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What makes long-term monitoring convenient? A parametric analysis of value of information in infrastructure maintenance

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Summary

Information collected by monitoring systems can provide a significant economic benefit to the operation and maintenance of infrastructure components only under specific conditions. The information has to be precise, not redundant, related to relevant decision problems under uncertainty as, for example, the appropriate scheduling of maintenance actions, and the decision maker needs to be able to process that information and react timely. All these considerations can be naturally embedded in the value of information (VoI), a utility-based metric for assessing the impact of the additional information in decision making under uncertainty. In this paper, we investigate the relation between the VoI and key features of the monitoring system, of the component deterioration and of the decision-making process, including measure accuracy and availability, deterioration rates, damage predictability, reaction time, maintenance costs, and the economic discount factor. By leveraging previous work, we model the maintenance process as a partially observable Markov decision process, and we compute the VoI of long-term monitoring. Our proposed framework allows for a detailed quantitative analysis on the joint effects of these features and can be useful to identify conditions when the benefit of monitoring is high, to assign priorities among components that deserve to be instrumented or to optimize the allocation of resources to monitoring efforts.

KEYWORDS

condition-based maintenance, long-term monitoring, POMDP, value of information

1 | INTRODUCTION

Information collected by inspectors and monitoring systems can support the operation and maintenance (O&M) of infrastructure components, reducing uncertainty in the assessment of their current condition state and in the prediction of their future evolution and allowing for a closed-loop control scheme, which can be economically effective. In the absence of external constraints,[1] if the monitoring effort is free of cost, all components should be permanently monitored according to the principle "information never hurts."[2] However, due to the cost of acquiring, installing, and operating monitor devices, of collecting and analyzing data, and of identifying decision strategies to react to new information, it is nontrivial to assess if it is worth implementing a specific monitoring process in a given circumstance. Collecting information is useful only under uncertainty, and the future economic benefit of the monitoring efforts cannot be deterministically predicted. However, its expected value can be assessed by Bayesian pre-posterior analysis,[3] which predicts, in the prior condition before the information is available, how the information can affect the posterior belief.

The expected benefit of embedding a specific monitoring system in an O&M process is a complicated function of many parameters modeling the performance of the sensing devices, the prior information, the available maintenance actions, the immediate and long-term costs, etc. Intuitively, many features tend to make the implementation of a monitoring process convenient. Some of them are related to the O&M of the specific component. One is its economic relevance: if the costs of a component's malfunction and those related to performing maintenance are high. Also, if malfunctions can occur frequently, possibly because the component's condition evolves quickly, and if the time horizon of the O&M process is remote. Another feature is the uncertainty about the component's condition: if, without the monitoring process, the current condition cannot be accurately assessed and its evolution cannot be predicted with confidence. Other features are related to the monitoring process itself: if frequent and precise measures are collected. Also, if its implementation is cheap. Finally, some features are related to interaction between measures and decisions: if measures can be promptly processed to inform and affect the control policy.

The value of information (VoI) is a utility-based metric introduced by the seminal work of Raiffa and Schlaifer,[3] it is related to decision making under uncertainty, and it measures the benefit of additional information in reducing economic costs. The VoI of a long-term monitoring campaign can be defined as the difference between the expected discounted long-term cost of the O&M process with and without using the data collected by that campaign. Applications of VoI analysis to Structural Health Monitoring are presented by Pozzi and Der Kiurehian,[4] Straub,[5] Schweckendiek and Vrouwenvelder,[6] Qin et al.,[7] and Zonta et al.[8] Evaluation of VoI in sequential infrastructure management is illustrated by Srinivasan and Parlikad.[9] The recent interest in evaluating the economic impact of integrating sensors in infrastructure management is testified by many researches.[10-14] Assessing the VoI allows for an appropriate calibration of investments in inspections, sensors, and monitoring systems, as the overall investment for collecting information should not exceed its value, In addition, it allows for comparing expensive explorative actions (as installing monitoring systems) with "exploitative" actions (i.e., actions that change the actual condition of the system, as retrofitting a component) on an equal ground.

In this paper, expanding our preliminary results,[15] we adopt a computation framework proposed by Memarzadeh and Pozzi[16] to model O&M under uncertainty by a partially observable Markov decision process (POMDP) and assess the long-term VoI of permanent integration of monitoring systems. We make use of that framework to investigate how the features of the monitoring system and of the O&M process influence the VoI. In Section 2, we show how to discretize the O&M process in continuous time. In Section 3, we introduce the POMDP framework, and we illustrate how to compute the VoI. In Section 4, we investigate how the VoI depends on the features of the monitoring system and of the O&M process. In Section 5, we show how to apply our analysis on an example of bridge maintenance, before drawing conclusions in Section 6.

2 | CONTINUOUS AND DISCRETE MODELING OF THE MAINTENANCE PROCESS

2.1 | Problem statement

A wide range of O&M processes can be modeled as a Markov process in continuous time, by selecting an appropriate set of state variables (e.g., these variables describe the complex evolution of damage patterns in structural systems).[17-19] This process can be converted into an approximate POMDP, by discretizing the time and domain of possible states and by defining costs, available actions, and corresponding transitions.[14,20] In turn, the POMDP can be solved by implementing an appropriate numerical scheme to identify the optimal control policy. However, the computational complexity of this optimization depends strongly on the dimensions of the discrete process,[21,22] that in turn depends on the complexity of the original problem and the required quality of the approximation. The VoI analysis outlined below can be applied to any POMDP and, consequently, at least in principle, to any O&M process following a Markov process.

Here, we focus our analysis on a specific class of O&M processes where, at any time, an infrastructure component is in one among a set of M possible conditions, orderable from the "best" one to the "worst" one. Typical examples of such processes are the deterioration of road pavements,[23] the crack growth,[24] the water pipe infrastructures,[25] or the railway tracks.[26] In these examples, the condition is the discretized version of a continuous variable that quantifies the amount of degradation.

If the deterioration process in continuous time visits all the discrete conditions in the sequence, we can define this stochastic process by the probability distribution of the transition times $\{\Delta t_1, \Delta t_2, \ldots, \Delta t_{M-1}\}$, where Δt_k is the time between the visits of condition k and that of condition $k+1$. We also assume these transition times are independent, so their joint distribution is completely defined by the set $\{p_1, p_2, \ldots, p_{M-1}\}$ of their marginal distributions, where p_k is the marginal distribution of Δt_k. Hence, the degradation process starting from a condition is independent of the past path that took the component to that condition. This is a typical assumption in infrastructure management.[27-29] If such assumption does not hold true, one needs to define another, more fundamental state, affecting the transitions. That more complex formulation induces a higher computational cost.[14,30,31]

In this and the following two sections, we limit our analysis to a component with three possible conditions. In Section 5, we refer to a case with a higher number of states. Here, we focus on this simple setting because it is general enough to be applied, at least as a first model, to a wide range of problems and because this setting allows us to perform an extensive parametric investigation. We note that the low number of conditions can be appropriate if the number of possible maintenance actions is also low as, in this case, the damage condition can be intended as the union of all heterogeneous physical conditions rising significant concerns and calling for an intervention.

To introduce our analysis, we start defining a typical case of O&M of an infrastructure component in continuous time. Following the assumption in Khaleghei and Makis,[32] in any moment, the component can be in one of three conditions: intact, damaged, failed. We assume a new component starts in the intact condition and, if left uncured, stays in that condition for duration $\Delta t_1 = \Delta t_d$, after which it becomes damaged and, after additional duration $\Delta t_2 = \Delta t_f$, it fails. These two durations can be modeled as independent random variables, defined by marginal distributions $p_1 = p_d$ and $p_2 = p_f$, respectively. We also assume that, at any time, the decision maker managing the component, who we will refer to as the "agent," can repair the component, and if so, the process starts anew in the intact condition. The costs are related to two sources: the repair actions and the component's malfunction. Cost C_r is incurred once the component is repaired, and the agent has to pay cost C_f in case of its failure. Moreover, following the assumption in Memarzadeh and Pozzi,[16] no lack of functioning or cost derives from the appearance of the damage: Only the failure induces a direct economic loss. Costs are assumed to be discounted using factor γ_1 per unit time.[33]

Without adopting the monitoring effort, the agent cannot distinguish between the intact and damaged condition. However, any failure event is immediately detectable.[32] The time to failure is obtained as $\Delta t_T = \Delta t_d + \Delta t_f$, whose distribution p_T derives from p_d and p_f by convolution. The agent has to select a repair time interval Δ_d before hand, trading off the risk of failure with the cost of periodic repairs. The policy is "open-loop," except for the reaction to failure events. Suppose that, as alternative, the agent can periodically take measures of the component's condition, update the probability of the component being damaged, and adapt her plans, postponing or anticipating the repair accordingly. This is a "closed-loop" policy. If we assume optimality of both open and closed-loop policies according to the stochastic process, and we do not include the cost of the monitoring process, the closed-loop behavior will be economically convenient in the expected sense. Our aim is to evaluate this expected economic convenience of long-term monitoring via the VoI. This benefit can be compared with corresponding long-term costs for installing and operating the monitoring system, to decide about its adoption. Figure 1 shows a scheme of the open-loop and closed-loop policies, that is, without and with the availability of monitoring measures, respectively.

