An LFT Approach for Distributed Control of Nonstationary LPV Systems

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***Abstract*— We develop an operator theoretic framework for heterogeneous, discrete-time, nonstationary linear parameter-varying systems in linear fractional representation. These systems are interconnected over arbitrary directed graphs and subjected to a communication latency of one sampling period. We give results, based on the ℓ_2-induced norm performance measure, for analysis and synthesis of distributed controllers that have the same structure as the plant. The analysis and synthesis conditions are convex, but infinite dimensional in general. They become finite dimensional in the case of distributed eventually time-periodic systems over finite graphs.**

I. Introduction

This paper is on the distributed control of heterogeneous subsystems interconnected over arbitrary directed graphs. Each subsystem has its own sensing and actuating capabilities, and is modeled as a discrete-time, nonstationary linear parameter-varying (NSLPV) system [1]–[4], formulated in a linear fractional transformation (LFT) framework. This class of systems generalizes stationary linear parameter-varying (LPV) systems in the sense that the state-space matrix-valued functions can have an explicit dependence on time, in addition to the scheduling parameters. Data transfer between the subsystems is subject to a one-step time-delay. We aim at constructing a feedback distributed NSLPV controller, with the same topological structure as the plant, that renders the closed-loop system asymptotically stable, and further guarantees some ℓ_2-gain performance level, i.e., an upper bound on the ℓ_2-induced norm of the closed-loop input-output map, for all permissible parameter trajectories.

A survey of the literature reveals numerous works on distributed control. The works of [5]–[8] deal with homogeneous, i.e., identical, subsystems, whereas, the works of [9]–[14] treat heterogeneous subsystems. The classification can also be based on the complexity of the interconnection structure. For instance, [6], [10] consider highly structured networks, whereas, [9], [11] study arbitrary networks. The aforementioned references assume either linear time-invariant or linear time-varying models for the subsystems. Other works [15]–[20] address the problem of distributed control for stationary LPV interconnected subsystems.

In this work, we focus on subsystems with NSLPV models. We generalize the operator theoretic description of [14], along with the analysis results on well-posedness, stability, and performance. This framework permits the representation of the distributed system in a way reminiscent of standard state-space systems, thus allowing an immediate extension of standard results such as [21]–[23]. Additionally, this work generalizes the synthesis results of [1] to the context of distributed systems. The derived analysis and synthesis conditions are convex in nature, yet they are, in general, infinite dimensional. These conditions become finite dimensional in the case of eventually time-periodic subsystems interconnected over a finite graph, i.e., when the a priori known time-varying terms in the state-space matrices are aperiodic for an initial amount of time and then become periodic afterwards, and the interconnection graph has a finite number of vertices and edges.

In Section II, we gather the relevant notations. Then, in Section III, we present the operator based description of the systems. We dedicate Sections IV and V for the development of the analysis and the synthesis results, respectively.

II. Notations

We denote by $\mathbb{N}_0$, $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{R}$ the sets of nonnegative integers, positive integers, integers, and real numbers, respectively. The notation $\operatorname{diag}(M_i)$ refers to the block-diagonal augmentation of the sequence of matrices M_i.

$\mathcal{G}(V, E)$ denotes a directed graph with set of vertices V and set of directed edges E. We restrict our discussion to directed graphs with a countable number of vertices. We use the ordered-pair (i, j) to represent an element of E directed from vertex $i \in V$ to vertex $j \in V$. We define $m(k)$ and $p(k)$ as the indegree and outdegree of vertex $k \in V$. The vertex degree $v(k) = \max\{m(k), p(k)\}$. We assume that $v(k)$ is uniformly bounded and define $s(\mathcal{G}) = \max_{k\in V}\{v(k)\}$. A graph is said to be d-regular if, for each $k \in V$, $m(k)=p(k)=d$. An arbitrary directed graph can be turned into an $s(\mathcal{G})$-regular graph by the addition, when necessary, of virtual edges and/or nodes. So, without loss of generality, we assume that the graph structure under consideration is d-regular. Thus, we can define d permutations, namely, $\rho_1, \ldots, \rho_d$, of the set of vertices according to the interconnections. See [14] for more details. The left diagram of Fig. 1 shows a directed graph with 4 vertices, 5 edges, and $s(\mathcal{G}) = 2$. The right diagram shows the same graph rendered 2-regular after the addition of the needed virtual edges, along with the permutations ρ_1 and ρ_2.

Let J be a vector space. We say that a linear mapping $P : J \to J$ has an algebraic inverse on J if there exists a linear mapping $P^{-1} : J \to J$ such that $PP^{-1} = P^{-1}P = I$, where I denotes the identity map on J. Now, let H, W, and F be Hilbert spaces. We denote by $\langle\cdot,\cdot\rangle_H$ and $\|\cdot\|_H$, the inner product and the norm associated with H, respectively. We drop the subscript when the corresponding Hilbert space is clear from context. The Hilbert space direct sum of H

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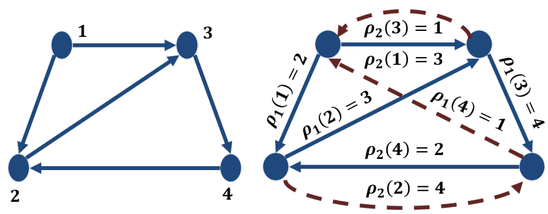

Fig. 1. Left: Example of a directed graph. Right: Directed graph rendered regular via the addition of the virtual edges (red, dashed arrows).

and W is written as $H \oplus W$. We represent the space of bounded linear operators mapping H to F by $\mathcal{L}(H, F)$, a notation we simplify to $\mathcal{L}(H)$ when F is equal to H. The space of bounded linear causal operators mapping H to F is denoted by $\mathcal{L}_c(H, F)$. We define the algebra $\mathcal{L}_e(H, F)$ as the space of linear causal operators mapping H to F and equipped with the point-wise topology with respect to the standard matrix representation. We also write $\mathcal{L}_c(H)$ and $\mathcal{L}_e(H)$ when $H = F$. Let X be an element in $\mathcal{L}(H, F)$. The notation $\|X\|$ refers to the H to F induced norm of X. Operator X^* denotes the adjoint of X. A self-adjoint operator $X \in \mathcal{L}(H)$ is said to be negative definite if, for all $x \in H$, there exists $\alpha > 0$ such that $\langle x, Xx \rangle < -\alpha \|x\|^2$.

