D-FROST: Distributed Frequency Reuse-Based Opportunistic Spectrum Trading via Matching With Evolving Preferences

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***Abstract*—Spectrum trading creates more accessing opportunities for secondary users (SUs), and economically benefits the primary users (PUs). Compared with centralized spectrum trading designs, e.g., spectrum auction, distributed spectrum trading captures instantaneous spectrum trading opportunities better over large geographical regions without incurring extra infrastructure deployment and has no network scalability issues. However, the existing distributed spectrum trading designs have limited concern regarding spectrum reuse. Considering spatial reuse, in this paper, we propose a novel distributed frequency reuse-based opportunistic spectrum trading (D-FROST) scheme, which can further improve spectrum utilization, provide more accessing opportunities for SUs, and increase the revenues of PUs. In this paper, we employ conflict graph to characterize the SUs' co-channel and radio interferences, and mathematically formulate a centralized PUs' revenue maximization problem under multiple wireless transmission constraints. Due to the NP-hardness to solve the problem and the non-existence of centralized trading entity, we develop the D-FROST algorithms based on matching with evolving preferences, and prove its stability. Through extensive simulations, we show that the proposed D-FROST algorithm is superior to other distributed spectrum trading algorithms without considering spectrum reuse, yields results close to the centralized optimal one, and is effective in increasing PUs' revenue and improving spectrum utilization.**

***Index Terms*—Distributed spectrum trading, frequency reuse, matching with evolving preferences, spectrum utilization, revenue.**

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I. INTRODUCTION

IN THE past few years, hand-held wireless devices and wireless services have gradually become an indispensable part of people's daily life. The recent escalating demand of smartphones and tablets especially activates the enlargement of wireless communication applications (e.g., live video meeting, group messaging, online gaming, on-demand video streaming, etc.), which leads to a rapidly increase of the requirement for radio spectrum [1]–[5]. On the other hand, current static spectrum trading policy of Federal Communications Commission (FCC) [1], [2] utterly exhausts the scarce spectrum resources. However, even in the most crowed region of bustling urban, many licensed spectrum bands are extremely under-utilized in certain geographical areas and are idle most of the time [6]. The dilemma between the proliferation of wireless users and the depletion of spectrum motivates FCC to open up licensed spectrum bands and seek new dynamic spectrum access methods [1]. As one of the most promising solutions, cognitive radio (CR) technology releases the spectrum from shackles of authorized licenses, and enables secondary users (SUs) to opportunistically access to the vacant licensed spectrum bands in either temporal or spatial domain [2], [3].

Prior work has investigated spectrum trading issues from different aspects. To be specific, due to the economic values of frequency, the idea of opportunistic spectrum accessing has initiated the spectrum market, in which primary users (PUs) can sell/lease/auction their vacant spectrum for monetary gains, and SUs can purchase/rent/bid the available licensed spectrum if they suffer from the lack of radio resources to support their traffic demands [7]–[9]. In our paper, we define revenue is the monetary gain from trading/lending spectrum to the SUs. Different from common commodities or resources, spectrum can be spatially reused, and this special feature of spectrum has promoted a lot of research on the centralized designs of spectrum trading. For example, Zhou *et al.* in [7] have proposed a strategy-proof spectrum auction considering frequency reuse. Jia *et al.* in [8] have investigated how to design the trading and price to achieve the maximal revenue and enforce truthfulness as well. In [10], Zhou and Zheng have extended their work in [7], and presented a truthful double spectrum auction, called TRUST, where multiple PUs and SUs can trade bands according to their own demands. Beyond the per SU or per SU transmission pair based spectrum trading, Li *et al.* in [11] have proposed a per transmission link based spectrum trading design, and shown its economic-robustness. Although the centralized spectrum trading design has a joint consideration of spectrum reuse and the guarantee of economic properties, it needs the infrastructure deployment with extra economic and control cost, and the designated centralized spectrum traders (e.g., base stations or accessing points) may add huge energy consumption in existing networks. Besides, the centralized spectrum trading designs may not capture instantaneous accessing opportunities well, and have scalability issues, when the network size of SUs increases.

Beyond centralized designs, there are also some interesting distributed spectrum trading schemes in existing literature [12]. For example, Xing *et al.* in [13] and Niyato and Hossain in [14] investigated the spectrum pricing issues in the spectrum market, where multiple PUs, whose goal is to maximize the monetary gains with their vacant spectrum, compete with each other to offer spectrum access to the SUs. Peng *et al.* in [12] implemented the celebrated Vickrey-Clarke-Groves mechanism to two distributed spectrum auction mechanism, which achieve the maximization of the agents' individual utilities. Leveraging models in game theory, Wang *et al.* in [15] proposed to construct spectrum trading systems with desired properties, such as power efficiency, trading fairness, Pareto efficiency, collusion resistance and so on. Zhang *et al.* in [16] employed many-to-one/student-project matching to share the spectrum trying to maximize the social welfare in CR networks/LTE-Unlicensed systems, respectively. However, most existing distributed spectrum trading designs have little consideration of frequency reuse, which might cost PUs to lose some monetary gains and SUs to miss many valuable spectrum accessing opportunities, and limit the improvement of spectrum utilization.

To address the issues above, in this paper, we propose a distributed frequency reuse based opportunistic spectrum trading (D-FROST) scheme, which considers spectrum's special feature, spatial reuse, and allows spectrum trading between PUs and SUs in distributed manners. Briefly, we employ matching theory to trade the spectrum with the objective to maximize PUs' revenues. Different from traditional matching [16], the PU's preference list evolves, which depends on both SUs' evaluated valuation and SUs' interference relationship observed by the PU. We mathematically model the problem, develop D-FROST matching algorithm, prove its stability, and conduct performance evaluations. We find that the proposed D-FROST not only saves the cost for additional infrastructure deployment with extra power consumption as in centralized spectrum trading designs, but also provides more accessing opportunities for SUs, increases the revenues of PUs, and improves spectrum utilization compared with existing distributed designs. Compared to the conference version [17], we add subsection "Matching Preliminaries" to explain the basic concept of matching. We also consider different evaluated value of SUs in this paper, compared to unified evaluated value in conference version. We also prove our proposed algorithm is pairwise stable by contradiction, and add analysis part of computational complexity of our algorithm. Our salient contributions are listed as follows.

