate is referred to as the “constellation-constrained capacity.” Bellorado *et al.* [127] studied the constellation-constrained capacity of a MIMO Rayleigh fading channel and showed that for a given information rate, the SNR gap between the uniformly distributed signal and the constellation-constrained capacity when the constellation size tends to infinity is 1.53 dB. This is the same as the QAM asymptotic shaping gap for the scalar Gaussian channel in [128]. It was also conjectured in [127] that when the elements of the input signal vector are chosen from a Cartesian product of 1-D PAM con-

stellations, the constellation-constrained capacity can be achieved by optimizing the distribution of each element of the signal vector independently. This reduces the computational complexity significantly. This conjecture was proved in [129]. Yankov *et al.* [130] derived a low-computational-complexity lower bound on the constellation-constrained capacity for a MIMO Gaussian channel based on QR decomposition. For a 1-D channel with memory and finite input state, Arnold *et al.* [131] proposed a practical algorithm to compute the information rate. For a 2-D channel, tight lower and upper bounds on the information rate were derived in [132]. Shental *et al.* [133] proposed a more accurate computational method for the information rate of a 2-D channel with memory and finite input state based on generalized belief propagation.

III. POINT-TO-POINT MIMO SYSTEMS

In this section, we focus on transmission designs for point-to-point MIMO systems with finite input signals. We classify these designs into four categories: 1) designs based on mutual information; 2) designs based on MSE; 3) diversity-driven designs; and 4) adaptive transmission.

A. Designs Based on Mutual Information

Mutual information maximization yields the maximal achievable rate of the communication link with the implementation of reliable error control codes [134]. For point-to-point MIMO systems, maximization of the mutual information was achieved in [1]. It was proved in [1] that the mutual information is maximized by a Gaussian-distributed signal with a covariance structure satisfying two conditions. First, the signal must be transmitted along the right eigenvectors of the channel, which decomposes the original channel into a set of parallel subchannels. Second, the power allocated for each subchannel is based on a water-filling strategy, which depends on the strength of each subchannel and the SNR.

Although the Gaussian signal is capacity-achieving, it is too idealized for implementation in practical communication systems. The finite input signals are taken from a finite discrete constellation set, such as PSK, PAM, or QAM. Therefore, some work has begun to investigate MIMO transmission designs based on discrete constellation input mutual information.

1) *Mutual Information Maximization:* For a scalar Gaussian channel with amplitude-constrained input signals and a discrete-time Rayleigh fading scalar channel with an average power constraint, the capacity-achieving inputs have been shown to be discrete random variables over a finite number of values [135], [136]. For MIMO Gaussian channels with amplitude-constrained input signals, finding the capacity is still an open problem. The technical challenges lie in the difficulty of extending the identity theorem from an N -dimensional real-valued domain to an N -dimensional complex-valued domain. Dytso *et al.* [137] derived novel upper and lower bounds

for a MIMO Gaussian channel with amplitude-constrained input signals. These bounds have been proved to lie within a constant gap for invertible channel matrices and are tight in the high-amplitude regime for arbitrary channel matrices.

By exploiting the I-MMSE relationship in [18], Lozano *et al.* [105] proposed a mercury water-filling power allocation policy that maximizes the input-output mutual information for diagonal MIMO Gaussian channels with arbitrary inputs. The mercury water-filling policy pours mercury onto each of the subchannels to a certain height before filling the water (allocated power) to the given threshold, which is different from the conventional water-filling policy. Because of the poured mercury, the classical conclusion for a Gaussian input signal that the stronger subchannel receives more filling water (allocated power) no longer holds. The asymptotic expansions of the MMSE function and the mutual information in the low- and high-SNR regimes⁴ were obtained in [105]. Based on these expansions, it has been proved that allocating power only to the strongest channel is asymptotically optimal in the low-SNR regime, as with the conventional water-filling allocation. By contrast, however, for optimal power allocation in the high-SNR regime, less power should be allocated to the stronger subchannel for equal constellation inputs. This is significantly different from the conventional water-filling allocation. Also, it has been shown that more power should be allocated to the richer constellation inputs for equal channel strength at high SNR.

For a nondiagonal MIMO Gaussian channel with arbitrary inputs, an iterative numerical routine to search for the optimal linear precoder based on a gradient descent update of the precoder matrix was proposed in [19]. Moreover, necessary conditions for the optimal linear precoding matrix maximizing the mutual information of a nondiagonal MIMO Gaussian channel with finite alphabet inputs were presented in [139], and a numerical algorithm was formulated to search for the optimal precoder. Simulations indicate that the nondiagonal and nonunitary precoder obtained in [139] outperforms the mercury water-filling design for a nondiagonal MIMO Gaussian channel. The design presented in [139] has been extended to the case where only statistical CSI (SCSI) is available at the transmitter [140]. Meanwhile, through Karush–Kuhn–Tucker (KKT) analysis, Pérez-Cruz *et al.* [106] also established necessary conditions to be satisfied by the optimal precoder for a nondiagonal MIMO Gaussian channel with finite alphabet inputs. The MMSE function and the mutual information of a nondiagonal MIMO Gaussian channel with finite alphabet inputs have been expanded in the asymptotic low- and high-SNR regimes. It has been proved that in the low-SNR regime, the optimal precoder design for finite alphabet inputs is exactly the same as the optimal design for a Gaussian input. In the high-SNR regime, lower

⁴It should be noted that the optimality of finite inputs in the high-SNR regime for SISO channels was shown in [138].

and upper bounds on the mutual information have been derived. Based on these bounds, it has been shown that the precoder that maximizes the mutual information is equivalent to the precoder that maximizes the minimum Euclidean distance of the receive signal points in the high-SNR regime. This result indicates that the three important criteria in MIMO transmission design, namely, maximizing the mutual information, minimizing the symbol error rate (SER), and minimizing the MSE, are equivalent in the high-SNR regime. Similarly, the strong version of the Ozarow–Wyner bound in [137] also supports the intuition that designing optimal constellations for MIMO systems in the high-SNR regime is equivalent to minimizing the MSE. The optimal power allocation scheme for a general MIMO Gaussian channel was also obtained in [106].

For real-valued nondiagonal MIMO Gaussian channels, Payaró *et al.* decomposed the design of the optimal precoder into three components: the left singular vectors, the singular values, and the right singular vectors [141]. The optimal left singular vectors were shown to be the right singular vectors of the channel matrix. Necessary and sufficient conditions on the optimal singular values were established, and from these the optimal singular values can be found based on standard numerical algorithms such as Newton’s method. The optimal solution of the right singular vectors is a difficult problem. Based on standard numerical algorithms, a local optimum design can be found. Moreover, the optimal designs in the low- and high-SNR regimes were analyzed in [141]. For complex-valued nondiagonal MIMO Gaussian channels, Lamarca [142] proved that the optimal left singular vectors of the precoder are still the right singular vectors of the channel matrix. More importantly, Lamarca also proved that when the entries of the channel matrix and the input signals are all real-valued, the mutual information is a concave function of a quadratic function of the precoder matrix and proposed an iterative algorithm to maximize the mutual information by updating this quadratic function along the gradient descent direction. For a complex-valued nondiagonal MIMO Gaussian channel with finite alphabet inputs, Xiao *et al.* [17] proved that the mutual information is still a concave function of a quadratic function of the precoder matrix, and they designed a parameterized iterative algorithm to find the optimal precoder that achieves the global maximum of the mutual information. Moreover, they simulated the coded bit-error rate (BER) performance of the precoder thus obtained. From a comparison with conventional designs, it can be seen that the BER gains of this precoder coincide with the mutual information gains, which indicates that maximizing the mutual information is also an effective criterion for optimizing the coded BER in practical systems.

Yuan *et al.* [143] considered a joint linear precoding and linear MMSE detection scheme for finite alphabet input signals with perfect CSIT (PCSIT) and perfect CSI at the receiver (PCSIR). The linear precoder was taken as the product of three matrices: the right singular vector matrix

of the channel, the diagonal power allocation matrix, and the normalized discrete Fourier transform matrix. When the detector and decoder satisfy the curve-matching principle [143, eq. (23)], an asymptotically achievable rate of the linear precoding and linear MMSE detection scheme can be derived as the dimension of the transmit signal tends to infinity. This rate is a concave function of the power allocation matrix. Consequently, an optimal power allocation scheme for maximizing the achievable rate can be obtained based on the standard convex optimization method.

Very recently, Cao *et al.* [144] have proposed a precoding design in the high-SNR regime by transforming the maximization of the minimum Euclidean distance into a semi-definite programming optimization problem. Moreover, when the number of antennas is high, the implementation of digital beamforming may be of significant complexity. To reduce this complexity, MIMO beamforming can be implemented in both the analog and digital domains, which is referred to as “hybrid beamforming” [145]. Rajashekar *et al.* [146] have investigated hybrid beamforming designs to maximize mutual information for millimeter-wave MIMO systems with finite alphabet inputs. A brief summary of above-mentioned work is given in Table 5.

2) Low-Complexity Design: Although the algorithm proposed in [17] finds the global optimal precoder capable of maximizing the mutual information for a complex-valued nondiagonal MIMO Gaussian channel with finite alphabet inputs, its implementation is usually too computationally complex for practical applications. There are two main reasons for this high computational complexity. First, both the mutual information and the MMSE function lack closed-form expressions. Therefore, the Monte Carlo method has to be used to evaluate both terms, and this requires an average over an enormous number of noise realizations in order to achieve acceptable accuracy. More importantly, the evaluation of the mutual information [139, eq. (5)] and the MMSE function [139, eq. (8)] involves a number of additions over the modulation signal space that rises exponentially with the number of transmit antennas. This results in a prohibitive computational complexity when there are many transmit antennas.

By employing Jensen’s inequality and an integral over the exponential function, a lower bound on the single-input single-output (SISO) mutual information with finite alphabet inputs was derived in [147, eq. (9)]. The evaluation of this lower bound does not require calculation of the expectation over the noise. Consequently, Zeng *et al.* [148] were able to extend this lower bound to the case of MIMO systems and thereby deal with the first aspect of the high computational complexity related to the use of the Monte Carlo average. An algorithm was proposed to maximize this lower bound. Numerical results indicate that the design proposed in [148] reduces the computational complexity without loss of performance. Zeng *et al.* [26]

Table 5 MIMO Transmissions With Finite Alphabet Inputs Based on Mutual Information Maximization

Paper	Model	Main contribution
Lozano <i>et al.</i> [105]	Diagonal MIMO	Obtains the optimal power allocation
	PCSIT, PCSIR	Obtains the optimal power allocation in low- and high-SNR regimes
Palomar <i>et al.</i> [19]	Non-diagonal MIMO	Proposes a gradient descent algorithm to search for the optimal precoder
	PCSIT, PCSIR	
Xiao <i>et al.</i> [139]	Non-diagonal MIMO	Establishes necessary conditions for the optimal precoder
	PCSIT, PCSIR	Proposes a numerical algorithm to search for the optimal precoder
Xiao <i>et al.</i> [140]	Non-diagonal MIMO	Establishes necessary conditions for the optimal precoder
	SCSIT, PCSIR	Proposes a numerical algorithm to search for the optimal precoder
Pérez-Cruz <i>et al.</i> [106]	Non-diagonal MIMO	Establishes necessary conditions for the optimal precoder
	PCSIT, PCSIR	Analyzes the optimal precoder in low- and high-SNR regimes
Payaró <i>et al.</i> [141]	Real-valued non-diagonal MIMO	Obtains the left singular vectors and the singular values of the optimal precoder
	PCSIT, PCSIR	Analyzes the optimal precoder in low- and high-SNR regimes
Lamarca [142]	Non-diagonal MIMO	Obtains the left singular vectors of the optimal precoder in complex-valued channels
	PCSIT, PCSIR	Proves that the mutual information is a concave function of a quadratic function of the precoder matrix for real-valued non-diagonal MIMO channels Proposes an iterative algorithm to maximize the mutual information
Xiao [17]	Non-diagonal MIMO	Proves that the mutual information is still a concave function of a quadratic function of the precoder matrix for complex-valued non-diagonal MIMO channels
	PCSIT, PCSIR	Proposes a global optimal iterative algorithm to maximize the mutual information Examines the coded BER performance of the obtained precoder
Yuan <i>et al.</i> [143]	Non-diagonal MIMO	Designs the linear precoder as a product of three matrices
	PCSIT, PCSIR	Obtains an asymptotic achievable rate of this linear precoder and linear MMSE detection
Kao <i>et al.</i> [144]	Non-diagonal MIMO	Transforms the maximization of the minimum Euclidean distance in the high-SNR regime into a semi-definite programming optimization problem
	PCSIT, PCSIR	
Rajashekar <i>et al.</i> [146]	Non-diagonal MIMO	Proposes hybrid beamforming designs to maximize
	PCSIT, PCSIR	mutual information for millimeter-wave MIMO systems

further investigated a linear precoder design with SCSIT over the Kronecker fading model. A lower bound on the ergodic mutual information that avoids the need for expectations over both the channel and the noise was derived. Simulations over various fading channels showed that with a constant shift, this lower bound provides an accurate approximation of the ergodic mutual information. Moreover, it was proved that maximizing this lower bound is asymptotically optimal in the low- and high-SNR regimes. An iterative algorithm was further developed by maximizing this lower bound. Numerical results indicate that the design proposed in [26] achieves a near-optimal mutual information and BER performance with reduced complexity. The lower bound in [26] was utilized in [149] to design a linear precoder for practical MIMO-orthogonal frequency division multiplexing (OFDM) systems.

When perfect instantaneous CSI is available at the transmitter, the MIMO channel can be decomposed into a set of parallel subchannels. To deal with the second aspect of the high computational complexity, Mohammed *et al.* [25] proposed X-Codes to pair two subchannels into a group. X-Codes are fully characterized by the pairing and a 2×2 single-angle parameterized rotation matrix for each pair. Based on the X-Code structure, the mutual information can be expressed as the sum of the mutual information of all the pairs. In this case, the calculation of the mutual information only involves additions over the modulation signal space, and the number of these additions goes polynomially with the number of transmit antennas. This significantly reduces the computational complexity of evaluating the mutual information. Designs of the rotation angle and power allocation within each pair and the optimization pairing and power allocation

Table 6 Number of Additions Required for Calculating the Mutual Information and the MSE Matrix

N_t	4	8	16	32
Complete-search design in [26]	65 536	4.29 e+009	1.84 e+019	3.4 e+038
Algorithm in [152]	512	1024	2048	4096

among pairs have been proposed to increase the mutual information performance. Ketseoglou *et al.* [27] extended this idea to pair multiple subchannels based on a per-group precoding (PGP) technique. The PGP technique decomposes the design of the power allocation matrix and the right singular vector matrix into the design of several decoupled matrices of much smaller dimension. An iterative algorithm was proposed to find the optimal solution of these decoupled matrices. Numerical results indicate that the proposed algorithm achieves almost the same performance as the design in [17], but with a significant reduction in computational complexity. Moreover, Ketseoglou *et al.* [150] proposed a novel and efficient method to evaluate the mutual information of a complex-valued nondiagonal MIMO Gaussian channel with finite alphabet inputs based on the Gauss–Hermite quadrature rule. This approximation method can be combined with the PGP technique to further reduce the design complexity.

For the case when only SCSIT is available at the transmitter, a low-complexity iterative algorithm to find the optimal design over the Kronecker fading model was proposed in [151]. This algorithm relies on an asymptotic approximation of the ergodic mutual information in the large-system limit, which avoids the need for a Monte Carlo average over the channel. Moreover, based on this approximation, the design of the power allocation matrix and the right singular vector matrix of the precoder can also be decoupled into the design of small-dimension matrices within different groups. Simulations indicate that the algorithm proposed in [151] radically reduces the computational complexity of the design in [26] by orders of magnitude, with only minimal losses in performance. Wu *et al.* [152] proposed a unified low-complexity precoder design for a complex-valued nondiagonal MIMO Gaussian channel with finite alphabet inputs. An asymptotic expression for the ergodic mutual information over the jointly correlated Rician fading model was derived in large-system limit. Combined with the approximation given in [26], the asymptotic expression thus obtained can provide an accurate approximation of the ergodic mutual information without the need for a Monte Carlo average over the channel and the noise. The left singular matrix of the optimal precoder for this general model has been obtained. The structures of the power allocation matrix and the right singular matrix of the precoder have been established. The proposed structures decouple the independent data streams over parallel equivalent subchannels, which avoids the need for a complete search of the entire signal space for the precoder design. Based on this approach, a novel low-computational-complexity iterative algorithm has been proposed to search for the optimal precoder with general CSI availability at the transmitter, which includes the cases of PCSIT, imperfect CSIT (IPC-SIT), and SCSIT. Here we provide Examples 1 and 2 to analytically and numerically compare the complexities of the low-complexity algorithm in [152] and the complete-search design in [26]. As can be seen from Tables 6–9, the computational complexity of the complete-search design in

Table 8 Running Time (Seconds) per Iteration With QPSK

N_t	$N_s = 2$	$N_s = 4$	$N_s = N_t$
4	0.1149	21.5350	21.5350
8	0.2029	23.3442	×
16	0.3001	48.1725	×
32	0.7094	98.7853	×

otic expression for the ergodic mutual information over the jointly correlated Rician fading model was derived in large-system limit. Combined with the approximation given in [26], the asymptotic expression thus obtained can provide an accurate approximation of the ergodic mutual information without the need for a Monte Carlo average over the channel and the noise. The left singular matrix of the optimal precoder for this general model has been obtained. The structures of the power allocation matrix and the right singular matrix of the precoder have been established. The proposed structures decouple the independent data streams over parallel equivalent subchannels, which avoids the need for a complete search of the entire signal space for the precoder design. Based on this approach, a novel low-computational-complexity iterative algorithm has been proposed to search for the optimal precoder with general CSI availability at the transmitter, which includes the cases of PCSIT, imperfect CSIT (IPC-SIT), and SCSIT. Here we provide Examples 1 and 2 to analytically and numerically compare the complexities of the low-complexity algorithm in [152] and the complete-search design in [26]. As can be seen from Tables 6–9, the computational complexity of the complete-search design in

Table 7 Running Time (Seconds) per Iteration With BPSK

N_t	$N_s = 2$	$N_s = 4$	$N_s = N_t$
4	0.0051	0.0190	0.0190
8	0.0112	0.0473	11.6209
16	0.0210	0.1939	×
32	0.0570	0.4111	×

Table 9 Running Time (Seconds) per Iteration With 16-QAM

N_t	$N_s = 2$	$N_s = N_t$
4	28.0744	×
8	58.3433	×
16	106.6022	×
32	233.2293	×

Table 10 Low-Complexity MIMO Transmissions With Finite Alphabet Inputs

Paper	Model	Main contribution
Zeng <i>et al.</i> [148]	MIMO PCSIT, PCSIR	Design based on a lower bound without Monte Carlo average over the noise
Zeng <i>et al.</i> [26]	MIMO SCSIT, PCSIR	Design based on a lower bound without Monte Carlo average over the channel and the noise
Zheng <i>et al.</i> [149]	MIMO-OFDM practical channel estimation	Evaluates MIMO transmissions with finite alphabet inputs in a practical MIMO test-bed
Mohammed <i>et al.</i> [25]	MIMO PCSIT, PCSIR	Decouples the mutual information into a sum of the mutual information of small-dimension matrices
Ketseoglou <i>et al.</i> [27]	MIMO PCSIT, PCSIR	Decomposes the design of the power allocation matrix and the right singular vector matrix into the design of several decoupled matrices with a much smaller dimension
Ketseoglou <i>et al.</i> [150]	MIMO PCSIT, PCSIR	Proposes a novel and efficient method to evaluate the mutual information based on the Gauss–Hermite quadrature rule
Wu <i>et al.</i> [151]	MIMO SCSIT, PCSIR	Proposes a design that reduces the computational complexity of the design in [26] by orders of magnitude with only minimal losses in performance
Wu <i>et al.</i> [152]	MIMO general CSIT, PCSIR	Proposes a unified low-complexity precoder design that applies for the general CSI availability at the transmitter

[26] grows exponentially and quickly becomes unwieldy. Simulations indicate that the algorithm proposed in [152] drastically reduces the complexity of implementation for finite alphabet precoder design, but in such a way that the loss in performance—established based on the 3GPP spatial channel model [153]—is minimal. Moreover, some novel insights into precoder design with SCSIT have been revealed in [152]. A brief summary of the above-mentioned low-complexity MIMO transmissions with finite alphabet inputs is given in Table 10.