2.2 | Discretized modeling of the maintenance process

To identify the optimal policy and the corresponding maintenance costs in a computationally tractable way, we discretize the time in steps of duration δt. Actions can be implemented, costs are paid, and observations are available only at the end of each time step. The effect of the discretization is numerically investigated in Appendix B.

We define a Markov process for a component with $n = n_1 + n_2 + 1$ possible states. The first n_1 states refer to the intact condition, the following n_2 ones to the damaged one and the last one to the failure. We assume that the

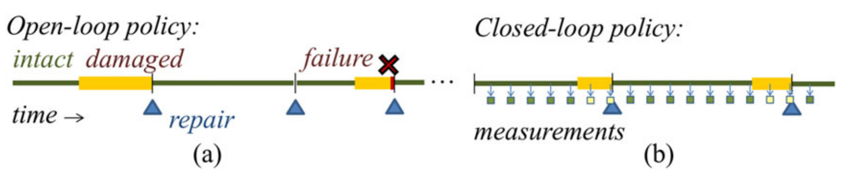

FIGURE 1 Scheme of the (a) open- and (b) closed-loop O&M processes: Measurements are available in the latter case

transition time between conditions without any repair can be modeled by the sum of exponentially distributed random variables, with identical or different rates. By adding $n_1 - 1$ and $n_2 - 1$ intermediate states, the transition time between each condition can be modeled as sum of random variables. Specifically, transition time τ_i between the state $s = i$ and $s = i + 1$ is exponentially distributed with rate λ_i. A component takes transition time $\tau_{i,j} = \tau_i + \ldots + \tau_{j-1}$ to deteriorate to state j from state i for $1 \leqslant i < j \leqslant n$, and $\tau_{i,i} = 0$.

For homogeneous rates, when rates are identical for all states (i.e., $\lambda_i = \lambda$ for $i = 1, 2, \ldots, n - 1$), the transition time $\tau_{i,j}$ is represented by the Erlang distribution,[34] a special case of the Gamma distribution. Erlang distributions are unimoldal. The transition time from condition h to h+1 has mean $\frac{n_h}{\lambda}$ and coefficient of variation $\frac{1}{\sqrt{n_h}}$, where n_h, again, is the number of intermediate states, for h equal to 1 or 2. Using nonhomogeneous rates allows for even higher flexibility in modeling the transition times. Note that $\tau_{1,n_1+1} = \Delta t_d$ and $\tau_{n_1+1,n} = \Delta t_f$, so a vast class of distributions p_d and p_f can be approximated using an appropriate set of rate values.

2.3 | Transition process modeling

The agent can choose between two actions: *Do-Nothing* ($a = 1$) and *Repair* ($a = 2$). We define the transition probability $T_c(i, j) = \mathbb{P}[s' = j | s = i, a = c]$, as the probability of reaching state j after one step of duration δt if the current state is i and action c is selected. Hence, $T_1(i, j) = \mathbb{P}[\tau_{i,j} \leqslant \delta t < \tau_{i,j+1}]$. In the homogeneous case, we can compute the probability of random variable k, indicating the number of state increments in time interval δt and distributed according to the Poisson distribution with rate $\lambda\delta t$, as

$$f(k) = \frac{(\lambda\delta t)^k e^{-\lambda\delta t}}{k!} \quad \text{for} \quad \delta t, \lambda > 0, k \geqslant 0. \tag{1}$$

In this model, the component deterioration follows a Poisson process, and the damage and failure condition occur when the corresponding counting process reaches n_1 and n, respectively. When the repair action ($a = 2$) is selected, the evolution starting from state i follows that of a new component, so $T_2(i,j) = T_1(1,j)$ for every value of i. The transition probability matrices under the two maintenance actions are shown in Table 1.

For nonhomogeneous rates, the distribution of transition time $\tau_{i,j}$ is[35]

$$p(\tau_{i,j}) = \left(\prod_{l=i}^{j-1} \lambda_l\right) \sum_{n=i}^{j-1} \frac{e^{-\lambda_n \delta t}}{\prod_{m=i,m\neq n}^{j-1} (\lambda_m - \lambda_n)} \quad \text{for} \quad \delta t > 0, j > i. \tag{2}$$

Then the probability of at least $j - i$ increments of state for a component starting from state i during the time interval δt can be obtained by integrating Equation 2,

$$F_i(j - i) = \begin{cases} \left(\prod_{l=i}^{j-1} \lambda_l\right) \sum_{n=i}^{j-1} \frac{1-e^{-\lambda_n \delta t}}{\lambda_n \prod_{m=i,m\neq n}^{j-1} (\lambda_m - \lambda_n)} & (j - i > 1) \\ 1 - e^{-\lambda\delta t} & (j - i = 1) \end{cases} \tag{3}$$

TABLE 1 Transition probability matrices for homogeneous rates

$$\mathbf{T}_1 = \begin{bmatrix} f(0) & f(1) & \ldots & f(n-2) & 1 - \sum_{i=0}^{i=n-2} f(i) \\ 0 & f(0) & \ldots & f(n-3) & 1 - \sum_{i=0}^{i=n-3} f(i) \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & \ldots & \ldots & f(0) & 1 - f(0) \\ 0 & 0 & \ldots & 0 & 1 \end{bmatrix}_{n\times n} \quad \mathbf{T}_2 = \begin{bmatrix} f(0) & \ldots & f(n-2) & 1 - \sum_{i=0}^{i=n-2} f(i) \\ \vdots & \ddots & \ddots & \vdots \\ f(0) & \ldots & f(n-2) & 1 - \sum_{i=0}^{i=n-2} f(i) \end{bmatrix}_{n\times n}$$

TABLE 2 Transition probability matrices for nonhomogeneous rates

$$\mathbf{T}_1 = \begin{bmatrix} f_1(0) & f_1(1) & \ldots & f_1(n-2) & f_1(n-1) \\ 0 & f_2(0) & \ldots & f_2(n-3) & f_2(n-2) \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & \ldots & \ldots & f_{n-1}(0) & f_{n-1}(1) \\ 0 & 0 & \ldots & 0 & 1 \end{bmatrix}_{n\times n} \quad \mathbf{T}_2 = \begin{bmatrix} f_1(0) & \ldots & f_1(n-2) & f_1(n-1) \\ \vdots & \ddots & \ddots & \vdots \\ f_1(0) & \ldots & f_1(n-2) & f_1(n-1) \end{bmatrix}_{n\times n}$$

TABLE 3 Ordinary and additional observation matrices and cost matrix

$$\mathbf{O}_{1-2}=\begin{bmatrix}1&0\\\vdots&\vdots\\1&0\\0&1\end{bmatrix}_{n\times2}\quad \mathbf{E}=\begin{bmatrix}1-\epsilon&\epsilon\\\vdots&\vdots\\1-\epsilon&\epsilon\\\epsilon&1-\epsilon\\\vdots&\vdots\\\epsilon&1-\epsilon\\0&1\end{bmatrix}_{(n_1+n_2+1)\times2}\quad \mathbf{C}=\begin{bmatrix}0&C_r\\\vdots&\vdots\\0&C_r\\C_f&C_r+C_f\end{bmatrix}_{n\times2}$$

When $j-i=1$, Equation 3 is an exponential distribution. The probability of ending in state j can be expressed as

$$f_i(j-i)=\begin{cases}1-F_i(1) & (j=i<n)\\ F_i(j-i)-F_i[(j+1)-i] & (i+1\leqslant j<n)\\ F_i(j-i) & (j=n)\end{cases}\tag{4}$$

We derive the transition matrix under the two available actions in Table 2.

2.4 | Observation modeling

At each time step, the agent receives a perfect observation about the failure or survival of the component, independent on the action. Variable z has two possible values: $z=1$ indicates that the component has not failed (i.e., $s<n$), whereas $z=2$ indicates that it has failed ($s=n$) and the corresponding observation function $O_c(i,j)=\mathbb{P}[z=j|s=i,a=c]$ is reported in Table 3. When the monitoring system is adopted, the agent also receives an additional observation h at each step, related to the current condition of the component: $h=1$ suggests that the condition is intact, whereas $h=2$ that it is damaged. Observations are not necessarily perfect, and inaccuracy value ϵ, between 0 and 50%, defines the probability of an incorrect suggestion. The outcome is perfect if $\epsilon=0$, and it is unrelated to the component condition if $\epsilon=50\%$. We assume that the performance of the monitoring system is time invariant. Conversely, to model a degrading monitoring system, one should define additional states describing the condition of that system: that is certainly possible, but at higher computational cost. The corresponding observation function $E(i,j)=\mathbb{P}[h=j|s=i]$ is reported as a matrix in Table 3. In addition, we also assume this monitoring observation is not always available at each step, and the availability P defines the probability of the observation being available. The availability of the additional observation at any step is independent of those at other steps. Observations are always available if $P=1$, and they are never if $P=0$.

