Spectrum Measurement Markets for Tiered Spectrum Access

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***Abstract*—The recent framework for tiered spectrum sharing in the 3.5 GHz band establishes rules in which multiple firms called Environment Sensing Capability operators (ESCs) may measure spectrum occupancy and sell these measurements to other firms to help facilitate spectrum access. Motived by this we consider a scenario in which two spectrum access firms (SAs) seeks to access a shared band of spectrum and must in turn purchase spectrum measurements from one of two ESCs. Given the measurements they purchase, the SA firms then compete on price to serve customers in a shared band of spectrum. We study how differences in the quality and price of the spectrum measurements impact the resulting market equilibrium between the SAs and find that having different qualities of measurements available to different SAs can lead to better economic welfare.**

I. Introduction

Recently, the FCC in the U.S. has finalized plans for the new Citizens Broadband Radio Service (CBRS) which will enable commercial users to share the 3.5GHz band with federal incumbent users [1], which include naval radar and satellite services. Sharing of this band in a given location is to be controlled by one or more Spectrum Access Systems (SASs), which are geographical databases that coordinate usage of the band. It is envisioned that in many areas multiple companies will operate approved SASs. Companies wishing to offer service in that band must then register with one SAS. Additionally, each SAS can contract with an environmental sensing capability (ESC) operator. An ESC will consist of a network of sensors used to detect the presence (or absence) of federal incumbent users. An ESC that can deliver high quality measurements will enable a SASs to allow its customers to access the spectrum band more frequently. For example, without any such sensing, an SAS may be forced to adopt overly conservative exclusions zones to prevent interference with incumbents.

An interesting feature of the CBRS ecosystem is that there are multiple levels of competition that may emerge. Multiple ESCs may compete to sell their spectrum measurements to different SAS's, who in turn may compete for registering different users in a given area (and these users in turn may be competing for offering wireless services to end users). Furthermore, different ESCs may offer different qualities of sensing, in which case the choice of ESC will in turn impact the quality of service offered by the downstream firms. There are many questions that arise in such a setting. In general, it is not clear if multiple such ESCs would be able to co-exist in the market, and if so, what is the impact of the quality of their information on their market share? Likewise, would multiple SASs or wireless service providers exist in the market? Does encouraging such competition improve economic welfare?

In this paper, we consider a stylized model motivated by the CBRS ecosystem to gain insights into the above questions. We consider a model with two tiers and two firms at each tier. At one tier are two ESCs, who offer spectrum measurement data to Spectrum Access firms (SAs).[1] Given this information, two SAs in turn compete to serve end-users in a given area whenever the ESC data tells them the spectrum is available. We focus on a single geographic area and assume both SAs only use a single shared band of spectrum[2] We focus on the case where the ESCs have different information regarding the presence of the incumbent because of different sensing capabilities. The SAs can obtain information from at most one of the ESCs. If a SA does not obtain information from any of the ESCs, it can not offer service to the customers. We then analyze a multi-stage game in which the SAs first decide on contracting with a ESC. Given these decisions, the then SAs compete for users.

To model the competition among the SAs we adopt a similar framework as that used in [2], [3], [4], [5] to study competition among wireless service providers using shared spectrum, which in turn is based on models used in the economics and operations literature to study competition with congestible resources (e.g. [6]). As in these models, we assume that when a SA learns from a ESC that the spectrum is available, it competes by offering a price for its service. There is a continuum of non-atomic users who in turn select a SA based on the *delivered price* given by the sum of the announced service price and a congestion cost which increases in the number of users using the shared spectrum.

Our analysis shows that interestingly the two SAs never chose to obtain information from the same ESC. If both ESC's offer the same quality only one SA will purchase this information and the other will stay out of the market. However, the SAs may obtain information from different ESCs if they

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[1]We can view an SA as either a SAS provider or a wireless service provider in a market in which the ESC and SAS provider are a single firm.

[2]As the spectrum band is shared, this would correspond to the Generalized Authorized Access (GAA) tier in the CBRS system. CBRS also defined a Priority Access (PA) tier, which we leave to future work.

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offer different qualities of information. Moreover, in the later case, more customers will be served.