Example 1: Denote by N_t and N_s the number of transmit antennas and the number of streams in each subgroup, respectively. Assume $N_s = 2$ and QPSK modulation. The numbers of additions required for calculating the mutual information and the MSE matrix in the complete-search design in [26] and the algorithm in [152] are listed in Table 6 for different numbers of transmit antennas.

Example 2: Let us evaluate the complexity of the algorithm in [152] for different values of N_s . MATLAB is used on an Intel Core i7-4510U 2.6GHz processor. Tables 7–9 provide the running time per iteration, for various numbers of antennas and constellations, with $\times$ indicating that the time exceeds one hour.

B. Design Based on MSE

1) *Linear Transmission Design:* The MSE is an important criterion to measure the quality of the communication link with finite input signals. With PCSIT and PCSIR, an

optimal linear transceiver design to minimize the weighted sum MSE of all channel substreams under a total average transmit power constraint was proposed in [154], based upon early work in [155]–[158]. The minimization of the sum MSE of all channel substreams under a maximum eigenvalue constraint was investigated in [159]. A unified framework to optimize the MSE function of all channel substreams under a total average transmit power constraint for a multi-carrier MIMO system and linear processing at both ends of the link was established in [29]. The system model in [29] is illustrated in Fig. 4, where a multi-carrier MIMO system is considered

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{B}_k \mathbf{x}_k + \mathbf{n}_k, \quad 1 \leq k \leq L \quad (10)$$

where L is the number of carriers. $\mathbf{y}_k$, $\mathbf{H}_k$, $\mathbf{B}_k$, $\mathbf{x}_k$, and $\mathbf{n}_k$ denote the received signal vector, the channel matrix, the precoder matrix, the transmit signal, and the zero-mean Gaussian noise vector with covariance matrix $\mathbf{R}_{n_k}$, respectively.

The received signal $\hat{\mathbf{x}}_k$ at the k th carrier is given by

$$\hat{\mathbf{x}}_k = \mathbf{A}_k^H \mathbf{y}_k \quad (11)$$

where $\mathbf{A}_k$ is the equalizer at the receiver. The optimal linear MMSE receiver $\mathbf{A}_k^{\text{opt}}$ is given by

$$\mathbf{A}_k^{\text{opt}} = (\mathbf{H}_k \mathbf{B}_k \mathbf{B}_k^H \mathbf{H}_k^H + \mathbf{R}_{n_k})^{-1} \mathbf{H}_k \mathbf{B}_k. \quad (12)$$

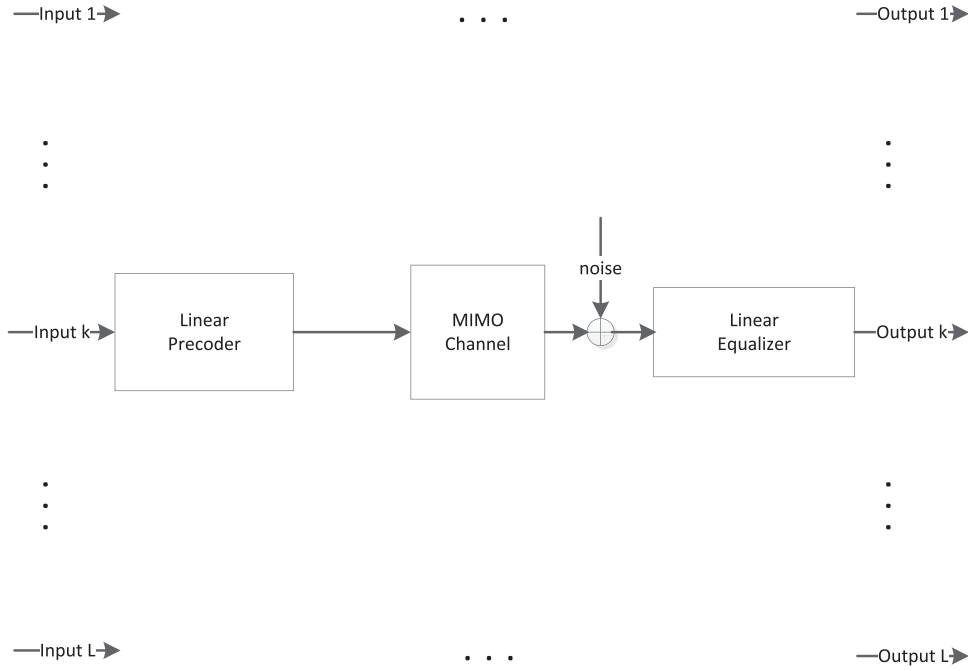


Fig. 4. Multicarrier MIMO system with linear processing and carrier-noncooperative scheme.

Based on $\mathbf{A}_k^{\text{opt}}$, the covariance matrix $\mathbf{E}_k(\mathbf{B}_k)$ of the error vector $\mathbf{e}_k = \hat{\mathbf{x}}_k - \mathbf{x}_k$ can be written as

$$\mathbf{E}_k(\mathbf{B}_k) = (\mathbf{I} + \mathbf{B}_k^H \mathbf{R}_{H_k} \mathbf{B}_k)^{-1} \quad (13)$$

where $\mathbf{R}_{H_k} = \mathbf{H}_k^H \mathbf{R}_{n_k}^{-1} \mathbf{H}_k$.

For the design of the precoder $\mathbf{B}_k$, the following optimization problem is considered:

$$\min_{\mathbf{B}_k} f_0(\mathbf{d}(\mathbf{E}_k(\mathbf{B}_k))) \quad (14)$$

$$\text{s.t. } \text{Tr}(\mathbf{B}_k \mathbf{B}_k^H) \leq P_T \quad (15)$$

where $\mathbf{d}(\mathbf{E}_k(\mathbf{B}_k))$ returns the vector containing the diagonal elements of $\mathbf{E}_k(\mathbf{B}_k)$.

If f_0 is Schur-concave, it can be proved that the optimal $\mathbf{B}_k$ has the following structure:

$$\mathbf{B}_k = \mathbf{U}_{H,k} \mathbf{\Sigma}_{B,k} \quad (16)$$

where $\mathbf{U}_{H,k}$ is the eigenvector matrix corresponding to nonzero eigenvalues of $\mathbf{R}_{H_k}$, and $\mathbf{\Sigma}_{B,k}$ has zero elements except along the right-most main diagonal.

If f_0 is Schur-convex, it can be proved that the optimal $\mathbf{B}_k$ has the following structure:

$$\mathbf{B}_k = \mathbf{U}_{H,k} \mathbf{\Sigma}_{B,k} \mathbf{V}_{B,k}^H \quad (17)$$

where $\mathbf{V}_{B,k}^H$ is a unitary matrix such that $\mathbf{E}_k(\mathbf{B}_k)$ has identical diagonal elements.

It should be noted that the above two families of objective functions embrace most of the popular criteria used in communication systems, including the MSE, the SINR, and also the uncoded BER with identical constellations. Based on the precoder structures in (16) and (17), the original matrix optimization problem in (14) can be transformed into a much simpler scalar optimization problem, the solution of which can easily be obtained using classic optimization methods. The design in [29] was extended to a general shaping precoder constraint in [160]. Moreover, Palomar [161] employed a primal decomposition approach to decompose the original optimization problem over multiple (e.g., multicarrier) MIMO channels into several subproblems, each of which can be independently solved over a single MIMO channel. These subproblems are coordinated by a simple master problem. As a result, the optimization problem in [29] can be solved in a simple and efficient way. Meanwhile, the transmission design that satisfies the specific MSE constraints with minimum transmit power was investigated in [162]. The optimal solution is to prerotate the transmit signal by a particular unitary matrix and then diagonalize the channel. A similar problem was also considered in [163] under the uncoded BER constraints instead of the MSE constraints. It was assumed that the constellation used for each substream is different. Then, the uncoded BER is neither a Schur-convex nor a Schur-concave function of the MSE. This optimization problem can be solved by the primal

decomposition approach. Moreover, Ordóñez *et al.* [164] proposed a transmission scheme with an adaptive number of substreams to minimize the uncoded BER under the constraints of a fixed transmission rate and a fixed total average transmit power. A widely linear MMSE transceiver that processes the in-phase and quadrature components of the input signals separately was designed in [165]. The widely linear processing is implemented using a real-valued representation of the input signals. More recently, a linear precoder design to minimize the sum MSE of all channel substreams under a joint total average transmit power constraint and maximum eigenvalue constraint was proposed in [166].

However, perfect CSI is only reasonable for the slow fading scenario where the channel remains deterministic for a long period. Otherwise, the CSI estimated at the transmitter and the receiver may have mismatches with the actual channel. Therefore, Serbetli *et al.* [30] investigated the optimum transceiver structure in the sense of minimizing the sum MSE of transmit data streams via imperfect CSI at both ends. A MIMO system with correlated receive antennas was considered and the effect of channel estimation was investigated in [30]. Zhang *et al.* [167] investigated a statistically robust design of linear MIMO transceivers based on minimizing the MSE function under a total average transmit power constraint. The statistically robust design exploits the channel mean and channel covariance matrix (equivalently, the channel estimate and the estimation error) to design linear MIMO transceivers. Two cases were considered in [167]: 1) IPCSIT and imperfect channel state information at the receiver (IPCSIR) with no transmit antenna correlation; and 2) IPCSIT and PCSIR. For the former case, the optimal solution has the same structure as the perfect CSI case on replacing the instantaneous channel realization matrix and the noise covariance matrix with the channel mean matrix and an equivalent covariance matrix (e.g., the channel covariance matrix plus the noise covariance matrix), respectively. For the latter case, a lower bound on the average MSE of all substreams was derived and the linear precoder minimizing this lower bound was obtained. A statistically robust design for the IPCSIT and IPCSIR case with transmit antenna correlation was investigated in [168]. By finding the optimal solution that satisfies the KKT conditions, the structures of the optimal precoder and equalizer minimizing the sum MSE of transmit data streams under a total average transmit power constraint were obtained in [168]. Meanwhile, Wang *et al.* [169] investigated a deterministic robust design for a MIMO system under a total average transmit power constraint. The deterministic robust design assumes the estimated imperfect CSI is within a neighborhood of actual channel. The case of IPCSIT and IPCSIR was considered in [169]. Given a prefixed linear receiver structure, the optimal transmission direction and the optimal power allocation for the linear precoder matrix minimizing the MSE of transmit data streams were found in [169]. For the same scenario, the optimal linear precoder structure given

a prefixed linear receiver and the optimal linear equalizer structure given a prefixed linear precoder were obtained in [170]. Therefore, the precoder and the equalizer can be designed alternatively by an iterative algorithm. For the scenario considered in [169], the optimal linear MIMO transceiver design was found in [171]. On the other hand, to reduce the cost of analog-to-digital converter (ADC) chains at the receiver, MIMO analog beamformers were designed in [172] to minimize both the output MSE of the digital beamformer and the power consumption of the ADC by exploiting statistical knowledge of the channel. A brief summary of linear transmission designs based on MSE is given in Table 11, where “LP” and “LE” indicate linear precoder and linear equalizer, respectively.

2) Nonlinear Transmission Design: The approaches described above are based on linear processing at both ends of the link. However, research has shown that nonlinear processing schemes can offer additional advantages over linear processing schemes [31], [173]–[177]. Two typical techniques are Tomlinson–Harashima precoding (THP) at the transmitter and decision-feedback equalization (DFE) at the receiver. A unified transceiver structure of a MIMO system with THP and DFE is illustrated in Fig. 5. Assuming PCSIT and PCSIR, a THP precoding with linear equalization that minimizes the sum MSE of transmit data streams was proposed in [31], which focused on the design of the feedback precoder in Fig. 5. The permutation matrix and the precoder matrix in Fig. 5 are taken to be identical.

The THP design with linear equalization was extended to a MIMO-OFDM system in [173], where the input signal was preordered by a permutation matrix and an additional precoder matrix was designed to improve the overall MSE performance. Two linear precoding designs with DFE at the receiver were proposed in [174] and [175], optimizing the sum MSE and the MSE function of the transmit data streams, respectively. A unified framework applicable both to THP with linear equalization and to linear precoding with DFE was established in [176]. The designs proposed in [176] optimize the MSE function of the transmit data streams. It was proved that if the MSE function is Schur-convex, then the optimal design forces the MSE of each transmit data stream to be equal, while if the MSE function is Schur-concave, then the optimal THP leads to linear precoding and the optimal DFE leads to linear equalization. For the case when only statistical CSI is available at the transmitter, Simeone *et al.* [177] proposed a THP with a linear equalization design based on minimizing a lower bound on the sum MSE of the transmit data streams. A brief summary of nonlinear transmission designs based on MSE is given in Table 12, where the “NLP” and “NLE” indicate nonlinear precoder and nonlinear equalizer, respectively.

3) Performance Analysis: Following the publication of [29], attempts were made to analyze the performance of the uncoded MIMO systems within a linear processing framework (also referred to as MIMO multichannel-

Table 11 Linear Transmission Designs With Finite Input Signals Based on the MSE

Paper	Systems	Criterion	Transceiver	CSI	Constraint
Sampath <i>et al.</i> [154]	MIMO	Weighted sum MSE	LP LE	PCSIT PCSIR	Sum power
Scaglione <i>et al.</i> [159]	Multi-carrier MIMO	Sum MSE	LP LE	PCSIT PCSIR	Maximum eigenvalue
Palomar <i>et al.</i> [29]	Multi-carrier MIMO	MSE function	LP LE	PCSIT PCSIR	Sum power
Palomar [160]	MIMO	MSE function	LP LE	PCSIT PCSIR	Covariance matrix shaping
Palomar [161]	Multi-carrier MIMO	MSE function efficient solution	LP LE	PCSIT PCSIR	Sum power
Palomar <i>et al.</i> [162]	MIMO	Transmit power	LP LE	PCSIT PCSIR	MSE
Palomar <i>et al.</i> [163]	Multi-carrier MIMO	Uncoded BER different constellations	LP LE	PCSIT PCSIR	Sum power
Ordóñez <i>et al.</i> [164]	MIMO	Uncoded BER adaptive streams	LP LE	PCSIT PCSIR	Sum power fixed rate
Sterle [165]	MIMO	Sum MSE	Widely LP Widely LE	PCSIT PCSIR	Sum power
Dai <i>et al.</i> [166]	MIMO	Sum MSE	LP LE	PCSIT PCSIR	Sum power maximum eigenvalue
Serbetli <i>et al.</i> [30]	MIMO	Average sum MSE channel estimation effect	LP LE	IPCSIT and IPCSIR no transmit correlation	Sum power
Zhang <i>et al.</i> [167]	MIMO	Average MSE function	LP LE	IPCSIT and IPCSIR no transmit correlation	Sum power
Zhang <i>et al.</i> [167]	MIMO	Average MSE function lower bound	LP LE	IPCSIT PCSIR	Sum power
Ding <i>et al.</i> [168]	MIMO	Average sum MSE	LP LE	IPCSIT and IPCSIR with transmit correlation	Sum power
Wang <i>et al.</i> [169]	MIMO	Sum MSE	LP fixed LE	IPCSIT IPCSIR	Sum power
Wang <i>et al.</i> [170]	MIMO	Sum MSE not optimal	LP LE	IPCSIT IPCSIR	Sum power
Tang <i>et al.</i> [171]	MIMO	Sum MSE optimal	LP LE	IPCSIT IPCSIR	Sum power
Venkateswaran <i>et al.</i> [172]	MIMO	MSE, ADC power optimal	LP LE	SCSIT SCSIR	No

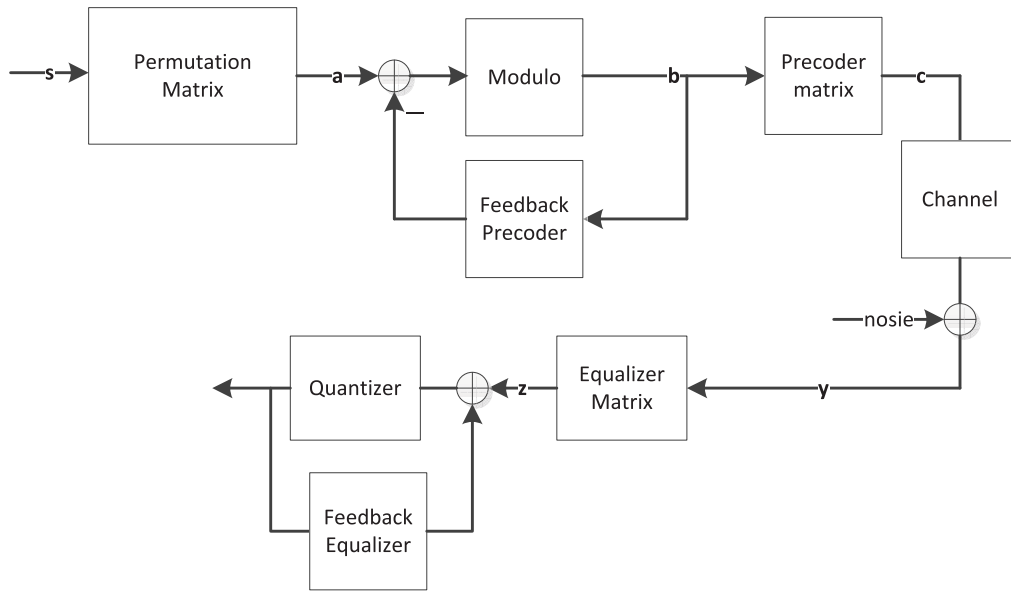


Fig. 5. MIMO transceiver with THP and DFE.

