Contracts as Investment Barriers in Unlicensed Spectrum

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***Abstract*—By not requiring expensive licenses, unlicensed spectrum lowers the barriers for firms to offer wireless services. However, incumbent firms may still try to erect other entry barriers. For example, recent work has highlighted how customer contracts may be used as one such barrier by penalizing customers for switching to a new entrant. However, this work did not account for another potential benefit of unlicensed spectrum, having access to this open resource may incentivize entrants to invest in new and potentially better technology. This paper studies the interaction of contracts and the incentives of firms to invest in developing new technology. We use a game theoretic model to study this and characterize the effect of contracts on economic welfare. The role of subsidies or taxes by a social planner is also considered.**

I. Introduction

Recently, there has been much interest in expanding the amount of unlicensed spectrum available for wireless communications. For example the TV white spaces [2] and the Generalized Authorized Access (GAA) tier in the 3.5 GHz band [3] are both recent examples of policy in the U.S. that supports unlicensed usage. Unlicensed spectrum lowers the barriers faced by new firms seeking to enter the market for wireless services and to deploy new technologies. However, this spectrum also has a greater risk of becoming over congested and yielding lower profits to firms offering service in such a band [1], [8]. Though a license is not needed to offer service in such a band, there are other forms of entry barriers that firms using the band can employ. In this paper we explore one such example, the use of customer contracts, a common practice in the wireless industry.

The use of contracts as an entry barrier has been studied in the economic literature in work such as [4], which shows that an incumbent firm in a commodity market can effectively use a simple contract as an entry barrier, resulting in a loss of welfare. In recent work [5], we adopted the model from [4] and applied it to a market for service using unlicensed spectrum. A key difference between the spectrum market in [5] and the commodity market in [4] is that spectrum is a congestible resource, i.e., a user's value for service will degrade as more users share this resource. Due to this, in [5] it is shown that an incumbent firm can effectively use a contract as an entry barrier, but that doing this can increase both consumer welfare and overall social welfare. The reason for this is that by reducing entry, contracts also help to reduce congestion. The work in [5] focuses on the entry decision of a wireless service provider with a given technology. This does not account for another key feature of unlicensed spectrum, namely, that it can help incentivize SPs to invest in the development and deployment of new technology. In this paper we consider this dimension and seek to understand the impact of customer contracts on the incentives of a SP to invest in the development of new technology. Contracts in wireless markets have also been studied in other contexts such as [11], [13], [14].

As in [4], [5], we consider a simplified contract consisting of only two variables: the service price and the liquid damage, which is the price a customer pays for breaking the contract. We combine this with a model as in [1], [8] for competition among service providers (SPs) with unlicensed spectrum. As in [1], [8], we assume that customers choose a SP based on the *delivered price*, which is the sum of a service price and a congestion cost. Similar models are also studied in [6], [7], [9]. The congestion cost is increasing in the number of customers served in the band modeling the congestible nature of this resource. Different from [5], we consider a model in which given the customers under contract, an entrant firm must decide on how much to invest in the development of a new technology. We assume that the outcome of this development is uncertain and if the resulting technology is not "good enough," the entrant will fail to profit in the market. A contract can discourage this investment as it makes the threshold for the technology's performance higher as the entrant needs to be able to compensate users for breaking their contract and paying the liquid damages. Adding this investment consideration significantly complicates the model compared to [5] and leads to different types of conclusions.

We adopt a multi-stage Bayesian game to model this situation. We assume that there is one SP who is the first to offer service using a new unlicensed band. We refer to this first SP as the incumbent. This SP may offer a long-term contract to its customers as well as give them the opportunity to buy service without a contract. Subsequently, one new entrant SP has the opportunity to invest to develop technology for offering service in this band. If the entrant invests and offers service in the band, the customers under contract have the option of breaking the contract and buying service from the new entrant. At the time that customers are offered the contract, we assume that both the customers and the incumbent are

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uncertain about the future Quality of Service (QoS) that will be offered by an entrant. This QoS will in turn depend on the amount the incumbent SP invests as well as an additional random parameter, whose distribution is common knowledge. Note in this setting, at times the incumbent may profit more from the entrant entering the market and receiving the liquid damages from each customer who breaks their contract, and so in designing its contract it must weigh these potential gains against the gain it makes from discouraging investment and maintaining its status as a monopolist. Customers must also weigh the lower service price they obtain from a contract and the potential benefits they obtain by being able to more easily switch to a new entrant. Finally, from a social planner point-of -view, it is not clear if such contracts are beneficial as in [5] or if they will result in a loss of welfare.

Our findings in this work include the following.

- As in [5], the incumbent SP should always offer an *exclusive contract* meaning that the liquid damage fee is greater than the contract service price. We call the difference of these two the *strictness* of the contract. Our results show that the contract strictness is increasing with how much the new technology can improve the spectrum efficiency and the incumbent's uncertainty about the entrant's spectrum efficiency, and decreasing with the incumbent's spectrum utilization efficiency. The contract service price is more complicated, but it will decrease if the entrant's investment efficiency improves.
- The customers should sign the contract if they have a better expected payoff. Our results show that more customers will sign the contract as investment efficiency increases. This might be counter-intuitive. The reason is that the incumbent will make the contract more attractive to encourage more customers to sign.
- The entrant provider will invest as much as possible whenever he decides to enter the market. There exists a threshold for the entrant SP's spectrum efficiency above which the entrant SP will enter.
- Contracts can still improve the expected social welfare although it might lower the probability of improved technology being adopted depending on the parameters of the incumbent's technology. A social planner can either subsidize customers switching service or restrict the strictness of the contract to improve the overall social welfare.
- If the incumbent's technology has a constant investment efficiency, then improving the limit of the technology's impact always increases social welfare, though it may not benefit the incumbent's expected profit. However, when there exists a constant upper bound for the technology's impact, improving the investment efficiency is not always beneficial for the social welfare when contracts are allowed.

II. Competition Model

In this paper, we focus on the situation where there is one incumbent service provider, SP1, in the unlicensed market facing the possible future entrance of one new entrant service provider, SP2. We model this as a four-stage game among the SPs and the customers.

Stage 1: The incumbent offers a contract to every customer.
Stage 2: The customers decide whether to accept or decline the contract.
Stage 3: The entrant SP decides whether to enter the market and how much to invest in a new technology.
Stage 4: If the entrant enters the market, both the entrant and incumbent compete on price for customers.
We will discuss more details of these stages later.