Given an integer sequence $n : (t, k) \in \mathbb{Z} \times V \to n(t, k) \in \mathbb{N}_0$, we define $\ell\left(\{\mathbb{R}^{n(t,k)}\}\right)$, or simply ℓ, as the vector space of mappings $w : (t, k) \in \mathbb{Z} \times V \to w(t, k) \in \mathbb{R}^{n(t,k)}$. We define the Hilbert space ℓ_2 as the subspace of ℓ that consists of mappings w having a finite norm $\|w\|_2 := \sqrt{\sum_{(t,k)} w(t,k)^T w(t,k)}$. We also define ℓ_{2e} as the subset of ℓ with elements w satisfying $\sum_k w(t,k)^T w(t,k) < \infty$, for each $t \in \mathbb{Z}$. We use the notations ℓ, ℓ_2, ℓ_{2e} irrespective of the associated integer sequence $n(t, k)$.

III. Operator Based Description

We represent the interconnection structure of the distributed system using a directed graph, where each subsystem $G^{(k)}$ corresponds to a vertex $k \in V$ and the interconnections between the subsystems are described by the directed edges in E. Each subsystem is modeled as a discrete-time NSLPV system, with state-space equations of the form

$$\begin{bmatrix} x(t+1) \\ z(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A(\delta(t), t) & B_1(\delta(t), t) & B_2(\delta(t), t) \\ C_1(\delta(t), t) & D_{11}(\delta(t), t) & D_{12}(\delta(t), t) \\ C_2(\delta(t), t) & D_{21}(\delta(t), t) & D_{22}(\delta(t), t) \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix},$$

for $w \in \ell_2$. The variable t denotes discrete time, and $\delta(t) = (\delta_1(t), \ldots, \delta_r(t))$ is a vector of real scalar parameters. $x(t)$ is the state vector, $w(t)$ and $z(t)$ correspond to the exogenous disturbances and errors, respectively, and $u(t)$ and $y(t)$ denote the control inputs and the output measurements. The state-space matrix-valued functions are known a priori, and are assumed to be uniformly bounded. To formulate the model in an LFT framework, we assume that the dependence of these functions on the parameters is rational. This assumption is not restrictive as it is possible to approximate an irrational function with a rational one.

The states of the subsystems are called temporal states. We denote by $x_0(t, k)$ the temporal state associated with subsystem $G^{(k)}$, and write its possibly time-varying dimension as $n^s(t, k)$. The interconnections between the subsystems are also modeled as states, which we refer to as spatial states. We associate the spatial state $x_i(t, k)$ with the interconnection $(\rho_i^{-1}(k), k)$, and denote the corresponding dimension by $n_i^g(t, k)$. If virtual edges and/or nodes are added to make the directed graph regular, the corresponding temporal and spatial states will have zero dimensions, which is a slight abuse of notation permitted in our framework. Due to communication latency, the data sent by a subsystem at the current time step reaches the target subsystem at the next time step. Before giving the state-space equations of the distributed system, we introduce the following notations:

$$x_g(t,k) = \begin{bmatrix} x_1(t,k)^T & \cdots & x_d(t,k)^T \end{bmatrix}^T,$$

$$x_g(t,\rho(k)) = \begin{bmatrix} x_1(t,\rho_1(k))^T & \cdots & x_d(t,\rho_d(k))^T \end{bmatrix}^T.$$

Then, for all $(t, k) \in \mathbb{Z} \times V$, we have

$$\begin{bmatrix} x_0(t+1,k)^T x_g(t+1,\rho(k))^T \alpha(t,k)^T z(t,k)^T y(t,k)^T \end{bmatrix}^T = \begin{bmatrix} A_{ss}(t,k) & A_{sg}(t,k) & A_{sp}(t,k) & B_{s1}(t,k) & B_{s2}(t,k) \\ A_{gs}(t,k) & A_{gg}(t,k) & A_{gp}(t,k) & B_{g1}(t,k) & B_{g2}(t,k) \\ A_{ps}(t,k) & A_{pg}(t,k) & A_{pp}(t,k) & B_{p1}(t,k) & B_{p2}(t,k) \\ C_{1s}(t,k) & C_{1g}(t,k) & C_{1p}(t,k) & D_{11}(t,k) & D_{12}(t,k) \\ C_{2s}(t,k) & C_{2g}(t,k) & C_{2p}(t,k) & D_{21}(t,k) & D_{22}(t,k) \end{bmatrix}$$

$$\times \begin{bmatrix} x_0(t,k)^T & x_g(t,k)^T & \beta(t,k)^T & w(t,k)^T & u(t,k)^T \end{bmatrix}^T, \quad (1)$$

$$\begin{aligned} \beta(t,k) &= \mathrm{diag}\big(\delta_1(t,k)I, \ldots, \delta_r(t,k)I\big)\alpha(t,k) \\ &= \underline{\Delta}(t,k)\alpha(t,k). \end{aligned}$$

