TAKING PROOF INTO SECONDARY CLASSROOMS – SUPPORTING FUTURE MATHEMATICS TEACHERS

Orly Buchbinder
University of New Hampshire
orly.buchbinder@unh.edu

Sharon McCrone
University of New Hampshire
sharon.mccrone@unh.edu

For reasoning and proof to become a reality in mathematics classrooms, it is important to prepare teachers who have knowledge and skills to integrate reasoning and proving in their teaching. Aiming to enhance prospective secondary teachers' (PSTs) content and pedagogical knowledge related to proof, we designed and studied a capstone course Mathematical Reasoning and Proving for Secondary Teachers. This paper describes the structure of the course and illustrates how PSTs' interacted with its different components. The PSTs first strengthened their content knowledge, then developed and taught in local schools a lesson incorporating proof components. Initial data analyses show gains in PSTs' knowledge for teaching proof and dispositions towards proving, following their participation in the course.

Keywords: Reasoning and Proof, Teacher Education-Preservice, Design-Based Research

Mathematics education researchers (e.g., Hanna & deVillers, 2012) and policy documents (e.g., NGA & CCSSO, 2010) emphasize the importance of teaching mathematics in ways that promote sense making, reasoning and proving across grade levels and mathematical topics. Yet, the reality of many mathematics classrooms rarely reflect this vision; even teachers who recognize the importance of reasoning and proof, in principle, often struggle to enact them, and tend to choose skills-oriented activities over proof-oriented ones for their own classrooms (Kotelawala, 2016). Preparing teachers who are capable of implementing such teaching practices and cultivating positive attitudes towards reasoning and proving is a critical objective of teacher preparation programs (AMTE, 2017). We used a design-based-research (DBR) approach to develop and study a novel capstone course *Mathematical Reasoning and Proving for Secondary Teachers*. Our goal was to explore how PSTs' knowledge and dispositions towards the teaching and learning of proof develop as a result of participating in the course, and to identify design principles that afford PSTs' learning.

Theoretical Framework and the Course Design

Researchers have conjectured that engaging students in reasoning and proving, that is, exploring, generalizing, conjecturing and justifying, might require a special type of teacher knowledge: Mathematical Knowledge for Teaching of Proof (MKT-P) (e.g., Lesseig, 2016). Building on their work, we theorize that MKT-P consists of four interrelated types of knowledge (Buchbinder & Cook, 2018). Teachers must have robust Subject Matter Knowledge (SMK) of (a) mathematical concepts and principles (Ball, Thames & Phelps, 2008), and (b) knowledge of the logical aspects of proof, such as proof techniques, valid and invalid arguments, the functions of proof and the role of examples and counterexamples in proving (Hanna & deVillers, 2012). Teachers also need strong Pedagogical Content Knowledge (PCK) specific to proving, such as (c) knowledge of students' proof-related conceptions and misconceptions, and (d) knowledge of pedagogical strategies for supporting students' proof activities (Buchbinder & Cook, 2018). Thus, we created opportunities for PSTs to develop these four types of knowledge.

The course comprises four three-week long modules, each module addressing one proof theme: quantified statements, conditional statements, direct proof, and indirect reasoning. These themes were identified in the literature as challenging for students and PSTs (e.g., Weber, 2010). Our first goal was to help PSTs to enhance their knowledge of the four themes of the course. Figure 1 shows the structure of the course (top) and a structure of one course module (bottom).

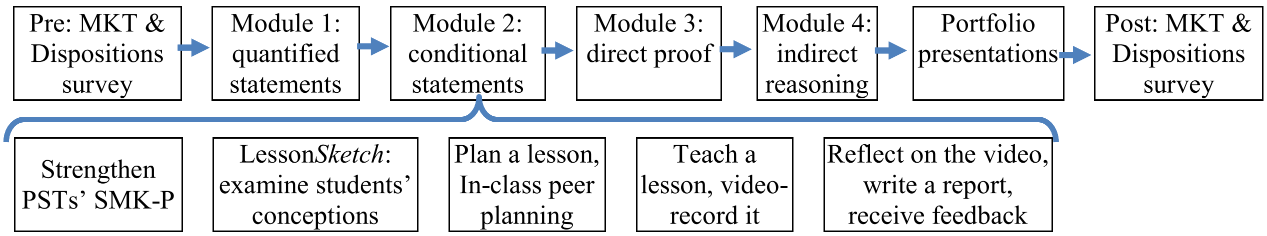

Figure 1. Design of the course *Mathematical Reasoning and Proving for Secondary Teachers*.

