Anomalous negative magnetoresistance of two-dimensional electrons

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Effects of temperature T (6–18 K) and variable *in situ* static disorder on dissipative resistance of two-dimensional electrons are investigated in GaAs quantum wells placed in a perpendicular magnetic-field $B_\perp$. Quantum contributions to the magnetoresistance, leading to quantum positive magnetoresistance (QPMR), are separated by application of an in-plane magnetic field. QPMR decreases considerably with both the temperature and the static disorder and is in good quantitative agreement with theory. The remaining resistance R decreases with the magnetic field exhibiting an anomalous polynomial dependence on $B_\perp$: $[R(B_\perp) - R(0)] = A(T,\tau_q)B_\perp^\eta$ where the power is $\eta \approx 1.5 \pm 0.1$ in a broad range of temperatures and disorder. The disorder is characterized by electron quantum lifetime τ_q. The scaling factor $A(T,\tau_q) \sim [\kappa(\tau_q) + \beta(\tau_q)T^2]^{-1}$ depends significantly on both τ_q and T where the first term $\kappa \sim \tau_q^{-1/2}$ decreases with τ_q. The second term is proportional to the square of the temperature and diverges with increasing static disorder. Above a critical disorder the anomalous magnetoresistance is absent, and only a positive magnetoresistance, exhibiting no distinct polynomial behavior with the magnetic field, is observed. The presented model accounts memory effects and yields $\eta = 3/2$.

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I. INTRODUCTION

Within Boltzman-Drude kinetic theory the magnetoresistance of two-dimensional (2D) electrons is absent [1]. In practice 2D electron systems exhibit both a positive magnetoresistance (PMR) [2,3] and a negative magnetoresistance (NMR) [4–16], which are attributed to non-Markovian processes in the dynamics of classical electrons moving in a static disorder potential [17–29]. In high mobility samples the negative magnetoresistance is strong and depends considerably on the temperature and sample size [11,13,14]. A recent theoretical model relates this NMR to a reduction of the electron viscousity in magnetic fields [30]. Despite significant efforts to understand both PMR and NMR, a quantitative agreement between the experiments and the theory remains to be quite illusive.

Mentioned above, positive and negative magnetoresistances are observed in the high-temperature domain $kT \gg \hbar\omega_c$, where ω_c is the cyclotron frequency. At this condition quantum (Shubnikov–de Haas) oscillations of the resistance are completely suppressed by the temperature, and the classical electron transport is often assumed to be dominating. It has been shown, however, that Landau quantization of the electron spectrum affects significantly the electron scattering already in the high-temperature domain [31]. The spectrum quantization leads to a considerable quantum positive magnetoresistance (QPMR), which was observed recently at small magnetic fields in high quality samples [32,33]. Thus, the magnetoresistance may contain contributions from different classical effects mixed with the quantum contributions. The mix may lead to a significant discrepancy between the experimental data and the theoretical models in the high-temperature domain. An extraction (separation) of the classical effects from this mix presents a challenge especially in the high magnetic-field sector where QPMR is strong.

Recently we have observed that application of a magnetic field parallel to the 2D layer suppresses the quantum contributions to the magnetoresistance [34,35]. The suppression correlates with the spin splitting of Landau levels Δ_Z and reaches an extremal value at $\Delta_Z \approx \hbar\omega_c/2$ at which the density of states (DOS) is nearly constant in small quantizing magnetic fields. This observation opens a way to quantitatively study contributions to the magnetoresistance which are not related to the DOS quantization. Below we label these contributions as a classical magnetoresistance. We note, however, that an absence of the quantization of the density of states may not be sufficient to eliminate completely quantum mechanical outcomes as has been shown recently [36].

The paper presents an experimental investigation of effects of temperature and a static disorder on both quantum and classical magnetoresistances. In the experiment the static disorder is controlled *in situ* and varies continuously. We study a gated remotely doped GaAs single quantum well of width $d =$ 13 nm sandwiched between AlAs/GaAs superlattice barriers. The superlattice barriers contain X electrons screening charged dopants [37]. This screening enhances both the electron transport mobility and, to a larger extent, the electron quantum lifetime τ_q since the dopants are localized at a distance of $L_d \approx$ 36 nm from the conducting 2D layer and predominantly induce a small-angle electron-impurity scattering [38]. A negative

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gate voltage depopulates first the nearest screening electron layer leading to a strong reduction of the electron quantum lifetime, which is sensitive to the small-angle scattering. The transport time τ_{tr}, which is determined by the large-angle scattering, shows significantly smaller absolute variations [39]. The 2D electron density stays nearly the same in this regime [40]. A further decrease in the gate voltage depopulates completely the screening layer, and 2D electron density starts to follow the gate voltage [39,40].