Table 12 Nonlinear Transmission Designs With Finite Input Signals Based on the MSE

Paper	Systems	Criterion	Transceiver	CSI	Constraint
Fischer <i>et al.</i> [31]	MIMO	Sum MSE	NLP	PCSIT	Sum power
			LE	PCSIR	
Amico <i>et al.</i> [173]	MIMO	Sum MSE	NLP, LP	PCSIT	Sum power
	OFDM		LE	PCSIR	
Xu <i>et al.</i> [174]	MIMO	Sum MSE	LP	PCSIT	Sum power
			LE, NLE	PCSIR	
Jiang <i>et al.</i> [175]	MIMO	MSE function	LP	PCSIT	Sum power
			LE, NLE	PCSIR	
Shenouda <i>et al.</i> [176]	MIMO	Sum MSE	NLP, LP, LE	PCSIT	Sum power
			LP, LE, NLE	PCSIR	
Simeone <i>et al.</i> [177]	MIMO	Average sum MSE	NLP, LP, LE	SCSIT	Sum power
		lower bound	LP, LE, and NLE	PCSIR	

beamforming (MB) systems [178]) of [29]. In [179], first-order polynomial expansions for the marginal pdfs of all of the individual ordered eigenvalues of the uncorrelated central Wishart-distributed matrix were derived. Based on these results, closed-form expressions for the outage probability and the average uncoded BER performance for each subchannel of MIMO MB systems with fixed power allocation over an uncorrelated Rayleigh flat-fading channel in high-SNR regime were obtained [179]. These expressions reveal both the diversity gain and the

array gain in each subchannel. Also, tight lower and upper bounds on the performance with nonfixed power allocation such as water-filling power allocation have been obtained. For the correlated central Wishart-distributed matrix, exact pdf expressions for the ordered eigenvalues were derived in [180]. These expressions allow analysis of MIMO MB systems over a semi-correlated Rayleigh flat-fading channel under various performance criteria, including the outage probability and the average uncoded BER. Simple closed-form first-order polynomial expansions for

Table 13 Performance Analysis of MIMO MB Systems

Paper	User channel	Interferer channel	Interferer power	Outage probability
Ordóñez <i>et al.</i> [179]	Uncorrelated Rayleigh fading	No	No	Asymptotic
Ordóñez <i>et al.</i> [180]	Semicorrelated Rayleigh fading	No	No	Exact
	Uncorrelated Rician fading			
Ordóñez <i>et al.</i> [164]	Semicorrelated Rayleigh fading	No	No	Asymptotic
	Uncorrelated Rician fading			
Jin <i>et al.</i> [178]	Uncorrelated Rician fading	No	No	Exact, asymptotic
Zhang <i>et al.</i> [181]	Rayleigh-product fading	No	No	Asymptotic
Sun <i>et al.</i> [182]	Semicorrelated Rayleigh fading	Semicorrelated Rayleigh fading	Unequal	Exact
Jin <i>et al.</i> [183]	Uncorrelated Rician fading	Uncorrelated Rayleigh fading	Equal	Exact, asymptotic Interference-limited
Wu <i>et al.</i> [184]	Rayleigh-product fading	Uncorrelated Rayleigh fading	Arbitrary	Exact, asymptotic

these marginal pdfs were derived in [164], from which the diversity gain and the array gain of the MIMO MB systems can be obtained. Meanwhile, exact pdf expressions for the uncorrelated noncentral Wishart-distributed matrix were independently derived in [178] and [180]. First-order polynomial expansions for these marginal pdfs were independently obtained in [164] and [178]. The performance of MIMO MB systems over uncorrelated Rician fading channels can be analyzed similarly based on these expressions. For MIMO MB over a more general Rayleigh-product fading channel, Zhang *et al.* [181] derived first-order polynomial expansions of the ordered eigenvalues of the channel Gram matrix. The diversity gain and array gain for MIMO MB systems with fixed power allocation over the Rayleigh-product fading channel were found in [181]. Later, the analysis for MIMO MB systems was extended to scenarios in the presence of cochannel interference. With fixed power allocation, Sun *et al.* [182] obtained exact and asymptotic (low outage probability) expressions for the outage probability of MIMO MB systems in the presence of unequal power interferers, where both the desired user and the interferers experience semi-correlated Rayleigh flat-fading. Also, Jin *et al.* [183] obtained exact and asymptotic expressions for the outage probability of MIMO MB systems in interference-limited (without noise) scenarios with equal-power interferers. It was assumed in [183] that the desired user experiences uncorrelated Rician fading and the interferers experience uncorrelated Rayleigh fading. Wu *et al.* [184] obtained exact and asymptotic expressions for the outage probability of MIMO MB systems with arbitrary power interferers, where the desired user experiences Rayleigh-product fading and the interferers experience uncorrelated Rayleigh fading. A brief summary of the performance analysis of MIMO MB systems is given in Table 13.

C. Diversity-Driven Designs

1) *Early Research on Open-Loop Systems:* For an open-loop MIMO system, it is usually assumed that CSI is available at the receiver but not at the transmitter. To acquire receive diversity, the exploitation of diversity-combining schemes at the receiver, such as equal-gain combining, selection combining, and maximum ratio combining (MRC), can be traced back 60 years ago via references in [185]. In the absence of CSIT, space-time techniques have been designed to combat the deleterious effects of small-scale fading. These techniques can acquire additional transmit diversity and thereby improve the reliability of the link with finite input signals. A simple and well-known space-time technique is the Alamouti orthogonal space-time block code (OSTBC) for a system with two transmit and two receive antennas [32], as illustrated in Fig. 6. For the OSTBC in Fig. 6, in time slot 1, the symbols s_1 and s_2 are transmitted by antennas 1 and 2, respectively. In time slot 2, the symbol $-s_2^*$ and $-s_1^*$ are transmitted by antennas 1 and 2, respectively. After passing the channel and the combiner in Fig. 6, the signals $s_{1,c}$ and $s_{2,c}$ sent to the maximum-likelihood (ML) detector are given by

$$s_{1,c} = (|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2) s_0 + h_{11}^* n_{11} + h_{12} n_{12}^* + h_{22}^* n_{22} + h_{21} n_{21} \quad (18)$$

$$s_{2,c} = (|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2) s_1 - h_{11} n_{11}^* + h_{12}^* n_{12} - h_{22} n_{22}^* + h_{21}^* n_{21}. \quad (19)$$

Equations (18) and (19) are equivalent to those for four-branch MRC. Therefore, a diversity of 4 is achieved for the ML detector. Additional transmit diversity can be obtained by utilizing the OSTBC. A construction of both real and

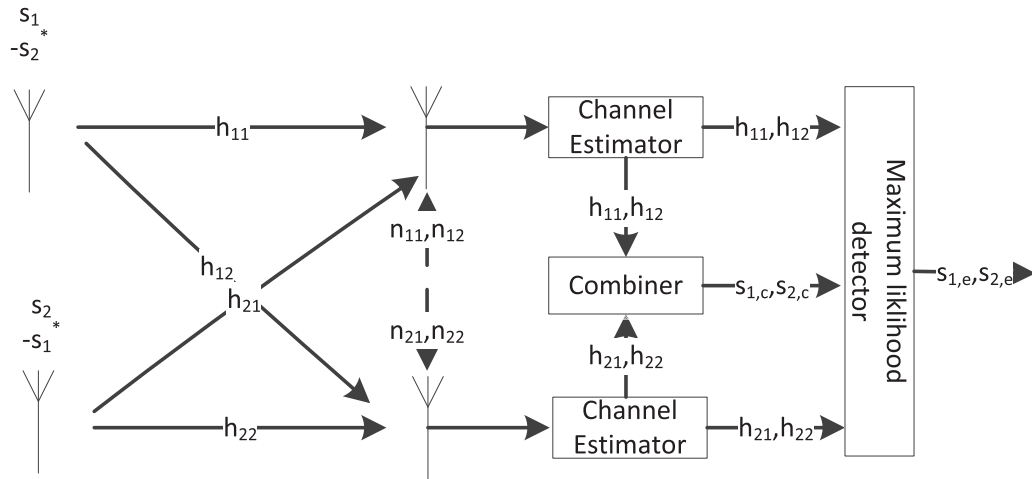


Fig. 6. The OSTBC for a 2×2 system.

complex OSTBC for any number of transmit and receive antennas was proposed in [33]. The proposed design achieves the maximum possible spatial diversity order with a low-complexity linear decoding. The space-time block code (STBC) was extended to a single-carrier MIMO system in [34] and to a MIMO-OFDM system in [186]. The proposed designs achieve the maximum diversity, but at the price of an increase in decoding complexity, since the detector needs to jointly decode all subchannels. To solve this problem, a subchannel grouping method with the maximum diversity performance and reduced decoding complexity was proposed in [186]. The transmission rate of the above OSTBC design is less than 1 symbol per channel use (pcu) for more than two transmit antennas. Therefore, a quasi-orthogonal space-time block code (QOSTBC) that provides a 1 symbol pcu transmission rate and half of the maximum possible diversity for four transmit antennas was proposed in [187]. The QOSTBC divides the transmission matrix columns into different groups and makes the columns in different groups orthogonal to each other. This quasi-orthogonal structure increases the transmission rate. Also, based on this structure, the ML decoding at the receiver can be done by searching symbols

in each group individually instead of searching all transmit signals jointly. By rotating half of the transmit symbol constellations, QOSTBC can achieve a 1 symbol pcu rate and maximum diversity for four transmit antennas [188], and a higher rate than OSTBC and maximum diversity for MIMO systems [189]. Based on a different rotation method, Yuen *et al.* [190] constructed a class of QOSTBCs for MIMO systems, whose ML decoding can be reduced to a joint detection of two real symbols. On the other hand, a space-time trellis code (STTC) that achieves both maximum spatial diversity and large coding gains was designed in [191]. However, its decoding complexity grows exponentially with transmit rate and becomes significantly high for high-order modulation.

A space-time linear constellation precoding (ST-LCP) was proposed in [192] as an alternative scheme to obtain transmit diversity. The baseband equivalent framework of ST-LCP is illustrated in Fig. 7. The key idea of ST-LCP is to perform a precombination of the transmit signal with a linear precoder matrix as in Fig. 7. Then, the preprocessed data is mapped to the transmit antenna by a diagonal matrix and a unitary matrix. Based on the average PEP expression, linear precoders that achieve the maximum

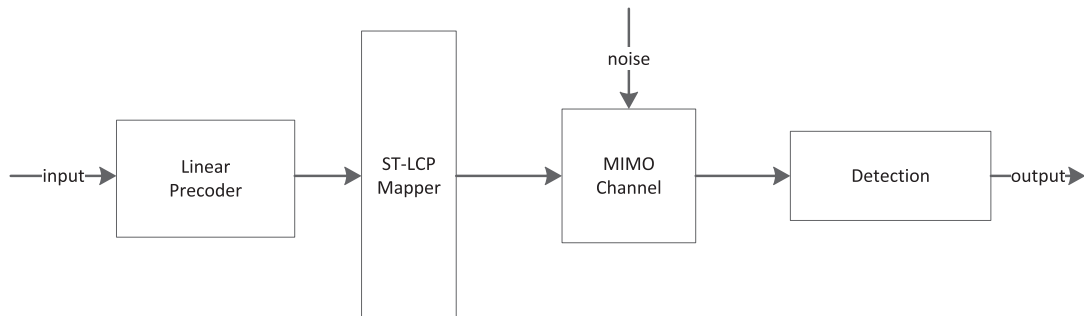


Fig. 7. The baseband equivalent framework of ST-LCP.

diversity and perform close to the upper bound of the coding gain are designed in closed form. Near-optimal sphere decoding algorithms are employed to reduce the decoding complexity of ST-LCP. Simulations indicate that ST-LCP achieves a better BER performance than classic OSTBC. On the other hand, Liu *et al.* [193] studied ST-LCP for a MIMO-OFDM system. They combined ST-LCP with a subcarrier grouping method to realize both maximum diversity and low decoding complexity.

Although the above-mentioned approaches achieve good BER performance, their transmit rate cannot exceed 1 symbol pcu. This limits the transmission efficiency of a MIMO system. Sethuraman *et al.* [194] constructed a high-rate full-diversity STBC using division algebras with high decoding complexity. There have also been a number of attempts to construct full-rate (N_t symbols pcu) full-diversity space-time codes. One approach is to extend ST-LCP to the full-rate full-diversity design [195]. The key idea is to replace the linear precoder in Fig. 7 with a linear complex-field encoder (LCFE). The LCFE divides the N_t^2 transmit symbols into N_t layers, where N_t is the number of transmit antenna. In each layer, a closed form linear precoder matrix is used to combine the N_t symbols. Then, the ST-LCP mapper in Fig. 7 is replaced with a space-time LCFE (ST-LCFE) mapper, where the preprocessed N_t^2 data is mapped into a $N_t \times N_t$ matrix. Finally, the mapped data is transmitted by N_t antennas in N_t time slots. In this case, an N_t symbols pcu transmission rate is achieved. It has been proved that the ST-LCFE design also achieves the maximum spatial diversity for a MIMO system. The price of the ST-LCFE is an exponential increase in decoding complexity. Therefore, both diversity-complexity tradeoff and modulation-complexity tradeoff schemes were proposed in [195]. Moreover, the ST-LCFE design was extended to frequency-selective MIMO-OFDM and time-selective fading channels in [195]. Another approach is to employ a threaded algebraic space-time (TAST) code [196]. When using a TAST code, the size of the information symbol alphabet is increased since the degree of the algebraic number field increases. Therefore, full-rate N_t symbols pcu and maximum spatial diversity can be achieved simultaneously. A Golden STBC with nonvanishing determinants (NVD) property when the signal constellation size approaches infinity was first constructed for 2×2 , 3×3 , and 6×6 antenna systems in [197] and [198], and extended to MIMO systems with an arbitrary number of antennas in [199]. The NVD property guarantees that the STBC performs well irrespectively of the transmit signal alphabet size. The above-mentioned works have established some important initial space-time techniques with finite input signals and PCSIR. These space-time techniques focus on improving the transmit diversity, coding gain, and transmission rate of the link. A brief comparison among them is given in Table 14.

For the noncoherent scenario where CSI is available neither at the transmitter nor at the receiver, Tarokh *et al.* [200], [201] combined differential modula-

tion with OSTBC to formulate transmission schemes that can still obtain transmit diversity with PSK signal. Based on unitary group codes, differential STBCs to achieve the maximum transmit diversity of MIMO systems were proposed in [202] and [203], and an optimal modulation scheme for two transmit antennas was obtained in [203]. A double differential STBC was designed for time-selective fading channels in [204]. In addition, a differential STBC using QAM signals was constructed in [205]. For the partially coherent scenario where the CSI is not available at the transmitter and only imperfect CSI is available at the receiver, robust STBCs were designed in [206]–[208]. The effects of mapping on the error performance of the coded STBC were studied in [209] and [210]. A rate-diversity tradeoff for STBC under finite alphabet input constraints was obtained in [211], and the corresponding optimal signal constructions for BPSK and QPSK constellation were provided. The space-time techniques have also been extended to MIMO CDMA systems [212]–[214].

2) *Recent Advances for Open-Loop Systems:* Recent space-time techniques with finite input signals have focused on designing high-rate full-diversity STBCs with NVD and low decoding complexity. For a multiple-input single-output (MISO) system equipped with a linear detector, a general criterion for designing full-diversity STBC was established in [36]. A STBC with Toeplitz structure satisfies this criterion. Also, the rate of this Toeplitz STBC approaches 1 symbol pcu when the number of channel users is sufficiently large. For a MISO system equipped with ML detector, this Toeplitz STBC achieves the maximum coding gain. A new 2×2 full-rate full-diversity STBC was proposed by reconstructing the generation matrix of OSTBC [215]. The proposed STBC reduces the complexity of ML decoding to the order of the standard sphere decoding. Another 2×2 full-rate full-diversity STBC was designed in [216]. During the ML decoding exhaustive search process for this STBC, the Euclidean distance is divided into groups, with each group being minimized independently. Hence, this STBC leads to an obvious reduction in decoding complexity compared with the Golden code in [197], with a slight performance loss. A unified design framework for full-rate full-diversity 2×2 fast-decodable STBC was provided in [217]. The ML decoding complexity of the designed code is reduced to the complexity of standard sphere decoding. For a 4×2 system, two QSTBCs are combined to formulate a new 2 pcu symbol rate and half-diversity STBC with simplified ML decoding complexity [217]. Srinath *et al.* [218] constructed a 2×2 STBC that has the same performance as the Golden code in [197] and a lower ML decoding complexity for nonrectangular QAM inputs. Srinath *et al.* [218] further constructed a 2 pcu symbol rate and full-diversity 4×2 STBC offering a large coding gain with reduced ML decoding complexity. Wang *et al.* [219] constructed a QOSTBC that exhibits the same low decoding complexity as the design in [190], but has a better coding gain for rectangular QAM inputs.