- We consider a spectrum trading market consisting of PU and SU transmission pairs as shown in Fig. 1. Similar to [18] and [19], we employ the conflict graph to characterize the SU transmission pairs' interference relationships, i.e., co-band interference and radio interference. Based on the constructed conflict graph, we formulate the centralized spectrum trading optimization problem with the objective of maximizing PUs' revenues under both frequency reuse and wireless transmission constraints. There is a lack of centralized spectrum trader and the formulated problem is a mixed integer nonlinear programming (MINLP).
- To pursuit feasible solutions in distributed manners, we exploit matching with preferences [20] to propose a novel D-FROST scheme, which jointly considers interference mitigation, frequency reuse, and spectrum trading benefits in matching process. In D-FROST, by opportunistically accessing to a certain band, the SU, who targets at maximizing its transmission rate, lists its preferences over PUs' bands based on his potential transmission rates over those bands. Considering frequency reuse, the PU, who targets at maximizing its revenues, will accommodate as many SUs as possible, in case that those SUs have no mutual interferences. A PU lists its preferences over SUs based on the SUs' evaluated valuation and its observations of SUs' conflicts, and the preference list evolves during the matching process. We mathematically present the PUs' and SUs' utility functions, develop the D-FROST, a two-phase matching algorithm with PUs' evolving preferences, and prove its pairwise stability [21].
- By carrying out extensive simulations with different numbers of PU and SU transmission pairs, we demonstrate that the proposed D-FROST has great advantages over other distributed spectrum trading algorithms without considering frequency reuse, and show that the feasible solutions obtained by the proposed algorithm are close to the optimal one in terms of the PUs' revenues, the aggregated network throughput of SUs, and the spectrum utilization.

The rest of paper is organized as follows. In Section II, we introduce the network model and related CR transmission

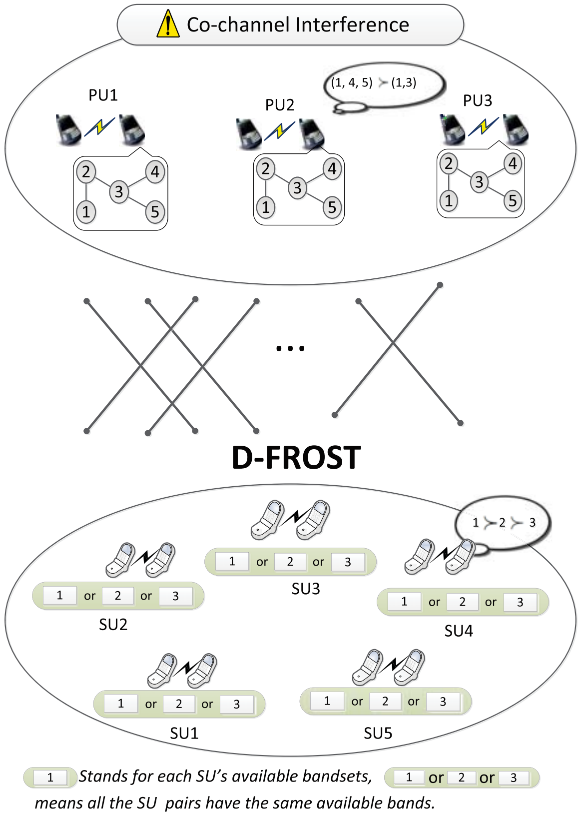


Fig. 1. Network Architecture for D-FROST.

models in FROST. In Section III, we mathematically formulate the FROST problem in centralized way and show its hardness to solve. We come up with a D-FROST solution, prove its stability and present its complexity in Section IV. In Section V we evaluate the performances, and draw conclusion remarks in Section VI.

II. Network Model

A. Network Configuration

We consider a spectrum trading plaza consisting of $\mathcal{N} = \{1, 2, \cdots, n, \cdots, N\}$ SU transmission pairs, and $\mathcal{M} = \{1, 2, \cdots, m, \cdots, M\}$ PU transmission pairs operating on different spectrum bands. For the simplicity of presentation, we use the words SU transmission pairs/SU pairs/SUs, and PU transmission pairs/PU pairs/PUs, interchangeably in the rest of this paper. Assuming each SU transmitter/receiver has only one radio interface, and each PU pair owns one spectrum band, i.e., PU $k \in \mathcal{M}$ owns band k.[1] Denote the unequal sized bandwidths[2] of PUs' bands by $\mathcal{W} = \{W^1, W^2, \cdots, W^m, \cdots, W^M\}$. In addition, all available spectrum bands at one SU are the same as those at another SU in the network, i.e., every SU has opportunity to access any PUs band in the overlapped coverage area of PUs. To put it in a mathematical way, let $\mathcal{M}_i \subseteq \mathcal{M}$ represent the set of available bands at SU pair $i \in \mathcal{N}$, then $\mathcal{M}_i = \mathcal{M}_j$ if $j \in \mathcal{N}$ and $i \neq j$.

In such a spectrum trading market, PUs sell available bands for monetary gains, and the SUs choose available bands of PUs to purchase for opportunistic accessing to deliver their traffic. Here, SU $i \in \mathcal{N}$ is allowed to opportunistically access to a licensed band $k \in \mathcal{M}$ and transmit with full power, when the services of PU_k are not active, but SU i has to reduce their transmission power to make sure that the aggregated interferences from SUs are below the "interference temperature" of PU_k's receiver [2], when primary services become active over band k. Suppose that the evaluated valuation of SUs for opportunistic spectrum accessing are $\mathcal{B} = \{b_1, b_2, \cdots, b_N\}$.[3] From the SU's perspective, SU_i would like to select a band k to maximize its data transmission rate; from the PU's perspective, considering spatial reuse [23]–[25], a PU would like to accommodate as many SUs as possible to maximize its revenue, as long as there are no co-channel interferences among those SUs as shown in Fig. 1.

B. Other Related Models in FROST

1) SU's Transmission Range/Interference Range: When primary services are not active over a certain band, SUs can transmit with full power over that band. Suppose all SUs have the same full transmission power P. The power propagation gain [22], [26] is

$$g_i = \gamma \cdot d_i^{-\alpha} \quad (i \in \mathcal{N}), \tag{1}$$

where α is the path loss factor, γ is an antenna related constant, and d_i is the distance between transmitter and receiver of SU pair i.[4] We assume that the data transmission is successful only if the received power at the SU pair's receiver exceeds the receiver sensitivity, i.e., a threshold P_{Tx}. Meanwhile, we assume interference becomes non-negligible only if it is over a threshold of P_{In} at the SU pair's receiver. Thus, the transmission range for a SU is $R_{Tx} = (\gamma P / P_{Tx})^{1/\alpha}$, which comes from $\gamma \cdot (R_{Tx})^{-\alpha} \cdot P = P_{Tx}$. Similarly, based on the interference threshold $P_{In} (P_{In} < P_{Tx})$, the interference range for a SU is $R_{In} = (\gamma P / P_{In})^{1/\alpha}$. It is obvious that $R_{In} > R_{Tx}$ since $P_{In} < P_{Tx}$. Typically, the interference range is 2 or 3 times of the transmission range [22], [23], [27], i.e., $\frac{R_{In}}{R_{Tx}} = 2$ or 3. These two ranges may vary with frequency. The conflict relationship between two SU pairs over the same frequency band can be determined by the specified interference range. In addition, if the interference range is properly set, the protocol model can be accurately transformed into the physical model as illustrated in [25].