Our experiments indicate strong effects of the temperature and the static disorder on both the QPMR and the remaining classical magnetoresistance, which is found to be negative and demonstrating an anomalous polynomial behavior at small magnetic fields. The focus of this paper is this negative magnetoresistance. To characterize the static disorder we use the quantum scattering time obtained from QPMR [33,34].

II. EXPERIMENT AND RESULTS

A studied GaAs quantum well was grown by molecular beam epitaxy on a semi-insulating (001) GaAs substrate. Samples were etched in the shape of a Hall bar. The width and the length of the measured part of the samples are $W = 50$ and $L = 250$ μm. AuGe eutectic was used to provide electric contacts to the 2D electron gas. Two samples were studied in magnetic fields up to 9 T applied *in situ* at different angles α relative to the normal to 2D layers and perpendicular to the applied current. Angle α has been evaluated using Hall resistance $R_H = B_\perp/(en)$, which is proportional to the perpendicular component $B_\perp = B\cos(\alpha)$ of the total magnetic-field B. The electron-density n was evaluated from the Hall measurements taken at $\alpha = 0^\circ$ in classically strong magnetic fields. Sample resistance was measured using the four-point probe method. We applied a 133-Hz ac excitation $I_{ac} = 1$ μA through the current contacts and measured the longitudinal (in the direction of the electric current, the x direction) and Hall ac (along the y direction) voltages (V^{ac}_{xx} and V^{ac}_H) using two lock-in amplifiers with 10-MΩ input impedance. The measurements were performed in the linear regime in which the voltages are proportional to the applied current. Both samples have demonstrated a similar magnetoresistance. Below we present data obtained on Sample N1.

Figure 1(a) presents magnetic-field dependencies of the dissipative resistance R_{xx} of 2D electrons taken at two different angles α between the magnetic-field B and the normal to 2D electron layer at different temperatures. At angle $\alpha = 0^\circ$ the magnetic-field B is perpendicular to the 2D layer, and in GaAs quantum wells the Zeeman spin splitting $\Delta_Z = \mu g B$ is negligibly small in comparison with the cyclotron energy $\Delta_c = \hbar\omega_c$, where μ is the Bohr magneton and g is the electron g factor. At $B_\perp$ exceeding $\sim$0.2 T the electron spectrum is quantized leading to the QPMR [33].

An application of the in-plane magnetic field suppresses QPMR [34]. The suppression correlates with the increase in the Zeeman term Δ_Z in tilted magnetic fields. At $\alpha = 82.5^\circ$ the QPMR suppression is in a vicinity of the extremum related to the condition: $\Delta_Z = \Delta_c/2$, corresponding to nearly constant electron DOS in small quantizing magnetic fields [34]. In accordance with a proposed model, QPMR is absent at this condition [34]. The difference between curves taken at these two angles is in quantitative agreement with QPMR theory [31], yielding the quantum scattering time τ_q [34]. An increase in the temperature decreases the QPMR considerably. The QPMR reduction is related to the decrease in the quantum scattering time τ_q at high temperatures due to the enhancement of electron-electron scattering [33].

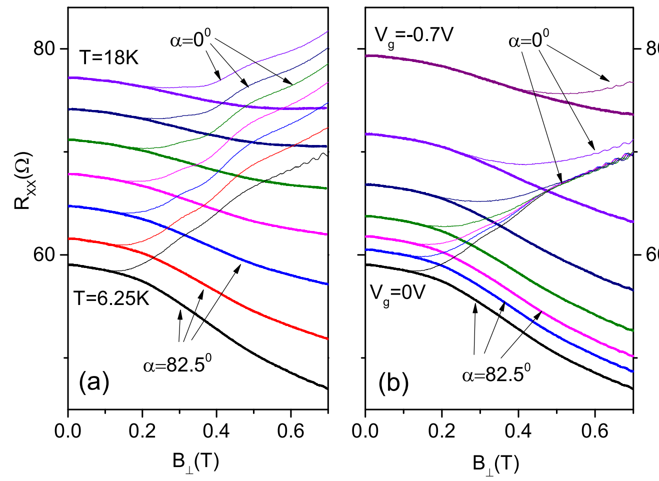

FIG. 1. Dependencies of the dissipative resistance R_{xx} of 2D electrons on a perpendicular magnetic field taken at two different angles between the magnetic-field B and the normal to the 2D layer: $\alpha = 0^\circ$ and $\alpha = 82.5^\circ$. (a) R_{xx} vs $B_\perp$ at gate voltage $V_g = 0$ V and different temperatures T: 6.25, 8, 10, 12, 14, 16, and 18 K. (b) R_{xx} vs $B_\perp$ at temperature $T = 6.25$ K and different gate voltages $V_g = 0$ V and between -0.2 and -0.7 V with step -0.1 V.