$\beta(t, k)$ and $\alpha(t, k)$ are the states associated with the parameters, and $\delta_j(t, k)$ are scalar functions, for $j = 1, \ldots, r$. $\beta(t, k)$ and $\alpha(t, k)$ are partitioned into r vector-valued channels, e.g., $\alpha(t,k) = \begin{bmatrix} \alpha_1^T(t,k) & \alpha_2^T(t,k) & \cdots & \alpha_r^T(t,k) \end{bmatrix}^T$, where $\alpha_j(t, k)$ and $\beta_j(t, k)$ share the same dimension $n_j^p(t, k)$. The identity matrices in $\underline{\Delta}(t, k)$ have dimensions $n_j^p(t, k)$, for $j = 1, \ldots, r$, respectively. Note that it is permissible for $n_j^p(\cdot, k)$ to be zero for some j, k. Even if various subsystems depend on the same parameter, this formulation assumes an independent evolution of the parameter in each of the subsystems. We denote by $n^z(t, k)$, $n^y(t, k)$, $n^u(t, k)$, and $n^w(t, k)$ the dimensions of $z(t, k)$, $y(t, k)$, $u(t, k)$, and $w(t, k)$, respectively. It is convenient to define the following:

$$\bar{A}(t,k) = \begin{bmatrix} A_{ss}(t,k) & A_{sg}(t,k) & A_{sp}(t,k) \\ A_{gs}(t,k) & A_{gg}(t,k) & A_{gp}(t,k) \\ A_{ps}(t,k) & A_{pg}(t,k) & A_{pp}(t,k) \end{bmatrix},$$

$$\bar{B}(t,k) = \begin{bmatrix} B_1(t,k) & B_2(t,k) \end{bmatrix} = \begin{bmatrix} B_{s1}(t,k) & B_{s2}(t,k) \\ B_{g1}(t,k) & B_{g2}(t,k) \\ B_{p1}(t,k) & B_{p2}(t,k) \end{bmatrix},$$

$$\bar{C}(t,k) = \begin{bmatrix} C_1(t,k) \\ C_2(t,k) \end{bmatrix} = \begin{bmatrix} C_{1s}(t,k) & C_{1g}(t,k) & C_{1p}(t,k) \\ C_{2s}(t,k) & C_{2g}(t,k) & C_{2p}(t,k) \end{bmatrix},$$

$$\bar{D}(t,k) = \begin{bmatrix} D_{11}(t,k) & D_{12}(t,k) \\ D_{21}(t,k) & D_{22}(t,k) \end{bmatrix}.$$

Fig. 2 shows a distributed system having NSLPV subsystems formulated in an LFT framework. The figure also gives the spatial state associated with each interconnection.

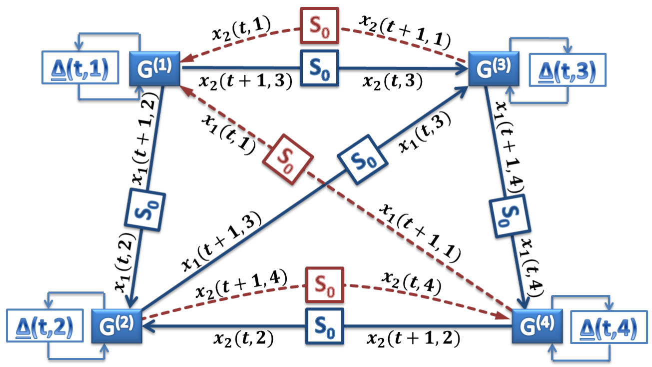

Fig. 2. Distributed System consisting of NSLPV subsystems formulated in an LFT framework, where S_0 denotes the one-step delay operator.

At this point, as in [14], we introduce two classes of operators that will be used in the sequel. An operator Q is said to be graph-diagonal if $(Qv)(t,k) = Q(t,k)v(t,k)$, for all $(t,k) \in \mathbb{Z} \times V$. Similarly, an operator $W = [W_{ij}]$ is said to be partitioned graph-diagonal if each partition W_{ij} is a graph-diagonal operator. We define the mapping $[\![W]\!](t,k) = [W_{ij}(t,k)]$, which is a homomorphism from the space of partitioned graph-diagonal operators to that of graph-diagonal operators. This mapping is isometric and preserves products, addition, and ordering. We use the notations 0 and I for both graph-diagonal and partitioned graph-diagonal zero and identity operators, respectively, and leave it to the leader to determine their associated dimensions from context. The blocks of the state-space matrices, e.g., $A_{ss}(t,k)$, define graph-diagonal operators, e.g., A_{ss}. These can in turn be used to construct partitioned graph-diagonal operators A, B_1, B_2, B, C_1, C_2, C, and D, e.g.,

$$A=\begin{bmatrix} A_{ss} & A_{sg} & A_{sp} \\ A_{gs} & A_{gg} & A_{gp} \\ A_{ps} & A_{pg} & A_{pp} \end{bmatrix}, \quad B=\begin{bmatrix} B_1 & B_2 \end{bmatrix}=\begin{bmatrix} B_{s1} & B_{s2} \\ B_{g1} & B_{g2} \\ B_{p1} & B_{p2} \end{bmatrix}.$$

These operators satisfy $[\![A]\!](t,k) = \bar{A}(t,k)$, $[\![B]\!](t,k) = \bar{B}(t,k)$, and so on. Also, we define graph-diagonal operators Δ_j, for $j = 1, \ldots, r$, such that $\Delta_j(t,k) = \delta_j(t,k)I$. These operators can be block-diagonally augmented to form the partitioned graph-diagonal operator $\underline{\Delta} = \operatorname{diag}(\Delta_1, \ldots, \Delta_r)$. It will be useful to group the operators associated with the temporal and spatial states as shown next.

$$\bar{A}_{11}=\begin{bmatrix} A_{ss} & A_{sg} \\ A_{gs} & A_{gg} \end{bmatrix}, \ \bar{A}_{12}=\begin{bmatrix} A_{sp} \\ A_{gp} \end{bmatrix}, \ \bar{A}_{21}=\begin{bmatrix} A_{ps} & A_{pg} \end{bmatrix}, \quad (2)$$

$$\bar{B}_1=\begin{bmatrix} B_{s1} \\ B_{g1} \end{bmatrix}, \bar{B}_2=\begin{bmatrix} B_{s2} \\ B_{g2} \end{bmatrix}, \bar{C}_1=\begin{bmatrix} C_{1s} & C_{1g} \end{bmatrix}, \bar{C}_2=\begin{bmatrix} C_{2s} & C_{2g} \end{bmatrix}.$$