A. Price competition Model

As in [1], [5], [8], we assume that all SPs in the market compete for a common pool of customers to maximize their revenue. The customers are modeled as non-atomic users with a total mass of 1. Each customer will choose a SP considering its *delivered price* given by the sum of its service price and a congestion cost, which characterizes the SP's QoS. We denote the congestion cost of SPi by a function of total number of customers in this band, $g_i(x_T)$, and assume that it is increasing in x_T, the total number of customers receiving service from any SP in the band. This means that the customers of all SPs will suffer congestion due to all other users in this band; this models the shared nature of the unlicensed band. For this paper, to simplify our analysis, we assume that the congestion costs are linear, i.e., $g_i(x_T) = k_i x_T$. Here, $k_i > 0$ determines the slope of SPi's congestion cost function and may vary across SPs to model differences in the technology they use and the amount of infrastructure they originally invest in. Note that a smaller slope indicates less congestion and thus a SP with a better service. Hence, having a smaller k_i enables a SP to better compete in the market. Each SP announces a price of p_i for their service and seeks to maximize their profit $p_i x_i$, where x_i is the mass of customers they serve. Each customer is identified as x, $x \in [0, 1]$, with a reservation price for the service denoted by an inverse demand function $v(x)$. We assume $v(x)$ is also linear and given by

$$v(x) = 1 - x.$$

A customer will only accept service if the delivered price is no greater than its reservation price and seeks to obtain service from the SP offering the lowest delivered price. Without a contract, it follows that the customers must be in a Wardrop equilibrium [10], which specifies that for an SP serving customers it must be that $g_i(x_T) + p_i = v(x_T)$. In the market model in [1], [8] (without contracts) a market equilibrium is a choice of price by each SP so that neither can unilaterally improve their profit, assuming for any choice of prices the customers are in a Wardrop equilibrium. In such an equilibrium, it can be easily shown that the SP with the larger value of k_i will make no profit.

B. Investment

Different from prior work, here we assume that the entrant firm is facing the option of investing in a new technology. We define β as the investment efficiency and I as the investment of SP2. We denote $k_2(I)$ as the slope of congestion function g_2 with investment I and assume this is given by:

$$k_2(I) = \begin{cases} k_2(0) - \beta I, & \text{if } I < I_0, \\ k_2(0) - \triangle, & \text{o.w.} \end{cases}$$

Here, Δ represents a maximum performance improvement obtainable by investing no less than I_0, so that $\Delta = \beta I_0$. $k_2(0)$ denotes the initial performance of the technology (with no investment). It is private information of SP2. The customers and SP1 only know that this is drawn from given distribution which is common knowledge (as are all other parameters). For simplicity in this paper we assume that this is a uniform distribution on $[(1-\alpha)k_1, (1+\alpha)k_1]$ for a given $\alpha > 0$, i.e., the expected value of the entrant's technology without investment is k_1 and α indicates the uncertainty in the new entrant's technology. We will focus on the case where $\triangle < \alpha k_1$. Otherwise, SP2 will always have a better technology than SP1 if it invests, in which case contracts are no longer interesting. We denote this as *Assumption 1*. We further assume that when making its investment decision the entrant is aware of the outcome of the two previous stages, i.e., the parameters of the contract offered and the number of customers that signed the contract.

C. Contract

As in [4], [5] we consider that the incumbent offers all customers a contract with service price P_1^C and liquid damage P_0. Note that if the incumbent makes $P_0 = 0$, it will be equivalent to not offering a contract. Such a contract is denoted as $\{P_1^C, P_0\}$. In [5], it is shown that only exclusive contracts are profitable for the incumbent, meaning that $P_0 > P_1^c$.[1] Hence, we focus on such contracts in the following and we denote the strictness of such a contract by $y = P_0 - P_1^C > 0$.

Since customers are uncertain about the future QoS of an entrant, they decide whether or not to sign a contract based on their expected future pay-off, or equivalently on the expected delivered price from the entrant. This in turn will depend on the distribution of $k_2(0)$, the investment decision of the entrant and the number of other customers that sign the contract.

In stage 4, if SP2 enters the market with a better technology, the customers may break the contract if SP2 is able to offer them a sufficiently low delivered price. Note that if x^C customers signed the contract, the delivered price for customers in the contract is $k_1x_T + P_1^C$, where $x_T \geq x^C$ is the total customers accepting service at the end of the game. So, as long as $k_2x_T + P_0 < k_1x_T + P_1^C$, SP2 is able to offer a positive price for customers to switch service. This inequality gives a threshold of k_2 smaller than which SP2 can make contract customers break the contract and switch service. Denote this threshold as k_b.

When $k_b \leq k_2 < k_1$, SP2 may be able to make a profit and attract customers who didn't sign, but will not be able to make contract customers switch. He will choose a price $\hat{P}_2$ to maximize his revenue, which is given by

$$\begin{aligned} \hat{P}_2 = \underset{p_2}{\text{argmax}} \quad & p_2(x_T - x^C) \\ \text{subject to} \quad & k_2x_T + p_2 \leq 1 - x_T, \\ & x_T \geq \frac{1}{1+k_1}. \end{aligned} \tag{1}$$

Note in this case, SP1 and SP2 are competing for the remaining customers on price and so to have a positive profit in equilibrium, SP2's price must be small enough so that SP1 can not undercut it. The second constraint in (1) ensures that this is true as the right-hand side of this is the maximum number of customers SP1 could serve when its price is zero. If the realized k_2 is smaller than k_1, this constraint should hold because of the competition between SP1 and SP2. To simplify our analysis we assume that α is small enough so that this is always true in the remainder of the paper, in which case $x_T = \frac{1}{1+k_1}$ if SP2 enters the market. We denote this as *Assumption 2*. With this assumption, $k_b = k_1 - \frac{y}{x_T}$ can be re-written as $k_1 - (1 + k_1)y$. In the subsequent sections we will give precise conditions for when this assumption holds. Otherwise, when $k_2 \geq k_1$, SP2 will not enter the market as is explained in [1]. In this case, SP1 will offer a new price P_1^{NC} to customers who didn't sign before to maximize his revenue, where

$$\begin{aligned} P_1^{NC} = \underset{p_1}{\text{argmax}} \quad & P_1^C x^C + p_1(x_T - x^C) \\ \text{subject to} \quad & k_1x_T + p_1 = 1 - x_T. \end{aligned} \tag{2}$$

It follows that $P_1^{NC} = \frac{1-k_1x^C-x^C}{2}$. We denote the amount of new customers of SP1 as x_1^{NC}, where $x_1^{NC} = \frac{1}{2(1+k_1)} - \frac{x^C}{2}$, and the total customers when SP2 doesn't enter is $x_T^{NE} = \frac{1}{2(1+k_1)} + \frac{x^C}{2}$.