In Fig. 1(a) thick lines demonstrate the magnetoresistance at $\alpha = 82.5^\circ$ obtained at $\Delta_Z \approx \Delta_c/2$ corresponding to a nearly constant DOS. At $V_g = 0$ V the magnetoresistance is negative in the studied temperature range. Higher temperatures make the NMR progressively weaker. The notable feature of the curves is the independence of the magnetoresistance on angle α at small magnetic-fields $B_\perp < 0.1$ T. At these fields the quantization of the electron spectrum is exponentially suppressed, and the DOS does not depend on Landau and Zeeman splittings. The progressively strong deviation between curves at higher $B_\perp$ indicates the progressively strong modulations of the DOS due to quantization of the electron spectrum [34].

Figure 1(b) presents magnetic-field dependencies of the resistance R_{xx} taken at two different angles α and different gate voltages V_g. This set of curves demonstrates the effect of the static disorder on both QPMR and NMR. Qualitatively, effects of the temperature and the static disorder on the magnetoresistance look similar: An increase in the temperature or disorder reduces both QPMR and NMR. Below we investigate these effects quantitatively.

Figure 2(a) presents the 2D electron-density n obtained from an analysis of the Hall resistance at different gate voltages V_g. In the range between 0 and -0.6 V the 2D electron density changes weakly with the gate voltage. In this regime the applied voltage depopulates the screening layer with X electrons leading to a substantial increase in the smooth electrostatic potential of the remote dopants. It results in a strong enhancement of the small-angle scattering and a significant increase in the quantum scattering rate $1/\tau_q$. A comparison of the $1/\tau_q$,

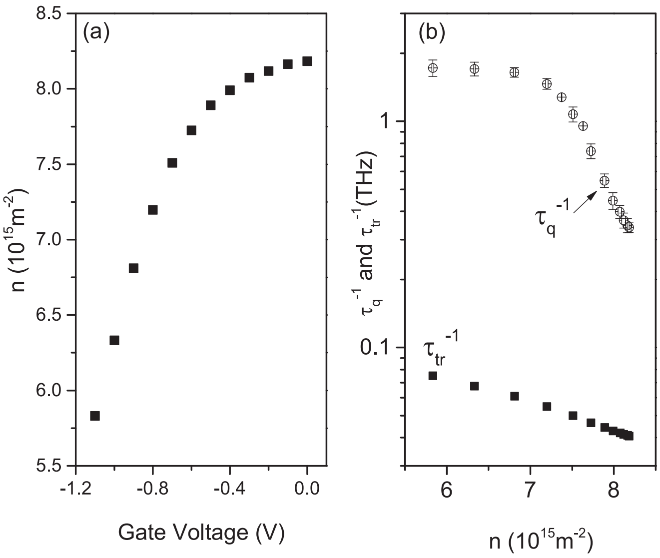

FIG. 2. (a) Dependence of 2D electron density obtained from Hall resistance on gate voltage V_g. (b) The open symbols present the quantum scattering rate $1/\tau_q$, obtained from analysis of the magnitude of the quantum positive magnetoresistance shown in Fig. 1(b) at different electron densities. The filled squares present the transport scattering rate $1/\tau_{\rm tr}$, obtained from resistance at $B = 0$ T using the Drude formula for the resistivity at different electron densities. $T = 6.25$ K.