The temporal shift operator S_0 and spatial shift operators S_i, for $i = 1, \ldots, d$, are defined as

$$(S_0 v)(t,k) = v(t-1,k) \text{ and } (S_i v)(t,k) = v(t, \rho_i^{-1}(k)),$$

for any $v \in \ell$. These operators are unitary and satisfy

$$(S_0^* v)(t,k) = v(t+1,k) \text{ and } (S_i^* v)(t,k) = v(t, \rho_i(k)).$$

The composite shift operator S and the conformably partitioned operator Δ can then be defined as

$$S = \operatorname{diag}(S_0, S_0 S_1, \ldots, S_0 S_d, I_r), \quad \Delta = \operatorname{diag}(I_{d+1}, \underline{\Delta}),$$

where I_q denotes the block-diagonal augmentation of q graph-diagonal identity operators I. We are only interested in operators Δ that satisfy $\|\Delta\| \leq 1$. We denote the corresponding set by $\mathbf{\Delta}$. Thus, (1) can be rewritten as

$$\begin{bmatrix} x \\ \beta \end{bmatrix} = \Delta S A \begin{bmatrix} x \\ \beta \end{bmatrix} + \Delta S B \begin{bmatrix} w \\ u \end{bmatrix}, \ \begin{bmatrix} z \\ y \end{bmatrix} = C \begin{bmatrix} x \\ \beta \end{bmatrix} + D \begin{bmatrix} w \\ u \end{bmatrix}, \quad (3)$$

where $x = \left[x_0^*, x_1^*, \ldots, x_d^*\right]^*$. From the previous equations, and assuming the relevant algebraic inverse exists, the input-output map of the system can be written as $G_\delta = \Delta \star G$, where

$$\Delta \star G = \Delta \star \left[\begin{array}{c|c} SA & SB \\ \hline C & D \end{array}\right] = C(I - \Delta SA)^{-1} \Delta SB + D, \quad (4)$$

for some $\Delta \in \mathbf{\Delta}$. Finally, the distributed NSLPV system $\mathcal{G}_\delta$ is defined as $\mathcal{G}_\delta = \mathbf{\Delta} \star G = \{\Delta \star G : \Delta \in \mathbf{\Delta}\}$.

IV. Analysis Results

We now give results on well-posedness, stability, and performance of the systems under consideration. We develop the results for the open-loop equations. For simplicity, we neglect the exogenous inputs w and the exogenous errors z. Then, (1) can be rewritten as

$$\begin{bmatrix} x_0\,(t+1,k)^T & x_g(t+1,\rho(k))^T & y\,(t,k)^T \end{bmatrix}^T =$$
$$\left(\begin{bmatrix} A_{ss}(t,k) & A_{sg}(t,k) & B_s(t,k) \\ A_{gs}(t,k) & A_{gg}(t,k) & B_g(t,k) \\ C_s(t,k) & C_g(t,k) & D(t,k) \end{bmatrix} + \begin{bmatrix} A_{sp}(t,k) \\ A_{gp}(t,k) \\ C_p(t,k) \end{bmatrix} \underline{\Delta}(t,k) \times \right.$$
$$\left. (I - A_{pp}(t,k)\underline{\Delta}(t,k))^{-1} \begin{bmatrix} A_{ps}(t,k) & A_{pg}(t,k) & B_p(t,k) \end{bmatrix} \right)$$
$$\times \begin{bmatrix} x_0\,(t,k)^T & x_g(t,k)^T & u\,(t,k)^T \end{bmatrix}^T.$$

One can see that, for the equations to be well-defined, $I - A_{pp}(t,k)\underline{\Delta}(t,k)$ must be invertible for all $(t,k) \in \mathbb{Z} \times V$.

Definition 1: A system $\mathcal{G}_\delta$ is well-posed if $G_\delta : u \to y \in \mathcal{L}_e(\ell_{2e}, \ell_{2e})$ for all $\Delta \in \mathbf{\Delta}$, i.e., G_δ defines a causal mapping on ℓ_{2e} for all permissible parameter trajectories.

From (4) and given that $\mathcal{L}_e(\ell_{2e}, \ell_{2e})$ is an algebra, the well-posedness of $\mathcal{G}_\delta$ is equivalent to the invertibility of $I - \Delta SA$ in $\mathcal{L}_e(\ell_{2e})$ for all $\Delta \in \mathbf{\Delta}$. But, using (2),

$$I - \Delta SA = \begin{bmatrix} I - \hat{S}_0 \hat{S} \bar{A}_{11} & -\hat{S}_0 \hat{S} \bar{A}_{12} \\ -\underline{\Delta} \bar{A}_{21} & I - \underline{\Delta} A_{pp} \end{bmatrix},$$

where $\hat{S}_0 = \operatorname{diag}(S_0, \ldots, S_0)$ ($d+1$ times) and $\hat{S} = \operatorname{diag}(I, S_1, \ldots, S_d)$. Then, we can state the following.

Lemma 1: $I - \Delta SA$ is invertible in $\mathcal{L}_e(\ell_{2e})$ for all $\Delta \in \mathbf{\Delta}$, and hence $\mathcal{G}_\delta$ is well-posed, if $\bar{A}(t,k) = 0$ for $t < 0$ and $I - \underline{\Delta} A_{pp}$ is invertible in $\mathcal{L}_e(\ell_{2e})$ for all $\Delta \in \mathbf{\Delta}$.

Proof: The proof parallels the ones of [2, Proposition 2] and [10, Lemma 8]. We omit it for space considerations. ■

Definition 2: A system $\mathcal{G}_\delta$ is ℓ_2-stable if $G_\delta \in \mathcal{L}_c(\ell_2, \ell_2)$ for all $\Delta \in \mathbf{\Delta}$.

Since $\mathcal{L}_c(\ell_2, \ell_2)$ is an algebra, stability of $\mathcal{G}_\delta$ reduces to $(I - \Delta SA)^{-1}$ being in $\mathcal{L}_c(\ell_2)$. Next, we give a sufficient condition that guarantees this. But first, we define the set

$$\mathcal{X}=\Big\{X : X=\text{diag}(X_0, X_1^g,\dots,X_d^g,X_1^p,\dots,X_r^p)=X^*,$$
$$X\succ 0, X^{-1}\in\mathcal{L}\left(\ell_2(\{\mathbb{R}^{n^s(t,k)}\})\oplus(\oplus_{i=1}^d\ell_2(\{\mathbb{R}^{n_i^g(t,k)}\}))\oplus\right.$$
$$\left.(\oplus_{j=1}^r\ell_2(\{\mathbb{R}^{n_j^p(t,k)}\}))\right), \text{where } X_0,\ X_i^g,\ X_j^p, \text{ for } i{=}1,\dots,d,$$
$$j=1,\dots,r, \text{ are bounded graph-diagonal operators}\Big\}.$$

Notice that $\mathcal{X}$ is a commutant of $\mathbf{\Delta}$. Throughout the paper, we overload the notation $\mathcal{X}$ to denote any similarly defined set even if the corresponding dimensions are different.