This paper adds to a growing literature that studies the role of information acquisition on competition. For example, other work on this topic includes [7], which considers acquiring information about competitor's supply, and [8], which studies firms that can acquire information about customer demand from the third party.

II. System Model

We consider a model in which there are two ESCs (denoted by ESC A and ESC B) and two SAs denoted by SA 1 and SA 2. Each SA seeks to serve users in a given band of spectrum at a given location. To do this, the SAs must acquire spectrum measurements from one of the ESCs and can only use the spectrum when the ESC indicates that it is available (i.e., not being used by a federal incumbent). If a SA does not acquire information from either ESC, we assume it can not serve any users. If both SAs receive information that the spectrum is available, then they both can utilize it as shared spectrum. We discuss the various participants in this market in more detail.

A. Information Selling from the ESC

Each ESC provides a binary indication of whether the spectrum is available for use over time based on their own sensing capabilities. We assume that each ESC must be certified to have a negligible probability of missed detection of the incumbent, i.e. if the incumbent is present, the ESC will never announce that the spectrum is available. However, we do allow the SPs to incur false alarms, i.e., if the incumbent is not present, an ESC may still announce that the spectrum is not available. ESC with better sensing capabilities will be less likely to make sure errors. We model this by identifying each ESC A and B with a probability q_A and q_B, respectively, that gives the probability that the ESC indicates that the spectrum is available (which in turn depends on the incumbent's usage patterns and the ESC's sensing capability). Without loss of generality, we assume that $q_A \geq q_B$ so that ESC A has the higher quality of information (unless they are identical). Further to simplify our exposition, we assume that ESC B's announcement is a degraded version of ESC A, so that whenever the ESC B indicates the channel is available, ESC A also does the same. However, when ESC A estimates the channel is available, ESC B may not estimate the same.[3]

Each ESC will incur a cost providing its service due for example to the cost of building and operating its sensors and communicating information to a SA. We denote the cost incurred by ESC A and B by c_A and c_B, respectively. Since ESC A provides information with a higher accuracy, we assume that $c_A \geq c_B$ with equality only if $q_A = q_B$.

We assume that the ESC A (B, resp.) sells its prediction to any of the SA at the price $\tilde{p}_A$ ($\tilde{p}_B$ resp.). Here, $q_A, q_B, \tilde{p}_A$, and $\tilde{p}_B$ are of common knowledge to both the ESCs and to the SAs. Throughout this paper, we also consider that $\tilde{p}_A$ and $\tilde{p}_B$ are exogenous parameters and focus on the strategic decisions of the SAs given these prices.

[3]Our analysis can easily be extended to the case where instead ESCs A and B make independent errors.

B. SAs Decisions

Each SA must make two decisions. First, it must decide whether to acquire information from ESC A, ESC B, or to not acquire any information at all. Second, if SA i acquires information, then it must decide on a price p_i that it will charge users for its service. We assume that these decisions are made in stages. In the first stage, both the SAs simultaneously make their decision about acquiring information. In the second stage, given both SAs first stage decisions, the SAs then simultaneously choose prices to compete for users to serve. Each SA i, seeks to maximize its profit given by

$$\pi_i = p_i \lambda_i - \tilde{p}_k \tag{1}$$

where λ_i indicates the number of users the SA serves and $\tilde{p}_k$ is the price it pays to acquire information from ESC k. If a SA decides not to acquire any information in the first stage, then we view $\tilde{p}_k$ and λ_i as both being equal to zero so that the overall profit is also zero, i.e., this models a case where a SA decides not to enter the market. This may occur when the revenue that a SA would generate is not sufficient to recover the cost of acquiring information from one of the ESCs.