Table 14 Some Important Initial Space–Time Techniques With Finite Input Signals and Perfect CSIR

Paper	System	Coding gain	Rate	Decoding complexity
Alamouti [32]	2×2	Not optimal	1 symbol pcu	Low
Tarokh <i>et al.</i> [33]	MIMO	Not optimal	< 1 symbol pcu	Low
Zhou <i>et al.</i> [34]	Single-carrier MIMO	Not optimal	< 1 symbol pcu	High
Liu <i>et al.</i> [186]	MIMO-OFDM	Large	< 1 symbol pcu	Lower than [34]
Jafarkhani [187]	4×4	Not optimal (half diversity)	1 symbol pcu	Reduced
Sharma <i>et al.</i> [188]	4×4	Not optimal	1 symbol pcu	Reduced
Su <i>et al.</i> [189]	MIMO	Not optimal	≤ 1 symbol pcu	Reduced
Yuen <i>et al.</i> [190]	MIMO	Not optimal	≤ 1 symbol pcu	Lower than [189]
Tarokh <i>et al.</i> [191]	MIMO	Large	< 1 symbol pcu	High
Xin <i>et al.</i> [192]	MIMO	Near-optimal	1 symbol pcu	High
Liu <i>et al.</i> [193]	MIMO-OFDM	Not optimal	1 symbol pcu	Lower than [34]
Sethuraman <i>et al.</i> [194]	MIMO	Near-optimal	> 1 symbol pcu	High
Ma <i>et al.</i> [195]	MIMO, MIMO-OFDM MIMO time selective channels	Not optimal	N_t symbols pcu	High
Gammal <i>et al.</i> [196]	MIMO	Not optimal	N_t symbols pcu	High
Belfiore <i>et al.</i> [197]	2×2	Large (NVD)	2 symbols pcu	High
Oggier <i>et al.</i> [198]	$2 \times 2, 3 \times 3, 6 \times 6$	Large (NVD)	N_t symbols pcu	High
Elia <i>et al.</i> [199]	MIMO	Large (NVD)	N_t symbols pcu	High

Kumar *et al.* [220] designed an STTC for MIMO systems based on lattice coset coding. The constructed code has a good coding gain and a reduced decoding complexity for large block length with a DFE and lattice decoding.

In [221], a general framework of multigroup ML decodable STBC was introduced to reduce the decoding complexity of MIMO STBC. Multigroup ML decodable STBC is a full-diversity STBC. It allows ML decoding to be implemented for the symbols within each individual group. As a result, multigroup ML decodable STBC provides a tradeoff between rate and ML decoding complexity. A specific multigroup ML decodable STBC based on extended Clifford algebras, called the Clifford unitary weight, was constructed to meet the optimal tradeoff between the rate and the ML decoding complexity in [222]. Moreover, the structure of a fast decodable code has been integrated into the multigroup code to formulate a fast-group-decodable STBC [223]. A new class of full-rate full-diversity STBCs that exploits the block-orthogonal property of STBC (BOSTBC) was proposed in [224]. Simulations show that this BOSTBC achieves good BER performance with reduced decoding complexity when a QR decomposition decoder with M paths (QRDM decoder) is employed at the receiver. Also, this block-orthogonal property can be utilized [225] to reduce the decoding complexity of the

Golden code in [197] with an ML decoder. Rigorous theoretical analyses of the block-orthogonal structure of various STBCs have been given in [226]. Some fast-decodable asymmetric STBCs have been designed in [227]–[229]. In particular, Vehkalahti *et al.* [227] established a family of fast-decodable STBCs from division algebras for multiple-input double-output (MIDO) systems. These codes are full-diversity with rate no more than $N_t/2$ symbols pcu. By an appropriate geometric construction of such codes, the ML decoding complexity can be reduced by at least 37.5%. In addition, explicit constructions for 4×2 , 6×2 , and 6×3 codes have been given in [227]. Markin *et al.* [228] proposed an iterative STBC design method for $N_t \times N_t/2$ MIMO systems. The iterative design constructs two $N_t/2 \times N_t/2$ original codewords, with algebraic codewords based on cyclic algebra, to create new $N_t \times N_t$ codewords. The resulting code achieves a rate of $N_t/2$ symbols pcu and full diversity with fast decoding complexity. This iterative design was used to construct new 4×2 and 6×3 STBCs in [228] based on some well-known STBCs such as the Golden code in [197]. Moreover, a generalized framework of this iterative design was used in [229] to construct a 2 symbol pcu rate and full-diversity STBC with large coding gain and fast decoding complexity for MIDO systems. Based on this framework, explicit 4×2 , 6×2 , 8×2 ,

Table 15 Recent Low-Decoding-Complexity Space-Time Techniques With Finite Input Signals

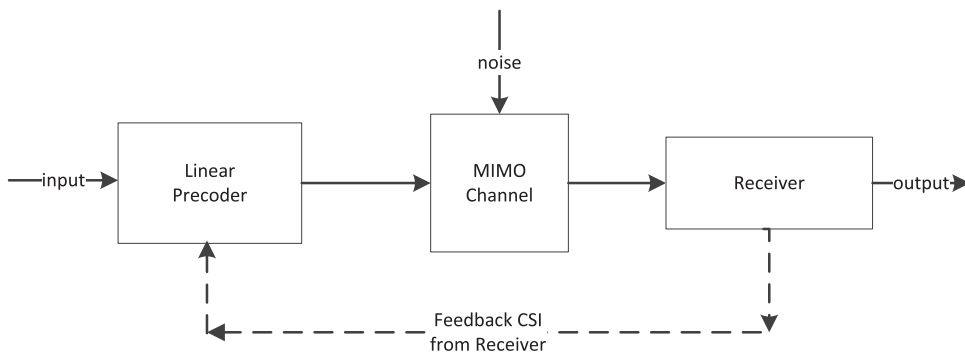
Paper	System	Coding gain	Rate	Decoding complexity
Liu <i>et al.</i> [36]	MISO	NVD	Approaches 1 symbol pcu	Linear
Paredes <i>et al.</i> [215]	2×2	NVD	2 symbols pcu	Reduced ML
Sezginer <i>et al.</i> [216]	2×2	Not optimal	2 symbols pcu	Reduced ML
Biglieri <i>et al.</i> [217]	2×2	Not optimal	2 symbols pcu	Reduced ML
	4×2	Not optimal (half diversity)	2 symbols pcu	Reduced ML
Srinath <i>et al.</i> [218]	2×2	NVD	2 symbols pcu	Lower than Golden code [197]
	4×2	NVD	2 symbols pcu	Reduced
Wang <i>et al.</i> [219]	MIMO	Larger than [190]	≤ 1 symbol pcu	Same as [190]
Kumar <i>et al.</i> [220]	MIMO	Good	High	Reduced ML
Karmakar <i>et al.</i> [221]	MIMO	Not optimal	Flexible	Trade-off with rate
Rajan <i>et al.</i> [222]	MIMO	Not optimal	Flexible	Trade-off with rate
Ren <i>et al.</i> [223]	MIMO	Not optimal	Flexible	Lower than [217]
Ren <i>et al.</i> [224]	MIMO	Not optimal	N_t symbols pcu	Reduced QRDM
Sinnokrot <i>et al.</i> [225]	2×2 Golden code [197]	NVD	2 symbols pcu	Reduced ML
Jithamithra <i>et al.</i> [226]	MIMO	Classic STBCs	Flexible	Reduced SD
Vehkalahti <i>et al.</i> [227]	MIDO	NVD	$N_t/2$ symbols pcu	Reduced ML
Markin <i>et al.</i> [228]	$N_t \times N_t/2$	NVD	$N_t/2$ symbols pcu	Reduced ML
Srinath <i>et al.</i> [229]	$4 \times 2, 6 \times 2$	NVD, large	2 symbols pcu	Reduced ML
	$8 \times 2, 12 \times 2$	NVD, large		

and 12×2 STBCs have been constructed in [229]. A brief comparison of the above low-decoding-complexity STBC designs is given in Table 15.

Besides those aimed at reducing the decoding complexity, some other space-time techniques have recently been developed. For example, a full-rate full-diversity improved perfect STBC was designed in [230] with larger coding gain than the perfect STBC in [198]. A full-rate STBC under the BICM structure was designed in [230] to optimize all components of the transmitter jointly, including an error-correcting code, an interleaver, and

a symbol mapper. A new spatial modulation technique that simultaneously activates only a few of the antennas comprising the entire antenna array has been described in [231]–[233]. Since the publication of [32], space-time techniques with finite input signals have been studied extensively [35], [234], [235].

3) *Closed-Loop Systems*: The space-time techniques achieve transmit diversity without CSI at the transmitter and are applicable to open-loop systems. Alternatively, for closed-loop MIMO systems as in Fig. 8, CSI is available at


Fig. 8. The baseband equivalent framework of a closed-loop system.

the transmitter either by a feedback link from the receiver (frequency division duplex systems) or a channel estimation from the pilot signal of the receiver (time division duplex systems). In this case, appropriate transmission designs can be formulated to exploit the closed-loop transmit diversity of MIMO systems with finite input signals. For example, a bit interleaved coded multiple beamforming technique was introduced in [236] to achieve full diversity and full spatial multiplexing for MIMO and MIMO-OFDM systems over i.i.d. Rayleigh fading channels. The decoding complexity of the bit interleaved coded multiple beamforming design was further reduced in [237]. An important criterion to enhance the transmit diversity for closed-loop MIMO systems is maximization of the minimum Euclidean distance of the receive signal points. This criterion is directly related to the SER of the MIMO system if the ML detector is used and has been proved to achieve the optimal mutual information performance in high-SNR regime [106]. For a 2×2 system with BPSK and QPSK inputs, an optimal solution for a nondiagonal precoder, which maximizes the minimum Euclidean distance of the receive signal points ($\max\text{-}d_{\min}$), was proposed in [238]. It was shown in [238] that for BPSK inputs, the optimal transmission is to focus the power on the strongest subchannel, which is the beamforming design. For QPSK inputs, the beamforming design is optimal when the channel angle is less than a certain threshold. When the channel angle is larger than this threshold, the optimal design can be found by numerically searching the angle of the power allocation matrix. In [37], a suboptimal precoder design for MIMO systems with an even number of transmit antenna and 4-QAM inputs was proposed in which the MIMO channels are transformed into parallel subchannels by a singular value decomposition (SVD). Each two subchannels are paired into one group to formulate a set of 2×2 subsystems. Then, 2×2 subprecoders are designed in a manner similar to that in [238]. The design in [37] is called “X-structure design.” It should be noted that this X-structure design enjoys both reduced precoding and decoding complexities. For M -QAM inputs, finding the optimal $\max\text{-}d_{\min}$ MIMO precoder is rather complicated, and the optimal solution varies for different modulations. Therefore, Ngo *et al.* [239] proposed a suboptimal design by selecting 2×2 subprecoders between two deterministic structures. These subprecoders are then combined together to formulate a MIMO precoder based on the X-structure. The suboptimal design in [239] has good uncoded BER performance with a reduced implementation complexity. An exact expression for the pdf of the minimum Euclidean distance under this suboptimal precoder design was derived in [240]. Also, a $\max\text{-}d_{\min}$ MIMO precoder design for a three parallel data-stream scheme and M -QAM inputs was proposed in [241]. This allows decomposition of arbitrary MIMO channels into 2×2 and 3×3 subchannels, which facilitates efficient design of the $\max\text{-}d_{\min}$ MIMO precoder. On the other hand, a suboptimal real-valued $\max\text{-}d_{\min}$

MIMO precoder for M -QAM inputs was proposed in [242]. This design has an order-of-magnitude lower ML decoding complexity than the precoders in [37], [239]. Moreover, it was proved in [242] that the $\max\text{-}d_{\min}$ precoder achieves full diversity for MIMO systems. Meanwhile, two precoders of low decoding complexity, derived from a rotation matrix and the X-structure, called the X-precoder and the Y-precoder, were proposed in [25]. The X-precoder has a closed-form expression for 4-QAM inputs. The Y-precoder has an explicit expression for arbitrary M -QAM inputs but with a worse error performance than the X-precoder. Overall, both the X- and Y-precoders are suboptimal in terms of error performance and lose out in word error probability when compared with the $\max\text{-}d_{\min}$ precoder [37], [239]. Two suboptimal $\max\text{-}d_{\min}$ MIMO precoder designs for M -QAM inputs, which work by optimizing the diagonal elements of an “SNR-like” matrix and the lower bound of the minimum Euclidean distance, were proposed in [243] and [244], respectively.

An alternative approach to exploiting closed-loop transmit diversity is to design the precoder based on lattice structure. A lattice-based linear MIMO precoder design to minimize the transmit power under fixed block error rate and fixed total data rate was proposed in [245]. The precoder design is decomposed into the design of four matrices. The optimal rotation matrix is given in closed form, the bit load matrix and the basis reduction matrix are optimized alternately via an iterative algorithm, and the lattice base matrix is selected based on a lower bound on the transmit power. Linear precoders that achieve full diversity of $N_t \times N_t$ MIMO systems and apply to integer-forcing receivers were designed in [246] based on full-diversity lattice generator matrices. Some approaches have also used lattice theory to design $\max\text{-}d_{\min}$ precoders. For a 2×2 system with a square QAM constellation input, the optimal real-valued $\max\text{-}d_{\min}$ precoder is found by expanding an ellipse within a 2-D lattice to its maximum extent [247]. For MIMO systems with 4-QAM inputs, a real-valued $\max\text{-}d_{\min}$ precoder has been directly constructed based on well-known dense packing lattices [248]. This precoder offers better minimum Euclidean distance and BER performance than the precoder based on the X-structure [37]. For $N_t \times 2$ systems with infinite signal constellation, an explicit structure of the $\max\text{-}d_{\min}$ precoder has been obtained from the design of the lattice generator matrix [249]. Numerical results indicate that the obtained precoder achieves good mutual information performance with large QAM inputs. This precoder design was extended to MIMO systems in [250]. A brief summary of transmission designs with finite input signals to optimize the transmit diversity for closed-loop MIMO systems is given in Table 16.

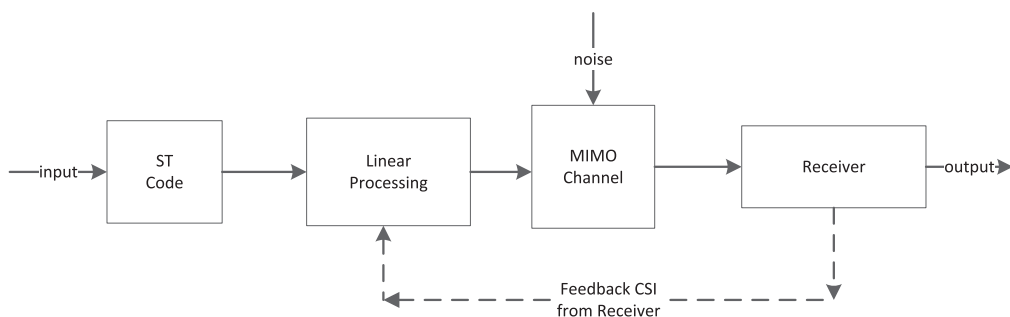
Some approaches have combined the STBC with the linear processing step in a closed-loop system to further improve the performance of MIMO systems. The structure of a closed-loop STBC system is illustrated in Fig. 9. It is important to note that the main difference between the linear processing in Fig. 7 and the linear precoder in

Table 16 Techniques for the Transmit Diversity of Closed-Loop MIMO Systems With Finite Input Signals

Paper	System	Modulation	Minimum Euclidean distance	Design complexity	Decoding complexity
Akay <i>et al.</i> [236]	MIMO MIMO-OFDM	M -QAM	Not optimal	Low	High
Li <i>et al.</i> [237]	MIMO-OFDM	M -QAM	Not optimal	Low	Reduced
Collin <i>et al.</i> [238]	2×2	BPSK, QPSK	Optimal	Low	Low
Vrigneau <i>et al.</i> [37]	MIMO	QPSK	Near-optimal	Reduced	Reduced
Ngo <i>et al.</i> [239]	MIMO	M -QAM	Suboptimal	Reduced	Reduced
Ngo <i>et al.</i> [241]	MIMO	M -QAM	Suboptimal	Reduced	Reduced
Srinath <i>et al.</i> [242]	MIMO	M -QAM	Smaller than [37]	Lower than [239]	Lower than [239]
Mohammed <i>et al.</i> [25]	MIMO	M -QAM	Not optimal	Lower than [239]	Lower than [239]
Ngo <i>et al.</i> [243]	MIMO	M -QAM	Larger than [239]	Reduced	High
Lin <i>et al.</i> [244]	MIMO	M -QAM	Close to [25]	Lower than [25]	High
Bergman <i>et al.</i> [245]	MIMO	M -QAM	Not optimal	Reduced	High
Sakzad <i>et al.</i> [246]	$N_t \times N_t$	M -QAM	Not optimal	Reduced	Reduced
Xu <i>et al.</i> [247]	2×2	Square M -QAM	Optimal	Low	Low
Xu <i>et al.</i> [248]	MIMO	QPSK	Larger than [239]	Reduced	High
Kapetanović <i>et al.</i> [249]	$N_t \times 2$	Infinite Constellation	Asymptotic optimal	Low	Low
Kapetanović <i>et al.</i> [250]	MIMO	Infinite Constellation	Asymptotic optimal	Reduced	High

Fig. 9 is the use of CSI. The linear precoder in Fig. 7 does not need any knowledge of CSI, whereas the linear processing in Fig. 9 requires some form of CSI. With PCSIT, Park *et al.* [251] exploited the constellation precoding scheme in [192] to design a full-diversity multiple beamforming scheme for uncoded systems. For both PCSTT and SCSIT, Gore *et al.* [252] investigated antenna selection techniques to minimize the SER for MIMO systems employing OSTBC. Jongren *et al.* [253] combined a beamforming design and the OSTBC with partial knowledge of the channel over a frequency-nonselective fading channel. A linear precoder design for an STBC system over a transmit-correlated Rayleigh fading channel was proposed in [254]. The transmitter utilizes the transmit

correlation matrix to design a linear precoding matrix, which minimizes an upper bound of the average PEP of the STBC system. This design was extended to a transmit-correlated Rician fading channel and arbitrary correlated Rician fading channels in [255] and [256], respectively. Linear precoding for a limited-feedback STBC system was proposed in [257]. For the noncoherent scenario where CSI is not available at the receiver, linear precoder designs for differential STBC systems over a Kronecker fading channel and arbitrary correlated Rayleigh fading channels were presented in [258] and [259], respectively. For the partially coherent scenario where imperfect channel estimation is performed at the receiver and the estimation error covariance matrix is perfectly known at the trans-

**Fig. 9.** Closed-loop STBC system.

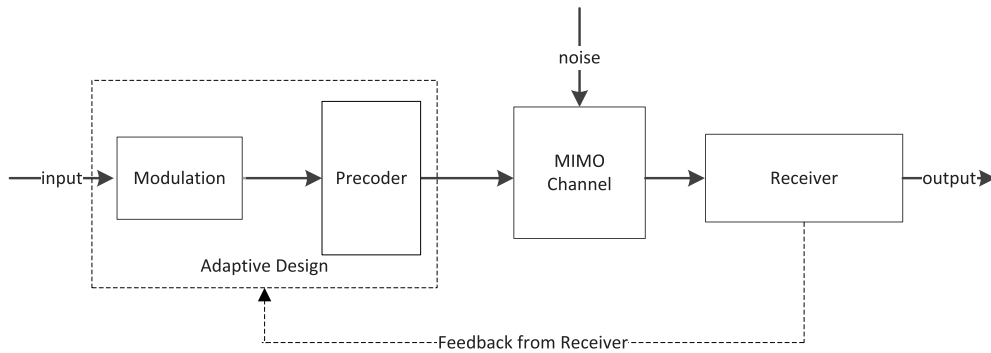


Fig. 10. Adaptive MIMO transmission for uncoded systems.

mitter via feedback, STBC designs were proposed in [260] and [261] for i.i.d. Rayleigh fading channels based on the criteria of the Kullback–Leibler distance and the cutoff rate, respectively. The design in [262] was extended to correlated Rayleigh fading channels where the transmit and receive correlation matrices are perfectly known at the transmitter. Moreover, a novel and efficient bit mapping scheme was proposed in [262]. A linear precoder was designed in [263] to adapt to the degradation caused by imperfect CSIR.