2) Link Capacity/ Achievable Data Rate: We employ the ON/OFF model [28] to represent the active/inactive status of primary services. Suppose that PU_k is "OFF" with probability β_k, and is "ON" with the probability $(1-\beta_k)$ over band k. [28]

[1]Since each PU owns only one band, PU set $\mathcal{M}$ can also be used to represent the band set.

[2]Taking the least-utilized spectrum bands introduced in [22] for example, we found that the bandwidth between [1240, 1300] MHz (allocated to amateur radio) is 60 MHz, while bandwidth between [1525, 1710] MHz (allocated to mobile satellites, GPS systems, and meteorological applications) is 185 MHz.

[3]Note that we assume all the SUs bid with their true evaluation values for spectrum accessing. Bidding strategy/incentive mechanism designs are beyond the scope of this paper.

[4]The capacity formulation is similar if we consider fading. The major procedure of proposed algorithms will not be changed.

also present the specific β_k formulation when PU activity is modeled as exponentially distributed interarrivals.

When band k is available, SU_i accessing to this band can transmit with full power P, while other SUs within SU_i's interference range keep silent. We assume that the channel is a slow-fading channel with channel gain H_i; the distance between transmission and receiver of SU_i is d_i, and the variance of the additive white Gaussian noise at the receivers side is σ^2. The signal-to-noise ratio (SNR) between the SU_i transmission pair are

$$SNR_i = \frac{PH_i}{\sqrt{d_i}\sigma^2}, \tag{2}$$

and the capacity of SU $i \in \mathcal{N}$ over band $k \in \mathcal{M}$ is

$$c_i^{k,OFF} = W^k \log_2\Big(1 + \frac{SNR_i}{\gamma}\Big). \tag{3}$$

When band k is not available, SUs accessing to this band have to reduce their transmission power to make sure that the aggregated interference is below the "interference temperature" of PU_k [2]. Suppose that the interference tolerance power sensitivity for only one SU access to PU_k is P_Δ^k at PU_k's receiver. To make sure the aggregated interference power is lower than interference temperature, we assume $sP_\Delta^k \leq P_{Ik}$, and $P_\Delta^k \leq P_{Ik}/s$. The maximum number of accepted SUs is S, thus we add the constraint: $P_\Delta^k = P_{Ik}/S$. Therefore, it can be assure that the aggregated interference at the primary receiver from SUs is below the sensitivity Let SU_i accessing to band k transmit with power $P_i^{k,\mathrm{ON}}$. Then, we have $P_\Delta^k = P_i^{k,\mathrm{ON}} \cdot g_{ik} = P_i^{k,\mathrm{ON}} \cdot \gamma \cdot d_{ik}^{-\alpha}$, where d_{ik} is the distance between SU_i and PU_k. Thus, when PU_k is "ON", the capacity of SU_i over band k is

$$c_i^{k,\mathrm{ON}} = W^k \log_2\Big(1 + \frac{SNR_i}{sP^k\gamma d_{ik}^{-\alpha} + \gamma\sigma^2}\Big) \tag{4}$$

$$= W^k \log_2\Big(1 + \frac{g_i P_\Delta^k \gamma^{-1} d_{ik}^{\alpha}}{sP^k\gamma d_{ik}^{-\alpha} + \gamma\sigma^2}\Big), \tag{5}$$

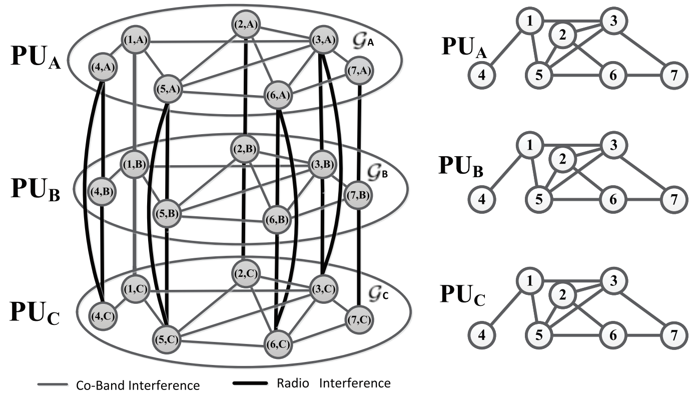
where P^k is the transmission power of PU_k, $k \in \mathcal{M}$, and $P^k\gamma d_{ik}^{-\alpha}$ is the PU_k's interference to SU_i over band k. Therefore, the expected capacity of SU_i over band k can be written as

$$c_i^k = \beta_k c_i^{k,\mathrm{OFF}} + (1 - \beta_k) c_i^{k,\mathrm{ON}}. \tag{6}$$

Depending on different modulation schemes, the achievable data rate is actually determined by the SNR at the receiver and receiver sensitivity [22], [27]. However, in most of existing literature [22], [24], the achievable data rate is approximated by Eq. (6), even though this data rate can never be achieved in practical. In this paper, we follow the same approximation. Note that this approximation will not affect the theoretical analysis or performance comparison in this work.

III. C-FROST: Centralized FROST Optimization Formulation

In this section, we first characterize the interferences among SUs by using conflict graph, and then we mathematically formulate the centralized FROST optimization problem with the objective of maximizing PUs' revenue under multiple wireless transmission constraints.



(a) Conflict graph $\mathcal{G}$. (b) Conflict graph $\mathcal{G}^k$, $k \in \{A, B, C\}$.

Fig. 2. Interference relationship represented by conflict graph in FROST.

A. Conflict Graph and Maximal Independent Sets

1) Construction of Conflict Graph: We introduce a conflict graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ to characterize the interferences relationship among SUs in FROST. Following the definitions in [24] and [29], we interpret the SU network in FROST as a two-dimensional resource space, with dimensions defined by the set of SUs, and the set of available bands. In $\mathcal{G}(\mathcal{V}, \mathcal{E})$, each vertex corresponds to a SU opportunistically accessing to certain band, i.e., a SU-band pair (i, k), where $i \in \mathcal{N}$ and $k \in \mathcal{M}$ [29], [30]. Each SU_i stands for a SU transmission pair, including a SU transmitter and a SU receiver from the same SU. Moreover, the distance between transmission pairs is much larger than the distance between transmitter and receiver of SU communication.

Similar to the interference conditions in [22], [24], and [30], there is interference if either of the following conditions is true: (i) if two different SUs are using the same band, the receiver of one SU transmission pair is in the interference range of the transmitter in the other SU pair; (ii) a SU pair transmits over two or more bands at the same time. Here, the first condition represents co-band interference, and the second condition represents the radio interface conflicts of SU itself, i.e., the single radio of SU transmitter/receiver cannot support multiple transmissions over multiple bands simultaneously.