2.5 | Maintenance costs and discount factor

The downtime cost C_f refers to all costs related to a component being unavailable, including physical damages and service disruption. After time discretization, cost C_r is incurred for repairing the component, and additional cost C_f is incurred if the component fails, as reported in Table 3. The values of these costs are highly dependent on the specific problem: For example, in the case of bridges, downtime cost C_f can range from about \$5,000 per day for a pedestrian bridge[8] to \$220,000 per day for a highway bridge.[36] Moreover, we are not including any monitoring expense in the VoI analysis, so that the result can be compared with the overall monitoring cost. Consequently, the cost model with and without the monitoring systems is the same. If, on the contrary, a specific non-negligible cost for re-installing the monitoring system is incurred at any failure, this can be captured by differentiating the failure costs for the cases with and without the monitoring system, adding an increment for re-installation in the former case. But this setting is not explored in the numerical investigation in Section 4. Furthermore, costs postponed of time interval δt are discounted with factor $\gamma=\gamma_1^{\delta t}$ (where γ_1 is the discount factor per unit time).

2.6 | Reaction time and delayed maintenance actions

In the previous formulation, we have implicitly assumed that the agent can repair the component as soon as the data from monitoring systems are available. However, the maintenance actions may take time to be executed. To model this, we assume that, if the agent orders the repair of a component, the actual action will be executed only after time interval Δ_d. After the time discretization, an additional parameter r is introduced to model the "reaction time," and it defines

TABLE 4 Transition probability matrices combined with the reaction time[a]

$$\mathbf{T}_1^{(r)} = \begin{bmatrix} \mathbf{T}_{a=1} & 0_{n\times nr} \\ 0_{nr\times n} & \mathbf{M} \\ \mathbf{T}_{a=2} & 0_{n\times nr} \end{bmatrix}_{n(r+1)\times n(r+1)} \qquad \mathbf{T}_2^{(r)} = \begin{bmatrix} 0_{nr\times n} & \mathbf{Q} \\ \mathbf{T}_{a=2} & 0_{n\times nr} \end{bmatrix}_{n(r+1)\times n(r+1)}$$

where

$$\mathbf{M} = \begin{bmatrix} 0_{n\times n} & \mathbf{T}_{a=1} & 0_{n\times n} & \dots & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} & \mathbf{T}_{a=1} & \dots & 0_{n\times n} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & \dots & \mathbf{T}_{a=1} \end{bmatrix}_{n(r-1)\times nr} \qquad \mathbf{Q} = \begin{bmatrix} \mathbf{T}_{a=1} & 0_{n\times n} & \dots & 0_{n\times n} \\ 0_{n\times n} & \mathbf{T}_{a=1} & \dots & 0_{n\times n} \\ \vdots & \ddots & \ddots & \vdots \\ 0_{n\times n} & 0_{n\times n} & \dots & \mathbf{T}_{a=1} \end{bmatrix}_{nr\times nr}$$

[a] $0_{m\times v}$ is a zero matrix of dimension m by v, n is the total number of states for a component, and r is the reaction time.

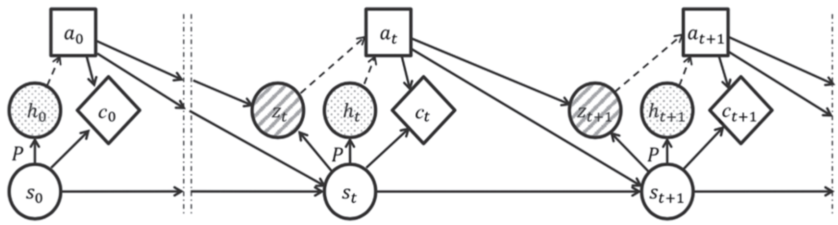

FIGURE 2 A decision graph for the SA model

the number of steps between the agent's order and the actual execution of a repair. So a component is repaired r steps after an agent takes a *Repair* action. If r is zero, the repair is instantaneous, and the transition and cost matrices are those in Tables 2 and 3. If r is positive, the transition and cost matrices are adjusted to model the delay in the repair. In the latter case, selecting the action *Repair* is equivalent to send an order of repair, which will be executed later. We introduce counting variable u that is zero until the agent orders a repair, and it grows of one unit per time step after that order (no matter what further action is selected), until it reaches value r: When this happens, the repair is executed, and the variable is set again to zero. The augmented states are summarized by variables $\{s, u\}$ in $s_+ = nu + s$, and the corresponding transition matrices are reported in Table 4. Actually, we want to give the possibility to repair a component immediately after it fails. To do so, we further modify the transition matrices in Table 4. For any augmented state s_+ when the physical state s is n, indicating a failure, the corresponding row s_+ in matrix $T_2^{(r)}$ for the augmented state is $[\mathbf{f}\ 0_{1\times n(r-1)}]$, where $\mathbf{f}$ is the last row in matrix T_2 as reported in Tables 1 and 2 (depending on the assumed deterioration model). Details on how the observation and cost matrices are affected by r are reported in Appendix C.

3 | VOI IN POMDPS

3.1 | Background and notation for POMDP framework

Long-term O&M under uncertainty can be modeled as a POMDP.[16,37] In this framework, the condition of a component evolves stochastically in time, following a Markov process, depending on the maintenance actions, and costs depend on the current condition and maintenance action. The condition state is not necessarily completely observable. Hence, the maintenance actions are based on a probabilistic distribution, defining the uncertain belief on the current condition state. By analyzing a POMDP, one can identify the optimal policy and the corresponding long-term expected cost.

A POMDP is defined as an 8-tuple $(S, A, Z, \mathbf{C}, \mathbf{T}, \mathbf{O}, \mathbf{b}_o, \gamma)$, where $S = \{1, 2, \dots, |S|\}$ and $A = \{1, 2, \dots, |A|\}$ are finite discrete sets of the condition states and available actions that the agent can select, respectively. Based on the current state $s \in S$, the agent pays a cost $C(s, a)$ after taking an action $a \in A$. The transition function $T_a(s, s')$ defines the probability of reaching the state $s' \in S$ from the previous state s after action a. The emission function $O_a(s, z)$ defines the conditional probability of observing one value z in set $Z = \{1, 2, \dots, |Z|\}$, given that the action a is taken in the previous step from state s, for a noisy and imperfect measure of the current state. $|S|$, $|A|$, and $|Z|$ are the number of possible state, actions, and observations, respectively. The size of matrices $\mathbf{T}$, $\mathbf{O}$, and $\mathbf{C}$ is $|S| \times |S| \times |A|$, $|S| \times |Z| \times |A|$, and $|S| \times |A|$, respectively.

A graphical model of a POMDP is reported in Figure 2 using the classical notation of dynamic Bayesian networks and influence diagrams from Barber's textbook.[38] Time is discretized in steps. The variables s_t, a_t, z_t, c_t represent state, action, observation, and cost at time t, respectively. The agent's goal is to minimize the total expected cost by selecting an optimal policy. In an infinite time horizon, the total cost is defined as $\mathbb{E}\left(\sum_{t=0}^{\infty}\gamma^t c_t\right)$, where γ is the discount factor for one step. The agent's knowledge about the current state s_t is represented by belief $\mathbf{b}_t$, where the ith entry is $b_t(i) = \mathbb{P}[s_t = i|z_1, \dots, z_t; a_0, \dots, a_{t-1}]$. The initial belief state is denoted as $\mathbf{b}_0$.