D. Adaptive MIMO Transmission

The above-mentioned multiplexing, hybrid, and diversity transmission schemes can be employed adaptively according to the channel condition. Adaptive MIMO transmission is an effective approach to increase the data rate and decrease the error rate for wireless communication systems [264]–[266]. Adaptive MIMO transmission techniques have been investigated for both uncoded and coded systems.

1) *Uncoded Systems*: The capacity-achieving coding usually has long (infinite) codewords and imparts delay to the system. Therefore, for delay-sensitive applications, the modulation part of the physical layer is usually designed separately before the outer error-correcting coding is applied, as shown in Fig. 10. For an uncoded SISO system subject to average power and the BER (instantaneous or average) constraints, maximization of the spectral efficiency was achieved by jointly optimizing the transmit rate and power over Rayleigh fading channels in [267], [268] and over Nakagami fading channels in [269], building upon previous adaptive transmission techniques [270]–[280]. Zhou *et al.* [38] investigated adaptive transmission schemes for i.i.d. Rayleigh flat-fading MIMO channels with perfect or imperfect CSIT and CSIR. With perfect CSIT and CSIR, the MIMO channels are decomposed into a set of parallel SISO subchannels by SVD. The transmit rate and power allocation of each subchannel must then be optimally designed to maximize the average spectral efficiency under average transmit power and instantaneous subchannel BER constraints. It has been

proved that this optimization problem can be transformed into several subproblems, each of which only involves one subchannel. The adaptive transmission designs for SISO channels in [267], [268] can then be exploited to find the optimal solution. The effect of a discrete rate constraint and imperfect CSIT on system performance were evaluated in [38]. For the case when only partial CSI, i.e., the channel mean feedback, is available at the transmitter, Zhou *et al.* [39] provided an adaptive 2-D beamformer to exploit the transmit diversity by incorporating the Alamouti code. Based on this beamformer, the beamvectors, power allocation policy, and modulation schemes that maximize the average discrete spectral efficiency under constant power and average BER constraints were designed for MIMO and MIMO-OFDM systems in [39] and [281], respectively. To further study the impact of CSI error on adaptive MIMO transmission, Zhou *et al.* [282] considered the case where the transmitter performs pilot-signal-assisted channel prediction and examined the effect of prediction error on BER performance for a 1-D adaptive beamformer. Numerical results showed that when the normalized estimation error is below a critical threshold, the channel prediction error has a minor impact on BER performance. For correlated MISO Rayleigh fading channels with a limited number of feedback bits at the transmitter, Xia *et al.* [283] developed a strategy to adaptively select the transmission mode to maximize the average spectral efficiency under average power and average BER constraints. For correlated MIMO fading channels where the correlation matrices are available at the transmitter and equal power is allocated to each transmit antenna, a strategy to determine the number of active transmit antenna and the corresponding transmit constellations was developed by maximizing a lower bound on the minimum SNR margin with a fixed data rate [284]. Heath *et al.* [285] proposed a simple MIMO adaptive transmission scheme by switching between spatial multiplexing and transmit diversity. With perfect CSIR, the minimum Euclidean distances of both a spatial multiplexing scheme and an STBC scheme that achieve a fixed transmit rate are computed at the receiver. The scheme with larger minimum Euclidean distance is cho-

Table 17 Adaptive MIMO Transmitter Designs for Uncoded Systems

Paper	System	Criterion	CSI	Constraint
Zhou <i>et al.</i> [38]	MIMO i.i.d. Rayleigh fading	Maximize average spectral efficiency	PCSIT, IPCSIT	Instantaneous BER
			PCSIR	Average transmit power
Zhou <i>et al.</i> [39]	MIMO i.i.d. Rician fading	Maximize average spectral efficiency	Channel mean feedback	Average BER
			PCSIR	Constant transmit power
Xia <i>et al.</i> [281]	MIMO-OFDM i.i.d. Rician fading	Maximize average spectral efficiency	Channel mean feedback	Average BER
			PCSIR	Constant transmit power
Zhou <i>et al.</i> [282]	MIMO i.i.d. Rayleigh fading	Maximize average spectral efficiency	Channel prediction error	Instantaneous BER
				Constant transmit power
Xia <i>et al.</i> [283]	MISO i.i.d. Rayleigh fading	Maximize average spectral efficiency	Limited feedback	Average BER
			PCSIR	Average transmit power
Narasimhan [284]	MIMO correlated Rayleigh fading	Maximize minimal SNR margin	SCSIT	Fixed data rate
			PCSIR	Equal power allocation
Heath [285]	MIMO	Spatial multiplexing	Feedback decision	Fixed data rate
		OSTBC switch	PCSIR	
Huang [286]	DIMO correlated Rayleigh fading	Spatial multiplexing	PCSIT, SCSIT	ZF receiver
		OSTBC switch	PCSIR	
Kongara [287]	MIMO-OFDM i.i.d. Rayleigh fading	Performance analysis	PCSIT	MRT
			PCSIR	MRC

sen and the decision is fed back to the transmitter via a low-rate feedback channel. For double-input multiple-output (DIMO) systems with both PCSIT and SCSIT, an adaptive scheme switching between OSTBC and spatial multiplexing with a zero-forcing (ZF) receiver based on the average spectral efficiency was proposed in [286]. For an adaptive MIMO-OFDM system employing maximum ratio transmission (MRT) and an MRC transceiver structure, the statistical properties of the number of bits transmitted per OFDM block and the number of outages per OFDM block over i.i.d. Rayleigh fading channels were derived in [287]. A brief comparison of the above adaptive MIMO transmitter designs is given in Table 17.

Some work has been done on adaptive MIMO transceiver designs. For perfect CSIT and CSIR, Palomar *et al.* [288] investigated transmit constellation selection and linear transceiver design to minimize the transmit power with fixed transmit rate, where each subchannel is under the same BER constraint. Assuming the symbol error probability of each subchannel to be the same, the gap approximation method was used to select the transmit constellations. Necessary and sufficient conditions were then established to examine the optimality of the subchannel diagonalization design. Moreover, an

upper bound on the transmit power loss incurred by using the subchannel diagonalization design was presented. An optimal bit loading scheme among subchannels was developed in [289]. The proposed scheme minimizes a lower bound of the average BER of the system subject to fixed transmit data, constant transmit power, and ZF transceiver structure constraints. Bergman *et al.* [290] presented an optimal transceiver design and bit loading scheme among subchannels with a DFE receiver. The p -norms of weighed MSEs are minimized under fixed transmit data and constant transmit power constraints. The subchannel diagonalization design was proved to be always optimal for the system model considered in [290]. Li *et al.* [291] established a duality relationship between the joint linear transceiver and bit loading design for maximizing the transmit rate and that for minimizing the transmit power. An algorithm can then be developed to find the optimal design for transmit rate maximization by using the solution of the transmit power minimization problem. A brief comparison of the above adaptive MIMO transceiver designs is given in Table 19.

For the case of perfect CSIR, Ko *et al.* [292] proposed a rate-adaptive modulation scheme combined with OSTBC for i.i.d. Rayleigh fading MIMO channels. They derived

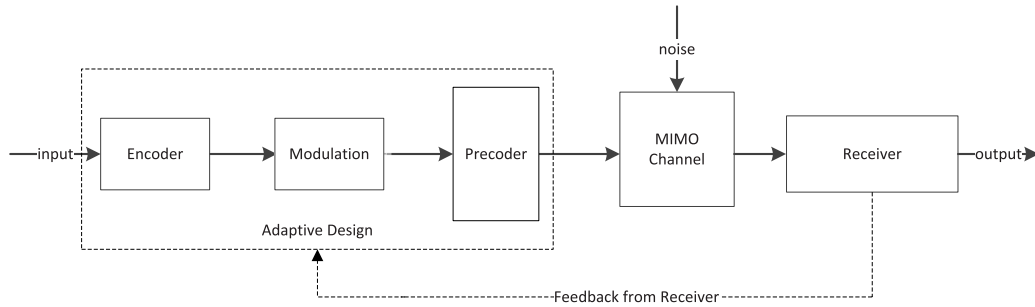


Fig. 11. Adaptive MIMO transmission for coded systems.

closed-form expressions for the average spectral efficiency and a tight upper bound on the average BER in the presence of feedback delay. Optimal SNR switching thresholds for adaptive modulation, which maximize the average spectral efficiency under average BER, outage BER, and constant transmit power constraints, were designed at the receiver. The constellation chosen at the receiver is fed back to the transmitter and the impact of feedback delay on the average BER, throughput, and outage probability are analyzed. The design in [292] was extended to spatially correlated Rayleigh fading channels in [293], where a low-complexity method to determine a set of near-optimal SNR switching thresholds was presented. The design in [292] was also extended to the case with channel estimation error and an average transmit power constraint in [294]. For MIMO-OSTBC systems, the channel estimation process of a rate-adaptive modulation scheme was investigated in [295]. The transmit constellation and the allocation of time and power between the pilot and data symbols were optimally designed to maximize the average spectral efficiency under an instantaneous BER constraint. An adaptive MIMO transmission scheme for energy-efficiency maximization of MIMO-OSTBC systems with channel estimation error subject to an instantaneous BER constraint was proposed in [40]. By utilizing imper-

fect CSIT, Kuang *et al.* [41] incorporated the design of spatial power allocation between different transmit antennas into the adaptive modulation scheme in [294] to further increase the average spectral efficiency. A brief comparison of the above adaptive transmission designs for MIMO-OSTBC systems is given in Table 19.

Other adaptive MIMO transmission schemes with outdated CSIT and imperfect CSIT were investigated in [296] and in [297], [298], respectively. Zhang *et al.* [299] and Delamotte *et al.* [300] considered the adaptive power allocation for each subchannel with an instantaneous BER constraint based on the modified water-filling and mercury water-filling algorithms, respectively. Haustein *et al.* [301] developed an adaptive power allocation strategy for each subchannel by minimizing the MSEs at the receiver under a maximum average BER constraint. Some adaptive MIMO transmission schemes providing reconfigurable operational modes of antenna arrays are discussed in [302] and [303].

2) *Coded Systems*: The capacity expression in [1] first provided a fundamental transmission limit for MIMO systems. Since then, adaptive modulation and coding (AMC) schemes have been extensively investigated in attempts to approach this performance limit. McKay *et al.* [42] consid-

Table 18 Adaptive MIMO Transceiver Designs for Uncoded Systems

Paper	System	Criterion	CSI	Constraint
Palomar <i>et al.</i> [288]	MIMO	Minimize transmit power	PCSIT	Instantaneous BER
			PCSIR	Fixed transmit rate
Yasotharan <i>et al.</i> [289]	MIMO	Minimize average BER	PCSIT	ZF transceiver, constant transmit power
			PCSIR	Fixed transmit rate
Bergman <i>et al.</i> [290]	MIMO	Minimize weighted MSE	PCSIT	DFE receiver, constant transmit power
			PCSIR	Fixed transmit rate
Li <i>et al.</i> [291]	MIMO	Maximize transmit rate	PCSIT	Instantaneous BER
			PCSIR	Constant transmit power

Table 19 Adaptive Transmission Designs for MIMO-OSTBC Uncoded Systems

Paper	System	Criterion	CSI	Constraint
Ko <i>et al.</i> [292]	MIMO i.i.d. Rayleigh fading	Maximize average spectral efficiency	Feedback decision with delay PCSIR	Average BER, outage BER Constant transmit power
Huang <i>et al.</i> [293]	MIMO correlated Rayleigh fading	Maximize average spectral efficiency	Perfect channel estimation Imperfect channel estimation	Instantaneous BER, average BER Constant transmit power
Yu <i>et al.</i> [294]	MIMO i.i.d. Rayleigh fading	Maximize average spectral efficiency	Imperfect channel estimation	Average BER Average transmit power
Duong <i>et al.</i> [295]	MIMO i.i.d. Rayleigh fading	Maximize average spectral efficiency	Channel estimation optimization	Instantaneous BER Equal power allocation
Chen <i>et al.</i> [40]	MIMO i.i.d. Rayleigh fading	Maximize energy efficiency	IPCSIT PCSIR	Instantaneous BER Equal power allocation
Kuang <i>et al.</i> [41]	MIMO i.i.d. Rayleigh fading	Maximize average spectral efficiency	IPCSIT PCSIR	Average BER Equal power allocation

ered AMC for correlated Rayleigh fading MIMO channels. BICM was used for coding. With SCSIT, two simple transmission schemes are switched: statistical beamforming or spatial multiplexing with a zero-forcing receiver. Then, the transmitter selects the best combination of code rate, modulation format, and transmission scheme to maximize the data rate subject to a general BER union bound. Adaptive DIMO switching between spatial multiplexing and STBC based on spectral efficiency maximization for a nonselective channel with practical constraints, impairments, and receiver designs was investigated in [43]. For spatial multiplexing or STBC, the spectral efficiency thresholds to select the best coding rate and modulation format are determined based on the packet error rate (PER) versus instantaneous capacity simulation curves (convolutional code). The selection is performed at the receiver with perfect CSIR and then feeds back to the transmitter. For a time- and frequency-selective channel, a new physical layer abstraction and switching algorithm was proposed in [43], in which a weighted sum of channel qualities is minimized to reduce the variance of the channel qualities. Tan *et al.* [44] investigated AMC for MIMO-OFDM systems in the presence of channel estimation errors in slow fading channels. In this approach, convolutional or low-density parity check codes (LDPC) coded BER at the output of the detector are evaluated via a curve-fitting method. The packet error rate (PER) for each modulation and coding scheme is accurately approximated based on the coded BER expression thereby obtained. Then, the code rate, modulation format, and number of transmit streams are selected by maximizing the system throughput under a fixed PER constraint. A supervised learning algorithm, called k -nearest neighbors, has been

used to select the AMC parameters by maximizing the data rate under frame error rate (FER) constraints for MIMO-OFDM systems with convolutional codes in frequency- and spatial-selective channels [304]. Assuming perfect CSIT and CSIR, Dorrance *et al.* [305] considered AMC for a vertical Bell Laboratories layer space-time (V-BLAST) MIMO system with rate-compatible LDPC codes and an incremental redundancy hybrid automatic repeat request (H-ARQ) scheme. A constrained water-filling algorithm for finite alphabet input signals is provided to allocate power for each subchannel. Then, the modulation format used for each subchannel is determined based on a look-up table. Zhou *et al.* [45] incorporated BICM into the adaptive transmission scheme in [38] to improve BER robustness against CSI feedback delay. The BICM technique was modified by using a multilevel puncturing and interleaving technique and combined with rate-compatible punctured code/turbo code to achieve nearly full multiplexing gain. An AMC strategy for BICM MIMO-OFDM systems with soft Viterbi decoding was proposed in [306]. In [306], a system performance metric called the expect goodput was defined that is a function of the code rate, the modulation format, the subchannel power allocation, and the space-time-frequency precoder. The transmitter determines these AMC parameters by maximizing the expect goodput via the feedback of the perfect CSI and the effective SNR from the receiver. Spectral efficiencies of adaptive MIMO transmission schemes in a real-time practical test-bed are presented in [307]. A brief comparison of the above AMC schemes for MIMO systems is given in Table 20.

Current wireless standards such as 3GPP LTE, IEEE 802.16e, and IEEE 802.11n define concrete implementation procedures for adaptive MIMO transmission in prac-

Table 20 AMC Schemes for MIMO Systems

Paper	System	Criterion	CSI	Constraint
McKay <i>et al.</i> [42]	MIMO correlated Rayleigh fading	Statistical beamforming spatial multiplexing switch	SCSIT PCSIR	BER union bound
Choi <i>et al.</i> [43]	DIMO, non-selective time- and frequency-selective	Spatial multiplexing STBC switch	Feedback decision PCSIR	Practical PER
Tan <i>et al.</i> [44]	MIMO-OFDM slow fading	Maximize system throughput	Channel estimation error	Fixed PER
Daniels <i>et al.</i> [304]	MIMO-OFDM frequency- and spatial-selective	Maximize average spectral efficiency	PCSIT PCSIR	Fixed FER
Dorrance <i>et al.</i> [305]	V-BLAST MIMO H-ARQ	Maximize average spectral efficiency	PCSIT PCSIR	Rate-constrained power, bit allocation
Zhou <i>et al.</i> [45]	MIMO i.i.d. Rayleigh fading	Maximize average spectral efficiency	IPCSIT PCSIR	Instantaneous BER Average transmit power
Stupia <i>et al.</i> [306]	MIMO-OFDM	Maximize expect goodput	PCSIT PCSIR	Soft Viterbi decoding
Haustein <i>et al.</i> [307]	Practical MIMO test bed	Maximize average spectral efficiency	Channel estimation in practical systems	Average BER

tice. We shall take downlink transmission in the LTE standard as an example. The code rate and modulation format are selected based on the received channel quality indicator (CQI) user feedback. For example, the 4-bit CQI indices and their interpretations in LTE [308] are given in Table 21. The modulated transmit symbols are then mapped into one or several layers. This antenna mapping depends on the rank indicator user feedback, which includes a single antenna port, transmit diversity, and spatial multiplexing [308, Table 7.2.3-0]. The precoding matrices for these transmission modes depend on the precoding matrix indicator user feedback, and are defined in detail in [309, Sec. 6.3.4]. The adaptive MIMO transmission schemes in IEEE 802.16e and IEEE 802.11n are similar. In IEEE 802.16e, 52 combinations of codes, code rates, and modulation formats are defined as in [310, Table 8.4]. The transmission mapping matrices include [310, eq. (8.7)] Matrix A, exploiting only diversity; Matrix B, combining diversity and spatial multiplexing; and Matrix C, employing spatial multiplexing. A transmission mode is then selected based on the link quality evaluation. In IEEE 802.11n, 16 mandatory modulation and coding schemes are defined [311, Table 1]. A spatial multiplexing matrix is applied to convert the transmitted data streams into the transmit antennas, and an additional cyclic delay can be applied per transmitter to provide transmit cyclic delay diversity [311, Fig. 1]. Some practical AMC simulation results in these standards are presented in [266], [304], [311]–[317].