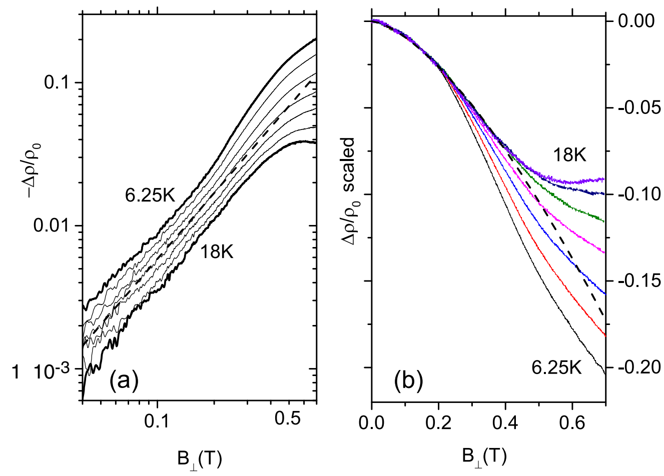

FIG. 3. (a) Variations of normalized resistivity $-\Delta\rho/\rho_0 = -[\rho(B_\perp) - \rho_0]/\rho_0$ with magnetic-field $B_\perp$, obtained from the dependencies $R_{xx}(B_\perp)$ presented in Fig. 1(a) at angle $\alpha = 82.5^\circ$ and different temperatures from $T = 6.25$ to $T = 18$ K. Plotted on a log-log scale these variations reveal a polynomial behavior: $\Delta\rho/\rho_0 = AB_\perp^\eta$ at small magnetic fields for all studied temperatures, where A is a coefficient and $\eta = 1.5 \pm 0.1$. The dashed line corresponds to $-\Delta\rho/\rho_0 \sim B_\perp^{3/2}$. (b) The resistivity variations $\Delta\rho/\rho_0$ at different temperatures scaled to the dependence at $T = 6.25$ K and presented on the linear scale. The dashed line corresponds to $\Delta\rho/\rho_0 \sim -B_\perp^{3/2}$. $V_g = 0$ V.

obtained from the analysis of QPMR [34], and the transport scattering rate $1/\tau_{\rm tr}$, obtained from Drude conductivity, is shown in Fig. 2(b). The figure demonstrates that the absolute variations of the transport scattering rate are much smaller than the $1/\tau_q$ variations, pointing toward the enhancement of the small-angle electron scattering in the system.

Furthermore, the relative variations of the transport scattering rate are also considerably smaller than the relative variations of the quantum scattering rate. This indicates the presence of a substantial amount of large-angle scatterers, such as rigid impurities localized inside the quantum well with a sharp scattering potential nearly independent of the X-electron screening. Thus Fig. 2(b) suggests that the static disorder contains sharp impurities embedded into a variable smooth electrostatic background. At gate voltages less than -0.8 V the X-electron layer is completely depopulated, and 2D electron density follows the gate potential. In this regime the quantum scattering time does not change significantly, indicating a weak variation of the static disorder. Below we use Fig. 2 to evaluate the disorder potential at different gate voltages.

Figure 3(a) presents variations of the normalized magnetoresistivity $-\Delta\rho/\rho_0 = -[\rho(B_\perp) - \rho_0]/\rho_0$ with magnetic-field $B_\perp$ at different temperatures, where ρ_0 is the resistivity at zero magnetic field. The figure demonstrates that at small magnetic fields the magnetoresistivity follows a polynomial law: $\Delta\rho/\rho_0 = A(T)B_\perp^\eta$ where power $\eta \approx 1.5 \pm 0.1$ and the scaling factor $A(T)$ depends on the temperature. The obtained polynomial decrease in the resistance is anomalous and, to the best of our knowledge, is beyond existing theories. The figure shows that at a higher temperature the polynomial behavior extends to a higher magnetic field. Thus the temperature promotes this anomalous magnetoresistance. An anomalous polynomial behavior of colossal negative magnetoresistance has been seen recently in a 2D electron system with the power of $\eta = 1.4$ at $T = 0.25$ K. This anomalous behavior, however, disappears at high temperatures [14].

Figure 3(b) presents the normalized negative magnetoresistance $\Delta\rho/\rho_0$ at different temperatures scaled vertically to the curve at $T = 6.25$ K using a scaling coefficient $K_Y(T) = A(T = 6.25\ \text{K})/A(T)$. The figure shows that at high temperatures (14–18 K) the scaling exists up to $B_\perp \approx 0.5$ T. A decrease in the temperature down to 6.25 K shrinks the range of the anomalous polynomial behavior inside the interval (0.03–0.2 T). At $T = 6.25$ K and $B_\perp > 0.2$ T the resistance decreases faster than $B_\perp^{1.5}$.

Figure 4(a) presents the normalized negative magnetoresistance $\Delta\rho/\rho_0$ taken at $T = 6.25$ K and different V_g's from 0 V (bottom curve) to -1 V (top curve). At $V_g < -0.8$ V a positive magnetoresistance grows up propagating to a smaller $B_\perp$ at $V_g = -1$ V. Figure 4(b) presents the anomalous negative magnetoresistance scaled to the curve at $V_g = 0$ V. The figure demonstrates that the anomalous negative magnetoresistance persists down to $V_g = -1$ V. In contrast to the temperature effect, the strong disorder reduces the magnetic-field range of the scaling behavior of NMR. At $V_g = -1$ V and higher temperatures the magnitude of NMR is very small making quantitative evaluations of the response not reliable. At $V_g = -1.1$ V the negative magnetoresistance is absent, and only a positive magnetoresistance with no distinct polynomial behavior is observed (not shown). This is out of the scope of this paper.