C. User's Subscription Model

We consider a mass Λ of non-atomic users, so that we have $\lambda_1 + \lambda_2 \leq \Lambda$. Each user obtains a value v for getting service from either SA. However, it also incurs the cost of service as well as a congestion cost. This congestion cost models the degradation in the quality of service due to congestion of network resources. As in [3], [4], [5], we assume that the average congestion cost during a time when each SA i has λ_i users that are able to use the spectrum is given by $g(\lambda_1 + \lambda_2)$ for some increasing, convex function $g(\cdot)$. Note that the congestion cost depends on the users served by both SA's, which models that they are both using the same band of shared spectrum. Further, as in [3], [4], [5], we assume that both the congestion cost and the service price are additive so that the pay-off seen by a user using SA i when each SA i has λ_i users able to use the spectrum is given by

$$v - p_i - g(\lambda_1 + \lambda_2). \tag{2}$$

The number of users of a given SA that are able to use the spectrum at any time in turn depends on the information obtained from the ESC selling information to that SA. In particular, if SA i obtains information from ESC k and has λ_i users, these users are only able to use the spectrum when the ESC k reports the spectrum is available (which occurs with probability p_k). When users can not use the spectrum, we assume their pay-off is zero. When users can use the spectrum, they receive a pay-off as in (2), where the traffic of the other SA will in turn depend on the information that SA receives from its ESC. Hence, the pay-off obtained will be a random variable. We assume that users seek to maximize the expected value of this quantity.[4] Further users can choose not to purchase service from either SA, in which case their pay-off is zero.

[4]For example, this is reasonable when users are purchasing service contracts with a long enough duration so that they see many realizations of the ESC reports.

The specific form of the average congestion will depend on the decisions the SAs make regarding which ESC to contract with. If both SAs obtain information ESC A, then both of their customers will be able to use the spectrum during the times ESC A specifies the spectrum is available (which occurs with probability q_A). In this case, the expected pay-off of any subscriber of SA i, $i = 1, 2$ is

$$q_A v - q_A g(\lambda_1 + \lambda_2) - p_i. \tag{3}$$

Similarly, if both the SAs obtain information from the ESC B, the expected pay-off of any subscriber of SA i, $i = 1, 2$ is $q_B v - q_B g(\lambda_1 + \lambda_2) - p_i$.

Now, we consider the payoff of the users when the SAs obtain information from different ESCs. Without loss of generality, assume that the SA 1 obtains information from ESC A and SA 2 obtains information from ESC B. Recall that when ESC B estimates that the channel is available (i.e., there is no incumbent), then ESC A also estimates that the channel is available. Thus, the subscribers of SA 2 always face congestion from the users SA 1. However, the subscribers of SA 1 do not always face congestion from SA 2's users. Namely, SA 1's subscribers only face congestion from SA 2's customers when ESC B also indicates that the incumbent is not present (which occurs with probability q_B). Thus, the subscribers of SA 1 enjoys an exclusive access to the spectrum with probability $q_A - q_B$. This results in the users of SA 1 obtaining an expected pay-off of

$$q_A v - (q_A - q_B) g(\lambda_1) - q_B g(\lambda_1 + \lambda_2) - p_1. \tag{4}$$

On the other hand, a user of SA 2 will obtain an expected pay-off of

$$q_B v - q_B g(\lambda_1 + \lambda_2) - p_2. \tag{5}$$

Finally, consider the case when one SA i $i \in \{1, 2\}$ has obtained information from an ESC k, while the other SA chooses not to acquire information from either ESC (and so does not serve any customers), In this case, the users of SA i obtain an expected pay-off of

$$q_k v - q_k g(\lambda_i) - p_i. \tag{6}$$

D. Market Equilibrium

We view the preceding model as a multi-stage game. In the first stage, each SA selects one of the ESCs by paying the fee $\tilde{p}_i$ $i = A, B$ or selects to stay out of the market. Knowing the choices of the first stage, in the second stage each SA i will select a price p_i to attract subscribers. If a SA does not obtain information from the ESCs, then, it can not offer any service to the customers. In the last stage, given the decisions in the first two stages, the subscribers will choose one of the SAs depending on the prices and the quality of service offered by the SAs or choose not to receive service. We study the sub-game perfect Nash equilibrium of this game.

III. Equilibrium Analysis

We now analyze the market equilibrium specified in the previous section via backward induction. We begin next with studying the final stage of the game, which we refer to as a user equilibrium.

A. User Equilibrium

In the final stage of the game, the user equilibrium will specify the mass of subscribers λ_i of each SA i for the given prices selected in the second stage and the ESC choices made in the first stage.