At time t, the agent takes an action a_t based on the belief b_t. Then, at time $t+1$, the agent receives an observation z_{t+1}, and the belief b_t is updated to b_{t+1} by using Bayes' rule:

$$b_{t+1}(s') = \frac{O_{a_t}(s', z_{t+1})\sum_{s=1}^{|S|}T_{a_t}(s,s')b_t(s)}{\sum_{s''=1}^{|S|}O_{a_t}(s'', z_{t+1})\sum_{s=1}^{|S|}T_{a_t}(s,s'')b_t(s)}. \tag{5}$$

The agent's behavior is defined by a policy π, which is a mapping from the domain of beliefs to actions. The optimal value V^*, that is, the cumulated expected discounted cost under the optimal policy π^*, is defined by the Bellman equation[39]:

$$V^*(\mathbf{b}_t) = \min_{a\in A}\left[\sum_{s=1}^{|S|}b_t(s)C(s,a) + \gamma\sum_{z=1}^{|Z|}P(z|\mathbf{b}_t, a)V^*(\mathbf{b}_{t+1})\right], \tag{6}$$

where the conditional probability $P(z|\mathbf{b}_t, a)$ is calculated as

$$P(z|\mathbf{b}_t, a) = \sum_{s'=1}^{|S|}O_a(s', z)\sum_{s=1}^{|S|}T_a(s,s')b_t(s), \tag{7}$$

and the optimal policy π^* can be identified using "argmin" instead of "min" in Equation 6. We refer to a sequential decision problem with the infinite horizon, so that the optimal policy is stationary (i.e., time-invariant), and the discount factor has to be strictly less than one, for the value to be finite.[18]

An efficient method called successive approximations of the reachable space under optimal policies (SARSOP)[40] is used to solve the POMDP, identifying the expected cost under the optimal policy, in the analysis of Sections 4 and 5.

3.2 | Integrating additional monitoring information and VoI assessment

Although the previous setting defines the default maintenance process, we now model the effect of the monitoring system. We consider an additional flow of observations $\{h_0, h_1, \dots\}$, one per time step, so that observation $h_t \in \{1, 2, \dots, |H|\}$ is collected at time t, and $|H|$ is the number of possible additional observations. The probabilistic relation between state s and observation h is modeled by emission function E, as defined in Section 2.4. At time t, belief $\mathbf{b}_t$ is updated to $\mathbf{b}'_t$ by processing observation h_t using Bayes' formula:

$$b'_t(s) = \frac{E(s, h_t)b_t(s)}{\sum_{s'=1}^{|S|}E(s', h_t)b_t(s')}. \tag{8}$$

However, as anticipated in Section 2.4, we assume the additional observations are available only with probability P at each step, for example, due to malfunction of the monitoring system occurring with probability $(1-P)$ at each step. To account for the possibility of not receiving any additional observation, we include the possibility of a dumb observation, and the adjusted emission probability E_P of dimension $|S| \times (|H|+1)$ is

$$\mathbf{E}_p = P[\mathbf{E} \quad 0_{|S|\times 1}] + (1-P)[0_{|S|\times|H|} \quad 1_{|S|\times 1}], \tag{9}$$

where $0_{m\times n}$ is a zero matrix and $1_{m\times n}$ a matrix of ones of size $m \times n$.

Ordinary observation z and additional observation h are combined in observation $y = \{z, h\}$, with emission matrix obtained as

$$\mathbf{O}_{a,P} = \mathbf{O}_a \times \mathbf{E}_P, \tag{10}$$

where $\mathbf{O}_a$ indicates the emission matrix corresponding to the action a and $\times$ means the cross product of two matrices' column. The full matrix $\mathbf{O}_P$ can be obtained by combining all possible actions, and its size is $|S| \times [|Z| \times (|H+1|)] \times |A|$. By using the POMDP formulation in Section 4.2, with emission matrix $\mathbf{O}_P$, we can identify the corresponding long-term optimal expected discounted cost, which we call V_w^*, to highlight that it is the cost *with* the monitoring system, whereas we can call $V_{w/o}^*$ the corresponding cost *without* the monitoring system, as computed in Section 4.2.

As shown in the decision graph of Figure 2, at time t_0, variable h_0 can also be observed. To include its contribution, we define the expected cost U^* as

$$U^*(\mathbf{b}_0) = \sum_{h_0=1}^{|H|+1} P(h_0|\mathbf{b}_0) V_w^*(\mathbf{b}_0), \tag{11}$$

where $P(h_0 \mid \mathbf{b}_0) = \sum_{s=1}^{|S|} E_P\,(s, h_0)\, b_0\,(s)$. This is the so-called stochastic allocation (SA) model, proposed by Memarzadeh and Pozzi.[16] The VoI analysis can be obtained by comparing the results of the cost analysis *with* and *without* the monitoring system as

$$VoI = V_{w/o}^* - U^*. \tag{12}$$

The resulting VoI is a function of the belief, the deterioration model, and the effectiveness of the repair action, embedded in the transition matrix, the costs and discount factor, the availability and precision of the monitoring system, and the agent's reaction time. The VoI, which is guaranteed to be non-negative as "information never hurts,"[2] quantifies the net overall economic benefit of the monitoring effort, as a discounted present value, not including the monitoring costs. A risk neutral agent should install the monitoring system if its cost is less than the VoI. Because of some constrains, if only a subset of the infrastructure components can be monitored, priorities should be assigned to those with higher information gains (i.e., difference between the VoI and monitoring cost).[41]

4 | PARAMETRIC INVESTIGATION

To introduce our analysis, we define a basic parameter setting for the O&M process of a deteriorating component. We have derived the parameters from the deterioration process of a wind turbine component.[16] Time is discretized in years, so $\delta t = 1$ year. The expected time to damage $\mathbb{E}[\Delta t_d]$ is 12.5 years, and expected further time to failure $\mathbb{E}[\Delta t_f]$ is three times higher and equal to 37.5 years, so that the expected time to failure $\mathbb{E}[\Delta t_T]$ is 50 years. The standard deviation of Δt_d is 12.5 years and that of Δt_f is 21.65 years. Specifically, p_d is an exponential distribution, with rate λ equal to 0.08 year^{-1} and p_f is a Gamma distribution, with same rate λ, and shape parameter 3. This deterioration process can be modeled, in the discretized version, with $n_1 = 1$ and $n_2 = 3$ (so that $n = 5$) and adopting the matrices in Table 1 for the homogeneous case. Cost of failure C_f is \$500K, and repair cost C_r is \$10K (i.e., 2% of C_f), whereas annual discount factor γ_1 is 95%. For a perfect annual condition monitoring (i.e., availability P is one and inaccuracy ϵ is zero), and an agent that can react immediately (i.e., reaction time r is zero), following the approach of previous section, the resulting VoI of the long-term monitoring is about \$10.4K, corresponding to the 2.08% of C_f and 104% of C_r.

4.1 | Varying the time to damage

With respect to the previous setting, we now investigate how changes in the assumed settings can affect the VoI. We repeat the computation of VoI for different parameter sets and plot the outcomes. For varying the assumptions on the deterioration, we modify λ, n, and n_1 and, consequently, the distributions p_d and p_f. For the homogeneous case, expected time to damage $\mathbb{E}[\Delta t_d]$ is $\frac{n_1}{\lambda}$. Figure 3a shows how p_d changes depending on n_1, when the rate λ is selected to keep $\mathbb{E}[\Delta t_d]$ equal to 50 years. The distribution is exponential when n_1 is one and Gamma for a higher value of n_1, and it becomes narrower when n_1 grows. The corresponding standard deviation and coefficient of variation (that is equal to $\frac{1}{\sqrt{n_1}}$) decrease with n_1 and are reported in Figure 3b. Identical graphs describe the relation between p_f and n_2. By selecting appropriate values of n_1 and n_2, we can model the uncertainty in predicting time to damage and failure.

4.2 | VoI versus measure availability

The first parametric analysis investigates the relation between the availability P and VoI, for different deterioration models. Costs, discount factor, and inaccuracy ($\epsilon = 0$, modeling a perfect sensor) are the same as those in the default setting. We start considering four deterioration models, with time to failure $\mathbb{E}[\Delta t_T]$ equal to 30, 50, 100, and 200 years, respectively. For all these models, n_1 is 1, and n_2 is 3, so the expected time that an unrepaired component stays undamaged is one third of the expected time that it stays damaged. Distributions p_d , p_f, and p_T are plotted in Figure 4d, for $\mathbb{E}[\Delta t_T]$ equal

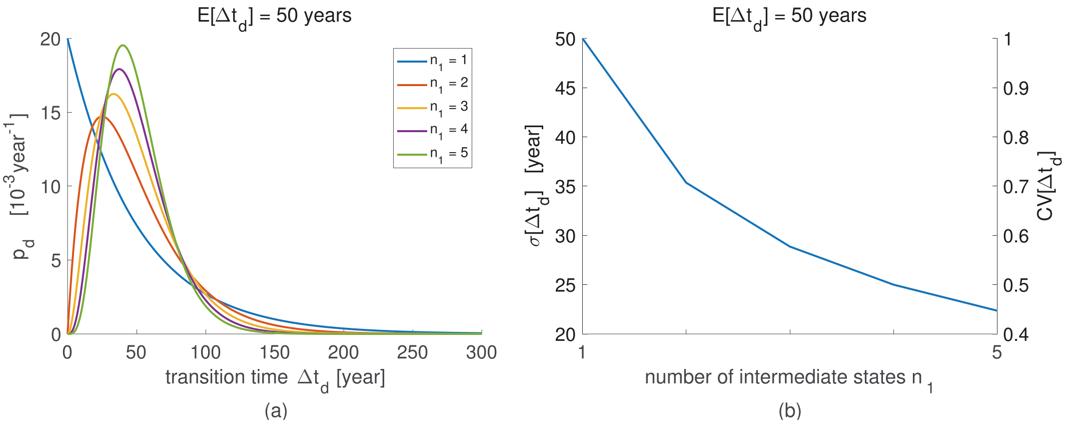

FIGURE 3 (a) Probability of time to damage depending on n_1. (b) Corresponding standard deviation and coefficient of variation