III. Method

The proposed model leads to a multi-stage Bayesian game, in which the players are SP1, SP2 and the customers. The definitions of key terms and symbols are included in Table I, and we will introduce the payoff functions and optimal

TABLE I
KEY TERMS AND SYMBOLS

Symbol	Definition
k_i	Slope of congestion cost function of SPi.
$\triangle$	Technology impact. The maximum shift on the slope of congestion cost function.
β	Investment efficiency that indicates the shift on the congestion cost slope given 1 unit investment.
I	Investment of SP2.
P_1^C	Contract service price
P_0	Liquid damage fee paid to incumbent if contract is broken.
x^C	Number of customers sign the contract.
y	Defined as strictness of the contract, given by $P_0 - P_1^C$.
k^*	The threshold for SP2's congestion cost slope to investment.
k_b	Tht threshold for SP2's congestion cost slope to make contract customers switch service.

[1]These are termed exclusive because they require customers to break them before getting service form a different SP modeling for example a market with locked devices.

strategies in the following subsections. We omit some detailed proofs due to space limit and they can be found in [15].

A. SP2

As mentioned, in stage 3, SP2 needs to decide entry and investment. There are three choices available: not entering, entering without investment, or entering with investment. If SP2 chooses not to enter, his payoff is 0. If SP2 enters, then his pay-off is given by

$$u_2(k_2(0), I; P_0, P_1^C, x^C) = \begin{cases} -I, & \text{if } k_2(0) - \beta I \geq k_1, \\ (\frac{1}{1+k_1} - x^C)\frac{k_1 - k_2(0) + \beta I}{k_1 + 1} - I, & \text{if } k_b \leq k_2(0) - \beta I < k_1, \\ \frac{k_1 - k_2(0) + \beta I}{(1+k_1)^2} - x^C y - I, & \text{if } k_2(0) - \beta I < k_b. \end{cases}$$

In the first case, SP2's technology is worse than SP1 so he will lose the investment. In the second case, SP2's can only win the customers who didn't sign the contract in stage 1. In the third case, SP2 can attract customers to break contracts.

For sake of succinctness, we define $h = (\frac{\beta}{(1+k_1)^2} - 1)I_0 > 0$ as the maximum investment gain. It should be greater than 0, otherwise, SP2 will never invest. Equivalently, we assume $\frac{\beta}{(1+k_1)^2} > 1$.

Note that at the stage SP 2 makes its decisions, it will know P_0, P_1^C, x^c and $k_2(0)$. Hence, given these parameters it will seek to optimize u_2 over I and compare this with the pay-off of 0 obtained by not entering. Also, note that u_2 is a convex function of I, which means that SP2 will only choose to invest 0 or the maximum amount I_0.

Based on this, we define an investment judgment function $f(k_2(0)) = u_2(k_2(0), I_0) - u_2(k_2(0), 0)$. Since $u_2(k_2(0), 0) \geq 0$, SP2 will enter and invest I_0 if $f(k_2(0)) > 0$ and not invest if $f(k_2(0)) \leq 0$. This leads to Lemma 1.

Lemma 1. *If there exists $k^* \in [(1-\alpha)k_1, (1+\alpha)k_1]$, such that $f(k^*) = 0$ and $f(k^* - \epsilon) > 0$ for all $\epsilon > 0$, then SP2 will invest I_0 when $k_2(0) < k^*$, and not invest when $k_2(0) \geq k^*$.*

Denote I^* as the optimal investment for SP2. It follows from Lemma 1 that

$$I^* = \begin{cases} 0 & \text{if } k_2(0) \geq k^* \\ I_0 & \text{if } k_2(0) < k^* \end{cases}.$$

Given k^*, we can determine the probability of SP2 entering the market and beating SP1 to take the whole market. Including SP2's investment decision, the realized $k_2(I^*)$ in stage 3 is uniformly distributed on $[(1-\alpha)k_1 - \beta I_0, k^* - \beta I_0] \cup [k^*, (1+\alpha)k_1]$. Denote the probability that $k_2(I^*) < k_1$, i.e., the probability of entry, as $\Phi^E(k^*)$ and the probability that SP2 will not enter as $\Phi^{NE}(k^*) = 1 - \Phi^E(k^*)$. From the assumed distribution of $k_2(0)$ we then have:

$$\Phi^E(k^*) = \begin{cases} \frac{1}{2}, & \text{if } k^* \leq k_1, \\ \frac{(1+\alpha)k_1 - k^*}{2\alpha k_1}, & \text{if } k^* > k_1. \end{cases}$$

Denote probability that $k_2(I^*) < k_b$ as $\Phi^B(k^*, y)$. This is the probability that SP2 enters and makes contract customers switch. This is given by

$$\Phi^B(k^*, y) = \begin{cases} \frac{k^* - \beta I_0 - (1-\alpha)k_1}{2\alpha k_1}, & \text{if } k^* - \beta I_0 < k_b, \\ \frac{k_1 - (1+k_1)y - (1-\alpha)k_1 + \beta I_0}{2\alpha k_1}, & \text{if } k^* - \beta I_0 > k_b. \end{cases}$$

B. Customers

Uncertain about SP2's future performance, customers need to decide whether to sign the contract offered by SP1 or wait for SP2's entrance. To do this, they will maximize their expected payoff at the end of the game. For customer x with service value $1 - x$, his expected delivered price if signing is:

$$D^S(P_1^C, k^*, x^C) = \Phi^{NE}(k^*) \cdot (k_1 x_T^{NE}(x^C) + P_1^C) + \Phi^E(k^*) \cdot (k_1 x_T^E + P_1^C).$$

Note that x_T^E and x_T^{NE} are the total number of customers in stage 3 when SP2 enters and does not enter, respectively. The delivered price is not changing whenever $k_2 < k_b$ since to maximize SP2's revenue, SP2 only need to offer a delivered price a little bit lower than SP1 for the contract customers to switch.[2]

If this customer doesn't sign, his expected delivered price is:

$$D^W(y, k^*, x^C) = \Phi^{NE}(k^*, y)(1 - x_T^{NE}(x^C)) + \Phi^E(k^*, y)(1 - x_T^E).$$

The customer will choose the option with lower expected delivered price. We know that x_T under different cases depends on the number of customers that have already signed, and the probability Φ of each case depends on SP2's investment strategy k^*. We will show that customers will keep signing the contract until $D^S > D^W$. We will study this in detail in Sect. V.