Recall, that each user is seeking to maximize its expected pay-off. Given our assumption of identical non-atomic users, the user equilibrium can be characterized similar to a *Wardrop equilibrium* [9]. More precisely, if in equilibrium both SAs are serving customers, then it must be that the expected pay-offs for both SA's are the same (since, otherwise some customers would switch to the other SA). If one SA is not serving any customers, then its expected pay-off must be larger than that of the other SA. Additionally, this expected pay-off must be non-negative as otherwise some customers would be better off not purchasing service. Finally, if fewer than Λ customers are receiving service, then it must be that the expected pay-off is equal to zero as otherwise, some customers not receiving service would choose to receive service. We will refer to these properties as the *Wardop equilibrium conditions*. It can be shown that these are necessary and sufficient for λ_1 and λ_2 to be a user equilibrium.

Suppose that both the SAs obtain information from the same ESC. In this case, customers of both SAs experience the same expected congestion cost, and so the expected pay-offs of the two SAs only differ in the announced prices. Using this and the Wardrop equilibrium condition we have the following characterization of the user equilibrium.

Theorem 1. *Assume that both SAs obtain information from ESC j ($j \in \{1, 2\}$).*

1) *If $p_1 = p_2$ and $q_j v - q_j g(\Lambda) - p_1 \geq 0$, then any choice of λ_1 and λ_2 such that $\lambda_1 + \lambda_2 = \Lambda$ is a user equilibrium;*
2) *If $p_1 = p_2$ and $q_j v - q_j g(2\alpha) - p_1 = 0$, then any choice of λ_1 and λ_2 such that $\lambda_1 + \lambda_2 = 2\alpha$ is a user equilibrium;*
3) *If $p_i > p_k$ (for $i \neq k$) and $q_j v - q_j g(\Lambda) - p_k \geq 0$, then the unique user equilibrium is $\lambda_k = \Lambda$ and $\lambda_i = 0$.*
4) *If $p_i > p_k$ (for $i \neq k$) and $q_j v - q_j g(\alpha) - p_k = 0$ for some $\alpha < \Lambda$, then the unique user equilibrium is $\lambda_k = \alpha$ and $\lambda_i = 0$.*

Remark 1. *If $p_1 = p_2$, then as noted in this theorem the user equilibrium is not unique. However, the total number of subscribers in an equilibrium is unique.*

If one of the SAs sets a higher price i.e., $p_i > p_j$, then that SA will not receive any customers. Hence, the SA that selects a higher price will not have any revenue and so have a negative profit due to the payment it makes to the ESC.

If the total number of customers served is less than Λ, then fixing the prices, the market coverage (given by the parameter α) will be higher if the SAs obtain information from ESC A rather from ESC B. This is intuitive as ESC A provides information of superior quality.

Later, we will show that in an equilibrium path, both the SAs obtaining information from the same ESC is not sustainable.

Next, consider that SA 1 and 2 obtain information from different ESCs. Without loss of generality, we assume that SA 1 obtains information from ESC A and SA 2 obtains information from ESC B in the first stage. We can again obtain

λ_1 and λ_2 from the Wardrop equilibrium conditions. This is summarized in the following.

Theorem 2. *Assume SA* 1 *obtains information from ESC* A *and SA* 2 *obtains information from ESC* B*, the* unique *user equilibrium* (λ_1, λ_2) *satisfies:*