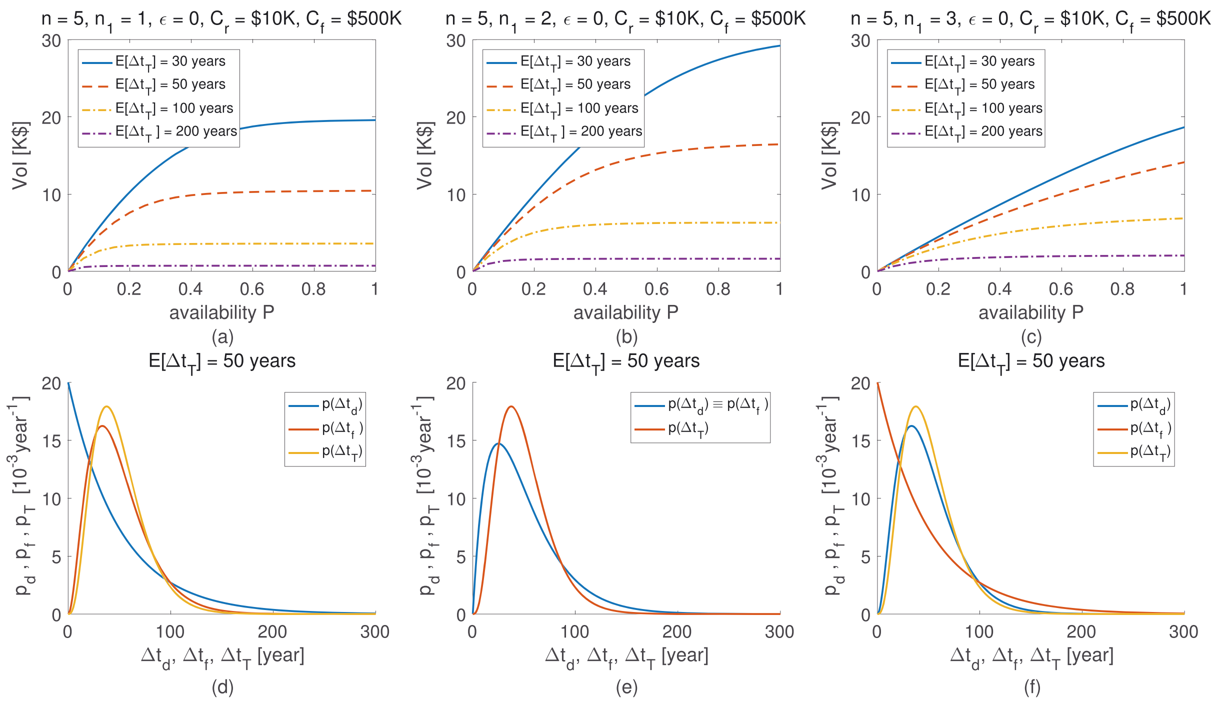

FIGURE 4 VoI versus measure availability under different deterioration modeling when (a) $n_1 = 1$, (b) $n_1 = 2$, and (c) $n_1 = 3$

to 50 years. Figure 4a shows how the VoI varies with availability P. For P equal to zero, no measure is available, and the corresponding VoI is zero. VoI is monotonically increasing with P as, according to the principle that "information never hurts,"[2] any increment in the probability of receiving observations gives a non-negative increment of VoI. In the analyzed problem, the second derivative of VoI with respect to P is negative, meaning that the benefit of a given increment in the availability is lower if P is higher. This indicates that the VoI is submodular in this setting[42]: A piece of information is highly valuable when few other pieces are available and less valuable when many other pieces are already available. Also, the graph shows that the VoI is higher when the deterioration is faster (i.e., when $\mathbb{E}[\Delta t_T]$ is lower) as, in that case, the component has to be repaired more frequently and the present cost of the maintenance process is higher. For $\mathbb{E}[\Delta t_T]$ equal to 30 years, the VoI is about \$19.6K. Moreover, we notice that the VoI is approximately constant above a specific value of P: This value is lower when the deterioration is slower. For example, when $\mathbb{E}[\Delta t_T]$ is 100 years, the VoI is almost constant for any P higher than 20%: This indicates that inspecting the component more frequently than, in the expected sense, every $\frac{1}{20\%} = 5$ steps (i.e., 5 years) does not give any significant additional value. If the deterioration is faster, there is an additional value in observing the condition more frequently.

We repeat the analysis by keeping all parameters, including n, as before, except for n_1 that is now 2 (so, n_2 is also 2). The corresponding distributions p_d , p_f, and p_T are plotted in Figure 4e, and we note that p_T is identical to the previous

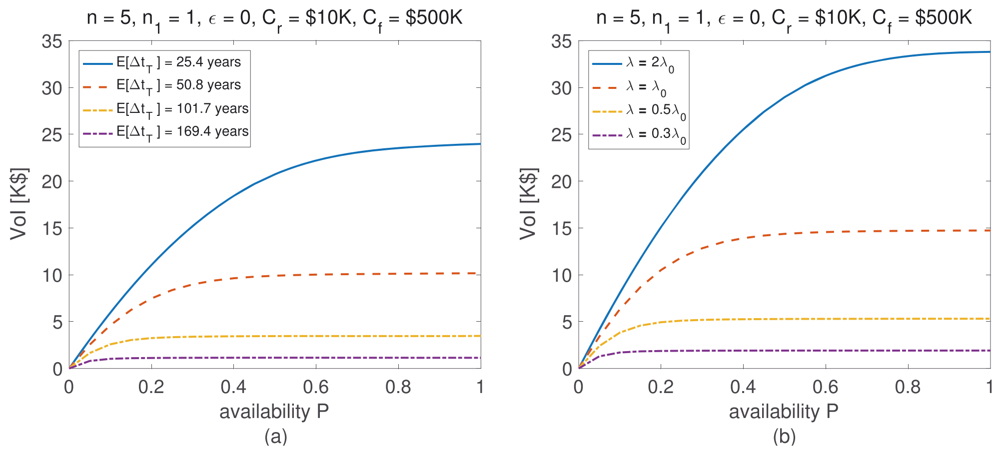

FIGURE 5 VoI versus measure availability under (a) homogeneous rates and (b) nonhomogeneous rates

case. Now, the unrepaired component spends an equal time (in the expected sense) in the intact and damaged conditions. Figure 4b illustrates the impact of this change on the VoI. The VoI is higher, as the detection of a damaged condition is now more informative for predicting the time to future failure.

Figure 4c,f refers to a further variation of the same analysis, where $n_1 = 3$ (so, $n_2 = 1$). The unrepaired component spends a period in the intact condition three times shorter than that spent in the damaged condition. The VoI is now lower than before when the deterioration is fast (for $\mathbb{E}[\Delta t_T]$ equal to 30 and 50 years) because, in that case, there is a significant chance that an intact component fails within 1 year, without the monitoring system detecting any damage, so the agent has to adopt a conservative policy even when using a monitoring system. However, for a slower deterioration (for $\mathbb{E}[\Delta t_T]$ equal to 100 and 200 years), the VoI is higher than before for the same argument outlined above.

By comparing Figure 4a–c, we also note that the VoI is less flat when n_2 (i.e., the period spent in the damaged condition) becomes smaller as, in that case, the penalty for lacking full availability is higher. Also, we note that, for a given value of $\mathbb{E}[\Delta t_T]$ and n, the optimal open-loop policy without the monitoring system is invariant with respect to n_1, and the difference among Figure 4a–c is only due to the difference in the closed-loop cost U^*. A variation of this setting is reported in Appendix A.

In Figure 5, we compare the relation between VoI and measure availability P for the homogeneous and nonhomogeneous case. Figure 5a,b refers to four values of $\mathbb{E}[\Delta t_T]$: 25.4, 50.8, 101.7, and 169.4 years. Figure 5a plots the corresponding curves for the homogeneous case, when $n_1 = 1$ and $n = 5$. In Figure 5b, all parameters are the same, except for the nonhomogeneous rates $\lambda = [\lambda_1 \lambda_2 \lambda_3 \lambda_4]$, where λ_2, λ_3, and λ_4 are equal to 1.6, 2, and 2.4 times λ_1. If λ_0 is the rate when $\mathbb{E}[\Delta t_T]$ = 50.8 years, the corresponding values of λ in each case are reported in Figure 5b. Again, when the expected damaged period is shorter, the VoI is higher.