According to these conditions, we connect two vertices in $\mathcal{V}$ with an undirected edge in $\mathcal{G}(\mathcal{V}, \mathcal{E})$, if their corresponding SU-band pairs interfere with each other. Given $\mathcal{G}(\mathcal{V}, \mathcal{E})$, we describe the impact of vertex $i \in \mathcal{V}$ on vertex $j \in \mathcal{V}$ as follows,

$$\delta_{ij} = \begin{cases} 1, & \text{if there is an edge between vertex } i \text{ and } j \\ 0, & \text{if there is no edge between vertex } i \text{ and } j, \end{cases} \tag{7}$$

where two vertices correspond to two SU-band pairs, respectively.

2) Maximal Independent Sets: Provided that there is a vertex set $\mathcal{I} \subseteq \mathcal{V}$ and a SU-band pair $i \in \mathcal{I}$ satisfying $\sum_{j \in \mathcal{I}, i \neq j} \delta_{ij} < 1$, the transmission at SU-band pair i will be successful even if all the other SU-band pairs in the set $\mathcal{I}$ are transmitting at the same time. If any $i \in \mathcal{I}$ satisfies the condition above, we can reuse the spectrum frequency, and allow the transmissions over all these SU-band pairs in $\mathcal{I}$ to be active simultaneously. Such a vertex/SU-band pair set $\mathcal{I}$ is called an independent set. If adding any one more SU-band pair into an independent set $\mathcal{I}$ results in a non-independent one, $\mathcal{I}$ is defined as a maximal independent set (MIS) [24], [29].

B. The Formulation of C-FROST Optimization

Let x_i^k denote the accessing status of SU $i \in \mathcal{N}$ to band $k \in \mathcal{M}$, where $x_i^k = 1$ indicates that SU_i is opportunistically transmitting over band k, otherwise 0. Given $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ constructed from FROST, suppose we can list all MISs as $\mathscr{I} = \{\mathcal{I}_1, \mathcal{I}_2, \cdots, \mathcal{I}_q, \cdots, \mathcal{I}_Q\}$, where Q is $|\mathscr{I}|$, and $\mathcal{I}_q \subseteq \mathcal{V}$ for $1 \leq q \leq Q$. Based on the definitions, assumptions and mathematical representations of interference relationship among SUs above, the revenue maximization optimization problem in FROST can be formulated as follows.

$$\text{Maximize} \sum_{k\in\mathcal{M}} \sum_{i\in\mathcal{N}} x_i^k b_i \tag{8}$$

$$\text{s.t.: } x_i^k \in \{0,1\}, \quad (i \in \mathcal{N}, k \in \mathcal{M}), \tag{9}$$

$$\sum_{k\in\mathcal{M}} x_i^k \leq 1, \quad (i \in \mathcal{N}), \tag{10}$$

$$x_i^k \cdot x_j^k = 0, \quad (i,j \in \mathcal{N}, k \in \mathcal{M}, (i,k) \in \mathcal{I}_u, (j,k) \in \mathcal{I}_{u'}, \mathcal{I}_u, \mathcal{I}_{u'} \in \mathscr{I} \text{ and } u \neq u') \tag{11}$$

where x_i^k is optimization variable, and b_i is deterministic value when SU_i is given. Here, binary value x_i^k indicates the accessing status of SU_i to band k, Eq. (10) means that SU_i can only access to one band at a time due to the radio interference, and Eq. (11) presents the co-band interference constraint.[5]

In the FROST optimization formulation above, we just take revenue maximization from the PUs' side as an example. The optimization objective can easily be converted into others. For example, the aggregated SU network throughput maximization from SUs' side, i.e.,$\sum_{k\in\mathcal{M}} \sum_{i\in\mathcal{N}} x_i^k c_i^k$, or the social welfare maximization from the overall network side, i.e., $\sum_{k\in\mathcal{M}} \sum_{i\in\mathcal{N}} x_i^k c_i^k r_i - \sum_{k\in\mathcal{M}} \sum_{i\in\mathcal{N}} x_i^k b_i + \sum_{k\in\mathcal{M}} \sum_{i\in\mathcal{N}} x_i^k b_i = \sum_{k\in\mathcal{M}} \sum_{i\in\mathcal{N}} x_i^k c_i^k r_i$, where r_i is the reward per Mbps for SU_i for traffic delivery.

The complexity of solving the optimization above arises from three parts: (i) how to identify all the MISs, (ii) how to fix the binary x_i^k variables, and (iii) who plays the role of centralized entity and conducts C-FROST optimization. As we know [24], [29], to find all the MISs in $\mathcal{G}(\mathcal{V}, \mathcal{E})$ itself is NP-complete. Besides, due to the binary values and the product constraints of x_i^k variables, the formulated optimization is a mixed-integer nonlinear programming (MINLP) problem, which has no classical optimal solution. Last but the most important, there is no centralized entity who can capture all the instantaneous spectrum accessing opportunities across different geographical areas, and conduct the C-FROST optimization as explained in Sec. I. To address those issues, we present a distributed FROST scheme in the following section.

[5]Actually, constraints Eq. (10) and Eq. (11) can be combined and equally substituted by one constraint, i.e., $x_i^k \cdot x_j^l = 0$, where $i,j \in \mathcal{N}, k,l \in \mathcal{M}, (i,k) \in \mathcal{I}_u, (j,l) \in \mathcal{I}_{u'}$, and $u \neq u'$. Note that this substitution does not change the type/property of the optimization, and it is still MINLP problem. We list two constraints instead of one for illustration purposes only.

IV. D-FROST: Distributed FROST via Matching With Evolving Preferences

In this section, we first present some preliminary of matching theory. Then, we describe some important definitions in D-FROST. After that, we develop a D-FROST scheme via matching with PUs' evolving preferences. Finally, we prove the pairwise stability of the proposed D-FROST and analyze the computational complexity.

A. Matching Preliminaries

1) Stable Marriage Matching: The marriage matching problem $(\mathcal{M}an, \mathcal{W}oman, \succ)$ is a bipartite one-to-one matching problem with two-sided preferences, which involves a finite set of men and a finite set of women.

Each man ranks the women from the most favourite to the least favourite based on his preferences, i.e., $man_i :\succ_{man_i}$. Such ranking is called men's preference list. Similarly, each woman has her preferences over man. Once the preference lists are built, men/women take actions according to the lists. The final result of this SM matching consists of man-woman pairs.