C. SP1

For SP1, his payoff u_1 is the expected revenue $u_1(P_1^C, y; k^*, x^C)$, depending on the contract he offers, the distribution of k_2 and number of customers who sign the contract, given by:

$$u_1 = \Phi^{NE} \cdot x_1^{NC} P_1^{NC} + \Phi^B \cdot x^C P_0 + (1 - \Phi^B) x^C P_1^C.$$

SP1's decision is then to chose the contract parameters P_0, P_1^C in stage 1 and its service price P_1^{NC} in stage 3. Note that in order to specify its contract parameters, it needs to know k^* and x^{c^*}, which it can determine via backward induction. Specifically, in a sub-game perfect equilibrium, if

[2] To be precise, and ensure an equilibrium exists, we assume that if SP2 offers the same price as SP1, then it gets all customers (in this case the customer equilibrium is not unique).

a pure strategy equilibrium $y^*, P_1^{C*}, k^*, x^{c*}$ exists, then these values should satisfy the best response conditions:

$$y^*, P_1^{C*} = \arg\max_{y, P_1^C} \ u_1(P_1^C, y; k^*, x^{C*})$$
$$k^* = \min_{f(k_2(0); P_0^*, P_1^{C*}, x^{c*}) \leq 0} k_2(0)$$
$$x^{C*} = \max_{D^S(P_1^{C*}, k^*, x^C) \leq D^W(y^*, k^*, x^C)} x^C.$$

We proceed in the following sections to solve for these. We first analyze SP2 strategy in Sect. IV given the contract and amount of customers signed. SP2's strategy determines the prior distribution of k_2. Then we analyze the customers decision in stage 2 in Sect. V, and finally consider what contract will SP1 offer in Sect. VI. We then give conditions for an equilibrium to exist in Sect. VII and characterize the equilibrium welfare in Sect. VIII and the role of a social planner in Sect. IX.

IV. Stage 3: SP2's Entry and Investment

In this section, we characterize SP2's investment and entry strategy given P_1^c, P_0 and x^C. In Sect. III, we derived that there is a threshold of $k_2(0)$ for SP2 to invest I_0. To further derive the dependence of this threshold k^* on y and x^C, we classify k^* by its relationship to k_1 and k_b as follows:[3]

$$\text{type A: } k^* - \beta I_0 \leq k_b \leq k^* < k_1,$$
$$\text{type B: } k_b \leq k^* - \beta I_0 < k_1 \leq k^*,$$
$$\text{type D: } k^* - \beta I_0 \leq k_b < k_1 \leq k^*.$$

It can be proved that these are the only possible relationships of these variables. Next, we give $f(k_2(0))$ for each of these different types:

$$f(k_2(0)) = \begin{cases} x^C \frac{k_1 - k_2(0)}{1+k_1} - x^C y + h, & \text{type A,} \\ (\frac{1}{1+k_1} - x^C)\frac{k_1 - k_2(0) + \beta I_0}{k_1+1} - I_0, & \text{type B,} \\ \frac{k_1 - k_2(0)}{(1+k_1)^2} - x^C y + (\frac{\beta}{(1+k_1)^2} - 1) I_0, & \text{type D.} \end{cases}$$

Recall, Lemma 1 specifies that $f(k^*) = 0$. This yields:

$$k^*(y, x^C) = \begin{cases} k_b + \frac{h(1+k_1)}{x^C}, & \text{type A,} \\ \beta I_0 + k_1 - \frac{I_0(1+k_1)^2}{1-(1+k_1)x^C}, & \text{type B,} \\ k_1 - x^C y(1+k_1)^2 + h(1+k_1)^2, & \text{type D.} \end{cases} \quad (3)$$

When k^* increases, SP2 is more likely to invest from SP1's and the customers' perspectives. Equation (3) shows that k^* is decided by the number of customers that sign and the strictness of the contract (i.e., y). It can be seen from (3) that $k^*(y, x^C)$ is decreasing with x^C and y. This is intuitive since as more customers sign the contract or the contract is more strict, SP2 will be less likely to invest.

Using the conditions defining these types, we have:

- if $x^C \geq \frac{h}{\min(y, \frac{\beta I_0}{1+k_1})}$, SP2 is in type A;
- if $x^C \leq \frac{1}{1+k_1} - \frac{I_0}{\min(y, \frac{\beta I_0}{1+k_1})}$, SP2 is in type B;
- if $\frac{1}{1+k_1} - \frac{I_0}{y} \leq x^C \leq \frac{h}{y}$, SP2 is in type D.

[3]We use type D instead of type C to avoid confusion with the subscript for "Contract".

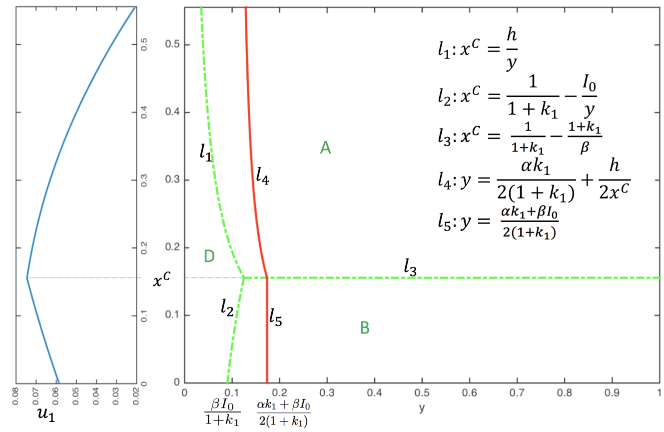

Fig. 1. The green lines separating the space to 3 areas shows how SP2's type depends on the number of customers signed and strictness of the contract. The red line shows the equilibrium path, and the sub-figure on the left side shows the value of the incumbent's expected revenue on the equilibrium path.