Lemma 2: A system $\mathcal{G}_\delta$ is ℓ_2-stable if there exists $X\in\mathcal{X}$ such that

$$A^*S^*XSA - X \prec 0. \qquad (5)$$

Proof: The proof is based on [24]. From (5), one can see that $\|X^{\frac{1}{2}}SAX^{-\frac{1}{2}}\| < 1$. But, $X^{\frac{1}{2}}\in\mathcal{X}$, and so, it commutes with every $\Delta\in\mathbf{\Delta}$. By the submultiplicative property, $\|X^{\frac{1}{2}}\Delta SAX^{-\frac{1}{2}}\| < 1$, i.e., ΔSA has a spectral radius less than 1. Thus, $(I-\Delta SA)^{-1}=\sum_{i=0}^{\infty}(\Delta SA)^i$ is a well-defined quantity in $\mathcal{L}(\ell_2)$. This quantity is is a sum of products of causal terms, and so, is causal. ■

Hereafter, we refer to systems that satisfy this condition as strongly stable. The next result guarantees the strong stability of $\mathcal{G}_\delta$ and further gives an upper bound on the ℓ_2-induced norm of G_δ for all $\Delta\in\mathbf{\Delta}$.

Lemma 3: A system $\mathcal{G}_\delta$ is strongly stable, and satisfies $\|G_\delta\| < \gamma$ for all $\Delta\in\mathbf{\Delta}$, if there exists $X\in\mathcal{X}$ such that

$$\begin{bmatrix} SA & SB \\ C & D\end{bmatrix}^*\begin{bmatrix} X & 0 \\ 0 & I\end{bmatrix}\begin{bmatrix} SA & SB \\ C & D\end{bmatrix}-\begin{bmatrix} X & 0 \\ 0 & \gamma^2 I\end{bmatrix}\prec 0. \quad (6)$$

Proof: The proof is based on [10]. From the $(1,1)$ term of (6), we get $A^*S^*XSA-X\prec 0$, i.e., the system is strongly stable. To prove the second part, we pre- and post-multiply (6) by $\begin{bmatrix}B^*S^*\Delta^*(I-A^*S^*\Delta^*)^{-1} & I\end{bmatrix}$ and its adjoint, respectively. After some mathematical manipulations, we get that $G_\delta^* G_\delta-\gamma^2 I\prec 0$, i.e., $\|G_\delta\|<\gamma$ for all $\Delta\in\mathbf{\Delta}$. ■

In the previous lemmas, we required the solutions to (5) and (6) to be in $\mathcal{X}$. In fact, the solutions just need to be positive definite and in the commutant of $\mathbf{\Delta}$. But, the added structure does not introduce conservatism, as shown in the next lemma, which is a generalization of [23, Theorem 11].

Lemma 4: A positive definite solution $\bar{X}$, belonging to the commutant of $\mathbf{\Delta}$, exists to (6) if and only if a solution $X\in\mathcal{X}$ exists.

The proof of this lemma is involved and will be given in the journal version of this work.

V. Synthesis Results

In this section, we develop a controller that guarantees a certain performance level for the closed-loop system. We assume that the plant $\mathcal{G}_\delta$ is well-posed. We also assume that all the state-space matrices are zeros for $t<0$, and that $D_{22}(t,k)=0$ for all $(t,k)\in\mathbb{Z}\times V$. The controller $\mathcal{K}_\delta$, as in Fig. 3, is a distributed NSLPV system that inherits the network structure of the plant. The parameters that affect the controller are the same ones that affect the plant. Thus, the controller state-space equations are in the form of (1) with

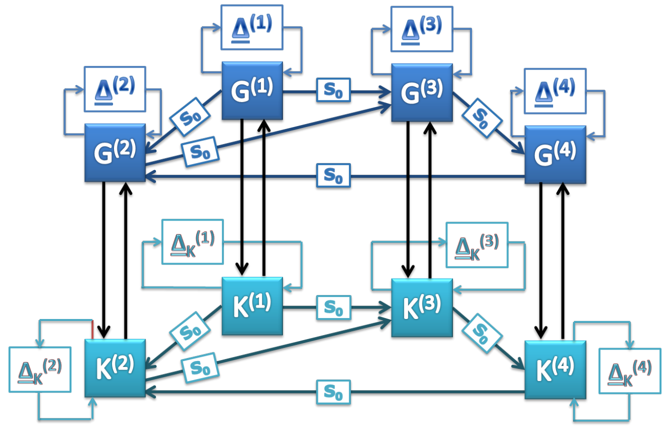

Fig. 3. Controller inheriting the network structure of the plant.

the additional superscript K. The controller has zero state-space matrices for $t<0$. The dimensions of the controller are not known at this point, but will be specified later.