2) College Admissions Matching: The many-to-one college admissions matching model $(\mathcal{S}tudent, \mathcal{C}ollege, q, \succ)$ consists of a finite set of colleges, a finite set of students and a finite non-negative quota $q_{college_i}$ for $college_i \in \mathcal{C}ollege$. Each college has its preferences over the studentss, and based on the college's preferences over students, each college accept a group of students below the college's quota.

Note that the proposed D-FROST matching between PUs and SUs is different from both matching models above. The D-FROST matching is different from the one-to-one stable marriage matching in that each PU can access multiple SUs who have no mutual interferences. The D-FROST matching is also different from the many-to-one college admission matching, since the PU has no quota limitation as long as the accepted SUs can transmit simultaneously without interfering one another.

B. Technical Challenges for Matching in Spectrum Trading

To implement matching theory into spectrum trading, we need to consider a modified many-to-one matching problem. Since in our assumption, each PU can accept several SUs while each SU can only propose to one PU at a time. This modified many-to-one matching is more complicated, since we jointly considerate dynamic matching with evolving preferences (i.e., the preference lists have to evict the interference of accepted SUs) and spatial reuse (modeling by conflict graph) under multiple wireless communication constraints in our scheme.

C. Definitions in D-FROST Matching

1) The PU's Observations on SUs' Interferences: An individual PU is not able to capture the multi-dimensional conflict graph, which covers both co-band and radio interferences of SUs, as shown in Fig 12(a), since a PU cannot have the complete information about all the SUs' preferences and accessing status. To maximize its revenues while avoiding co-band interferences among the accessed SUs, the PU has to build up its preferences based on its own observations on SUs' interference relation. Given the SUs' locations as *priori* information, PU_k can observe the co-band interferences among SUs, who may potentially access to band k as shown in Fig 12(b). That is, by dividing $\mathcal{G}(\mathcal{V},\mathcal{E})$ into $\mid\mathcal{M}\mid$ layers, where $\mathcal{G}^k(\mathcal{V}^k,\mathcal{E}^k)$ is the conflict graph over band $k\in\mathcal{M}$, $\mathcal{G}^k(\mathcal{V}^k,\mathcal{E}^k)$ is the conflict graph observed by PU_k.

Similar to the definition of $\mathcal{I}_u\in\mathcal{G}$ and $\mathscr{I}\subseteq\mathcal{G}$ in last section, we can define $\mathcal{I}_u^k\in\mathcal{G}^k$ and $\mathscr{I}^k\subseteq\mathcal{G}^k$, which represents the MISs observed by PU_k, and all the SUs in $\mathcal{I}_u^k$ can transmit simultaneously over band k.

2) The Preferences of SUs and PUs: The objective for the SU_i is to maximize its data transmission rate, i.e.,

$$\begin{aligned}&\text{Maximize}\ \sum_{k\in\mathcal{M}}x_i^kc_i^k\\&\text{s.t.: } x_i^k\in\{0,1\},\quad(i\in\mathcal{N},k\in\mathcal{M}),\\&\qquad\sum_{k\in\mathcal{M}}x_i^k\le 1,\quad(i\in\mathcal{N}),\end{aligned}\tag{12}$$

where c_i^k is defined in Sec. II-B. Thus, for SU_i, $i\in\mathcal{N}$, we can construct a complete, reflexive and transitive preference relation $\succ_i$ over all PUs as follows

$$k\succ_i l\Leftrightarrow c_i^k\succ_i c_i^l,\quad k,l\in\mathcal{M}.\tag{13}$$

As for PU_k, the goal is to maximize its revenue by accessing as many SUs with high evaluated valuation as possible to transmit over band k at the same time. Given $\mathcal{G}^k(\mathcal{V}^k,\mathcal{E}^k)$ observed by PU_k, we have

$$\begin{aligned}&\text{Maximize}\ \sum_{i\in\mathcal{N}}x_i^kb_i\\&\text{s.t.: } x_i^k\in\{0,1\},\quad(i\in\mathcal{N},k\in\mathcal{M}),\\&\quad x_i^k\cdot x_j^k=0,\quad(i,j\in\mathcal{N},k\in\mathcal{M},(i,k)\in\mathcal{I}_u^k,\\&\qquad(j,k)\in\mathcal{I}_{u'}^k,\mathcal{I}_u^k,\mathcal{I}_{u'}^k\in\mathscr{I}^k\text{ and }u\neq u').\end{aligned}\tag{14}$$

So, given a group of SUs in $\mathcal{I}_u^k$ and another group of SUs in $\mathcal{I}_v^k$, the preferences of PU_k over those SUs can be represented as

$$\mathcal{I}_u^k\succ_k\mathcal{I}_v^k\Leftrightarrow\sum_{i\in\mathcal{I}_u^k}b_i\succ\sum_{j\in\mathcal{I}_{u'}^k}b_j.\tag{15}$$

3) Individual Rationale and Pairwise Block: Let $\mathbb{PL}(\cdot)$ denote the preference list. According to the preferences of SUs and PUs, we define

- For SU_i, $\forall i\in\mathcal{N}$, $\mu(i)$ denotes the matching result of SU_i. $\mu(i)=k$, if SU_i can access band k owned by PU_k, and $\mu(i)=0$, if SU_i cannot access any band.
- For PU_k, $\forall k\in\mathcal{M}$, $\mu(k)$ denotes the matching result of PU_k, and $\mu(k)=\mathcal{I}_u^k,\mathcal{I}_u^k\subseteq\mathscr{I}^k$, if PU_k can accommodate every SU_i over band k, where $i\in\mathcal{I}_u^k$; $\mu(k)=\Phi$, if all SUs are denied by PU_k over band k.
- For PU_k and SU_i, $\mu(i)=k$, if and only if $i\in\mu(k)$.

Based on those definitions in D-FROST, we further define *individual rationale* [21] as:

Definition 1: Given an user $a\in\mathcal{M}\cup\mathcal{N}$ (i.e., a can either be PU or SU) and a set of partners $\mathcal{S}$ of user a, let $\Omega(\mathcal{S},\mathbb{PL}(a))$ denotes[6] user a's most favorite subset of $\mathcal{S}$ according to a's preference lists $\mathbb{PL}(a)$.[7] A D-FROST matching is defined as *individually rational* if and only if $\mu(a)=\Omega(\mu(a),\mathbb{PL}(a))$, $\forall a\in\mathcal{M}\cup\mathcal{N}$.