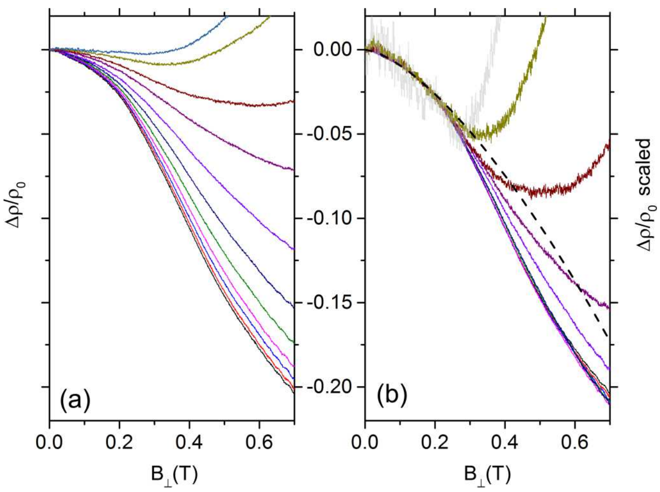

FIG. 4. (a) Dependence of the normalized resistivity $\Delta\rho/\rho_0 = [\rho(B_\perp) - \rho_0]/\rho_0$ on magnetic-field $B_\perp$ at angle $\alpha = 82.5^\circ$ and different V_g's from 0 V (bottom curve) to -1 V (top curve) with step -0.1 V. (b) The normalized magnetoresistivity $\Delta\rho/\rho_0$ at all different gates V_g shown in (a), scaled to the dependence at $V_g = 0$ V, indicates the robustness of the anomalous polynomial behavior at small $B_\perp$ with respect to variations of both the electron density and the static disorder. The dashed line corresponds to $\Delta\rho/\rho_0 \sim -B_\perp^{3/2}$. $T = 6.25$ K.

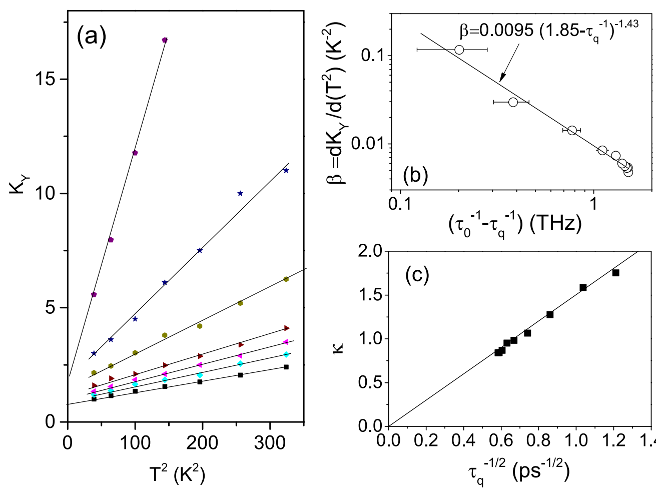

FIG. 5. (a) A linear dependence of the scaling factor $K_Y(T,V_g) = A(T = 6.25\text{ K}, V_g = 0\text{ V})/A(T,V_g)$ on the square of the temperature at different gate voltages V_g from the bottom to the top lines: 0, -0.4, -0.5, -0.6, -0.7, -0.8, and -0.9 V. The dependence is approximated by the following relation: $K_Y = \kappa[\tau_q(T = 0\text{ K})] + \beta[\tau_q(T = 0\text{ K})]T^2$ where the first term $\kappa[\tau_q(T = 0\text{ K})]$ describes effects of the static disorder only and the second term $\beta[\tau_q(T = 0\text{ K})]T^2$ describes both temperature and disorder effects. (b) Dependence of the parameter β on the quantum scattering rate $1/\tau_q$ agrees with a critical behavior: $\beta \approx \beta_0(1/\tau_0 - 1/\tau_q)^\gamma$, where $\beta_0 \approx 0.0095 \pm 0.0015$, $1/\tau_0 \approx 1.85 \pm 0.15$ (THz) and $\gamma \approx -1.43 \pm 0.3$. (c) Dependence of the temperature-independent term κ on the disorder reveals the following relation: $\kappa \sim \tau_q^{-1/2}$ [41].

Figure 5(a) presents the dependence of the scaling coefficient $K_Y = A(T = 6.25\text{ K}, V_g = 0\text{ V})/A(T,V_g)$ on the square of the temperature for different gate voltages. The figure shows that the temperature variations of K_Y are proportional to T^2 suggesting the electron-electron interaction as the origin of the temperature dependence of the scaling factor $A(T)$. We have approximated the scaling coefficient K_Y by the following relation:

$$K_Y = \kappa(\tau_q) + \beta(\tau_q)T^2, \tag{1}$$

where $\kappa(\tau_q)$ is the intersect of the straight lines with the K_Y axis whereas the coefficient $\beta(\tau_q)$ describes the strength of the temperature-dependent term.