4.3 | VoI versus measure accuracy

In the second campaign, we fix the deterioration rate (by selecting $\mathbb{E}[\Delta t_T]$, n_1, and n_2 equal to 50 years, 1 and 3, respectively), and we explore the impact of the monitoring inaccuracy. Figure 6a shows the VoI as a function of inaccuracy ϵ for three availability values: P equal to 30%, 10%, and full availability. As expected, the VoI is monotonically decreasing with inaccuracy, again as “information never hurts,”[2] and it is zero when inaccuracy is 50%, as in that case the monitoring observations are independent of state and condition.

This analysis illustrates the trade-off between the availability and accuracy of measures. In this example, the VoI of perfect measure available with 30% probability is nearly equivalent to that of measures with 20% inaccuracy, but always available. Availability and inaccuracy of measures define the quality of the monitoring effort: Depending on the parameters of the O&M process, VoI can be more sensitive to availability or to inaccuracy.

4.4 | VoI versus damage predictability

In the third analysis, we investigate how the VoI depends on the predictability of the deterioration process. To do so, we fix both $\mathbb{E}[\Delta t_d]$ and $\mathbb{E}[\Delta t_f]$ to 50 years. By increasing the number of intermediate states $n_1 = n_2$ from 1 to 8, in the homogeneous case, we reduce the standard deviation σ of Δt_d and Δt_f from about 40 years to about 14 years, according to the principle illustrated above. Figure 6b shows how the VoI monotonically increases with σ under three availability values of perfect measures. This trend follows the intuitive relation between the prior knowledge and VoI: When the deterioration process can be accurately predicted even without the monitoring support, the benefit of additional measures is low, because the open-loop policy based on the prior prediction is effective enough. This indicates that the prior information and that provided by the monitoring system are submodular, in this setting. However, this is not necessarily the case in all settings: Sometimes, the benefit of the monitoring system is high only in combination with the accurate prior information.[43]

4.5 | VoI versus reaction time

Figure 6c illustrates how the VoI changes with the reaction time r. Following the numerical approach in Section 2.6, the number of states in the POMDP goes higher up to 65 in some analyses. As expected, the VoI is monotonically decreasing with r, as a higher value of r poses more stringent constraints on the closed-loop policy, whereas the cost related to the open-loop policy is invariant when $r\delta t < \Delta_d$. Under the fully available monitoring (i.e., when $P = 1$), the VoI does not decrease when r is up to 3 years, indicating that, as soon as the damaged condition is perfectly detected, the agent can wait some years before repair. When the availability is less than one, the VoI is strictly decreasing even for the small reaction time. In all cases, the VoI falls to zero when the reaction time is more than 10 years.

If the reaction time is high, the agent cannot react promptly to the information provided by the monitoring system. In that case, even if the agent asks for repair as soon as she is informed that the component is damaged, there is a high chance that that action will not be executed before the failure happens. Being aware of this, the agent has to adopt a conservative close-loop policy, managing the component without relying much on the collected measures.

4.6 | VoI versus repair cost

Figure 6d reports the VoI as a function of the repair cost C_r when the failure cost C_f is \$500K. This is the first non-monotonic relation we encounter. If C_r is zero, the agent always repairs, and the VoI is zero, so the monitoring system is useless. For a low cost C_r, the higher C_r, the higher the VoI. However, after a maximum value, the VoI is decreasing with C_r, and it is zero when C_r is so high that it is never convenient to repair at that point, again, the monitoring system becomes useless. For the investigated parameters, the maximum VoI of a fully available system occurs when C_r is about \$40K, and it is about \$13.6K.

4.7 | VoI versus discount factor

We also investigate the effect of the discount factor γ_1 on the VoI. For the selected parameters, the VoI is almost negligible when γ_1 is less than 75%. This happens because the process starts with certainty of the initial condition being intact: Given the relatively small deterioration rate, an agent not caring much about future costs gets no value from collecting measures. Above that lower bound, as shown in Figure 6e, the relation between the VoI and the discount factor is almost linear in the log-scale, up to 97%. A high discount factor in the infinite-horizon case acts similarly to a distant finite horizon. Hence, by considering a higher factor, we are considering an equivalent longer (undiscounted) management period. Following this interpretation, we can read the monotonicity as indicating that the benefit of a monitoring system grows with the duration of the management process. When the factor goes to one, the equivalent duration is infinite and so is the VoI. As stated before, the analysis is based on the implicit assumption that no costs are due for renovating the monitoring system after failure or repair of the component.

4.8 | VoI versus belief

Although previous analyses refer to an intact component, we now investigate how the VoI is affected by the agent's belief on the component's current condition. During the O&M process, the agent can either know that the component is failed or know it can be intact or damaged, but not failed. After discretization, this means that the discrete belief distribution on the n states assigns 100% to the last entry, or the last entry is zero. In the latter case, the prediction of the time to damage and failure is encoded in a specific distribution on the first $(n - 1)$ entries: the first n_1 referring to the intact condition and

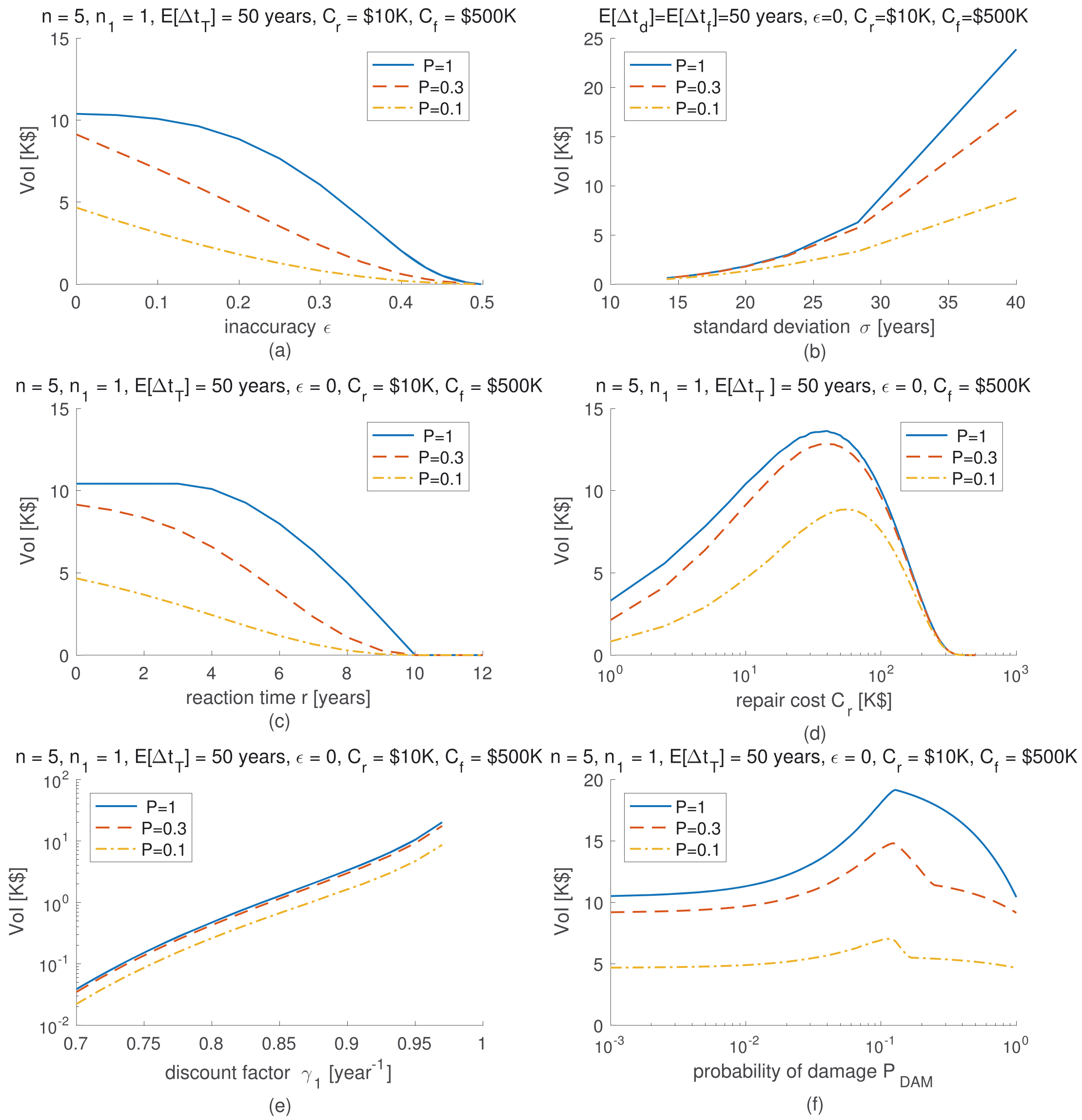

FIGURE 6 VoI versus (a) inaccuracy of measure, (b) standard deviation of Δt_d and Δt_f, (c) reaction time of repair, (d) repair cost, (e) discount factor, and (f) belief about damage condition

the rest to the damaged one. We consider $n_1 = 1$ and $n_3 = 3$, so the first entry is the probability of the intact condition, and we assume that the agent assigns a uniform probability to the three states modeling the damaged condition. Hence, the initial belief vector $\mathbf{b}_0$ has form $\left[1 - P_{DAM} \quad \frac{P_{DAM}}{3} \quad \frac{P_{DAM}}{3} \quad \frac{P_{DAM}}{3} \quad 0\right]$, where parameter P_{DAM} indicates the probability of damage. When P_{DAM} is zero, the initial belief describes an intact component, as described in the previous campaigns.