Using definitions as in (2), we write the controller equations as

$$\begin{bmatrix} x^K \\ \alpha^K \\ u\end{bmatrix}=\begin{bmatrix}\hat{S}_0\hat{S}\bar{A}_{11}^K & \hat{S}_0\hat{S}\bar{A}_{12}^K & \hat{S}_0\hat{S}\bar{B}^K \\ \bar{A}_{21}^K & A_{pp}^K & B_p^K \\ \bar{C}^K & C_p^K & D^K\end{bmatrix}\begin{bmatrix} x^K \\ \beta^K \\ y\end{bmatrix},$$
$$\begin{bmatrix} x^K \\ \beta^K\end{bmatrix}=\begin{bmatrix} I & 0 \\ 0 & \underline{\Delta}^K\end{bmatrix}\begin{bmatrix} x^K \\ \alpha^K\end{bmatrix}=\Delta_K\begin{bmatrix} x^K \\ \alpha^K\end{bmatrix}, \qquad (7)$$

where $\Delta^K\in\mathbf{\Delta}^K=\{\Delta^K : \|\Delta^K\|\le 1\}$. Writing the plant equations as in (7), and combining them with (7) yields

$$\begin{bmatrix} x \\ \alpha \\ x^K \\ \alpha^K \\ \hline z\end{bmatrix}=\begin{bmatrix}\tilde{S}A_{cl} & \tilde{S}B_{cl} \\ C_{cl} & D_{cl}\end{bmatrix}\begin{bmatrix} x \\ \beta \\ x^K \\ \beta^K \\ \hline w\end{bmatrix}, \qquad \begin{bmatrix}\alpha \\ \alpha_K\end{bmatrix}=\underline{\Delta}_{cl}\begin{bmatrix}\beta \\ \beta_K\end{bmatrix},$$

where A_{cl}, B_{cl}, C_{cl}, and D_{cl} are appropriately defined operators, $\underline{\Delta}_{cl}=\text{diag}(\underline{\Delta},\underline{\Delta}^K)$, and

$$\tilde{S}=\text{diag}(S,S)=\text{diag}\left(\hat{S}_0\hat{S}, I_r, \hat{S}_0\hat{S}, I_r\right).$$

If we define $x_{cl}=\begin{bmatrix}x^* & \beta^* & (x^K)^* & (\beta^K)^*\end{bmatrix}^*$, and $\tilde{\Delta}=\text{diag}(\Delta,\Delta^K)=\text{diag}\left(I_{d+1},\underline{\Delta},I_{d+1},\underline{\Delta}^K\right)$, then

$$x_{cl}=\tilde{\Delta}\tilde{S}A_{cl}x_{cl}+\tilde{\Delta}\tilde{S}B_{cl}w, \quad z_{cl}=C_{cl}x_{cl}+D_{cl}w. \quad (8)$$

Equations (8) describe the dynamics of the closed-loop system, but are not in the form of (3). As a remedy, we define the partitioned graph-diagonal operator $\underline{\Delta}^L=\text{diag}\left(\Delta_1^L,\dots,\Delta_r^L\right)$, where each Δ_j^L satisfies $\Delta_j^L(t,k)=\delta_j(t,k)I$ with the identity matrix having dimension $n_j^p(t,k)+n_j^{p,K}(t,k)$. We then define the permutation operator P such that $P^*\tilde{\Delta}P=\Delta^L=\text{diag}\left(I_{d+1},\underline{\Delta}^L\right)$, and operators A^L, B^L, C^L, and D^L according to

$$SA^L=P^*\tilde{S}A_{cl}P,\ SB^L=P^*\tilde{S}B_{cl},\ C^L=C_{cl}P,\ D^L=D_{cl}.$$

Thus, the closed-loop equations can be expressed as

$$\begin{bmatrix} x^L \\ \beta^L\end{bmatrix}=\Delta^L SA^L\begin{bmatrix} x^L \\ \beta^L\end{bmatrix}+\Delta^L SB^L w, \quad z=C^L\begin{bmatrix} x^L \\ \beta^L\end{bmatrix}+D^L w.$$

We are interested in operators Δ^L that satisfy $\|\Delta^L\| \leq 1$ and denote the corresponding set by $\boldsymbol{\Delta}^L$.

Definition 3: A controller $\mathcal{K}_\delta$ is an admissible synthesis for a plant $\mathcal{G}_\delta$ if the closed-loop system is strongly stable and, for all $\Delta^L \in \boldsymbol{\Delta}^L$, the input-output map satisfies

$$\|w \to z\| = \|C^L(I - \Delta^L S A^L)^{-1}\Delta^L S B^L + D^L\| < 1.$$

We now proceed onto designing an admissible synthesis for a given plant $\mathcal{G}_\delta$. We consider the following closed-loop system parametrization [1], [21], [23]:

$$A_{cl} = \bar{A} + \underline{B}J\underline{C}, \qquad B_{cl} = \bar{B} + \underline{B}J\underline{D}_{21},$$
$$C_{cl} = \bar{C} + \underline{D}_{12}J\underline{C}, \qquad D_{cl} = D_{11} + \underline{D}_{12}J\underline{D}_{21},$$

where

$$\bar{A} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} & 0 & 0 \\ \bar{A}_{21} & A_{pp} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} \bar{B}_1 \\ B_{1p} \\ 0 \\ 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} \bar{C}_1^* \\ C_{1p}^* \\ 0 \\ 0 \end{bmatrix}^*,$$

$$J = \begin{bmatrix} \bar{A}_{11}^K & \bar{A}_{12}^K & \bar{B}^K \\ \bar{A}_{21}^K & A_{pp}^K & B_p^K \\ \bar{C}^K & C_p^K & D^K \end{bmatrix}, \underline{B} = \begin{bmatrix} 0 & 0 & \bar{B}_2 \\ 0 & 0 & B_{p2} \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}, \underline{D}_{12} = \begin{bmatrix} 0 \\ 0 \\ D_{12}^* \end{bmatrix}^*,$$

$$\underline{C} = \begin{bmatrix} 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ \bar{C}_2 & C_{2p} & 0 & 0 \end{bmatrix}, \quad \underline{D}_{21} = \begin{bmatrix} 0 \\ 0 \\ D_{21} \end{bmatrix}.$$

This parametrization allows us to develop an affine condition in J to check for the admissibility of a given controller.