These constraints show that when x^C and y are big, k^* will be small so that SP2 will invest to make customers switch service. When x^C is small, SP2 will invest when $k_2(0) > k_1$ to win customers who didn't sign. In this case, when y is big, SP2 cannot make customers switch service, but when y is small enough, SP2's investment will enable it to win all customers in service.

These three types are displayed in Fig. 1 with green dash lines.

V. Stage 2: Customers' Signing Process

In this section, we characterize how many customers will sign the contract given P_0 and P_1^C. When SP2 enters the market with a good technology ($k_2(I) < k_b$), the customers who signed the contract have an option of breaking the contract and paying the liquid damage to SP1. This option affects the customers' consideration of signing the contract in stage 1. As introduced in Sect. III, the condition that customer x will sign is that his expected delivered price when signing is lower than his expected payoff of waiting given that x^C customers have signed, i.e.,

$$D^S(P_1^C, k^*, x^C) \leq D^W(y, k^*, x^C). \quad (4)$$

Starting with $x^C = 0$, x^C will keep increasing as long as this inequality holds. In other words, we need to find the largest x^{C*}, such that for every $x^C \in [0, x^{C*}]$, this inequality holds (see the left-hand side of Fig. 2). [4] Applying equation (3), we have the following two lemmas:

Lemma 2. *If $k^* < k_1$ (type A), $x^{C*} = \frac{1-4P_1^C}{1+k_1}$.*

[4]It is possible for (4) to hold with equality for other values of x^C, any such choice would be a customer equilibrium; Here, we focus on x^{C*} as the most plausible of these.

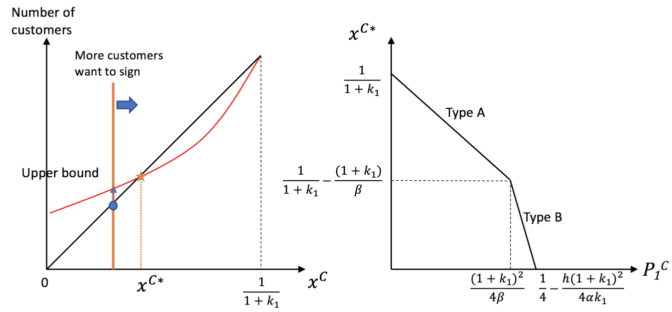

Fig. 2. The figure on the left shows an example of the customers who sign the contract. The figure on the right shows relationship between x^{C*} and P_1^C

Lemma 3. *If $k^b \leq k^* - \beta I_0 < k_1 \leq k^*$, i.e., SP2 is in type B, $x^{c*} = \frac{1}{1+k_1} + \frac{I_0(1+k_1)}{\alpha k_1 - \beta I_0} - \frac{4\alpha k_1 P_1^C}{(\alpha k_1 - \beta I_0)(1+k_1)}$.*

After applying the type constraints, x^{C*} has the following forms:

$$x^{C*}(P_1^C) = \begin{cases} \frac{1-4P_1^C}{1+k_1}, & \text{if } 0 \leq P_1^C \leq \frac{(1+k_1)^2}{4\beta}, \\ 0, & \text{if } P_1^C > \frac{1}{4} - \frac{h(1+k_1)^2}{4\alpha k_1}, \\ \frac{1}{1+k_1} + \frac{I_0(1+k_1)^2 - 4\alpha k_1 P_1^C}{(\alpha k_1 - \beta I_0)(1+k_1)}, & \text{o.w.} \end{cases} .$$

This is shown in Fig. 2 (right). The number of customers that sign the contract has nothing to do with the liquid damage P_0 but decreases with the contract service price P_1^C. The reason is that if SP2 comes with a better technology that attracts customers to switch service, SP2 will need to set its price low enough to compensate for P_0. However, P_1^C has a significant effect that increases the delivered price of those customers. The two lemmas above do not consider type D, and we will show the reason in Sect. VI. For now, we just focus on type A and B. We know that, when $P_1^C < \frac{(1+k_1)^2}{4\beta}$, SP2 will be in type A, in which SP2 invest to attract contract customers. Customers are less sensitive to P_1^C in this case.

VI. Stage 1: Incumbent's Contract Design

Our last step of backward induction is analyzing how SP1 will decide the contract offered to maximize his expected revenue. We first apply x^{C*} to $k^*(x^C, y)$, so that k^* is fully determined by the contract. We use $k^*(P_1^C, y)$ to denote k^* after applying the customers signing equilibrium and then continue to substitute k^*, x^C in $u_1(P_1^C, y, k^*, x^C)$ with $k^*(P_1^C, y)$ and $x^{C*}(P_1^C)$, so that we have

$$\begin{aligned} u_1(P_1^C, y) &= \Phi^B(k^*(P_1^C, y)) \cdot x^C \cdot (P_1^C + y) \\ &+ \Phi^{NE}(k^*(P_1^C, y)) \cdot x_1^{NC}(x^{C*}(P_1^C)) \cdot P_1^{NC}(x^{C*}(P_1^C)) \\ &+ (1 - \Phi^B(k^*(P_1^C, y), y)) \cdot x^C \cdot P_1^C. \end{aligned}$$

Note SP1 will have different calculations of his expected revenue under different types. In each type, we solve:

$$\begin{aligned} \max_{P_1^C, y} \quad & u_1(P_1^C, y) \\ \text{subject to:} \quad & \text{type constraints.} \end{aligned} \tag{5}$$

For example, when SP2 is type A, applying the type constraint $x^C \geq \frac{h}{\min(y, \frac{\beta I_0}{1+k_1})}$ to $x^{C*}(P_1^C)$ yields

$$P_1^C \leq \frac{1}{4} - \frac{h(1+k_1)}{4 \cdot \min(y, \frac{\beta I_0}{1+k_1})}.$$

The expected payoff of SP1 when SP2 is type A can be written as

$$\begin{aligned} u_1(P_1^C, y) = &\frac{1}{2}\left(\frac{1 - k_1 x^C - x^C}{2(1+k_1)} \cdot \frac{1 - k_1 x^C - x^C}{2}\right) \\ &+ \frac{((1+\alpha)k_1 - k^*}{2\alpha k_1} x^C P_1^C \\ &+ \frac{k^* - (1-\alpha)k_1}{2\alpha k_1} x^C (y + P_1^C), \end{aligned} \tag{6}$$

where

$$\begin{aligned} k^* &= k_1 - (1+k_1)y + \frac{h(1+k_1)}{x^C}, \\ x^C &= \frac{1 - 4P_1^C}{1+k_1}. \end{aligned}$$