Figure 5(a) shows that the parameter β increases strongly at low gate voltages indicating a tendency for a divergence. Furthermore the experiments demonstrate no NMR at $V_g = -1.1$ V indicating $\beta = \infty$. These observations suggest a critical behavior of the coefficient β at the low gate voltages. Variations of QPMR with angle α indicate a g factor enhanced by the e-e interaction [34], but no divergence of the g factor is observed in the studied range of the gate voltages. This suggests that the density-dependent electron-electron interaction by itself does not diverge. Below we propose that the critical behavior of the parameter β is induced by variations of the static disorder characterized by the quantum scattering rate $1/\tau_q$,

$$\beta = \beta_0\left(\tau_0^{-1} - \tau_q^{-1}\right)^\gamma, \tag{2}$$

where parameter τ_0^{-1} characterizes the strength of a critical disorder and γ is the critical exponent. Figure 5(b) presents the coefficient β plotted vs quantum scattering rate τ_q^{-1} in accordance with Eq. (2) on a log-log scale using $\beta_0 = 0.0095 \pm 0.0015$, $\tau_0^{-1} = 1.8 \pm 0.15$ (THz) and $\gamma = -1.43 \pm 0.3$ as fitting parameters. The obtained agreement supports the proposal.

Figure 5(c) presents the evolution of the coefficient κ with the static disorder. In a broad range of the disorder the coefficient is $\kappa \sim \tau_q^{-1/2}$. This finding suggests that the anomalous NMR should be significantly enhanced in systems with a long quantum lifetime. This outcome agrees with the observation of a large NMR in high mobility samples [10–16].

III. DISCUSSION

Below we describe a qualitative model leading to the polynomial negative magnetoresistance $\Delta\rho \sim -B_\perp^{3/2}$. Theoretical investigations indicate a strong negative magnetoresistance in 2D electron systems with a static sharp disorder only [17,18,20–22,26–29]. The decrease in the resistance is related to a separation of the 2D electrons in two groups: wandering electrons performing both a diffusive motion and a Hall drift [17,18,20,21] and electrons which do not collide with impurities and participate only in the Hall transport [27]. An inclusion of the long-range smooth disorder leads to substantial modifications of the negative magnetoresistance [25,26]. Below we discuss the model presented in Ref. [25]. In this model, 2D electrons perform a diffusivelike motion in the presence of both a sharp disorder characterized by a transport scattering time τ_S and a long-range smooth disorder characterized by a transport scattering time τ_L. During the cyclotron period $2\pi/\omega_c$ the smooth disorder displaces the cyclotron guiding center by a value δ, which is assumed to be larger than the size of the sharp impurities a,

$$\delta^2 = 4\pi R_c^2/(\omega_c \tau_L) \gg a^2, \tag{3}$$

where R_c is the cyclotron radius. The negative magnetoresistance is related to a reduction of the electron exploration rate due to memory effects. To evaluate the exploration rate a strip of the width $2a$ is associated with the particle trajectory. The particle will hit a sharp impurity if the center of the latter is located within the strip. Due to the stochastic motion of the guiding center at $\delta \gg a$, there is a small probability $P_1 \sim a/\delta$ that after the first revolution the strip covers again the starting point. Taking into account the diffusive dynamics of the guiding center, its rms shift after n revolutions is $\delta_n = \delta\sqrt{n}$ so that the return probability decreases with n as $P_n = P_1/\sqrt{n}$. The total return probability $P = \Sigma_{n=1}^N P_n \approx (a/\delta)N^{1/2}$, where $N \approx (\omega_c \tau_S)/2\pi$, determines the fraction of the area explored twice leading to the reduction of the exploration rate and thus to a negative correction to the resistivity [25],

$$\Delta\rho_{xx}/\rho_0 \sim -(a/\delta)(\omega_c \tau_S)^{1/2} \sim -B_\perp^2. \tag{4}$$

The presented model assumes that the size of the sharp impurities is much larger than the electron wavelength λ_F: $a \gg \lambda_F$ and, thus, the strip of the width $2a$ is adequate in the counting of the area explored by an electron. In the opposite limit $a \ll \lambda_F$ a strip with the width of $2\lambda_F$ has to be used [42]. Furthermore, in a magnetic field the width of a quasiclassical cyclotron orbit is determined by the magnetic length $l_B = (\hbar/eB_\perp)^{1/2}$ [43]. Thus at $l_B > a$ a strip of the width $\sim l_B$ is more appropriate for the counting of the electron exploration rate. In this case the return probability is $P \sim l_B/\delta$ that leads to the following negative magnetoresistance:

$$\delta\rho_{xx}/\rho_0 \sim -(l_B/\delta)(\omega_c \tau_S)^{1/2} \sim -B_\perp^{3/2}. \tag{5}$$