For instance, $\mathbb{PL}(\mathrm{PU}_1)$=$\{\{\mathrm{SU}_1,\mathrm{SU}_3\},\{\mathrm{SU}_2\},\{\mathrm{SU}_2,\mathrm{SU}_4\}\}$, and we have $\Omega(\{\mathrm{SU}_2,\mathrm{SU}_4\},\mathbb{PL}(\mathrm{PU}_1))=\{\mathrm{SU}_2\}$, which is not equal to $\{\mathrm{SU}_2,\mathrm{SU}_4\}$. After the D-FROST matching, if $\mu(\mathrm{PU}_1)$=$\{\mathrm{SU}_2,\mathrm{SU}_4\}$, it is not individual rational since $\mu(\mathrm{PU}_1)\neq\Omega(\mu(\mathrm{PU}_1),\ \mathbb{PL}(\mathrm{PU}_1))$. However, if $\mu(\mathrm{PU}_1)$=$\{\mathrm{SU}_1,\mathrm{SU}_3\}$, the matching is individual rational since $\Omega(\{\mathrm{SU}_1,\mathrm{SU}_3\},\mathbb{PL}(\mathrm{PU}_1))$=$\{\mathrm{SU}_1,\mathrm{SU}_3\}$.

Furthermore, we define *pairwise block* as

Definition 2: Pairwise block of μ means that there is a SU-PU pair (i,k),

- $i\notin\mu(k)$, $i\in\Omega(\mu(k)\cup i,\mathbb{PL}(k))$;
- $k\neq\mu(i)$, $k=\Omega(\mu(i)\cup k,\mathbb{PL}(i))$.

If matching μ is *individually rational* and there is no *pairwise block* in μ, then μ is ***pairwise stable***.

For illustrative purposes, we give a simple example for the pairwise block. For example, after the matching process, we have $\mu(\mathrm{PU}_1)=\{\mathrm{SU}_1,\mathrm{SU}_3\}$ and SU_5 is matched with PU_2. However, in $\mathbb{PL}(\mathrm{SU}_5)$, $\mathrm{PU}_1\succ_{\mathrm{SU}_5}\mathrm{PU}_2$.Then, we have $\mathrm{PU}_1\neq\mu(\mathrm{SU}_5)$, and $\mathrm{PU}_1=\Omega(\mu(\mathrm{SU}_5)\cup\mathrm{PU}_1,\ \mathbb{PL}(\mathrm{SU}_5))$. Meanwhile, $\mathrm{SU}_5\notin\mu(\mathrm{PU}_1)$, and $\mathrm{SU}_5\in\Omega(\mu(\mathrm{PU}_1)\cup\mathrm{SU}_5,\mathbb{PL}(\mathrm{PU}_1))$=$\{\mathrm{SU}_1,\mathrm{SU}_3,\mathrm{SU}_5\}$, then $(\mathrm{SU}_5,\mathrm{PU}_1)$ is a pairwise block.

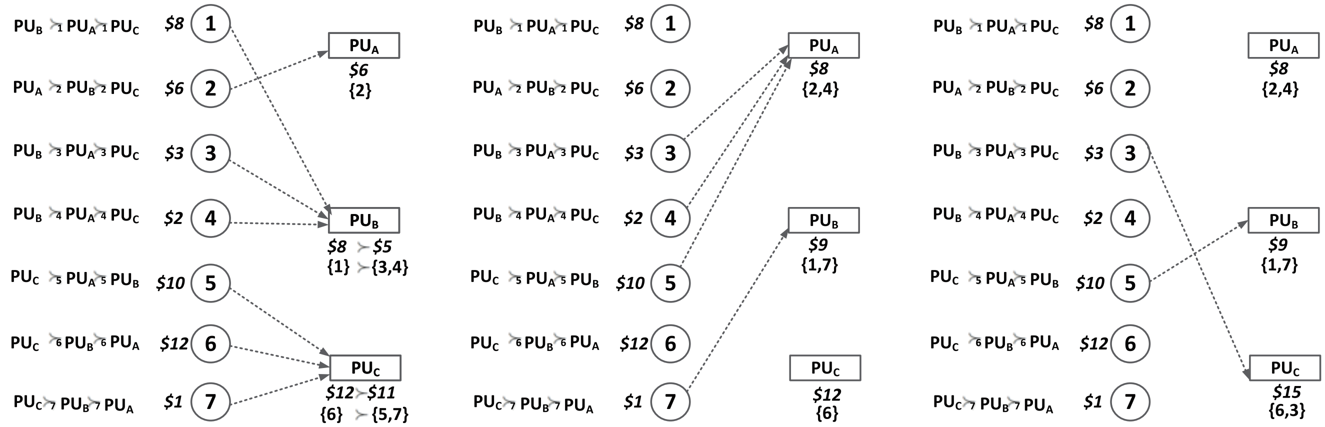
D. D-FROST Matching With Evolving Preferences

To avoid block and reach pairwise stable matching for spectrum trading, we illustrate the D-FROST matching procedure with PUs' evolving preferences in this subsection. Generally speaking, although a PU cannot have the complete information about all the SUs' preferences, SUs' evaluated valuation, and accessing status, the PU can build up its own SUs' conflict graph based on which SUs submitted proposals to it, evolve its preferences according to its observations, and make the decision of accessing the SUs or rejecting them rounds by rounds with the objective of maximizing its own revenues. The proposed D-FROST matching process can be carried out in two phases and five steps, which is shown in details as follows.

1) Phase I: Tentative Matching With PUs' Currently Observed MISs: There are four steps in *Phase I*: (i) preparing preference lists, (ii) SUs' proposal proposing, (iii) PUs'

[6]For the classical matching theory in economics, many researchers employ $Ch(\cdot)$ to represent such a subset [21]. In this paper, we use $\Omega(\cdot)$ instead to avoid causing any confusion, since Ch is always used to denote the channel in wireless communications and networking research community.

[7]For example, $\Omega(\mathcal{I}_u^k,\mathbb{PL}(k))$ denotes PU_k's most favorite subset of $\mathcal{I}_u^k$ according to preference lists $\mathbb{PL}(k)$, where $\mathcal{I}_u^k\subseteq\mathscr{I}^k$.



(a) 1st round proposal. (b) 2nd round proposal. (c) 3rd round proposal.

Fig. 3. **Phase I:** Tentative Matching with PUs' Currently Observed MISs.

tentative matching with (i.e., accessing/rejecting) SUs, and (iv) PUs' preferences evolving.

First of all, all PUs and SUs will initiate the procedure by preparing their preference lists. The SU_i constructs its preference list $\mathbb{PL}(i)$ according to Eq. (13). Since no SU submits evaluated valuation to PU_k yet, PU_i constructs the conflict graph $\mathcal{G}^k$ based on the *priori* location information of the SUs within its coverage, and lists its preferences $\mathbb{PL}(k)$ according to Eq. (15).

Then, having $\mathbb{PL}(i)$, SU_i proposes to the top PU of $\mathbb{PL}(i)$ in this round. Note that all the SUs propose to the PUs in the second step simultaneously, and a SU can only propose to one PU at a time due to the radio interference.