The first line of (6) denotes the expected profit from new customers in stage 4. Since $k^* < k_1$, the probability that k_1 is smaller than k_2 is $\frac{1}{2}$. The maximized profit is $\frac{1-k_1 x^C - x^C}{2(1+k_1)} \cdot \frac{1-k_1 x^C - x^C}{2}$, where the price for new customers is $\frac{1-k_1 x^C - x^C}{2}$. The second line of (6) denotes the expected profit from contract customers when the contract is not broken. The probability of this case is $\frac{((1+\alpha)k_1 - k^*}{2\alpha k_1}$, because $k^* > k_b$ while $k^* - \beta I < k_b$. The profit in this case is $x^C P_1^C$. Similarly, the third line of (6) denotes the expected profit from contract customers when the contract is broken. Optimizing this over y yields

$$\begin{aligned} y^* &= \frac{\alpha k_1}{2(1+k_1)} + \frac{h}{2x^C}, \\ k^* &= k_1 - \frac{\alpha k_1}{2} + \frac{h(1+k_1)}{2x^C}. \end{aligned}$$

Substituting these into (6) and taking derivatives gives

$$\frac{\partial u_1(P_1^C, y^*)}{\partial P_1^C} = \frac{h^2(1+k_1)^2}{2\alpha k_1(1-4P_1^C)^2} + \frac{1-4P_1^C}{1+k_1} - \frac{\alpha k_1}{2(1+k_1)^2}.$$

Assuming $h \geq \frac{\alpha^2 k_1^2}{3\sqrt{3}(1+k_1)^{9/2}}$ or $\beta \geq \frac{(1+k_1)^2}{1 - \frac{\alpha k_1}{2(1+k_1)}}$, it can be shown that $\frac{\partial u_1(P_1^C, y^*)}{\partial P_1^C} \geq 0$. We denote this assumption *Assumption 3*. With this assumption, $u_1(P_1^C, y^*)$ is increasing in P_1^C. So, P_1^{C*} should be the largest value allowed in this case, i.e., $P_1^{C*} = \frac{1}{4} - \frac{h(1+k_1)}{4 \cdot \min(y, \frac{\beta I_0}{1+k_1})}$. At the equilibrium, y should be greater than $\frac{\beta I_0}{1+k_1}$ to enlarge SP1's expected profit.

To sum up, if SP2 is type A, SP1 will choose the contract to be

$$\begin{aligned} y^* &= \frac{\alpha k_1 + \beta I_0}{2(1+k_1)}, \\ P_1^{C*} &= \frac{(1+k_1)^2}{4\beta}. \end{aligned}$$

Using these one can verify that $k^* - \beta I_0 \le k^b \le k^* < k_1$, as required to be type A.

Similarly when SP2 is type B, we can show that if $\frac{1}{4} - \frac{(\beta I_0+\alpha k_1)^2}{8\alpha k_1(1+k_1)} - \frac{\beta I_0}{4\alpha k_1} + \frac{I_0(1+k_1)^2}{8\alpha k_1} \ge \frac{(1+k_1)^2}{4\beta}$, SP1 will choose the contract to be

$$y^* = \frac{\alpha k_1 + \beta I_0}{2(1+k_1)},$$
$$P_1^{C*} = \frac{1}{4} - \frac{(\beta I_0 + \alpha k_1)^2}{8\alpha k_1(1+k_1)} - \frac{\beta I_0}{4\alpha k_1} + \frac{I_0(1+k_1)^2}{8\alpha k_1}.$$

If SP2 is type D, u_1's derivative of y can similarly be given by:

$$\frac{\partial u_1(P_1^C, y)}{\partial y} = \frac{h(1+k_1)^2}{4\alpha k_1} + \frac{1-4P_1^C}{4}.$$

Note that $\frac{\partial E(u_1)}{\partial y}$ is always greater than 0. In other words, u_1 is increasing in y. Thus y^* will hit the boundary of this type.

VII. Equilibrium

In this section, we characterize the equilibrium of this game.

Lemma 4. *$u_1(P_1^C, y)$ is a continuous function of P_1^C and y.*

Lemma 5. *No equilibria could be in type D.*

We illustrate this in Fig. 1. In area A, u_1 is maximized on l_4. In area C, u_1 is maximized on l_5. From Sect. VI, we know that in type D, $u_1(P_1^C, y)$ is maximized on the type boundary, l_1 and l_2. Since u_1 is a continuous function of x^C and y, the value of u_1 on l_1 and l_2 is smaller than u_1 on l_4 and l_5. So SP1 will never select a contract resulting in type D.

Theorem 1. *If Assumptions 1-3 hold, there exists a unique equilibrium, either in type A or type B, and*

$$y^* = \frac{\alpha k_1 + \beta I_0}{2(1+k_1)},$$
$$I^* = \begin{cases} 0 & \text{if } k_2(0) \ge k^* \\ I_0 & \text{if } k_2(0) < k^* \end{cases}.$$

Let $P_A^* = \frac{(1+k_1)^2}{4\beta}$, $P_B^* = \frac{1}{4} - \frac{(\beta I_0+\alpha k_1)^2}{8\alpha k_1(1+k_1)} - \frac{\beta I_0}{4\alpha k_1} + \frac{I_0(1+k_1)^2}{8\alpha k_1}$. *If* $P_A^* > P_B^*$, *the equilibrium is in type A, and*

$$P_1^{C*} = P_A^*,$$
$$x^{c*} = \frac{1}{1+k_1} - \frac{(1+k_1)}{\beta},$$
$$k^* = k_1 - \frac{\alpha k_1}{2} + \frac{\beta I_0}{2}.$$

If $P_A^* \le P_B^*$, *the equilibrium is in type B, and*

$$P_1^{C*} = P_B^*,$$
$$x^{c*} = \frac{(\beta I_0 + \alpha k_1)^2 + I_0(1+k_1)^3}{2(1+k_1)^2(\alpha k_1 - \beta I_0)},$$
$$k^* = \beta I_0 + k_1 - \frac{I_0(1+k_1)^2}{1-(1+k_1)x^{C*}}.$$

The equilibrium is unique because under our assumptions, $u_1(P_1^C, y^*)$ is monotonically increasing in P_1^C when $P_1^C \le P_1^{C*}$, and monotonically decreasing in P_1^C when $P_1^C > P_1^{C*}$. A numerical example of the relationship between u_1 and x^{C*} is shown on Fig. 1.