Theorem 1: The controller $\mathcal{K}_\delta$, parameterized by operator J, is an admissible synthesis for plant $\mathcal{G}_\delta$ if there exists $X^L \in \mathcal{X}$ such that

$$H + Q^*J^*R + R^*JQ \prec 0, \tag{9}$$

where $X_P^L = PX^LP^*$, $R = \begin{bmatrix} \underline{B}^* & 0 & 0 & \underline{D}_{12}^* \end{bmatrix}$, $Q = \begin{bmatrix} 0 & \underline{C} & \underline{D}_{21} & 0 \end{bmatrix}$, and

$$H = \begin{bmatrix} -\tilde{S}^*\left(X_P^L\right)^{-1}\tilde{S} & \bar{A} & \bar{B} & 0 \\ \bar{A}^* & -X_P^L & 0 & \bar{C}^* \\ \bar{B}^* & 0 & -I & D_{11}^* \\ 0 & \bar{C} & D_{11} & -I \end{bmatrix}.$$

Proof: From Lemma 3, the closed-loop system is strongly stable and satisfies $\|w \to z\| < 1$ for all $\Delta^L \in \boldsymbol{\Delta}^L$ if there exists an operator $X^L \in \mathcal{X}$ such that (6) holds. We pre- and post-multiply (6) by $\operatorname{diag}(P, I)$ and its adjoint, respectively, to get

$$\begin{bmatrix} A_{cl} & B_{cl} \\ C_{cl} & D_{cl} \end{bmatrix}^* \begin{bmatrix} \tilde{S}^*X_P^L\tilde{S} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_{cl} & B_{cl} \\ C_{cl} & D_{cl} \end{bmatrix} - \begin{bmatrix} X_P^L & 0 \\ 0 & I \end{bmatrix} \prec 0.$$

Applying the Schur complement formula and some permutations, one retrieves (9). ■

We now look closely at $X_P^L = PX^LP^*$, where $X^L = \operatorname{diag}(X_0, X_1^g, \ldots, X_d^g, X_1^p, \ldots, X_r^p) \in \mathcal{X}$. Consider operator X_0. For all $(t,k) \in \mathbb{Z} \times V$, we partition $X_0(t,k)$ as

$$X_0(t,k) = \begin{bmatrix} X_{0_{11}}(t,k) & X_{0_{12}}(t,k) \\ X_{0_{12}}^T(t,k) & X_{0_{22}}(t,k) \end{bmatrix},$$

where $X_{0_{11}}(t,k) \in \mathbb{S}^{n^s(t,k)}$ and $X_{0_{22}}(t,k) \in \mathbb{S}^{n^{s,K}(t,k)}$. These partitions define graph-diagonal operators, which we denote by $X_{0_{11}}$, $X_{0_{12}}$, and $X_{0_{22}}$. We repeat the partitioning process for X_i^g and X_j^p. Then, we construct $\hat{X}_{11} = \operatorname{diag}(X_{0_{11}}, X_{1_{11}}^g, \ldots, X_{d_{11}}^g, X_{1_{11}}^p, \ldots, X_{r_{11}}^p) \in \mathcal{X}$. $\hat{X}_{12}$ and $\hat{X}_{22}$ are constructed similarly. Then, $X_P^L = PX^LP^*$ and its inverse $(X_P^L)^{-1} = P(X^L)^{-1}P^*$ have the same structure:

$$X_P^L = \begin{bmatrix} \hat{X}_{11} & \hat{X}_{12} \\ \hat{X}_{12}^* & \hat{X}_{22} \end{bmatrix}, \quad \left(X_P^L\right)^{-1} = \begin{bmatrix} \hat{Y}_{11} & \hat{Y}_{12} \\ \hat{Y}_{12}^* & \hat{Y}_{22} \end{bmatrix}. \tag{10}$$

The following lemma follows immediately from the result [22, Lemma 6.2], and so, its proof is omitted.

Lemma 5: Suppose $\hat{X}_{11}$ and $\hat{Y}_{11} \in \mathcal{X}$. Then, there exists $X_P^L \succ 0$ satisfying (10) if and only if $\begin{bmatrix} \hat{X}_{11} & I \\ I & \hat{Y}_{11} \end{bmatrix} \succeq 0$, and

$$\operatorname{rank}\begin{bmatrix} X_{0_{11}}(t,k) & I \\ I & Y_{0_{11}}(t,k) \end{bmatrix} \leq n^s(t,k) + n^{s,K}(t,k),$$
$$\operatorname{rank}\begin{bmatrix} X_{i_{11}}^g(t,k) & I \\ I & Y_{i_{11}}^g(t,k) \end{bmatrix} \leq n_i^g(t,k) + n_i^{g,K}(t,k),$$
$$\operatorname{rank}\begin{bmatrix} X_{j_{11}}^p(t,k) & I \\ I & Y_{j_{11}}^p(t,k) \end{bmatrix} \leq n_j^p(t,k) + n_j^{p,K}(t,k),$$

for all $(t,k) \in \mathbb{Z} \times V$, $i = 1, \ldots, d$, and $j = 1, \ldots, r$.

The next result makes use of Theorem 1, and gives a means to check for the existence of an admissible synthesis. The proof is similar to that of a counterpart result in [23], and is hence omitted for brevity.

Lemma 6: There exists a partitioned graph-diagonal operator J that satisfies (9) if and only if

$$W_R^*HW_R \prec 0 \qquad \text{and} \qquad W_Q^*HW_Q \prec 0,$$

where W_R and W_R are any partitioned graph-diagonal operators that satisfy the following properties:

$$\operatorname{Im}W_R = \ker R, \qquad W_R^*W_R = I,$$
$$\operatorname{Im}W_Q = \ker Q, \qquad W_Q^*W_Q = I.$$

We now construct the needed W_R and W_Q. Let U_1 and V_1 be partitioned graph-diagonal operators such that

$$U_1 = \begin{bmatrix} U_0^* & (U_1^g)^* & \cdots & (U_d^g)^* & (U_1^p)^* & \cdots & (U_r^p)^* \end{bmatrix}^*,$$
$$V_1 = \begin{bmatrix} V_0^* & (V_1^g)^* & \cdots & (V_d^g)^* & (V_1^p)^* & \cdots & (V_r^p)^* \end{bmatrix}^*,$$

where $U_0(t,k) \in \mathbb{R}^{n^s(t,k)\times ?}$, $U_i^g(t,k) \in \mathbb{R}^{n_i^g(t,k)\times ?}$, $U_j^p(t,k) \in \mathbb{R}^{n_j^p(t,k)\times ?}$. The dimensions of the partitions of V_1 are defined similarly. Also, let U_2 and V_2 be graph-diagonal operators such that $U_2(t,k) \in \mathbb{R}^{n^w(t,k)\times ?}$ and $V_2(t,k) \in \mathbb{R}^{n^z(t,k)\times ?}$. These operators must also satisfy

$$\operatorname{Im}\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \ker\begin{bmatrix} C_2 & D_{21} \end{bmatrix}, \qquad U_1^*U_1 + U_2^*U_2 = I,$$
$$\operatorname{Im}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \ker\begin{bmatrix} B_2^* & D_{12}^* \end{bmatrix}, \qquad V_1^*V_1 + V_2^*V_2 = I.$$