After receiving the evaluated valuation from SUs, PU_k updates its $\mathcal{G}^k$, which includes the SUs proposals for PU_k for the 1st round, and includes the already accepted SUs and SUs newly proposal for PU_k from the 2nd round until the $|\mathcal{M}|$-th round. Based on the updated $\mathcal{G}^k$, PU_k will tentatively access/match with the SUs in $\mathcal{I}_u^k$, where $\mathcal{I}_u^k = \underset{\mathcal{I}_u^k \in \mathcal{G}^k}{\text{argmax}}\left(\sum_{i\in\mathcal{I}_u^k} b_i\right)$, and reject the SUs not in $\mathcal{I}_u^k$. If more than one MIS can reach the same maximal revenue of PU_k in the current round, PU_k will compare these MISs, and chooses the MIS which reaches the promising highest revenue in the future rounds.

After that, based on the accepted $\mathcal{I}_u^k$, PU_k evolves its preference list $\mathbb{PL}(k)$, which puts MISs/SUs not interfering $\mathcal{I}_u^k$ with higher priorities, and MISs/SUs interfering $\mathcal{I}_u^k$ with lower ones. Then, the process goes back to *Step 2*, where SUs start to propose to the second highest PUs in their preference lists.

The iterations continues until the tentative matching ends in the $|\mathcal{M}|$-th round.

Example of Phase I: Figure. 3 is an example for D-FROST with current MIS in *Phase I*. The conflict graph of each PU is based on Fig. 2(b). The SUs' preference lists are based on their utility functions. Each SU evaluated value in different prices to access band of PUs. Figure. 3 shows the process of matching.

In the first round, SU_2 proposes to PU_A; SU_1, SU_3 and SU_4 propose to PU_B; SU_5, SU_6 and SU_7 propose to PU_C as shown in Fig. 3(a). For PU_A, there is only one SU applies to access its band, so PU_C accepts SU_2. The SUs who propose to PU_B can divide into two maximum independent sets $\{\text{SU}_1\}$ and $\{\text{SU}_3,\text{SU}_4\}$. According to the algorithm, PU_B chooses proposed combination sets of SUs who has the highest revenue, which is $\{\text{SU}_1\}$ for \$8. For PU_C, SU_5, SU_6 and SU_7 interfere with each other, so PU_C chooses the SU who has a higher evaluated value, which is SU_6 for \$12.

In the second round, all PUs evict SUs which have impact on the SUs that are already accepted in the first round. Then PUs evolve their preference lists to new ones. The SUs who are rejected in the first round propose to their second favorite PUs. In our example, SU_3, SU_4 and SU_5 propose to PU_A. PU_A makes its decision by its new preference list. Then PU_A accepts SU_4 since SU_3 and SU_5 interference with SU_2, which is already accepted by PU_A in the first ground. PU_A earns the total revenue of \$8 in the second round. SU_7 proposes to PU_B and PU_B accept it since SU_7 does not conflict with SU_1.

Follow the similar procedure, SU_3 proposes to PU_C and is accepted. SU_5 proposes to PU_B and is rejected. At last, total revenue from allocated SUs is \$8+\$9+\$15=\$32. Each PU earns revenue from total the evaluated value of the accepted SUs.

2) Phase II: Block-Proof Matching With SUs' Swapping: In *Phase II*, SU_i will propose again to the PU_k, which SU_i prefers to its current matching $\mu(i)$, i.e., $k \succ_i \mu(i)$. Then, PU_k will check if it can generate more revenue by accessing SU_i, compared with PU_k's current revenue. If yes, SU_i will be swapped to PU_k, and PU_k will update MIS including SU_i, evict SUs who interfere with SU_i in the updated MIS, and evolve PU_k's preferences. The swapped-out/evicted SUs will repeat the same proposal procedure until no more swapping is needed. Since all PUs prefer to the MIS which has more evaluated value aggregation, there will not have a loop in the phase II. To be specific, if SU_i is evicted by PU, it means there is another SU_j which proposes more than SU_i, and PU can have higher monetary gain by accepting SU_j.

It should be noticed that the swapping is only proposed by SUs based on their individual rationale. PUs just need to make decision of accepting/rejecting the SUs' swaps based on PUs' preferences, and there is no requirement for PUs to

Algorithm 1 D-FROST With Current MIS

Phase I:Tentative Matching with PUs' Currently Observed MISs
1.Initialization
$\forall k \in \mathcal{M}, \mu(k) =$, the preference list of k, $\mathbb{PL}(k)$=$\mathscr{I}$. $\forall i \in \mathcal{N}, \mu(i) = i$, the preference list of i, $\mathbb{PL}(i)$=$\mathcal{M}$
2. SUs propose to PUs
for all $i \in \mathcal{N}$ **do**
  Propose to PU $k^* \in \mathbb{PL}(i)$,$\forall k' \in \mathbb{PL}(i)$, $k^* \succ_i k'$. $\mathbb{PL}(i)$=$\mathbb{PL}(i)\backslash\{k^*\}$
end for
3.PUs make decisions;
for all $k \in \mathcal{M}$ **do**
  $R(k)$ is the current Proposers for k. Select a subset of non-interfering SU $R \subseteq R(k)$, $\forall i$,$i' \in R$, $\delta^k_{i,i'} = 0$, $\sum_{i \in R} b^k_i x^k_i$ is maximized
  if $\exists R^*$ and R', $\sum_{i^* \in R^*} b^k_{i^*} x^k_{i^*}$ and $\sum_{i' \in R'} b^k_{i'} x^k_{i'}$ are both maximized **then**
    $\forall i^* \in R^* \cup \mu(k), i^* \in \mathcal{I}^k_u$. $\forall i' \in R' \cup \mu(k)$,$i' \in \mathcal{I}^k_v$
    if $\forall j^* \in \mathcal{I}^k_u, \forall j' \in \mathcal{I}^k_v$,$\sum b^k_{j^*} x^k_{j^*} > \sum b^k_{j'} x^k_{j'}$ **then**
      $R = R^*$
    else
      $R = R'$
    end if
  end if
  $\mathbb{PL}(k)$=$\mathbb{PL}(k)\backslash \left\{ i^* | i^* \in \mathbb{PL}(k), \exists i' \in R, e^k_{i^*,i'} = 1 \right\}$
  $\mu(k) = \mu(k) \cup R$
  $\mu(i) = k$
end for
if $\exists i \in \mathcal{N}, \mu(i) = i$, and $\mathbb{PL}(i) \neq$ **then**
  Go to step 2
else
  Go to step 4
end if
Phase II:Block-Proof Matching with SUs' Swapping.
if for i, $\exists k \succ \mu(i)$, according to $\mathbb{PL}(i)$, $i \in \mathcal{N}$ **then**
  i propose to k
  if $k \notin \mu(i)$, $k \in \Omega(\mu(i) \cup k$,$PL(i))$, and $i \notin \mu(k)$, $i \in \Omega(\mu(k) \cup i$,$\mathbb{PL}(k))$ **then**
    $\mu(k)^* = \mu(k)$,$\mathcal{N}^* = \mu(k) \cup i$. $\mu(k) = \Omega(\mu(k) \cup k$,$\mathbb{PL}(i))$,$\mathcal{N}^* = \mathcal{N}^* \backslash \mu(k)$
  else
    Repeat ***Phase II***
  end if
end if
4.End of algorithm;evaluated valuation