Figure 6f reports the VoI as a function of P_{DAM} under different availabilities. The VoI is not monotonically increasing with P_{DAM}: It is maximum around 12%, which indicates that, for the corresponding belief, the uncertainty between repair and do-nothing is highest, and monitoring information have a high impact. For higher values of P_{DAM}, the agent's prior decision is to repair the component, and the impact of the additional measure is relatively lower. We plot the relation VoI versus P_{DAM} in log-scale, despite this can mask piecewise linearity of this relation. Graphs similar to Figure 6f are also reported by Memarzadeh and Pozzi,[16] for a different setting.

4.9 | General properties and invariance of the VoI

In this section, we summarize the relation between VoI and properties of the O&M and monitoring process. VoI is monotonically increasing with measure availability, and monotonically decreasing with measure inaccuracy, according to the

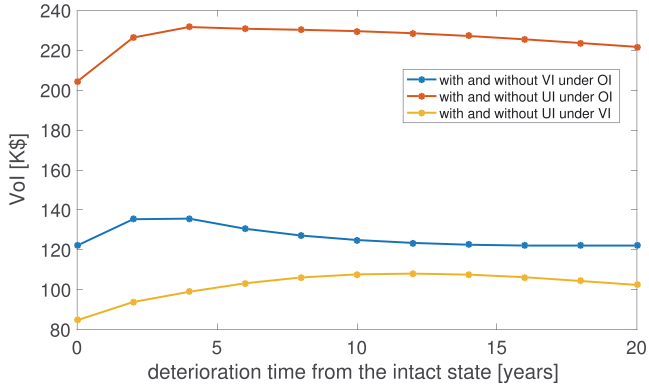

FIGURE 7 VoI versus different initial states

"information never hurts" principle,[2] and with the agent's reaction time. These properties can be proven analytically, while other properties hold in many applications, but one can find cases when they do not hold. Examples of such properties are the following: generally, the VoI tends to increase with the prior uncertainty in the component deterioration and with the discount factor.

One simple invariant property of the VoI is due to linearity of expectation: For example, if all costs (i.e., in the investigated setting, both C_r and C_f) are scaled by factor α, then the VoI is also scaled by the same factor α. Another approximate invariant property is related to the time discretization, discussed in Appendix B.

5 | VOI ANALYSIS ON A BRIDGE MODEL

Our VoI computation framework can be applied to a broad range of infrastructure models. In this section, we illustrate how to apply it to a bridge model derived from Corotis et al.[44] In that paper, a POMDP is introduced to determine optimal short-time inspection strategies. We take that bridge model with a few adjustments as described in Appendix D, and we compute the optimal maintenance cost and VoI in the following settings. We consider a baseline scenario when the bridge starts from an intact condition state under ordinary inspection (OI), and we assess the long-term cost of this scenario. The optimal expected long-term maintenance cost under no inspection, ordinary inspection (OI), visual inspection (VI), VI and ultrasonic inspection (UI), and perfect inspection is about \$2,490K, \$2,320K, \$2,200K, \$2,110K, and \$2,070K, respectively. Then we compare the baseline scenario with other scenarios under VI or UI and assess the VoI. The VoI is the reduction of optimal costs by using a more precise inspection strategy. For example, we assess the value of performing VI in additional respect to OI. We also assess how the VoI changes depending on the initial beliefs. These beliefs are selected by tracking an intact component deteriorating in time under doing nothing and recording the belief every time step. Figure 7 shows how the VoI changes as the initial state deteriorates under different scenarios. We can see that the VoI is lowest for the bridge is intact; this is because the inspection is less beneficial when the bridge is in intact condition. The VoI is not monotonically increasing as the initial state deteriorates, and its peak value shows where the uncertainty between two maintenance actions is the highest, as in the case described in Section 4.8.

6 | CONCLUSIONS

By modeling the long-term O&M of infrastructure components as POMDPs, this paper has illustrated how the benefit of integrating monitoring systems depends on key features of the component deterioration, the economic setting, and the performance of the monitoring system itself: The VoI arises from a complicate interplay among those features. Although it is intuitive that the VoI increases or decreases under some variations of the setting, our framework allows computing the specific sensitivity to all variations, including those with less predictable effects and to quantify the specific change of VoI under a finite change in the setting. For example, as shown in Section 4, detection of damage has a high value if a short damage period precedes any failure event, so that the detection indicates an incipient failure, and the agent can react promptly. However, in that context, the penalty for inaccuracy or imprecision of the monitoring system, or for the delayed maintenance action, is also high. Our framework allows for a detailed quantitative analysis of such circumstances. It also allows for investigating the trade-off between availability and inaccuracy: In the example of Section 4.2, the VoI hardly changes when the inaccuracy varies from 0% to 15% under full availability, and the VoI of perfect measure available with 30% probability is nearly equivalent to that of measures with 20% inaccuracy, but always available. The analysis can be adapted to model more complicated O&M processes than that taken as an example, and it can be the base for developing approximate heuristics for estimating the VoI and identifying conditions when it is high, to optimize the design and integration of monitoring systems.

The POMDP-based assessment of the VoI presented in this paper assumes that each component behaves independently, and information collected on one component is only relevant for the O&M of that component. Including system-level interaction and statistical interdependence among components poses highly computational challenges: Attempts to address these challenges can be found in Memarzadeh et al.[45] and Luque and Straub.[46]

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APPENDIX A: COMPARISON BETWEEN DIFFERENT TRANSITION MATRICES

In this appendix, we compare the parametric investigation using the transition matrix in Table 1 with that in previous work.[15,16] In Figure A1, we provide an example of a slight variation of the setting in Section 4, plotted in Figure 4a–c. In that setting, the transition matrix was that of Table 1, allowing for the occurrence of failure in one step from the intact condition. In this setting, the transition from the intact to failed condition has to pass at least one step through the damage. Thus, there is no danger of overlooking the symptoms of damage: The perfectly accurate and always available monitoring system will detect the damage before failure, and the agent will be able to react timely. Hence, in this setting, the shorter the damage period, the higher the VoI (as the expected time to failure $\mathbb{E}[\Delta t_T]$ is the same for all graphs). In Figure A1c the fast deterioration case (i.e., when $\mathbb{E}[\Delta t_T] = 30$ years) shows a non-submodular curve, with the positive second derivative: The same increment of availability gives a higher increment of VoI when the availability is already high.

APPENDIX B: INVARIANCE OF COSTS AND TIME DISCRETIZATION

To investigate the effect of the time discretization of continuous O&M processes, we repeat the analysis by varying the discretized period δt from 0.1 to 6 years. Depending on δt, the following quantities are adjusted: The transition matrix follows Table 1, where function f depends on δt, the discount factor is $\gamma_1^{\delta t}$, the availability $P = 1 - (1 - P_1)^{\delta t}$ where P_1 is the availability per year. We left cost matrix $\mathbf{C}$ invariant respect to δt.

Figure B1 shows how the expected costs, $V^*_{w/o}$ and U^*, and the VoI vary as a function of δt from 0.1 up to 6 years. Parameters of the analysis are reported in the graph caption. The costs and VoI are almost invariant respect to δt, even if, properly, large variations of δt can affect costs: For an infinitely large δt, for example, the cost of failure would be infinitely postponed, and discounted, so the present cost would consequently vanish.