1) $\lambda_1 = \lambda_2 = 0$, *if* $q_A v - (q_A - q_B)g(0) - q_B g(0) - p_1 < 0$, *and* $q_B v - q_B g(0) - p_2 < 0$*;*
2) $\lambda_1 = \Lambda$, *if* $q_A v - (q_A - q_B)g(\Lambda) - q_B g(\Lambda) - p_1 \geq 0$, *and* $q_A v - (q_A - q_B)g(\Lambda) - q_B g(\Lambda) - p_1 \geq q_B v - q_B g(\Lambda) - p_2$.
3) $\lambda_1 = \alpha$, *if* $q_A v - (q_A - q_B)g(\alpha) - q_B g(\alpha) - p_1 = 0$ *and* $q_A v - (q_A - q_B)g(\alpha) - q_B g(\alpha) - p_1 > q_B v - q_B g(\alpha) - p_2$.
4) $\lambda_2 = \Lambda$, *if* $q_B v - q_B g(\Lambda) - p_2 \geq 0$ *and* $q_A v - (q_A - q_B)g(0) - q_B g(\Lambda) - p_1 < q_B v - q_B g(\Lambda) - p_2$.
5) $\lambda_2 = \alpha$, *if* $q_B v - q_B g(\alpha) - p_1 = 0$ *and* $q_A v - (q_A - q_B)g(0) - q_B g(\alpha) - p_1 < q_B v - q_B g(\alpha) - p_2$.
6) $\lambda_1 > 0, \lambda_2 > 0$ *such that* $\lambda_1 + \lambda_2 = \Lambda$, *if* $q_A v - (q_A - q_B)g(\lambda_1) - q_B g(\Lambda) - p_1 = q_B v - q_B g(\Lambda) - p_2$*; and* $q_A v - (q_A - q_B)g(\lambda_1) - q_B g(\Lambda) - p_1 \geq 0$.
7) $\lambda_1 > 0, \lambda_2 > 0$*, such that* $\lambda_1 + \lambda_2 = \alpha < \Lambda$ *where* $q_A v - (q_A - q_B)g(\lambda_1) - q_B g(\alpha) - p_1 = q_B v - q_B g(\alpha) - p_2$ *and* $q_B v - q_B g(\alpha) - p_2 = 0$.

In the first case in Theorem 2, users do not subscribe to any of the SAs . This is because even 0 mass gives a negative expected payoff to the users for both the SAs.

Note that if $q_A v - (q_A - q_B)g(0) - q_B g(\lambda_2) - p_1 < 0$, then $\lambda_1 = 0$. Similarly, if $q_B v - q_B g(\lambda_2) - p_2 < 0$, then, $\lambda_2 = 0$. If $q_A v - (q_A - q_B)g(\Lambda) - q_B g(\Lambda) - p_1 > q_B v - p_2$, then $\lambda_1 = \Lambda$, and $\lambda_2 = 0$. Thus, in the cases 2 and 3 in Theorem 2, SA 1 has the market power as the subscribers only subscribe to the SA 1 as the expected payoff attained by the users is positive for SA 1, but it is negative for SA 2 even when $\lambda_2 = 0$. On the other hand, in cases 4 and 5 in Theorem 2. 4), SA 2 has the market power as the subscribers only subscribe to the SA 2 as the expected payoff attained by the users is positive for SA 2, but it is negative for SA 1 even when $\lambda_1 = 0$.

The number of subscribers is split between the two SAs when the expected payoff is the same in the Wardrop equilibrium. However, the market may or may not cover all the subscribers. It will depend on the prices p_1, p_2 and the probabilities q_A, q_B and the valuation v. Also in this case, the split of the market between the SAs is not arbitrary as in Theorem 1 - for a given set of prices there will now be a unique split that satisfies the Wardrop equilibrium conditions. In this unique split, as the quality of information from ESC A increases (i.e., q_A increases), the market share of the SA 2 will decrease. On the other hand, if p_i increases, the market share of SA i $i = 1, 2$ will decrease.

B. Price Equilibrium

Next we turn to the second stage in which given the ESC choices, each SA i decides on its service price p_i with the goal of maximizing their profit given as in (1). Note that in this stage any cost paid to an ESC in stage 1 is sunk and so equivalently in this stage, the SAs seek to maximize their revenue given by $p_i \lambda_i$. When doing this, λ_i will be specified by the corresponding user equilibrium determined in the previous section, which in turn depends on if the SAs obtain information from the same ESC or a different ESC, or if one SA does not obtain information. We treat each of these cases separately.

1) Both SAs obtain information from the same ESC: First, we describe the NE pricing strategy when both the SAs obtain information from the same ESCs.

Theorem 3. *If both the SAs obtain information from the same ESC, then in equilibrium* $p_1 = p_2 = 0$*.*

Essentially, in this case, both of the SAs are offering identical service using the same spectrum, which results in a "price war" leading the SAs to each try to undercut the other. At the resulting equilibrium, both the SAs set the price at 0. Note that in stage 1, both SAs will have incurred a cost of $\tilde{p}_j > 0$ to acquire information from the same ESC j, hence they will both have negative profits in such an equilibrium.[5]

Later, we show that the above equilibrium can not be sustained in an equilibrium path.