IV. MULTIUSER MIMO SYSTEMS

In this section, we introduce transmission designs with finite input signals for multiuser MIMO systems. We discuss the following four scenarios: 1) MIMO uplink transmission; 2) MIMO downlink transmission; 3) MIMO interference channel; and 4) MIMO wiretap channel.

A. MIMO Uplink Transmission

1) *Mutual Information Design*: For a multiple access channel (MAC) with amplitude-constrained inputs, the signals that maximize the mutual information have been shown to be discrete [318]. When the inputs are uniformly discretely distributed, the optimal rate region for

Table 21 Four-Bit CQI Table in the LTE Standard

CQI index	Modulation	Code rate $\times 1024$	Efficiency
0	Out of range		
1	QPSK	78	0.1523
2	QPSK	120	0.2344
3	QPSK	193	0.3770
4	QPSK	308	0.6016
5	QPSK	449	0.8770
6	QPSK	602	1.1758
7	16QAM	378	1.4766
8	16QAM	490	1.9141
9	16QAM	616	2.4063
10	64QAM	466	2.7305
11	64QAM	567	3.3223
12	64QAM	666	3.9023
13	64QAM	772	4.5234
14	64QAM	873	5.1152
15	64QAM	948	5.5547

Table 22 Mutual-Information-Based Precoder Designs for MIMO Uplink Transmission With Finite Input Signals

Paper	Model	Main contribution
Wang <i>et al.</i> [46]	MIMO MAC PCSIT, PCSIR	Proposes an iterative algorithm to find the optimal precoders which maximize the WSR
Wen <i>et al.</i> [118]	MIMO MAC, Kronecker fading SCSIT, PCSIR	Derives asymptotic expressions for the mutual information of the MIMO MAC with arbitrary inputs
Girnyk <i>et al.</i> [321]	MIMO MAC, Kronecker fading SCSIT, PCSIR non-Gaussian interferers	Proposes an iterative algorithm to find the optimal precoders that maximize the sum-rate
Wu <i>et al.</i> [323]	MIMO MAC, Weichselberger's fading SCSIT, PCSIR	Finds the asymptotic optimal left singular matrix of each user's optimal precoder Proposes an efficient iterative algorithm to maximize WSR

MAC is defined as the constellation-constrained capacity region [319], [320]. For a MIMO MAC with PCSIT, Wang *et al.* [46] derived the constellation-constrained capacity region for an arbitrary number of users and arbitrary antenna configurations. The boundary of this region can be attained by solving the weighted sum-rate (WSR) maximization problem with finite alphabet and individual power constraints.⁵ Necessary conditions for the optimal precoders of all users were established and an iterative algorithm was proposed to find these optimal precoders. Moreover, an LDPC coded system with iterative detection and decoding was provided to examine the BER performance of the obtained precoders. Numerical results indicate that the obtained precoders achieve considerably better performance than the nonprecoding design and the Gaussian input design in terms of both mutual information and coded BER.

For a MIMO MAC with SCSIT, asymptotic expressions for the mutual information of the MIMO MAC with arbitrary inputs over Kronecker fading channels have been derived in the large-system limit [118]. A linear precoder design for the MIMO MAC has been developed based on sum-rate maximization [321]. The proposed design includes the case where non-Gaussian interferers are present. For a more general Weichselberger fading model [322], Yu *et al.* [323] investigated linear precoder design based on WSR maximization. The asymptotic (in the large-system limit) optimal left singular matrix of each user's optimal precoder was shown to be the eigenmatrix of the user's transmit correlation matrix. This result facilitates the derivation of an efficient iterative algorithm to compute the optimal precoder for each user. A brief summary

⁵Based on [46, eq. (5)], we can observe that the mutual information expression for MIMO MAC with finite alphabet inputs is similar to that of a "big" point-to-point MIMO channel. The only difference is that this "big" MIMO channel conforms to the individual power constraint of each user. However, at high SNR when the transmit power becomes infinite, the individual power constraint and the total power constraint are identical. Therefore, for a MIMO MAC, the design of the constellation for the MMSE and mutual information at high SNR will also be very similar.

of the above mutual-information-based precoder designs for MIMO uplink transmission with finite input signals is given in Table 22.

2) *MSE Design*: For a MIMO MAC with PCSIT and PCSIR, Jorswieck *et al.* [48] designed transmit covariance matrices of the users to minimize the sum MSE with the multiuser MMSE receiver and individual user power constraints. A power allocation scheme among users was developed under a sum power constraint to further reduce the sum MSE. Based on the KKT conditions, the optimal transmit covariance matrices and the power allocation scheme can be found by an iterative algorithm. In addition, the achievable MSE region was studied in [48]. Serbetli *et al.* [324] investigated how to determine the number of transmit symbols for each user. Luo *et al.* [325] extended MMSE transceiver design to MIMO MAC OFDM systems. Lu *et al.* [326] studied the impact of practical per-antenna power and peak power constraints. MMSE transceiver design for a MIMO MAC with channel mean or covariance SCSIT feedback and PCSIR subject to individual user power constraints was investigated in [327] and [328]. For both sum and individual user power constraints, Zhang *et al.* [329] studied the average sum MSE minimization problem for a two-user MIMO MAC with correlated estimation error IPCSIT and IPCSIR. Layec *et al.* [330] considered the K -user MIMO MAC case with additional quantization noise and individual user power constraints. A near-optimal closed-form transceiver structure that minimizes the average sum MSE for a MIMO MAC with i.i.d estimation error IPCSIT and IPCSIR subject to a sum power constraint was presented in [331]. A brief summary of the above MMSE precoder designs for MIMO uplink transmission with finite input signals is given in Table 23.

3) *Diversity Design*: For open-loop systems, based on the framework for analyzing the dominant error event regions presented in [332], Gärtner *et al.* [333] derived space-time/frequency code design criteria for a two-user fading MIMO MAC. Moreover, an explicit two-user 2×2 coding

Table 23 MMSE Precoder Designs for MIMO Uplink Transmission With Finite Input Signals

Paper	Model	Main contribution
Jorswieck <i>et al.</i> [48]	MIMO MAC PCSIT, PCSIR Individual/sum power constraints	Proposes an iterative algorithm to minimize the sum MSE
Serbetli <i>et al.</i> [324]	MIMO MAC PCSIT, PCSIR Individual power constraints	Investigates how to determine the number of transmit symbols
Luo <i>et al.</i> [325]	MIMO MAC OFDM PCSIT, PCSIR Individual power constraints	Extends the MMSE transceiver design to OFDM systems
Lu <i>et al.</i> [326]	MIMO MAC PCSIT, PCSIR Per-antenna/peak power constraints	Proposes iterative algorithms to minimize the sum MSE
Jorswieck <i>et al.</i> [327], Zhang <i>et al.</i> [328]	MIMO MAC SCSIT, PCSIR Individual power constraints	Propose iterative algorithms to minimize the average sum MSE
Zhang <i>et al.</i> [329]	Two-user MIMO MAC IPCSIT, IPCSIR Individual/sum power constraints	Proposes iterative algorithms to minimize the average sum MSE
Layec <i>et al.</i> [330]	MIMO MAC IPCSIT, IPCSIR, Quantization noise Individual power constraints	Proposes iterative algorithms to minimize the average sum MSE
Huang <i>et al.</i> [331]	MIMO MAC IPCSIT, IPCSIR A sum power constraint	Provides closed-form near-optimal MMSE precoder designs

scheme was constructed by concatenating two Alamouti schemes with a column swapping for one user's codeword to achieve a minimum rank of three. A algebraic construction of STBC based on diversity and multiplexing tradeoff for a MIMO MAC was proposed in [334]. Similar STBC designs have also been provided for an asynchronous single-input multiple-output (SIMO) MAC in [335], a nonuniform user transmit power MIMO MAC in [336], and a two-user MIMO multiple-access AF relay channel in [337]. A space-frequency code for a MIMO-OFDM MAC that achieves full-diversity for every user was designed in [47]. However, the codes in [47] suffer from high large peak-to-average power ratios, since some elements in the codeword matrices are zero. Another group of STBCs for MIMO MACs was proposed in [338] based on minimization of a truncated union-bound approximation. Simulations show that the codes in [338] achieve better error probability than those in [47], [333]. Lu *et al.* [339] proposed sphere-decodable STBCs for a two-user MIMO MAC and provided an explicit construction for the 2×2 case. STBCs for a two-user MIMO MAC with reduced

average sphere decoding complexity were constructed in [340]. A differential STBC for a two-user MIMO MAC was proposed in [341]. Another differential STBC for a two-user MIMO MAC with low-complexity noncoherent decoders was designed in [342]. For closed-loop systems, by exploiting outdated CSI, Huang *et al.* [343] investigated linear precoding design for a MIMO MAC with OSTBC to minimize PEP subject to the individual transmit power constraints of each user. Kim *et al.* [344] proposed an STBC for a two-user MIMO MAC with linear detection by utilizing the phase feedback transmitted to one user. By exploiting PCSIT, Li and Jafarkhani proposed full-diversity precoder designs for two-user and K -user MIMO MACs in [345] and [346], respectively. A brief comparison of the above studies is given in Table 24.

4) *Adaptive Transmission:* For a MIMO-OFDM MAC with PCSIT and PCSIR, Zhang *et al.* [49] proposed an adaptive resource allocation scheme in which each user's signal is transmitted along the maximum singular of its own channel. The proposed scheme selects the subcarrier allocation,

Table 24 Space–Time Techniques for MIMO Uplink Transmission With Finite Input Signals

Paper	Model	Main contribution
Gärtner <i>et al.</i> [333]	MIMO MAC Two-user	Derives space–time/frequency code design criteria Constructs an explicit coding scheme for two-user 2×2
Badr <i>et al.</i> [334]–[337]	MIMO MAC, asynchronous SIMO MAC, unbalanced MIMO MAC Two-user MIMO multiple-access AF relay	Construct STBCs based on diversity and multiplexing trade-off
Zhang <i>et al.</i> [47]	MIMO-OFDM MAC	Designs a space–frequency code that achieves full-diversity for every user
Hong <i>et al.</i> [338]	MIMO MAC	Designs STBCs based on truncated union-bound approximation
Lu <i>et al.</i> [339], Harshan and Rajan [340]	MIMO MAC Two-user	Design sphere-decodable STBCs
Bhatnagar and Hjørungnes [341], Poorkasmaei and Jafarkhani [342]	MIMO MAC Two-user	Design differential STBCs
Huang <i>et al.</i> [343]	MIMO MAC Outdated CSIT	Investigates the linear precoding design subject to individual user’s transmit power constraint
Kim <i>et al.</i> [344]	DIMO MAC Two-user, feedback	Designs an STBC with a linear detection
Li and Jafarkhani [345], [346]	Two-user MIMO MAC, K -user MIMO MAC PCSIT	Propose full-diversity precoder designs

the power allocation, and modulation modes to minimize the total transmit power subject to each user’s uncoded BER and data rate constraints. It is assumed that these uplink transmission parameters are determined at the base station and then sent to users via the error-free channel. Simulation results indicate that within a reasonable region of Doppler spread, the proposed scheme is also robust to variations in channel time. A low-complexity design with a neighborhood search for the optimal resource allocation solution was proposed in [347]. For a MIMO-OFDM MAC in which each user employs the transmission design in [283] with its own SCSTI, Chemaly *et al.* [348] proposed an adaptive resource allocation scheme to maximize the sum data rate subject to each user’s uncoded BER and

total power constraints. By taking an additional fairness consideration among different users, another adaptive resource allocation scheme was given in [349] to maximize the sum data rate subject to each user’s uncoded BER and total power constraints. A brief comparison of the above studies is given in Table 25.

B. MIMO Downlink Transmission

1) *Mutual Information Design*: For a two-user MISO BC with PCSIT, PCSIR, and finite alphabet inputs, precoder designs to maximize the sum rate and the corresponding simplified receiver structure were studied in [350]–[352]. Wu *et al.* [50] investigated a linear precoder design to maximize the WSR of a general MIMO BC. They derived

Table 25 Adaptive Transmissions for MIMO Uplink Transmission With Finite Input Signals

Paper	Model	Main contribution
Zhang <i>et al.</i> [49]	MIMO-OFDM MAC, perfect CSI Each user’s uncoded BER and data rate constraints	Proposes an adaptive resource allocation to minimize the total transmit power
Zhang <i>et al.</i> [347]	MIMO-OFDM MAC, perfect CSI Each user’s uncoded BER and data rate constraints	Proposes a low-complexity adaptive resource allocation to minimize the total transmit power
Chemaly <i>et al.</i> [348]	MIMO-OFDM MAC, SCSTI, PCSIR Each user’s uncoded BER and total power constraints	Proposes an adaptive resource allocation to maximize the sum data rate
Maw <i>et al.</i> [349]	MIMO-OFDM MAC, perfect CSI Each user’s uncoded BER and total power constraints	Proposes an adaptive resource allocation considering users’ fairness and maximizing the sum data rate

Table 26 Mutual-Information-Based Precoder Designs for MIMO Downlink Transmission With Finite Input Signals

Paper	Model	Main contribution
Ghaffar <i>et al.</i> [350]–[352]	MISO BC, 2-user PCSIT, PCSIR	Propose precoder designs and the corresponding simplified receiver structures to maximize the sum-rate
Wu <i>et al.</i> [50]	MIMO BC PCSIT, PCSIR	Proposes an iterative algorithm to find the optimal precoders that maximize the WSR
Wu <i>et al.</i> [354]	Cooperative multi-cell MIMO downlink PCSIT, PCSIR	Proposes two low-complexity algorithms to find the optimal precoders that maximize the sum-rate
Wu <i>et al.</i> [355]	MISO downlink multicasting PCSIT, PCSIR	Proposes two transmit schemes to maximize the multicast rate

explicit expressions for the achievable rate region that are applicable to an arbitrary number of users with generic antenna configurations. It was found that sum-rate loss will occur in the high-SNR regime for an improper precoder design because of the nonuniquely decodable transmit signals. An iterative algorithm was proposed to optimize precoding matrices for all users. A downlink multiuser system with LDPC code and iterative detection and decoding was further developed. Simulation results indicate that the proposed precoding design provides substantial WSR and coded BER gains over the best rotation design for SISO BC [353], the nonprecoding design, and the Gaussian input design. Wu *et al.* [354] investigated linear precoder design for cooperative multi-cell MIMO downlink systems with PCSIT and finite alphabet inputs. To reduce the complexity of evaluating the non-Gaussian interferers, they used a Gaussian interferer to approximate the sum of non-Gaussian interferers at each user's receive side and proposed two iterative algorithms to maximize the approximated sum-rate under per base station power constraints. Compared with the algorithm in [50], the algorithms proposed in [354] reduce the complexity of implementation with negligible performance losses. Wu *et al.* [355] proposed two transmit schemes to maximize the multicast rate of MISO downlink multicasting systems with PCSIT and finite alphabet inputs. A brief summary of the above mutual-information-based precoder designs for MIMO downlink transmission with finite input signals is given in Table 26.

2) *MSE Design*: For linear processing, early work in [356], [357] considered extending the MMSE transceiver design in [29] to a MIMO BC. For a MIMO BC with PCSIT, PCSIR, and a sum power constraint, Shi *et al.* [52] established an uplink–downlink duality framework between the downlink and uplink MSE feasible regions. Based on this framework, downlink sum-MSE minimization can be transformed into an equivalent uplink problem. Two globally optimum algorithms were proposed to find the optimal MMSE transceiver design. Shi *et al.* [358]

further proposed near-optimal low-complexity iterative algorithms to minimize the maximum ratio of MSE and a given requirement under a sum power constraint and to minimize the transmit power under various MSE requirements. Hunger *et al.* [359] proposed a low-complexity approach to evaluate MSE duality that is able to support switched-off data streams and passive users correctly. The MSE duality in [52] was extended to the imperfect CSI case for a MISO BC in [360]. Murga *et al.* [361] presented a transceiver design framework for a MIMO-OFDM BC based on channel Gram matrix feedback, with the CSI quantization error also being taken into consideration. As an example, a robust precoder design with fixed decoder that minimizes the sum MSE subject to a sum power constraint was given in [361]. Ding *et al.* [362] proposed a robust transceiver design for network MIMO systems with imperfect backhaul links that minimizes the maximum substream MSE subject to per base station power constraints.

For a robust design in which the actual channel is considered to be within an uncertain region around the channel estimate, a robust power allocation scheme among users was proposed for a MISO BC to minimize the transmit power subject to each user's MSE requirement [363]. Vičić *et al.* extended this to robust transceiver design for a MISO BC and a MIMO BC in [364] and [365], respectively. He *et al.* [366] further investigated this robust transceiver design subject to the probabilistic MSE requirement of each user. A robust resource allocation design to optimize the function of worst case MSE subject to quadratic power constraints for multicell MISO downlink transmission was proposed in [367].