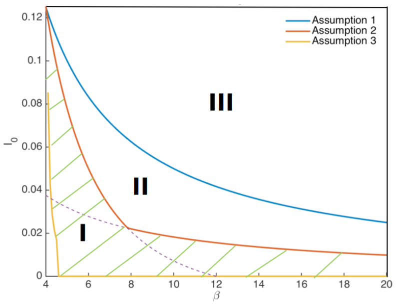

Fig. 3. An example of feasible region when $k_1 = 1$, $\alpha = 0.5$.

First, we discuss the assumptions in this theorem. Assumption 3 is a sufficient condition for monotonicity of $u_1(P_1^C, y^*)$ in type A. If this condition is violated, the maximum of $u_1(P_1^C, y^*)$ is difficult to characterize. Assumption 2 implies $\frac{1}{2(1+k_1-\alpha k_1-\beta I_0)} + \frac{x^{C*}}{2} \le \frac{1}{1+k_1}$, meaning that for any possible k_2, it is not possible for SP2 to attract more customers than $\frac{1}{1+k_1}$. As is shown on Fig. 3, the resulting feasible values of β and I_0 satisfying these assumptions is the region labeled I. Region II violates Assumption 2, making the equilibrium difficult to characterize. Region III violates Assumption 1, in which case no customer will sign the contract. This is easy to solve but not as interesting.

Using this result, we can then characterize how the parameters affect the equilibrium. As investment efficiency β or I_0 increases, SP1 will make the contract more strict and lower the contract service price so that more customers will sign the contract. If SP1 is more uncertain about of SP2's technology (i.e., α increases), SP1 will also make the contract more strict.

VIII. Welfare Analysis

Next we analyze the firm's expected profit, the expected customer surplus, defined as the sum of all customers' expected payoffs, and the expected social welfare defined as the expected sum of firms' profit and customer surplus.

A. Impact of new technologies maximum power.

Here, we set β, the investment efficiency, to be unchanged, so that the technology power $\triangle = \beta I_0$ is increasing with I_0. As is shown in Fig. 4, without contracts, SP1's expected revenue $SP1_{NC}$ is decreasing with I_0; SP2 expected revenue $SP2_{NC}$ is increasing with I_0. This is intuitive. However, in the market with contracts, when SP1's technology represented by k_1 is good enough, its revenue $SP1_C$ can be increasing with I_0 when the new technology can sufficiently improve the spectrum efficiency. The reason is that contracts enable SP1 to profit from the liquid damage, and the liquid damage profit, no mater paid by the customers or the entrant is because of the improved spectrum utilization. So, as the new technology can improve the spectrum efficiency more, SP1 is able to get more revenue. Interestingly, the two firms' profit can be both

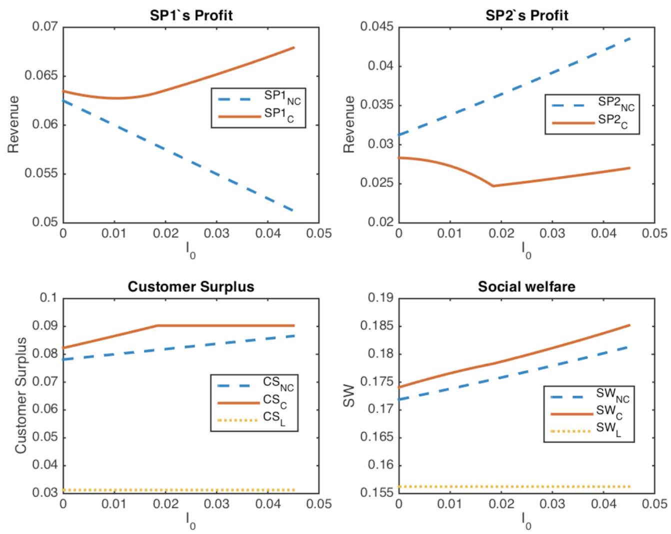

Fig. 4. An example of SP1, SP2's expected revenue, expected customer surplus and social welfare under contract, no contract and monopoly cases changing with I_0 when $k_1 = 1$, $\beta = 6$, $\alpha = 0.5$.

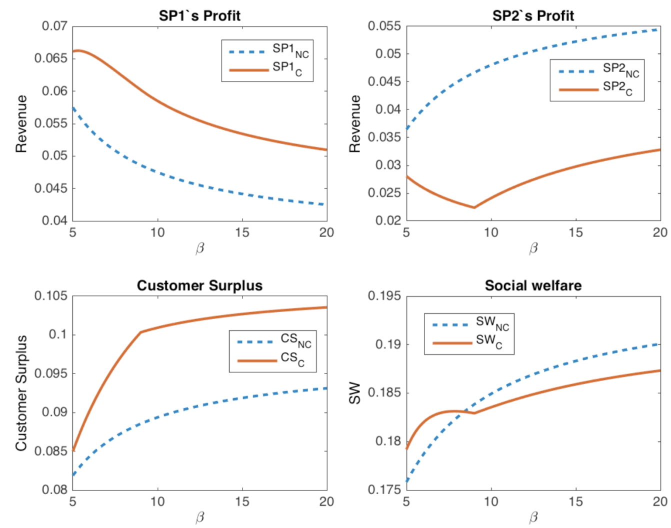

Fig. 5. An example of SP1, SP2's expected revenue, expected customer surplus and social welfare under contract and no contract cases changing with β when $k_1 = 1$, $\triangle = 0.2$, $\alpha = 0.5$.

decreasing with I_0 when I_0 is small. As I_0 increases, SP1 will make a stricter contract, decreasing SP2's revenue decrease.

We can observe from the lower left graph that the expected customer surplus is dramatically increased (compared to the monopoly case) due to the potential entry of SP2, and it is increasing linearly with I_0 if contracts are not allowed in the market. However, when contracts are allowed, the expected customer surplus will first increase with I_0, but as I_0 grows, the equilibrium moves from type B to type A and the expected customer surplus becomes constant. From the lower right graph, observe that the expected social welfare is also increasing with I_0.