2) Monopoly scenario: Next, we describe the scenario where only one of the SAs obtains information from one of the ESCs. Thus, the SA will essentially be a monopolist when making its pricing decision. The monopolistic price and profit is given in the following

Theorem 4. *If SA* i *obtains information from the ESC* j*,* $j \in \{A, B\}$*, while SA* $k \neq i$ *does not obtain information from either ESC, then the unique equilibrium price for SA* i *is*

$$p_i^* = \max\{q_j v - q_j \Lambda, \frac{q_j v}{2}\} \tag{7}$$

The third-stage Wardrop equilibrium *is*

$$\lambda_i = \min\{v - \frac{p_i^*}{q_j}, \Lambda\} \tag{8}$$

The monopolistic profit of the SAS i *is*

$$\pi_i = \max\{\frac{q_j v}{2} \min\{v/2, \Lambda\}, q_j (v - \Lambda)\Lambda\} - \tilde{p}_j \tag{9}$$

Note that though the first term in the expression of π_i is higher for $j = A$ as $q_A > q_B$, this does not necessarily mean that SA i will get a higher profit if it attains information from ESC A. This is because the price paid by the SA to obtain information from ESC A may be higher, i.e., $\tilde{p}_A > \tilde{p}_B$. Clearly, if $\tilde{p}_A \leq \tilde{p}_B$, the profit attained by the SA will be higher if it selects ESC A. The price selected by the SA i will be higher if it obtains information from the ESC A. Also note that the market share (λ_i) is independent of the ESC selected by the SA. Thus, *surprisingly*, in a monopolistic scenario the number of users which will receive the wireless service is independent of the choice of the ESC made by the SA.

3) The SAs obtain information from different ESCs: Earlier, we showed that the equilibrium where both the SAs obtain information from the same ESC renders a negative profit to each of the SAs. Now, we show that under some conditions there exists a price equilibrium where both the SAs can get positive profits if they obtain information from different ESCs. Later, we will show that such a price equilibrium is sustainable along an equilibrium path.

[5] If instead we assumed that each SA had to pay the ESC a marginal price for each customer (instead of a single flat price), then in equilibrium the prices must be set at the marginal price.

To facilitate our analysis in this section, we make the following common assumption regarding the congestion cost:

Assumption 1. *Assume that $g(\cdot)$ is linear.*

Without loss of generality, we again assume that SA 1 decides to obtain information from ESC A in the first stage and that SA 2 obtains information from ESC B.

Theorem 5. *Under Assumption 1, assume that SA 1 obtains information from ESC A and SA 2 obtains information from ESA B. If we have that*

$$v \geq \frac{2q_A + q_B}{q_A + 2q_B}\Lambda \tag{10}$$

then in the unique price equilibrium (p_1^*, p_2^*) is given by

$$p_1^* = (q_A - q_B)\frac{(v+\Lambda)}{3}$$
$$p_2^* = (q_A - q_B)\frac{(2\Lambda - v)}{3}. \tag{11}$$

The user equilibrium (third-stage) is then given by

$$\lambda_1 = \frac{v}{3} + \frac{\Lambda}{3}, \quad \lambda_2 = \frac{2\Lambda - v}{3}. \tag{12}$$

The profits of the SAs' are respectively

$$\pi_1 = (q_A - q_B)\left(\frac{\Lambda + v}{3}\right)^2 - \tilde{p}_A.$$
$$\pi_2 = (q_A - q_B)\left(\frac{2\Lambda - v}{3}\right)^2 - \tilde{p}_B. \tag{13}$$

Note that from the condition in (10) that $v > \Lambda$. Hence, the market share of SA 1 is higher than SA 2. The first term in the profit of SA 1 is also strictly higher compared to SA 2. However, SA 1 may have to pay more as $\tilde{p}_A$ maybe higher than $\tilde{p}_B$. Thus, SA 1's profit may be lower compared to that of SA 2. Also note that as the difference between q_A and q_B decreases, the profits of the SAs decrease. Intuitively, as the difference in the quality of the ESCs' information decreases, the SAs become competitive. When the qualities are equal, this becomes the same as if both SAs acquire information from a single ESC, which leads to a negative profit, as we have already seen in Theorem 3.