APPENDIX C: EFFECTS OF REACTION TIME ON COST AND OBSERVATIONS

In this appendix, we describe how to define the cost and observation matrices when the agent's reaction time is more than one step ($r > 0$). When r is positive, for any augmented state s_+ in variables $\{s, u\}$ where s is the actual physical condition state and u is the counting number, the costs only depend on the current physical condition and the selected action, no matter what the value of current counting number u is. We define the cost matrix $\mathbf{C}^{(r)}$ when $r > 0$, with size of $|S_+| \times |A|$ where $|S_+| = |S|(r+1)$. The sequence of rows in $\mathbf{C}^{(r)}$ starts with the physical state s in variables $\{s, u\}$. If $\mathbf{C}$ is the cost matrix without delayed actions, as reported in Table 3, and the ith row of $\mathbf{C}$ is denoted as $\mathbf{C}_i$, then the cost matrix is defined as $\mathbf{C}^{(r)}_{i+jn} = \mathbf{C}_i$, for $j = 1, 2, \ldots, r$.

Similarly, the observations also only depend on the current physical condition, not related to the counting number u. When $r > 0$, the additional observation matrix $\mathbf{E}^{(r)}$, with size of $|S_+| \times |A|$, is defined as $\mathbf{E}^{(r)}_{i+jn} = \mathbf{E}_i$, for $j = 1, 2, \ldots, r$,

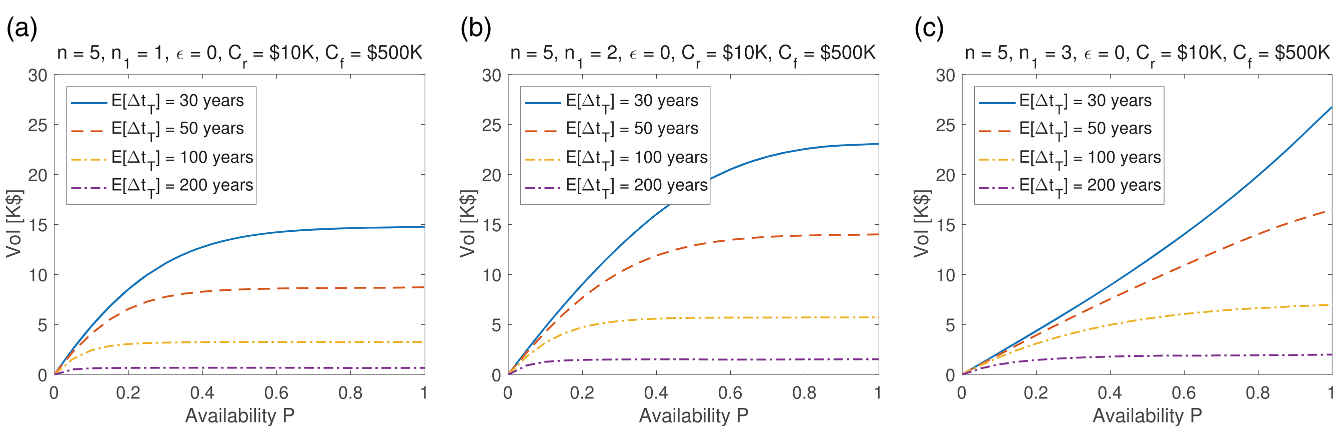

FIGURE A1 VoI versus measure availability under three types of condition states when (a) $n_1 = 1$, (b) $n_1 = 2$, and (c) $n_1 = 3$

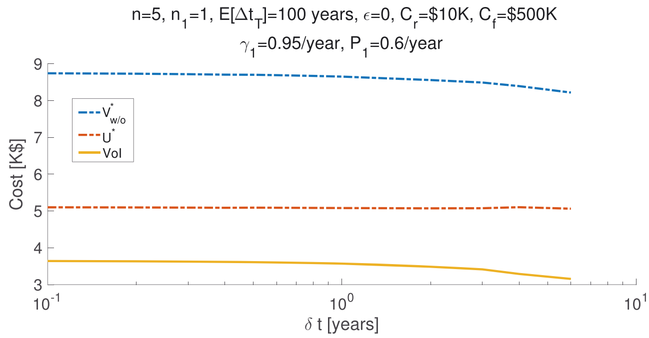

FIGURE B1 The costs $V^*_{w/o}$, U^* and VoI versus δt

where $\mathbf{E}_i$ is the ith row of $\mathbf{E}$ reported in Table 3. In addition, the ordinary observation $\mathbf{O}_a^{(r)}$ under any action a can be obtained by the same implementation on $\mathbf{E}^{(r)}$.

APPENDIX D: PARAMETER SETTING FOR THE BRIDGE EXAMPLE

Corotis et al.[44] modeled the deterioration, inspection, and maintenance process of a one-lane two-girder high way bridge as a POMDP. In this model, the performance of the bridge is described using five state ($n = 5$): $\leqslant 5\%$ ($s = 1$), $> 5\%$ but $\leqslant 15\%$ ($s = 2$), $> 15\%$, but $\leqslant 25\%$ ($s = 3$), $> 25\%$($s = 4$) deterioration of the initial strength of the girders, and the bridge failure ($s = 5$). It is assumed that initial strengths of the two girders have identical probability distribution, and that the loads on and strength deterioration of the two girders are perfectly correlated. The transition matrices under four maintenance actions are shown in Table D1, including do-nothing ($a = 1$), cleaning and repairing the corrosion surfaces ($a = 2$), repainting and strengthening the girders ($a = 3$), and extensive repair ($a = 4$).

TABLE D1 Transition probability matrices under four maintenance actions in the bridge example

$$\mathbf{T}_1 = \begin{bmatrix} 0.8 & 0.13 & 0.02 & 0 & 0.05 \\ 0 & 0.7 & 0.17 & 0.05 & 0.08 \\ 0 & 0 & 0.75 & 0.15 & 0.1 \\ 0 & 0 & 0 & 0.6 & 0.4 \\ 0 & 0 & 0 & 0 & 1.0 \end{bmatrix} \quad \mathbf{T}_2 = \begin{bmatrix} 0.8 & 0.13 & 0.02 & 0 & 0.05 \\ 0 & 0.8 & 0.1 & 0.02 & 0.08 \\ 0 & 0 & 0.8 & 0.1 & 0.1 \\ 0 & 0 & 0 & 0.6 & 0.4 \\ 0 & 0 & 0 & 0 & 1.0 \end{bmatrix}$$

$$\mathbf{T}_3 = \begin{bmatrix} 0.8 & 0.13 & 0.02 & 0 & 0.05 \\ 0.19 & 0.65 & 0.08 & 0.02 & 0.06 \\ 0.1 & 0.2 & 0.56 & 0.08 & 0.06 \\ 0 & 0.1 & 0.25 & 0.55 & 0.1 \\ 0 & 0 & 0 & 0 & 1.0 \end{bmatrix} \quad \mathbf{T}_4 = \begin{bmatrix} 0.8 & 0.13 & 0.02 & 0 & 0.05 \\ 0.8 & 0.13 & 0.02 & 0 & 0.05 \\ 0.8 & 0.13 & 0.02 & 0 & 0.05 \\ 0.8 & 0.13 & 0.02 & 0 & 0.05 \\ 0.8 & 0.13 & 0.02 & 0 & 0.05 \end{bmatrix}$$

TABLE D2 VI and UI observation matrices in the bridge example

$$\mathbf{O}_{1-4}^{VI} = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.2 & 0.6 & 0.2 \\ 0.05 & 0.7 & 0.25 \\ 0 & 0.3 & 0.7 \\ 0 & 0 & 1.0 \end{bmatrix} \quad \mathbf{O}_{1-4}^{UI} = \begin{bmatrix} 0.9 & 0.1 & 0 & 0 & 0 \\ 0.05 & 0.9 & 0.05 & 0 & 0 \\ 0 & 0.05 & 0.9 & 0.05 & 0 \\ 0 & 0 & 0.05 & 0.95 & 0 \\ 0 & 0 & 0 & 0 & 1.0 \end{bmatrix}$$

The duration of the time step is 2 years, and we assume the bridge is inspected at each time step. The available inspection strategies in this model include no inspection, visual inspection (VI), and ultrasonic inspection (UI). If no inspection is taken, no observations is obtained. The visual inspection gives three possible observations including "good," "fair," and "poor," whereas the ultrasonic one can distinguish each condition state but with some measurement errors. The emission matrices for visual and ultrasonic observations are shown in Table D2. In addition, we consider two more types of inspection. One is the ordinary inspection (OI), which can only detect the failure of the bridge and the other is the perfect inspection, which can detect the current condition state exactly. We assume all the inspections are free of cost, and the cost matrix for the four maintenance actions is defined in Table D3. The discount factor is 0.95 per year.

TABLE D3 Cost matrix in the bridge example. (unit: K$)

$$\mathbf{C} = \begin{bmatrix} 0 & 5 & 25 & 40 \\ 0 & 8 & 80 & 120 \\ 0 & 15 & 100 & 550 \\ 300 & 320 & 450 & 800 \\ 2000 & 2050 & 2500 & 4000 \end{bmatrix}$$