Some studies have considered MMSE precoder design with a fixed decoder. Dabbagh *et al.* [368] derived an MMSE-based precoding technique for a MISO BC with IPCSIT and a sum power constraint. They also investigated a limited feedback system model. Sung *et al.* [369] developed a generalized MMSE precoder for a MIMO BC with perfect and imperfect CSI. Designs for minimization of the sum MSE subject to a sum power constraint and

Table 27 Linear MMSE Precoder Designs for MIMO Downlink Transmission With Finite Input Signals

Paper	Model	Main contribution
Shi <i>et al.</i> [52]	MIMO BC PCSIT, PCSIR A sum power constraint	Establishes an uplink–downlink MSE duality framework
Shi <i>et al.</i> [358]	MIMO BC PCSIT, PCSIR	Minimizes the maximum ratio of MSE and given requirement Minimizes the transmit power under various MSE requirements
Hunger <i>et al.</i> [359]	MIMO BC PCSIT, PCSIR A sum power constraint	Establishes an MSE duality that is able to support the switched-off data streams and the passive users correctly
Ding <i>et al.</i> [360]	MISO BC IPCSIT, IPCSIR A sum power constraint	Extends the MSE duality in [52] to the imperfect CSI case
Murga <i>et al.</i> [361]	MIMO-OFDM BC Channel Gram matrix A sum power constraint	Provides a transceiver design framework based on the channel Gram matrices feedback
Ding <i>et al.</i> [362]	Network MIMO systems Imperfect backhaul links Per base station power constraints	Minimizes the maximum substream MSE
Payaró <i>et al.</i> [363]	MISO BC IPCSIT, IPCSIR Each user's MSE requirement	Proposes a robust power allocation scheme among users to minimize the transmit power
Vičić <i>et al.</i> [364]	MISO BC IPCSIT, IPCSIR Each user's MSE requirement	Proposes a robust transceiver design to minimize the transmit power
Vičić <i>et al.</i> [365]	MIMO BC IPCSIT, IPCSIR	Proposes robust transceiver designs for several MSE-optimization problem
He <i>et al.</i> [366]	MIMO BC IPCSIT, IPCSIR Each user's probabilistic MSE requirement	Proposes a robust transceiver design to minimize the transmit power
Björnson <i>et al.</i> [367]	MISO BC IPCSIT, IPCSIR Quadratic power constraints	Proposes a robust resource allocation design to optimize the function of worst case MSE
Dabbagh <i>et al.</i> [368]	MISO BC, fixed decoder IPCSIT, PCSIR A sum power constraint	Obtains an MMSE-based precoding technique
Sung <i>et al.</i> [369]	MIMO BC, fixed decoder Perfect and imperfect CSI	Minimizes the sum MSE subject to a sum power constraint Minimizes the interference-plus-noise power subject to an individual user's power constraints
Xiao <i>et al.</i> [370]	MIMO BC, fixed decoder Perfect and imperfect CSI A sum power constraint	Proposes an improved MIMO BC MMSE precoder for improper signal constellations

for minimization of the interference-plus-noise power at the receiver subject to individual users' power constraints were presented. Xiao *et al.* [370] proposed an improved MIMO BC MMSE precoder with perfect and imperfect CSI subject to a sum power constraint for improper signal constellations such as amplitude shift-keying and offset

quadrature phase shift-keying. A brief summary of the above linear MMSE precoder designs for MIMO downlink transmission with finite input signals is given in Table 27. There have also been some studies of nonlinear MMSE precoder designs for MIMO downlink transmission with finite input signals [52], [371]–[376].

Table 28 Space-Time Techniques for MIMO Downlink Transmission With Finite Input Signals

Paper	Model	Main contribution
Ma <i>et al.</i> [377]	MIMO BC	Proposes a trace-orthogonal space-time coding
Larsson [378], Kuo and Kuo [379]	MIMO BC	Propose differential STBC designs for data encoded using layered source coding
Chen <i>et al.</i> [380]	MIMO BC PCSIT, PCSIR	Combines a unitary downlink precoder with STBC
Chen <i>et al.</i> [51]	MIMO BC PCSIT, PCSIR	Combines a unitary downlink precoder with STBC and antenna selection
Wang <i>et al.</i> [381]	MIMO BC PCSIT, PCSIR	Combines THP with dominant eigenmode transmission and OSTBC
Clerckx <i>et al.</i> [382]	MISO BC two-user, outdated CSIT, PCSIR	Optimizes signal constellation to improve the average PEP
Lee <i>et al.</i> [383]	MIMO BC PCSIT, PCSIR	Analyzes opportunistic scheduling and diagonalization-precoding for MIMO BC with OSTBC
Li <i>et al.</i> [384]	MIDO BC PCSIT, no CSIR	Combines an interference cancellation scheme with the Alamouti code
Lin <i>et al.</i> [385]	MIMO BC PCSIT, PCSIR	Extends X-Structure precoder to MIMO BC

3) *Diversity Design*: In downlink transmission, mitigating multiuser interference is essential for reliable communication, and this normally requires that some form of CSI be available. Therefore, not much work has been done on open-loop downlink systems. Ma *et al.* [377] proposed a trace-orthogonal space-time coding that is suitable for a MIMO BC. This coding reduces the complexity of ML with little performance loss. Some differential STBC designs for data encoded using layered source coding have been proposed for MIMO BCs [378], [379]. For closed-loop systems, space-time techniques are usually combined with linear precoding designs to eliminate cochannel interference and improve diversity performance. Chen *et al.* [380] proposed a unitary downlink precoder design for multiuser MIMO STBC systems with PCSIT. With a sufficiently high antenna dimension, the proposed precoder can effectively cancel cochannel interference at each user's side. The additional antenna dimension can then be used to select the precoder matrix for the diversity gain. Chen *et al.* [51] presented a more general transmission design for MIMO BCs that combines a unitary downlink precoder for cochannel interference mitigation with a space-time precoder design and antenna selection technique for SER minimization. Simulation results indicate that the proposed design achieves significant diversity gains in terms of SER. Wang *et al.* [381] combined nonlinear THP with two diversity techniques, namely, dominant eigenmode transmission and OSTBC, to improve the BER perfor-

mance of a MIMO BC. Clerckx *et al.* [382] investigated the transmission strategy for a two-user MISO BC with outdated CSIT and obtained analytical expressions for the corresponding average PEP over a correlated Rayleigh fading channel. By exploiting SCSIT, signal constellations are optimized to improve the average PEP performance. Lee *et al.* [383] analyzed the outage probability of the average effective SNR for a MIMO BC employing OSTBC. The analytical results indicate that an opportunistic scheduling scheme provides an effective SNR and diversity gains compared with a block diagonalization-precoding scheme. With PCSIT and no CSIR, Li *et al.* [384] proposed an interference cancellation scheme for a MIMO BC in which the transmitter sends a precoded Alamouti code to every user simultaneously. Numerical results indicate that the design proposed in [384] has better diversity gains than the block diagonalization designs in [380], [383]. By using a regularized block diagonalization precoder to suppress multiuser interference in the first step, Lin *et al.* [385] extended the X-Structure precoder in [37] to a MIMO BC. A brief comparison of the above studies is given in Table 28.

4) *Adaptive Transmission*: For uncoded systems, most adaptive transmission studies have focused on adaptive resource allocation for multiuser MIMO-OFDM systems. Typically, with perfect CSIT and CSIR, Tsai *et al.* [53] devised an adaptive resource allocation algorithm for multiuser downlink MISO orthogonal frequency division

Table 29 Adaptive Transmissions for MIMO Downlink Transmission With Finite Input Signals

Paper	Model	Main contribution
Tsai <i>et al.</i> [53]	Uncoded MISO-OFDMA/SDMA, perfect CSI Various quality-of-service constraints	Devises an adaptive resource allocation algorithm to maximize the spectral efficiency
Yen <i>et al.</i> [386]	Uncoded MIMO-OFDMA, perfect CSI Various quality-of-service constraints	Defines a utility function to distinguish the real-time and non-real-time services
Hara <i>et al.</i> [390]	Coded MIMO, perfect CSI PER constraints	Designs an efficient downlink scheduling algorithm to maximize the overall system throughput
Esslaoui <i>et al.</i> [391]	Coded MISO-OFDM, perfect CSI Effective SNR constraints	Proposes a two-step adaptive transmission scheme to maximize the spectral efficiency
Esslaoui <i>et al.</i> [392]	Coded MIMO-OFDM, perfect CSI Effective SNR constraints	Fairness among users is taken into consideration of system designs
Wang <i>et al.</i> [393]	Coded MIMO-OFDMA, perfect CSI Effective SNR constraints	Proposes an adaptive scheme for power allocation, resource allocation, and the modulation and coding scheme
Chen <i>et al.</i> [394]	Coded MIMO-OFDM, feedback CSI Effective SNR constraints	Proposes an adaptive scheme to maximize the spectral efficiency
Alvariño <i>et al.</i> [395]	Coded MIMO-OFDM, feedback CSI FER constraints	Proposes an adaptive scheme to maximize the overall system throughput, which allows multiple spatial streams for each user

multiplexing access (OFDMA)/spatial division multiple access (SDMA) systems with multimedia traffic. A dynamical priority scheduling scheme was developed to provide service to urgent users based on time to expiration. Resources for power, subchannel, and bit allocation were then iteratively allocated to maximize spectral efficiency subject to various quality-of-service constraints. Yen *et al.* [386] extended this adaptive resource allocation design to multiuser downlink MIMO OFDMA systems, with a utility function being defined to distinguish more clearly between the real-time and nonreal-time services. Other resource allocation schemes for practical multiuser MIMO-OFDM systems can be found in [387]–[389].

For coded systems, Hara *et al.* [390] designed an efficient downlink scheduling algorithm for multiuser downlink MIMO systems by optimizing an equivalent uplink scheduling problem. With perfect knowledge of the channel, the proposed algorithm selects the user for each transmit beam, the transmit weight for the user, and the corresponding modulation and coding scheme to maximize the overall system throughput subject to PER constraints. Esslaoui *et al.* [391] proposed a two-step adaptive transmission scheme for multiuser downlink MISO-OFDM systems. In the first step, the users that should be simultaneously transmitted are selected based on a degree-of-orthogonality criterion. In the second step, the modulation and coding scheme for each user is determined adaptively by maximizing the throughput while satisfying the effective SNR constraint for each user. The proposed design was simulated for the IEEE 802.11ac. It is

worth mentioning that, as shown in [391, Fig. 4], even when an adaptive transmission technique is employed, there are still obvious gaps between the practical schemes and the theoretical capacity achieved by Gaussian input. The design in [391] was extended to multiuser downlink MIMO-OFDM systems and fairness among users was also taken into consideration in [392]. Wang *et al.* [393] proposed another transmission scheme for multiuser downlink MIMO-OFDMA systems in which the power allocation, the resource allocation policy, and the modulation and coding scheme for each user are adaptively determined to maximize the system throughput. The proposed design was simulated for IEEE 802.16e and was shown to exhibit good performance. Adaptive transmission for multiuser downlink MIMO-OFDM systems with limited feedback was presented in [394], with only a single spatial stream being transmitted for each user. Alvariño *et al.* [395] proposed an adaptive transmission scheme for multiuser downlink MIMO-OFDM systems with limited feedback that allows multiple spatial streams for each user. A greedy algorithm is used to select the users and the spatial modes. A machine learning classifier is used to select the modulation and coding schemes for users by maximizing the overall system throughput subject to FER constraints. A brief comparison of the above studies is given in Table 29.

C. MIMO Interference Channel

1) *Mutual Information Design*: By directly extending the classical interference alignment scheme used for a Gaussian input signal, Hari Ram *et al.* [396] and Fadlal-

Table 30 Mutual-Information-Based Precoder Designs for the MIMO Interference Channel With Finite Input Signals

Paper	Model	Main contribution
Hari Ram <i>et al.</i> [396], Fadlallah <i>et al.</i> [397]	MIMO interference channel Global PCSIT, PCSIR	Extend the interference alignment scheme to precoder design with finite alphabet inputs
Huang <i>et al.</i> [398]	MIMO interference channel Global PCSIT, PCSIR	Proposes a partial interference alignment and interference detection scheme
Wu <i>et al.</i> [54], Ganesan and Rajan [399]	MIMO interference channel Global PCSIT, PCSIR	Propose gradient-descent-based iterative algorithms to find optimal precoders of all transmitters
Hari Ram and Giridhar [400], [401]	MIMO interference channel Global PCSIT, PCSIR	Propose a novel low-complexity fractional interference alignment scheme

lah *et al.* [397] investigated precoder designs for a K -user MIMO interference channel by maximizing the mutual information of finite alphabet signal sets. Following an interference alignment scheme, the precoders in [396] and [397] only use half of the transmit antenna dimension to send the desired signals. Huang *et al.* [398] considered relaxing this dimension constraint by allowing the interfering signals to overlap with the desired signal and proposed a partial interference alignment and interference detection scheme for a K -user MIMO interference channel with finite alphabet inputs. For interfering signals that cannot be aligned at each receiver, the receiver detects and cancels the residual interference based on the constellation map. By treating the interference as noise, Wu *et al.* [54] and Ganesan *et al.* [399] independently derived mutual information expressions for a K -user MIMO interference channel with finite alphabet inputs. Gradient-descent-based iterative algorithms were proposed in [54] and [399] to find the optimal precoders of all transmitters. Instead of the degrees of freedom used for a Gaussian input signal, Hari Ram *et al.* [400] defined a new performance metric to analyze the transmission efficiency for a K -user

MIMO interference channel with finite alphabet inputs: the number of symbols transmitted per transmit antenna per channel user (SpAC). It was found in [54] and [399] that simple transmission schemes can achieve the maximum 1 SpAC in the high-SNR regime. However, joint detection of all the transmitter signals, including the desired signals and the interfering signals at each receiver, is needed in [54] and [399] to achieve 1 SpAC. Hari Ram *et al.* [401] proposed a fractional interference alignment scheme that does not decode any of the interfering signals. The fractional interference alignment can achieve any value of SpAC in the range $[0, 1]$ and a good error rate performance for both uncoded and coded BER. A brief summary of the above mutual-information-based precoder designs for a MIMO interference channel with finite input signals is given in Table 30.

2) *MSE Design*: For a MIMO interference channel with global perfect CSI, Shen *et al.* [55] investigated linear transceiver designs under a given and feasible degree of freedom. Two iterative algorithms were proposed to minimize both the sum MSE and the maximum per-user

Table 31 MMSE Transceiver Designs for MIMO Interference Channel With Finite Input Signals

Paper	Model	Main contribution
Shen <i>et al.</i> [55]	MIMO interference channel Global perfect/Imperfect CSI	Minimizes both the sum MSE and the maximum per-user MSE
Chen <i>et al.</i> [402]	MIMO interference channel Global perfect CSI	Minimizes the maximum per-stream MSE of all users
Sun <i>et al.</i> [403]	MIMO interference channel Global perfect CSI	Minimizes the MSE of the signal and interference leakage
Tseng <i>et al.</i> [404]	MIMO interference channel Random vector quantization feedback	Minimizes both the sum MSE and the maximum per-user MSE

Table 32 Space-Time Techniques With Finite Input Signals for MIMO Interference Channel

Paper	Model	Main contribution
Shi <i>et al.</i> [56]	Two-user MIMO interference channel No PCSIT, PCSIR	Designs full-diversity STBCs
Lu <i>et al.</i> [405]	MISO interference, reconfigurable antenna No PCSIT, PCSIR	Combines the blind interference alignment scheme with three STBCs
Sezgin <i>et al.</i> [406]	MIMO interference channel Global PCSIT, PCSIR	Combines STBC with the interference alignment design
Ning <i>et al.</i> [407]	MIMO interference channel Global PCSIT, PCSIR	Reveals that the separation of space-time coding and precoding design may not be optimal
Li <i>et al.</i> [408]	Three-user 2×2 interference channel Global PCSIT, PCSIR	Combines the antenna selection technique with the interference alignment design

MSE. A transceiver design for a MIMO interference channel with channel estimation error was also presented. With perfect CSI, Chen *et al.* [402] extended the design to minimize the maximum per-stream MSE of all users. Sun *et al.* [403] proposed a low-complexity transceiver design based on minimizing the MSE of the signal and interference leakage at each transmitter. A robust transceiver design for a MISO interference channel with random vector quantization feedback mechanism was proposed in [404], taking account of the impact of quantization error at the transmitter. The optimal transmit beamforming vectors and the receiver decoding scalars for both the sum MSE and the maximum per-user MSE minimization are found by iterative algorithms. A brief summary of the above studies is given in Table 31.

3) *Diversity Design*: For an open-loop system, Shi *et al.* [56] designed full-diversity STBCs for a two-user MIMO interference channel with a group ZF receiver. Lu *et al.* [405] combined a blind interference alignment scheme with three STBCs for a K -user MISO interference channel employing a reconfigurable antenna. These combinations provide a tradeoff between diversity, rate, and decoding complexity, where threaded algebraic space-time codes have full diversity and a high rate with high decoding complexity, OSTBCs have full diversity and a low rate with linear decoding complexity, and Alamouti codes have a high rate and low linear decoding complexity but with full-diversity loss. For a closed-loop system, when global CSI is available at all transmitters and receivers, Sezgin *et al.* [406] combined a STBC with interference alignment to achieve high diversity gains for a MIMO interference channel. However, an investigation of the feasibility condition for a MIMO interference channel revealed that separation of space-time coding and precoding design may not be

optimal in general [407]. The diversity gains of various interference alignment schemes for MIMO interference channels were analyzed in [407]. Li *et al.* [408] combined an antenna selection technique with an interference alignment design to improve the diversity gain of the worst case user for a three-user 2×2 interference channel. A brief comparison of the above studies is given in Table 32.

4) *Adaptive Transmission*: Xie *et al.* [57] investigated adaptive transmission for uncoded MIMO interference channels with imperfect CSIT. Interference-alignment-based bit-loading algorithms were proposed to minimize the average BER subject to a fixed sum-rate constraint. Based on this, an adaptive transmission scheme switching between two interference alignment algorithms and a basic time-division multiple access scheme to combat CSI uncertainty was presented. Taki *et al.* [409] investigated adaptive transmission for coded MIMO interference channels with imperfect CSIT. An interference-alignment-based adaptive transmission scheme was proposed that adaptively selects the coding, modulation, and power allocation across users to maximize the weighted sum-rate subject to a sum power constraint and BER constraints for every transmit stream.