We designed opportunities for PSTs to enhance their proof related pedagogical knowledge, by interpreting sample student work, identifying students' conceptions of proof, and envisioning responding to students' thinking in the virtual learning platform Lesson*Sketch* (Herbst & Chazan, 2015). We also engaged PSTs in planning and implementing, in local schools, lessons that combine the proof themes with mathematical content. As PSTs enacted their lesson, they recorded it with 360° video cameras, which captured simultaneously the PSTs' teaching and the students' engagement with proof-oriented lessons. PSTs then watched and analyzed their lesson, wrote a reflection report and received feedback from the course instructor.

Methods

Participants in the first iteration of the course were 15 PSTs in their senior year (4 middle-school, and 11 high-school track; 6 males and 9 females). The PSTs had completed the majority of their extensive mathematical coursework, and two educational methods courses.

Multiple measures were used to assess PSTs' learning, such as pre- and post- dispositions towards proof surveys, and an MKT-P instrument with sets of questions on the four types of MKT-P. We also collected PSTs' responses to home- and in-class assignments, video-recordings of in-class sessions, and PSTs' teaching portfolios containing four lesson plans, videos of the lessons taught, reflection reports and sample students' work. Below, we present initial results of our ongoing analysis of these data, focusing on the Conditional Statements (CS) module. The CS module comprised five activities as shown in Figure 1.

Results

The data analysis was guided by two main questions: (1) How did PSTs interact with the CS module?, and (2) What mathematical / pedagogical ideas addressed in the CS module got implemented in PSTs' lessons? In this section we present findings that respond to these questions and also describe evidence of the PSTs' learning.

PSTs' Interaction with the Conditional Statements Module

In the first activity, PSTs broke up into three groups and each group received one conditional statement, such as "A graph of an odd function, defined at zero, passes through the origin." The task was to identify its hypothesis P and conclusion Q, and use the P and Q to write statements in 11 logical forms such as: $P\ if\ Q$, $P\ only\ if\ Q$, and $If\ {\sim}P\ then\ {\sim}Q$. The PSTs were asked to differentiate between true and false statements, and to identify which statements are equivalent to $P \Rightarrow Q$. The most challenging form for PSTs to interpret was P *only if* Q. Some PSTs realized that it is equivalent to a contrapositive, $\sim Q \Longrightarrow \sim P$, and thus equivalent to the original statement. Other PSTs rejected this idea by arguing that P can be true "not only when Q is true." After a class discussion, all discrepancies appeared to be resolved.

The second activity utilized Lesson*Sketch*.org, a web-based platform for teacher education in which PSTs can analyze representations of classroom interactions depicted as cartoon sketches (Herbst & Chazan, 2015). The experience *Who is right?* first asked PSTs to decide whether the (false) statement: "If n is a natural number, then $n^2 + n + 17$ is prime," is true, false, sometimes true, or cannot be determined. Next, the PSTs viewed arguments of five pairs of students about the truth-value of this statement and were asked to evaluate the correctness of these arguments, identify instances of expertise and gaps in student reasoning and pose questions to advance or challenge that reasoning. The PSTs correctly identified invalid reasoning and proposed ways to acknowledge students' effort, while pointing them towards more valid modes of argumentation.

In the final activity, the PSTs analyzed current middle school and high school textbooks to identify where conditional statements appear in the secondary curriculum. PSTs also addressed students' difficulties with conditional statements and discussed ways to support student thinking.

PSTs' Implementation of a Lesson on Conditional Statements

After spending time developing lessons that involved conditional statements, PSTs implemented their lessons in the classrooms participating in the study. PSTs came up with many creative ways to integrate conditional statements in their lessons while appropriately adjusting them to the students' level. All PSTs used real-world examples, e.g., "If I do my homework, I will get good grades" to introduce the general structure of a conditional statement, and to identify their hypothesis (P) and the conclusion (Q). For example, Sam showed product advertisements, and asked students to turn them into conditional statements and analyze their structure.