B. Impact of the investment efficiency.

If we keep $\triangle$ as power of the new technology unchanged and increase the investment efficiency β, SP1's revenue is always decreasing with β and SP2's profit, customer surplus and social welfare are all increasing with β when contracts are not allowed in the market, as is shown in Fig. 5. This is intuitive, but it is more interesting when contracts are used. As is shown in Fig. 5, SP2's expected profit is decreasing with β when β is small and the equilibrium is in type A. The reason is that SP1 will set a more strict contract as the investment efficiency increases while the number of customers who sign in equilibrium increases. This lowers the expected profit of the entrant provider. However, as β grows, the equilibrium moves to type B. The positive effect of an increased investment efficiency is now dominating as the probability that SP2 enters the market and invests increases and the cost of investment decreases. We can also observe from this figure that when β is large enough, social welfare is decreased by the contract. The intuition is that as β increases, the new technology is more beneficial for improving welfare. However, the existence of

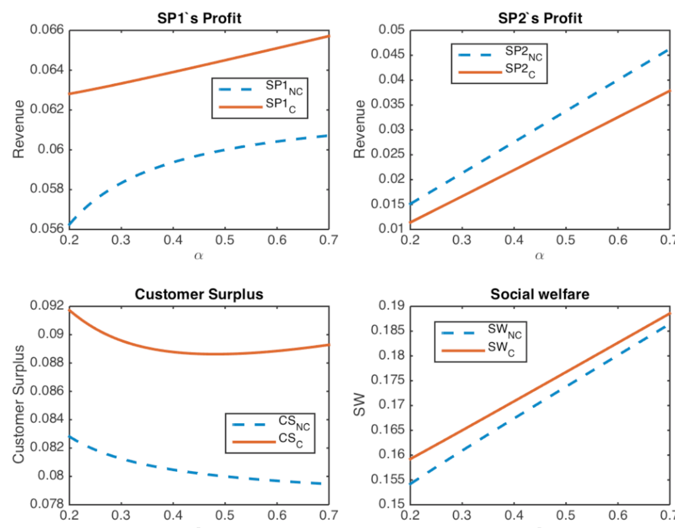

Fig. 6. SP1, SP2's revenue, customer surplus and social welfare under contract and no contract cases when $k_1 = 1$, $\beta = 5$, $I_0 = 0.02$.

the contract as an entry barrier lowers the possibility that the new technology is adopted. When β is small, social welfare with the contract is increased, and the social welfare is not always increasing with β when contracts are allowed in the market. The reason is similar as in the former section where I_0 decreases SP2's expected profit. When β is small, the increase of β urges SP1 to set a more strict contract which harms SP2's expected payoff and in turn decreases the social welfare.

C. Impact of the incumbent's uncertainty level.

Next, we analyze how the welfare will change when α increases, as is shown in Fig. 6. It is surprising that when there is no contract, SP1's expected revenue can be increasing with α. The reason is that as α increases, the probability that SP2 will not enter the market is actually increasing. We can observe from the figure that SP1's expected profit is increased when contracts are offered to customers. What is not shown in the figure but equally important is that the coefficient of variation (CV) is also greatly decreased by the contracts. In the example shown in the figure, the CV of SP1's profit without contracts is around 100% while the CV is reduced to around 45% if contracts are offered. So, by using contracts, SP1's profit is more stable in stage 3, which SP1 would prefer if he is risk adverse. Intuitively, customer surplus is decreased with α, as the customers have better payoff when SP2 enters the market. However, the increase of α decreases the probability that SP2 enters which hurts customers.

IX. Social Planer

For a social planner, a key question is how to regulate the market to increase social welfare. There are several options. One choice is add a subsidy or tax d on the liquid damage P_0. Another choice is to set an upper bound for the difference between the liquid damage and contract price, i.e. y.

A. Subsidy or tax

Suppose that the social planner can charge SP1 $-d$ tax for the liquid damage if $d < 0$ or compensate SP1 for the loss of customers by a subsidy d if $d > 0$. Then the payoff of SP1 is now given by

$$u_1 = \Phi^{NE} \cdot x_1^{NC} P_1^{NC} + \Phi^B \cdot x^C (P_0 + d) + (1 - \Phi^B) x^C P_1^C.$$

We can solve for the resulting equilibrium as before, yielding:

Lemma 6. *Social welfare can be increased if the incumbent SP is compensated with some subsidy $d > 0$.*

The intuition is that due to the subsidy, SP1 will offer a less strict contract increasing SP2's likelihood of investing.

B. Bounding y

The subsidy lowers the barrier to entry and investment indirectly. A more straightforward approach is restricting the strictness of the contracts offered, i.e., bounding y. It follows that y^* will be equal to the bound that social planner sets. Due to the space limit, here we just include a numerical example in Fig. 7, where the bound is set to be $\frac{1}{2}$ of the optimal y for SP1. We can see that the expected social welfare can also be improved this way.

X. Conclusion

We studied a game theoretical model of an unlicensed spectrum market with the option of contracts and investments in new technologies. The resulting model is a multi-stage game with incomplete information, which captures the behavior of entrant and incumbent SPs and customers. We are able to give conditions under which a unique market equilibrium exists. Further, we show how the welfare during the equilibrium changes with different factors. Finally, we characterize how social planners can better regulate the market.

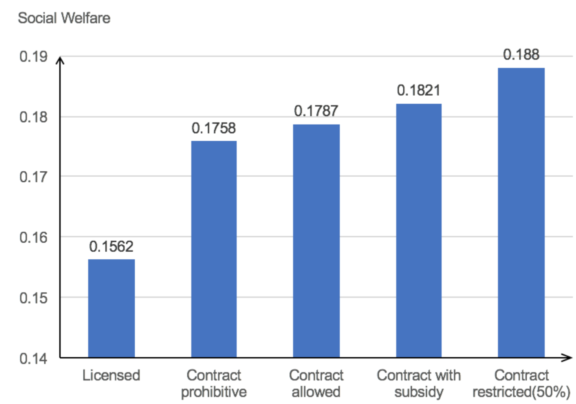

Fig. 7. An example of increased social welfare under different regulations when $k_1 = 1$, $\alpha = 0.5$, $\beta = 6$, $I_0 = 0.02$.

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