Then, we can take W_R and W_Q, respectively, as

$$W_R = \left[\begin{array}{c|ccc} V_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ \hline 0 & 0 & 0 & I \\ \hline V_2 & 0 & 0 & 0 \end{array}\right], \quad W_Q = \left[\begin{array}{c|ccc} 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ \hline U_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline U_2 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & I \end{array}\right].$$

The horizontal and vertical lines are just added to help in the computation of $W_R^* H W_R$ and $W_Q^* H W_Q$. Expanding $W_R^* H W_R \prec 0$ and $W_Q^* H W_Q \prec 0$, and then applying the Schur complement formula to each, we get, respectively,

$$
\begin{aligned}
&(V_1^* A + V_2^* C_1)\hat{Y}_{11}(A^* V_1 + C_1^* V_2) - V_1^* S^* \hat{Y}_{11} S V_1 \\
&\quad + (V_1^* B_1 + V_2^* D_{11})(B_1^* V_1 + D_{11}^* V_2) - V_2^* V_2 \prec 0, \quad (11)\\
&(U_1^* A^* + U_2^* B_1^*) S^* \hat{X}_{11} S (A U_1 + B_1 U_2) - U_1^* \hat{X}_{11} U_1 \\
&\quad + (U_1^* C_1^* + U_2^* D_{11}^*)(C_1 U_1 + D_{11} U_2) - U_2^* U_2 \prec 0. \quad (12)
\end{aligned}
$$

The following theorem combines Lemma 5, (11) and (12).

Theorem 2: There exists an admissible synthesis $\mathcal{K}_\delta$ for plant $\mathcal{G}_\delta$ if there exist $\hat{X}_{11}$, $\hat{Y}_{11} \in \mathcal{X}$ such that

$$
N_Y^* \left(\begin{bmatrix} A & B_1 \\ C_1 & D_{11} \end{bmatrix} \begin{bmatrix} \hat{Y}_{11} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A & B_1 \\ C_1 & D_{11} \end{bmatrix}^* - \begin{bmatrix} S^* \hat{Y}_{11} S & 0 \\ 0 & I \end{bmatrix} \right) N_Y \prec 0,
$$

$$
N_X^* \left(\begin{bmatrix} A & B_1 \\ C_1 & D_{11} \end{bmatrix}^* \begin{bmatrix} S^* \hat{X}_{11} S & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A & B_1 \\ C_1 & D_{11} \end{bmatrix} - \begin{bmatrix} \hat{X}_{11} & 0 \\ 0 & I \end{bmatrix} \right) N_X \prec 0,
$$

$$
\begin{bmatrix} \hat{X}_{11} & I \\ I & \hat{Y}_{11} \end{bmatrix} \succeq 0, \quad (13)
$$

where N_Y and N_X satisfy $\mathrm{Im} N_Y = \ker \begin{bmatrix} B_2^* & D_{12}^* \end{bmatrix}$,

$$
N_Y^* N_Y = I, \quad \mathrm{Im} N_X = \ker \begin{bmatrix} C_2 & D_{21} \end{bmatrix}, \quad N_X^* N_X = I.
$$

The controller dimensions are set as follows:

$$
\begin{aligned}
n^{s,K}(t,k) &= \mathrm{rank}\left(X_{0_{11}}(t,k) - Y_{0_{11}}(t,k) \right), \\
n_i^{g,K}(t,k) &= \mathrm{rank}\left(X_{i_{11}}^g(t,k) - Y_{i_{11}}^g(t,k) \right), \\
n_j^{p,K}(t,k) &= \mathrm{rank}\left(X_{j_{11}}^p(t,k) - Y_{j_{11}}^p(t,k) \right).
\end{aligned}
$$

After finding the solutions to (13) and determining the controller dimensions, we use Lemma 5 to construct an operator X_P^L that satisfies (10). Then, we solve (9) for the controller realization J. All the previous computations are convex, but infinite dimensional in general. However, when the subsystems are eventually time-periodic and the interconnection graph is finite, all the computations become finite dimensional. We briefly discuss this topic next.

We say that a system $\mathcal{G}_\delta$ with zero state-space matrices for $t < 0$ is (h,q)-eventually time-periodic (ETP), for some $h \in \mathbb{N}_0$ and $q \in \mathbb{N}$, if the state-space matrices of G are aperiodic for an initial amount of time h, and then become time-periodic with period q afterwards, e.g., $\bar{A}(t+h+zq,k) = \bar{A}(t+h,k)$, for all $t, z \in \mathbb{N}_0$ and $k \in V$. This class of systems G includes finite horizon, time-invariant, and time-periodic systems.

Lemma 7: For an (h,q)-ETP system $\mathcal{G}_\delta$, there exist solutions $\hat{X}_{11}$ and $\hat{Y}_{11} \in \mathcal{X}$ to (13) if and only if there exist (N,q)-ETP solutions, $\hat{X}_{11_{\mathrm{eper}}}$ and $\hat{Y}_{11_{\mathrm{eper}}}$, for some $N \geq h$.

The proof makes use of averaging techniques and convexity properties of LMIs, similar to the ones used in [3] and [23].

From Lemma 7, we conclude that for an (h,q)-ETP distributed NSLPV system $\mathcal{G}_\delta$, there exists an admissible synthesis if and only if there exists an (N,q)-ETP admissible synthesis $\mathcal{K}_\delta$. If, in addition, the interconnection graph is finite, i.e., the graph has a finite number of vertices and edges, then the computations involved in finding the desired synthesis become finite dimensional.

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