The most utilized types of tasks implemented by the PSTs were True or False, and Always-Sometimes-Never, where students had to identify the hypothesis and the conclusion, determine whether the statement is true or false, and provide justifications or counterexamples. Bill had his students match hypotheses (e.g., a triangle is not equilateral) with conclusions (e.g., a triangle is isosceles), written on index cards, to produce true conditional statements. Then the students switched the order of the cards to examine how the statement and its converse compare. Overall, except for four PSTs who only minimally addressed conditional statements, the majority of PSTs successfully and creatively integrated conditional statements in their lessons.

Evidence of PSTs' Learning

To trace changes in PSTs' knowledge and dispositions following the course, we administered Dispositions Towards Proof pre- and post- surveys and a 12 items MKT-P instrument (3 question in each of the four areas of MKT-P). The analysis revealed that PSTs had relatively high initial scores on three out of four types of MKT-P. Hence instead of examining the point increase, we calculated the percentage of possible growth. E.g., although the increase in the Knowledge of Logical Aspects of Proof was only 1.15 points, it constitutes 48% of possible 2.4 points needed to obtain the maximum score. The highest gain of 66% occurred in the Knowledge of Pedagogical Practices for supporting students' learning of proof. The items measuring PSTs performance in these areas called for analyzing, interpreting and responding to students' conceptions of proof, similar to activities described above in the CS module.

The Dispositions Survey had five categories of questions: (1) notions of a proof, (2) purpose of proof, (3) confidence and comfort with proving, (4) suitability of proof in the secondary curriculum, and (5) confidence in and knowledge about teaching proof. In general, the pre- and post-test results did not show much change in categories 1, 2 and 4; a few noteworthy results occurred in categories 3 and 5. The PSTs' confidence in their ability to prove results from the school curriculum increased from 65% to 92%; the percent of PSTs recording confidence of teaching proof to students, increased from 50% to 84%. The fact that PSTs' overall growth of dispositions was relatively modest may be attributed to an initial, perhaps false, feeling of confidence due to the PSTs' prior mathematical coursework.

Discussion and Implications

We reported on the first iteration of a 3-year DBR project aimed to enhance PSTs' knowledge and dispositions for teaching proof at the secondary level. We described the overall structure of the capstone course and the theoretical underpinnings of its design. We described how PSTs interacted with the CS module through practice-based activities of analyzing students' conceptions of proof; planning and sharing lessons; classroom implementation and reflecting on it, aided by video technology. These descriptions serve as a backdrop for understanding the gains in PSTs' content and pedagogical knowledge of proof (MKT-P). The areas in which this growth was most evident are: the logical aspects of proof and pedagogical knowledge specific to proving. Although the small sample size does not allow testing statistical significance of these outcomes, we find them encouraging, since the supporting evidence goes beyond self-report.

Our data analysis is still ongoing, as we examine video of on-campus sessions and of the PSTs' teaching to create more fine-grained description of how PSTs' content and pedagogical knowledge evolved throughout the course, and to match this growth to the design principles of the course. The results of this analysis will inform future iterations of our project, and, potentially advance the field's understanding of how to support PSTs' development of MKT-P.

Acknowledgments

This research was supported by the National Science Foundation, Award No. 1711163. The opinions expressed herein are those of the authors and do not necessarily reflect the views of the National Science Foundation.

References

Association of Mathematics Teacher Educators (2017). *Standards for preparing teachers of mathematics*. Available online at: amte.net/standards.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59* (5), 389 – 407.

Buchbinder, O., & Cook, A. (2018). Examining the mathematical knowledge for teaching of proving in scenarios written by pre-service teachers. In O. Buchbinder & S. Kuntze (Eds.). *Mathematics teachers engaging with representations of practice* (pp. 131-154). New York: Springer.

Hanna, G. & deVillers, M. D. (2012). *Proof and proving in mathematics education: The 19th ICMI study*. New York: Springer.

Herbst, P. and Chazan, D. (2015). Using Multimedia Scenarios Delivered Online to Study Professional Knowledge Use in Practice. *International Journal of Research and Method in Education,* 38(3), 272-287

Kotelawala, U. (2016). The status of proving among US secondary mathematics teachers. *International Journal of Science and Mathematics Education, 14*(6), 1113-1131.

Lesseig, K. (2016). Investigating mathematical knowledge for teaching proof in professional development. *International Journal of Research in Education and Science, 2*(2), 253-270.

National Governors' Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards: Mathematics.* Washington DC: Council of Chief State School Officers.

Weber, K. (2010). Mathematics majors' perceptions of conviction, validity, and proof. *Mathematical Thinking and Learning, 12*(4), 306-336.