D. MIMO Wiretap Channel

1) *Mutual Information Design*: For a wiretap channel, finite alphabet inputs may allow both the receiver and the eavesdropper to decode a transmitted message accurately if the transmit power is high enough [410]. Therefore, the secrecy rate may not be high at a high SNR. This phenomenon has been observed in simulations of SISO single-antenna eavesdropper wiretap channels [411]. Therefore, Bashar *et al.* [412] suggested that an optimum power control policy at the transmitter be found to maximize

Table 33 Mutual-Information-Based Precoder Designs for the MIMO Wiretap Channel With Finite Input Signals

Paper	Model	Main contribution
Bashar <i>et al.</i> [412]	MISOME SCSIT of the eavesdropper	Finds an optimum power control policy at the transmitter for maximizing the secrecy rate
Bashar <i>et al.</i> [413]	MIMOME PCSIT of the eavesdropper	Proposes a GSVD precoder structure to increase the secure rate performance
Wu <i>et al.</i> [58]	MIMOME PCSIT of the eavesdropper	Proposes iterative algorithms for secrecy rate maximization Constructs the jamming signal with the excess power
Aghdam <i>et al.</i> [414]	MIMOME SCSIT of the eavesdropper	Proposes iterative algorithms for secrecy rate maximization
Aghdam <i>et al.</i> [415]	MIMOME PCSIT of the eavesdropper	Proposes a joint precoder and artificial noise design based on the cut-off rate
Zeng <i>et al.</i> [416]	MIMO cognitive radio secure SCSIT of the eavesdropper	Proposes an iterative algorithm to maximize the secure rate of the secondary user
Vishwakarma <i>et al.</i> [417]	MIMO DF relay secure IPCSIT of all links	Designs the optimal source power, signal beamforming, and jamming signal matrix for worst-case secrecy rate maximization
Cao <i>et al.</i> [418]	MIMOME with a helper SCSIT of the eavesdropper	Designs a secure transmission scheme with the help of the source node

the secrecy rate. For a MISO single-antenna eavesdropper (MISOSE) wiretap channel, they investigated the power control optimization issue at the transmitter based on conventional beamforming transmission for the Gaussian input case and developed a numerical algorithm to find the optimal transmission power. This idea was extended to a MIMO multiple-antenna eavesdropper (MIMOME) wiretap channel by exploiting a generalized SVD (GSVD) precoding structure to decompose the MIMOME channel into a bank of parallel subchannels [413]. Numerical algorithms were proposed to determine the appropriate power to be allocated to each subchannel. Although the precoding design in [413] achieves significant performance gains over the conventional design that relies on the Gaussian input assumption, it is still suboptimal. The reasons are twofold. First, a proportion of the transmission symbols are lost by both the desired user and the eavesdropper according to the GSVD structure. This may result in a constant performance loss. Second, the power control policy compels the transmitter to utilize only a fraction of the power available to transmit in the high-SNR regime, which may impede further improvements in performance in some scenarios (see, e.g., [413, Fig. 3]). Consequently, Wu *et al.* [58] investigated a linear precoder design for a MIMOME wiretap channel when instantaneous CSI of the eavesdropper is available at the transmitter. An iterative algorithm for secrecy rate maximization was developed through a gradient method. For systems where the number

of transmit antennas is less than or equal to the number of eavesdropper antennas, only partial transmission power is necessary for maximizing the secrecy rate at high SNR. Accordingly, the excess transmission power is used to construct an artificial jamming signal to further improve the secrecy rate. By invoking the lower bound on the average mutual information in MIMO Kronecker fading channels with finite alphabet inputs [26], Wu *et al.* [58] also considered scenarios where the transmitter has only statistical CSI of the eavesdropper. Aghdam *et al.* extended the design to the scenarios where the transmitter has statistical CSI of both the receiver and eavesdropper [414]. Furthermore, Aghdam *et al.* [415] proposed a joint precoder and artificial noise design based on the cutoff rate. Zeng *et al.* [416] studied precoder design for MIMO secure cognitive radio systems with finite alphabet inputs. With the SCSIT of the eavesdropper, an iterative algorithm was proposed to maximize the secure rate of the secondary user under the control of power leakage to the primary users. Transmission design for MIMO decode-and-forward (DF) relay systems with IPCST and finite alphabet inputs in the presence of a user and J noncolluding eavesdroppers was investigated in [417]. Very recently, a secure transmission scheme with the help of source node was proposed in [418]. A brief summary of the above mutual-information-based precoder designs for a MIMO wiretap channel with finite input signals is given in Table 33.

Table 34 MMSE Transceiver Designs for MIMO Wiretap Channel With Finite Input Signals

Paper	Model	Main contribution
Reboredo <i>et al.</i> [60]	MIMOME Instantaneous/statistical CSI	Minimizes the receiver's MSE while guaranteeing the eavesdropper's MSE above a fixed threshold
Kong <i>et al.</i> [419]	MIMO interference channel secure Perfect CSI	Minimizes the receivers' sum MSE while guaranteeing the eavesdropper's MSE above a fixed threshold
Pei <i>et al.</i> [420]	MIMO BC secure, Perfect/imperfect CSI of receivers No CSI of the eavesdropper	Maximizes the transmit power to generate artificial noise subject to the MSE requirements of the desired receivers
Zhang <i>et al.</i> [421]	MIMOME, nonlinear THP design Perfect CSI of the receiver, no CSI of the eavesdropper	Maximizes the transmit power to generate artificial noise subject to the MSE requirements of the receiver

2) *MSE Design*: For a MIMOME where perfect instantaneous CSI is available to all parties, Reboredo *et al.* [60] investigated transceiver design minimizing the receiver's MSE while guaranteeing that the eavesdropper's MSE was above a fixed threshold. A linear zero-forcing precoder was used at the transmitter and either a zero-forcing decoder or an optimal linear Wiener decoder is designed at the receiver. Moreover, the design was generalized to scenarios where only the statistical CSI is available. Simulation results show that maintaining the eavesdropper's MSE above a certain level also limits the eavesdropper's error probability. This design was further extended to a MIMO interference channel with a multiple-antenna eavesdropper [419]. Pei *et al.* [420] investigated a masked beamforming design for MIMO BC in the presence of a multiple-antenna eavesdropper. It was assumed that the transmitter obtains perfect/imperfect CSI of the desired receivers, but no CSI of the eavesdropper. The transmit power used to generate the artificial noise was maximized subject to the MSE requirements of the desired receivers. Based on the same criterion, Zhang *et al.* [421] investigated nonlinear THP design for a MIMOME. A brief summary of the above work is given in Table 34.

3) *Diversity Design*: For two transmit antennas, two receive antennas, and a multiple-antenna eavesdropper wiretap channel, Fakoorian *et al.* [422] designed a secure STBC by exploiting the perfect CSI of the receiver but no CSI of the eavesdropper. By combining the QOSTBC in [187] with artificial noise generation, the design proposed in [422] led to an effective deterioration in the uncoded BER performance of the eavesdropper while maintaining the good uncoded BER performance of the receiver. For the same STBC system, Wen *et al.* [423] proposed a cross-layer scheme to further ensure secure communication. Allen *et al.* [424] investigated two/four transmit antennas, one receive antenna, and one antenna eavesdropper wiretap channel employing STBCs, with no CSI of the receiver or the eavesdropper being available at the transmitter. By randomly rotating the transmit

symbols at each transmit antenna based on the receive signal strength indicator, the proposed scheme provides full diversity of the receiver and enables the eavesdropper's diversity to be zero. For a MISOME wiretap channel with the same CSI assumption, Perreau [425] designed transmit antenna weights based on STBC, which formulates beamforming along the receiver's direction while disabling the eavesdropper's ability to discriminate the transmit signal. Under the assumption that the transmitter has perfect CSI of the receiver based on channel reciprocity, while both the receiver and the eavesdropper have no channel knowledge, Li *et al.* [59] investigated secure communication for a MISOME wiretap channel using M-PSK modulation and OSTBC. A phase-shifting precoder was designed to align the transmit signals at the receiver side so that the receiver can perform a noncoherent detection for the transmit signals. At the same time, the information rate of the eavesdropper can be reduced to zero by the proposed design. For the same channel model, another transmit antenna weights design method to deliberately randomize the eavesdropper's signal, but not that of the receiver, was proposed in [426]. For a MIMOME wiretap channel without the eavesdropper's CSI, Belfiore *et al.* [427] established a lattice wiretap code design criterion that minimizes the probability of correctly decoding at the eavesdropper's side. Nguyen *et al.* [428] analyzed the performance of a MISOME wiretap channel with perfect CSIT of the receiver and SCSIT of the eavesdropper. The SER of confidential information was derived and the corresponding secure diversity was obtained. A brief comparison of the above studies is given in Table 35.

V. OTHER SYSTEMS EMPLOYING MIMO TECHNOLOGY

In this section, we give an overview of transmission design with finite input signals for other systems employing MIMO technology. We consider the following three typical scenarios: 1) MIMO cognitive radio systems; 2) green MIMO communications; and 3) MIMO relay systems.

Table 35 Space-Time Techniques With Finite Input Signals for MIMO Wiretap Channel

Paper	Model	Main contribution
Fakoorian <i>et al.</i> [422]	Two transmit antennas and receive antennas Multiple-antenna eavesdropper Perfect CSI of the receiver, no CSI of the eavesdropper	Designs a secure STBC with low uncoded BER of the receiver and high uncoded BER of the eavesdropper
Wen <i>et al.</i> [423]	Same as [422]	Proposes a cross-layer scheme to further ensure secure communication
Allen <i>et al.</i> [424]	Two/four transmit antennas, one receive antenna one-antenna eavesdropper No CSI of the receiver and eavesdropper	Designs a secure STBC providing full-diversity of the receiver and zero diversity of the eavesdropper
Perreau [425]	MISOSE No CSI of the receiver and eavesdropper	Designs the transmit antenna weights based on STBC to prevent the eavesdropper from being able to decode the signal
Li <i>et al.</i> [59]	MISOME No CSI of the receiver and eavesdropper	Designs a secure communication scheme by using M-PSK modulation and OSTBC
Li <i>et al.</i> [426]	MISOME No CSI of the receiver and eavesdropper	Proposes a transmit antenna weight design method to deliberately randomize the eavesdropper's signal
Belfiore <i>et al.</i> [427]	MIMOME Perfect CSI of the receiver, no CSI of the eavesdropper	Establishes a lattice wiretap code design criterion minimizing the eavesdropper's correct-decoding probability
Nguyen <i>et al.</i> [428]	MISOME Perfect CSIT of the receiver, SCSIT of the eavesdropper	Analyzes the SER of confidential information

A. MIMO Cognitive Radio Systems

1) *Mutual Information Design*: Zeng *et al.* [61] investigated the spectrum sharing problem in MIMO cognitive radio systems with finite alphabet inputs. With the global PCSIT, a linear precoder was designed to maximize the spectral efficiency between the secondary-user transmitter and the secondary-user receiver with control of the interference power to primary-user receivers. This design was extended to secure communication of MIMO cognitive radio systems in [416]. However, these algorithms only apply for equiprobable finite alphabet inputs. By exploiting the I-MMSE relationship in [19], Zhu *et al.* [429] proposed a gradient-based iterative algorithm to maximize secondary users' spectral efficiency for arbitrary inputs.

2) *MSE Design*: Seo *et al.* [430] investigated transceiver design to minimize the MSE of the primary user subject to the secondary user's interference power constraint. For MIMO cognitive networks, Gong *et al.* [431] jointly designed the precoder and decoder to minimize the MSE of the secondary network subject to both transmit and interference power constraints. Transceiver designs for ad hoc MIMO cognitive radio networks based on total MSE minimization were proposed in [63] and [432]. For full-duplex MIMO cognitive radio networks, Cirik *et al.* [433] minimized both the total MSE and the maximum per secondary users' MSE.

3) *Diversity Design*: For cognitive radio networks, by considering multiple cognitive radios as a virtual antenna

array, Zhang *et al.* [62] designed space-time codes and space-frequency codes for cooperative spectrum sensing over flat-fading and frequency-selective fading channels, respectively. Liang *et al.* [434] designed a flexible STBC scheme for cooperative spectrum sensing by dynamically selecting users to form the clusters. A robust STBC scheme based on dynamical clustering was proposed in [435]. Differential STBCs for cooperative spectrum sensing without CSI knowledge were proposed in [436] and [437]. Kim *et al.* [438] analyzed the outage probability and diversity order of various spectrum sensing methods for cognitive radio networks. Letaief *et al.* [439] designed a space-time-frequency block code to exploit spatial diversity for cognitive relay networks with cooperative spectrum sharing. A distributed space-time-frequency block code was proposed to reduce detection complexity [440].

B. Green MIMO Communications

1) *Mutual Information Design*: Gregori *et al.* [64] studied a point-to-point MIMO system with finite alphabet inputs where the transmitter is equipped with energy harvesters. A total of N channels are used and noncausal knowledge of the channel state and harvested energy are available at the transmitter. By obtaining the optimal left singular matrix and selecting the right singular matrix to be the identity matrix, Gregori *et al.* [64] proposed an optimal offline power allocation solution to maximize the spectral efficiency among N channel users. This design was extended to the SCSIT case in [441], where an iter-

active algorithm to find the entire optimal precoder was proposed. Further exploitation of partial instantaneous CSI along with SCSIT was studied in [442]. A resource allocation algorithm for the downlink transmission of multiuser MIMO systems with finite alphabet inputs and an energy harvest process at the transmitter was proposed in [443]. With SCSIT and statistical energy arrival knowledge, the precoders, MCR selection, subchannel allocation, and energy consumption are jointly optimized by decomposing the original problem into an equivalent three-layer optimization problem and solving each layer separately.

2) *Diversity Design*: Tang et al. [444] proposed an STBC for geographically separated base stations that improves the energy efficiency at remote locations. Zhu et al. [445] designed a low-energy-consumption turbo-based STBC. A cooperative balanced STBC was proposed for dual-hop AF wireless sensor networks to reduce energy consumption [446]. Tomio et al. [447] compared the energy efficiency of a transmit beamforming scheme and a transmit antenna selection scheme and found that the transmit antenna selection scheme is more efficient for most transmit distances, since only a single radio-frequency chain is used at the transmitter. Novel noncoherent energy-efficient collaborative Alamouti codes have been constructed by uniquely factorizing a pair of energy-efficient cross QAM constellations and carefully designing an energy scale [65], [448].

C. MIMO Relay Systems

1) *Mutual Information Design*: Zeng et al. [66] investigated linear precoder design for dual-hop amplify-and-forward (AF) MIMO relay systems with finite alphabet inputs. Exploiting the optimal precoder structure similarly to the point-to-point MIMO case, they proposed a two-step algorithm to maximize the spectral efficiency. The precoder obtained in [66] also achieves good BER performance. Since the publication of [66], transmission design for spectral efficiency maximization under finite alphabet constraints has been investigated for various MIMO relay systems, including two-hop nonregenerative three-node MIMO relay systems [449], nonorthogonal AF half-duplex single relay systems [450], MIMO two-way relay systems [451], and MIMO DF relay secure systems [417].

2) *MSE Design*: Linear transceiver designs via MMSE criterion for MIMO AF relay systems have been proposed in [68] and [452]–[458]. In addition, nonlinear THP/DFE transceiver structures based on optimization of the MSE function have been designed for MIMO AF relay systems in [459]–[463]. The average error probability of AF relay networks employing MMSE-based precoding schemes was derived in [464]. MMSE-based transceiver designs for nonregenerative MIMO relay networks have been presented in [465]–[469]. MMSE transceiver designs have been further investigated in multiuser MIMO relay networks [470]–[474]. Two MMSE transceiver designs for filter-and-

forward MIMO relay systems and single-carrier frequency-domain equalization-based MIMO relay systems have been presented in [475] and [476], respectively.

3) *Diversity Design*: For AF relay systems with multiple antennas and finite alphabet inputs, various transmission schemes have been presented to optimize the diversity gain of the system [67], [477]–[480]. The diversity order and array gains for AF relay MIMO systems employing OSTBCs have been derived analytically [481], [482]. For DF relay systems with multiple antennas and finite alphabet inputs, various transmission schemes have been proposed based on minimizing the error probability of the system [478], [483], [484]. The error performance of DF relay MIMO systems has been analyzed [485], [486]. Further transmission schemes to increase the reliability of MIMO relay networks with finite alphabet inputs have been designed and analyzed [487]–[494]. Diversity designs for noncoherent MIMO relay networks have been investigated [495]–[497].

4) *Adaptive Transmission*: Abualhaol et al. [69] proposed an adaptive transmission scheme for MIMO-OFDM systems with a cooperative DF relay. The bit and power are adaptively allocated among different subchannels to maximize the receive signal power under the constraint that each subchannel is assigned either to the direct link or to the relay link. Munoz et al. [498] proposed an adaptive transmission scheme for MIMO-OFDMA systems with multiple relay nodes. Based on channel conditions, the transmission mode adaptively switches between joint relay and antenna selection and space-frequency block coding to minimize the sum BER of the direct link and the relay link subject to a total power constraint.

D. Others

The achievable information rate for an inter-symbol interference fading MIMO channel with finite input signals was studied in [499] and [500]. An optimal precoding technique for a coded MIMO inter-symbol interference MAC with joint linear MMSE detection and forward-error-correction codes was proposed in [501]. The geometric mean method, which obtains an equal SINR for all transmit streams, has been applied to single-user and multiuser MIMO systems [502]–[505]. There have been some precoder designs for multiuser MIMO systems based on the receiver side's SINR [506]–[513]. There have also been some transmission designs for the MIMO X-Channel based on diversity performance [514]–[519]. Diversity designs for cooperative systems, ad hoc networks, and distributed antenna systems have been presented in [520], [521], and [522], respectively.

VI. TECHNICAL CHALLENGES AND FUTURE TRENDS

Transmission design with finite input signals for the forthcoming massive MIMO systems in 5G wireless networks will become a more challenging problem. This paper has

presented a unified design framework with maximization of mutual information for point-to-point massive MIMO systems with finite input signals. However, a variety of problems remain open. A tractable mutual-information-maximization design for multiuser massive MIMO systems with finite input signals is still unknown. Robust MSE design for large-scale antenna arrays with channel uncertainty due to pilot contamination and other channel variation effects is still not well studied. High-rate low-detection-complexity STBCs for large MIMO systems need to be designed. Most importantly, an efficient adaptive transmission scheme with high integrity for practical massive MIMO systems has still to be found. This problem is extremely challenging, since it requires determination of optimal resource allocation (time/space/frequency), user scheduling, transmission mode selection, etc., for a significantly high dimension. The characteristics of massive MIMO channels need to be exploited, and heuristic methods from other research areas such as machine learning may be helpful in this context.

Another important future research trend concerns transmission design for massive-connectivity MIMO systems with finite input signals. The most challenging scenario for 5G wireless networks is the transmission of a large number of users in a limited spatial region. Driven by the emerging applications of the Internet of Things and machine-type communications, the support

of massive connectivity has become a key requirement for future wireless cellular networks. There have now been some studies of massive connectivity systems under ideal Gaussian input [523]–[525], but much work remains to be done on massive-connectivity MIMO systems with finite input signals. The potential challenges include a fundamental analysis for random access users with finite input signals, the prohibitive computational complexity involved in evaluating system performance, security concerns for massive accessible users, among other issues.

VII. CONCLUSION

This paper has provided the first comprehensive summary of research on MIMO transmission design with practical input signals. We have presented the current understanding of the situation for finite input signals. Then, for practical MIMO transmission techniques, we have discussed in detail transmission designs based on the criteria of mutual information, MSE, and diversity, and adaptive transmission switching among these criteria. These transmission schemes have been designed for point-to-point MIMO systems, multiuser MIMO systems, MIMO cognitive radio systems, green MIMO communication systems, MIMO relay systems, etc. Valuable insights and lessons have been extracted from these existing designs. ■

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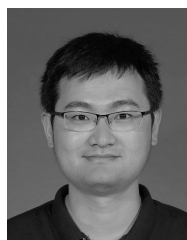
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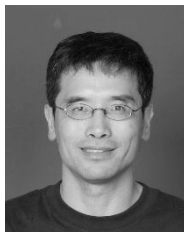
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