The obtained magnetic-field dependence agrees with the dependence shown in Fig. 3(a) at small magnetic fields. Below we provide further justification of the applicability of the model. At $B_\perp < 0.5$ T the magnetic length $l_B > 35$ nm and exceeds the typical size a of neutral impurities, which is a few nanometers. These impurities, embedded in the quantum well, provide a strong electron scattering at a large angle enhancing significantly the dissipative transport in magnetic fields. To evaluate the relative contributions of the smooth and sharp disorder to the resistivity we estimate below the correlation length of the smooth disorder ξ and the transport scattering times τ_S and τ_L. The distance $L_d \approx 36$ nm between the Si-doping layer and the quantum well dictates that the correlation length ξ of the smooth disorder potential is about 36 nm [24]. Another estimation of the correlation length ξ via the ratio between quantum and transport scattering times [31,33]: $\xi^* = (\lambda_F/2\pi)(\tau_{tr}/\tau_q)^{1/2}$ yields a considerably smaller value of $\xi^* = 13$ nm. We note that the estimation of ξ^* is based on the assumption that all scattering events produce small angular deviations of electron trajectories. In other words, only the smooth disorder is accounted in this estimation. The discrepancy between ξ and ξ^* suggests the presence of a sharp disorder with a correlation length of $a < 36$ nm. Assuming that $1/\tau_{tr} = 1/\tau_S + 1/\tau_L$ and using $\xi = (\lambda_F/2\pi)(\tau_L/\tau_q)^{1/2}$, we have found $\tau_L = 200$ and $\tau_S = 29$ ps. Thus the sharp disorder with a correlation length $a < l_B$ provides the dominant contribution to the electron dissipative transport in magnetic fields less than 0.5 T, whereas the smooth long-range disorder controls the electron quantum lifetime τ_q. Similar conclusions regarding the static disorder have been obtained from the comparison of variations of the quantum and transport scattering rates with the gate voltage shown in Fig. 2(b). The obtained estimates and conclusions support the applicability of the presented model to the studied 2D electron system.

A description of the temperature dependence of the anomalous magnetoresistance requires further development. Below we present an attempt in this direction. The obtained temperature behavior of the scaling factor K_Y, presented by Eq. (1), suggests the relevance of the electron-electron scattering. The temperature dependence of the quantum scattering time, extracted from QPMR shown in Fig. 1(a), indicates that the electron-electron scattering rate is $1/\tau_{ee} \approx 1$ (GHz)T^2 (K), which agrees with the rate in other samples [33]. Thus the electron-electron scattering time τ_{ee} is about 25 ps already at $T = 6.25$ K and $\approx$3 ps at $T = 18$ K. This time is shorter than the transport scattering time τ_{tr} indicating that electron-electron scattering may have a considerable impact on the electron transport. Due to the conservation of the total momentum in the electron-electron scattering, the latter does not contribute directly to the dissipative transport of electrons. However, these processes may change significantly both the return probability $P_1 = l_B/\delta$ via a modification of the parameter δ and the total return probability P via a modification of the number of the returns N.

A strong electron-electron scattering $1/\tau_{ee} \gg 1/\tau_S, 1/\tau_L$ produces an additional *strong* diffusivelike motion of electron cyclotron guiding centers mixing the diffusion in smooth and sharp disorder potentials. At these conditions it is reasonable to assume that the scattering rates, controlling parameters δ, N and, thus, the memory effects, are the same and have a form $1/\tau_m = 1/\tau_{st} + 1/\tau_{ee}^*$, where τ_m is a memory-breaking time, τ_{st} is a memory-breaking time due to the static disorder, and τ_{ee}^* is a memory-breaking time due to the electron-electron scattering. A substitution of $1/\tau_m$ instead of $1/\tau_S$ and $1/\tau_L$ in Eqs. (3) and (5) yields

$$\Delta\rho_{xx}/\rho_0 = -\frac{\hbar^{1/2} e^{3/2} B_\perp^{3/2}}{2\sqrt{2}\pi m p_F}\left[\frac{1}{\tau_{st}} + \frac{1}{\tau_{ee}^*}\right]^{-1}, \tag{6}$$

where m and p_F are the electron mass and momentum at Fermi energy. Equation (6) indicates that the anomalous magnetoresistance is proportional to the memory-breaking time τ_m. The obtained structure of the temperature- and disorder-dependent factor τ_m is compatible with the scaling coefficient K_Y: $K_Y = \kappa + \beta T^2 \sim 1/\tau_{st} + 1/\tau_{ee}^*$, providing $1/\tau_{ee}^* \sim T^2$.