The sum of λ_1 and λ_2 in the equilibrium is equal to the total number of subscribers. Thus, wireless service is provided to every user. Hence, when the SAs obtain information from different ESCs and the condition in (10) is satisfied, the SAs select prices such that it will cover the entire subscription base. The market share of the SA 1 is more than twice that of SA 2.

Also note that in contrast to the monopoly scenario, in this case the consumer surplus is positive.

Theorem 5 is valid if $q_B(v - \Lambda) \geq (q_A - q_B)(\frac{2\Lambda - v}{3})$. Next we characterize a price equilibrium when this condition is not satisfied.

Theorem 6. *Under Assumption 1, assume that SA 1 obtains information from ESC A and SA 2 obtains information from ESA B. If*

$$v < \frac{2q_A + q_B}{q_A + 2q_B}\Lambda$$

and $v > \frac{3q_A\Lambda}{4q_A - q_B}$, then in the unique price equilibrium (p_1^*, p_2^*) is given by

$$p_1^* = q_A v/2 - q_B\Lambda/2$$
$$p_2^* = q_B(v - \Lambda). \tag{14}$$

The third stage Wardrop equilibrium *is the following*

$$\lambda_1 = \frac{q_A v - q_B\Lambda}{2(q_A - q_B)}.$$
$$\lambda_2 = \frac{(2q_A - q_B)\Lambda - q_A v}{2(q_A - q_B)}. \tag{15}$$

The profits of the SA's are

$$\pi_1 = (q_A v/2 - q_B\Lambda/2)^2\frac{1}{q_A - q_B} - \tilde{p}_A.$$
$$\pi_2 = q_B(v - \Lambda)\frac{(2q_A - q_B)\Lambda - q_A v}{2(q_A - q_B)} - \tilde{p}_B. \tag{16}$$

Note that when $q_A = q_B$, this case never arises.

Similar to Theorem 5, the total market share of the SAs cover the whole subscription base Λ. The price set by the SA 1 is higher compared to SA 2. However, the consumer surplus is zero unlike in Theorem 5. The market share of SA 1 is higher compared to the SA 2, however in this case it is not double that of SA 2. The payoffs of the SAs are also lower compared to Theorem 5. This is because Theorem 6 is valid for a smaller range of v compared to Theorem 5.

Finally, we characterize the unique price equilibrium when $v \leq \frac{3q_A\Lambda}{4q_A - q_B}$.

Theorem 7. *Under Assumption 1, assume that SA 1 obtains information from ESC A and SA 2 obtains information from ESA B. If $v \leq \frac{3q_A\Lambda}{4q_A - q_B}$, then a price equilibrium (p_1^*, p_2^*) is given by*

$$p_1^* = \frac{(q_A - q_B)v2q_A}{4q_A - q_B}$$
$$p_2^* = \frac{(q_A - q_B)vq_B}{4q_A - q_B}. \tag{17}$$

The third stage Wardrop equilibrium *is given by*

$$\lambda_1 = \frac{v2q_A}{4q_A - q_B}, \lambda_2 = \frac{vq_A}{4q_A - q_B}. \tag{18}$$

The SAs' profits are

$$\pi_1 = (q_A - q_B)\left(\frac{v2q_A}{4q_A - q_B}\right)^2 - \tilde{p}_A,$$
$$\pi_2 = q_B q_A(q_A - q_B)\left(\frac{v}{4q_A - q_B}\right)^2 - \tilde{p}_B, \tag{19}$$

In this equilibrium, the subscribers are again split among the two SAs. However, in contrast to Theorems 5 and 6, in Theorem 7 the total market share of the SAs do not match the total number of users. Thus, there will be some users who will not subscribe to any of the SAs.

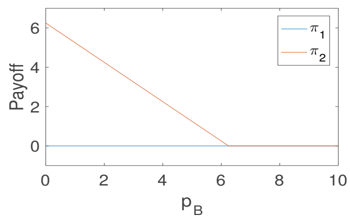

Fig. 1. The payoffs of the SAs with $v = 8, \Lambda = 5, q_A = 0.5, q_B = 0.25, \tilde{p}_A = 4$. The variations of the SAs with $\tilde{p}_B$. Note that only one SA

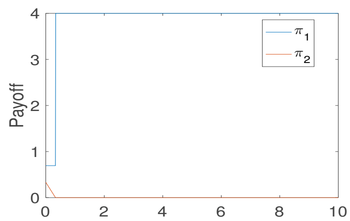

Fig. 2. The payoffs of the SAs with $v = 8, \Lambda = 5, q_A = 0.5, q_B = 0.25, \tilde{p}_A = 4$. The variations of the payoffs of the SAs with $\tilde{p}_B$. Note that both the SAs have positive payoff for smaller values of $\tilde{p}_B$. When $\tilde{p}_B$ is large, only SA1 has a positive payoff i.e. *monopoly power*.

Similar to Theorem 5, in Theorem 7 the price set by SA 1 is higher compared to SA 2 and SA 1's the market share is higher compared to SA 2. The profits of the SAs are lower compared to that obtained in Theorems 5 and 6 since the above result holds for smaller value of v.

Note that in Theorems 5 and 7 if the profit of one of the SA i is negative, then it will not be sustainable in the equilibrium path. Thus, we will observe the monopolistic scenario as in Section III-B2.

C. ESC Selection Equilibrium

Now, we discuss the first stage equilibrium. Specifically, we state an equilibrium strategy which prescribes which ESC should be chosen by each SA.

Theorem 8. *In the first stage only one of the following four equilibria are possible:*

1) *Only one of the SAs obtain information from the ESC A if* $(q_A - q_B)\left(\frac{2\Lambda - v}{3}\right)^2 < \tilde{p}_B$ *and* $\max\{\frac{q_A v}{2}\min\{v/2, \Lambda\}, q_A(v-\Lambda)\Lambda\} \geq \tilde{p}_A$.
2) *Only one of the SAs obtain information from the ESC B if* $\max\{\frac{q_B v}{2}\min\{v/2, \Lambda\}, q_B(v-\Lambda)\Lambda\} \geq \tilde{p}_B$ *and* $(q_A - q_B)\left(\frac{\Lambda + v}{3}\right)^2 < \tilde{p}_A$.
3) *Both the SAs obtain information from different ESCs if* $(q_A - q_B)\left(\frac{2\Lambda - v}{3}\right)^2 \geq \tilde{p}_B$ *and* $(q_A - q_B)\left(\frac{\Lambda + v}{3}\right)^2 \geq \tilde{p}_A$.
4) *Both the SAs choose not to obtain information from the ESCs if* $\max\{\frac{q_j v^2}{4}, q_j(v-\Lambda)\Lambda\} < \tilde{p}_j$ *for all* $j \in \{A, B\}$.

Hence, the scenario where the SAs will obtain information from the same ESC can not occur in an equilibrium path. *Thus, if the ESCs offer the same quality, then there will be no competition for the user market.* Fig. 1 depicts the monopoly situation when only one has the positive profit. The only scenario where both the SAs will be in the market is when they obtain information from different ESCs, which requires that both ESC offers different qualities of information. Fig. 2 shows a scenario where both the SAs have positive profits. Having such competition can provide positive consumer surplus. Note also that if two such ESC are operating in the market, improving the quality of the poorer ESC may in fact hurt the SA profits or lead to a monopolistic scenario, in which one SA stays out of the market. The monopoly profit is higher compared to the competitive one. However, if there is a monopoly, it never covers the entire subscription base in contrast to the competitive outcome. *From a regulatory point-of-view, these results suggest their may be a benefit in encouraging ESCs that offer different service qualities.*

IV. Conclusion

We have considered a simple model for competition in Shared spectrum motivated by the CBRS system to be deployed in the 3.5 GHz band. A key feature of our model is that firms offering wireless service must acquire information about spectrum availability from an ESC, where different ESCs may offer different qualities of information. Interestingly, we have shown that different information qualities are needed to sustain multiple firms in this market. There are many directions this work could be extended including considering more firms in the market, allowing for licensed shared access similar to the priority access tier in CBRS, and considering multiple bands of spectrum (e.g., in CBRS there are multiple 10 Mhz channels defined for spectrum access).

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