A direct comparison of Eq. (6) with the magnetoresistance, shown in Fig. 3(b), yields the memory-breaking time $\tau_m = 5.7$ ps at $T = 6.25$ K and $V_g = 0$ V. The obtained value is somewhat between an expected value of $\tau_m^{ex} \approx 12$ ps, following from the relation $1/\tau_m^{ex} = 1/\tau_{tr} + 1/\tau_{ee}$ and the quantum scattering time $\tau_q \approx$3 ps. An analysis of the temperature variations of the scaling coefficient K_Y, shown in Fig. 5(a), yields the following relation for the memory-breaking rate in gigahertz, $1/\tau_m = 141 + 0.89T^2$ (K) at $V_g = 0$ V, resulting in $1/\tau_{st} = 141$ (GHz) and $1/\tau_{ee}^*$ (GHz) $= 0.89T^2$ (K).

Far from the critical disorder $1/\tau_0$ at $V_g = 0$ V the (e-e)-induced memory-breaking rate $1/\tau_{ee}^*$ is close to the electron-electron scattering rate obtained from the analysis of the temperature dependence of QPMR shown in Fig. 1(a): $1/\tau_{ee}$ (GHz) $= (1 \pm 0.1)T^2$ (K) [33,34]. At the critical disorder, at $V_g = -0.9$ V the memory-breaking rate $1/\tau_{ee}^*$ is an order of magnitude stronger than the electron-electron scattering rate extracted from QPMR. It suggests that an effectiveness of e-e processes, which destroy the memory effects, increases significantly with the static disorder. Furthermore the experiment shows no divergency of the parameter $\kappa \sim 1/\tau_{\rm st}$ indicating again that the presence of the electron-electron scattering is required to suppress the anomalous magnetoresistance.

Figure 5(c) shows that the memory-breaking rate due to the static disorder $1/\tau_{\rm st}$ is proportional to the $\tau_q^{-1/2}$, suggesting that $1/\tau_{\rm st} = (\tau_q \tau_{\rm st}^*)^{-1/2}$ where the quantum scattering time τ_q accounts a contribution of the small-angle scattering whereas the time $\tau_{\rm st}^* \approx 13$ ps accounts contributions of the large-angle scattering events to the memory-breaking rate due to the static disorder. Obtained results suggest nontrivial mutual relations among the small-angle scattering, the large-angle scattering, and the electron-electron interactions leading to the reduction of the anomalous negative magnetoresistance.

IV. CONCLUSION

To summarize an anomalous polynomial negative magnetoresistance of the 2D electrons $\Delta\rho \sim A(\tau_q,T)B_\perp^\eta$ is observed, where $\eta \approx 1.5 \pm 0.1$. The factor $A(\tau_q,T) \sim [\kappa(\tau_q) + \beta(\tau_q)T^2]^{-1}$ depends on temperature T and static disorder characterized by the quantum scattering time τ_q. The temperature-dependent term is proportional to T^2 suggesting a dominant contribution of the electron-electron interactions to the temperature dependence of the magnetoresistance. The temperature-independent term $\kappa(\tau_q)$ is found to be proportional to $\tau_q^{-1/2}$ and describes the considerable reduction of the negative magnetoresistance by the static disorder. The factor β is found to be diverging with the static disorder: $\beta \sim (\tau_0^{-1} - \tau_q^{-1})^\gamma$ where the critical exponents are $\gamma \approx 1.4 \pm 0.3$ and $\tau_0^{-1} \approx 1.85 \pm 0.15$. Above the critical scattering rate $1/\tau_0$ the anomalous negative magnetoresistance is absent, and only a positive magnetoresistance, exhibiting no distinct polynomial behavior with the magnetic field, is observed.

The presented model of the phenomenon is based on memory effects accounting for the return probability of the semiclassical trajectories and leading to the polynomial magnetic-field dependence: $\Delta\rho \sim \tau_m B_\perp^{3/2}$. The temperature dependence of the anomalous magnetoresistance is compatible with the model, assuming that the memory-breaking time τ_m has a form: $1/\tau_m = 1/\tau_{\rm st} + 1/\tau_{ee}^*$, where $\tau_{\rm st}$ is the memory-breaking time due to static disorder and τ_{ee}^* is the memory-breaking time due to electron-electron scattering.

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