Algorithm 2 D-FROST With Expected MIS

1.Initialization
Same to Algorithm 1.
2. SUs propose to PUs
Same to Algorithm 1.
3.PUs make decisions;
for all $k \in \mathcal{M}$ **do**
  R_k is the current Proposers for k.
  if $\exists$ R, $R \subseteq R_k$, $\forall i$,$i' \in R$, $\delta^k_{i,i'} = 0$, $i \in \mathcal{I}^k_u$, **&** $\forall j \in \mathcal{I}^k_u$, $\sum b^k_j x^k_j$ is maximized. **then**
  $\mathbb{PL}(k)$=$\mathbb{PL}(k)\backslash \left\{ i^* | i^* \in \mathbb{PL}(k), \exists i' \in R, e^k_{i^*,i'} = 1 \right\}$
    $\mu(k) = \mu(k) \cup R$
    $\mu(i) = k$
  end if
end for
if $\exists i \in \mathcal{N}, \mu(i) = i$, and $\mathbb{PL}(i) \neq$ **then**
  Go to step 2
else
  Go to step 4
end if
4.End of algorithm;

share information, or even communicate with one another. That helps to keep the distributed feature of the proposed D-FROST.

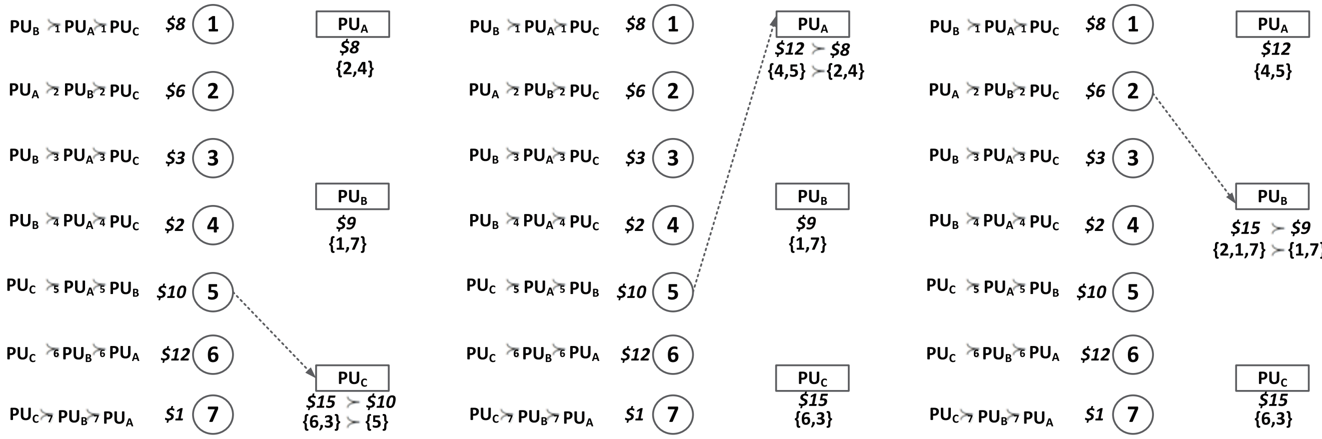
Example of Phase II: As shown in Fig. 4, after matching process, SU_5 is not allocated by any PU. Then SU_5 proposes again based on its preference list. It can be observed that PU_C will reject SU_5 since its revenue \$10 is less than the current revenue \$15. Then SU_5 proposes to PU_A, PU_A can get revenue \$12 from SU_5, which is more than \$8 from its current accepted set $\{SU_2$,$SU_4\}$. Then PU_A accepts SU_5 and evicts SU_2, since SU_2 has interference with SU_5. For the similar rules, SU_2 applies to PU_B and is matched because PU_B can have higher monetary gain when it accept SU_2, and SU_2 does not interfere with SU_1 and SU_7. At the end, the whole spectrum get revenue of \$42 after the whole matching process.

This algorithm is summarized in Algorithm 1. In this D-FROST with the current proposed MIS algorithm, we not only consider PUs' conflict graph but also consider the maximal revenue in SUs who propose to PUs. On the other hand, we try to propose another algorithm which value the revenue PUs in the future more than current time, which is called D-FROST with expected MIS algorithm. The detail is discussed in Sec .IV-D.5.

3) Pairwise Stability of D-FROST: The matching result of the proposed Algorithm 1 is pairwise stable.

Proof: It is proved by contradiction. Suppose the final matching result is not pairwise stable, i.e, $\exists k', \exists i, k' \notin \mu(i)$, $k' \in \Omega(\mu(i) \cup k')$,$\mathbb{PL}(i))$, and $i \notin \mu(k')$, $i \in \Omega(\mu(k') \cup i$,$\mathbb{PL}(k'))$. In other word, $\mu(k') \neq \Omega(\mu(k') \cup i$, $\mathbb{PL}(k'))$ and $\mu(i) \neq \Omega(\mu(i) \cup k'$, $\mathbb{PL}(i))$. It means that SU_i prefers to join another band of $PU_{k'}$ rather than its current matching results PU_k. Meanwhile, $PU_{k'}$ would like to accept it since it can help $PU_{k'}$ earn more revenue from SU_i. If the algorithm has pairwise block, it will transfer the element of block in *Phase II*. Then, after *Phase II*, $i \in \mu(k')$, $\mu(k') = \Omega(\mu(k')$,$\mathbb{PL}(k'))$, and $k' = \mu(i)$,$\mu(i) = \Omega(\mu(i)$,$PL(i))$. Hence, this matching result of the proposed algorithm is pairwise stable. ■

4) Computational Complexity of D-FROST: The complexity of the proposed D-FROST algorithm arises from two parts: (i) the complexity for the PU to list its preferences by finding enough MISs, and (ii) the complexity for the matching between PUs and SUs. To find all MISs is NP-complete. Thus, we use the greedy algorithm in [24] to find out a large number



(a) 1st round swapping. (b) 2nd round swapping. (c) 3rd round swapping.

Fig. 4. **Phase II:** Block-Proof Matching with SUs' Swapping.

of MISs (e.g., the number is $Z = 10000$) for approximation, whose complexity is $\mathcal{O}(M^4N^8)$. For the matching part, the complexity is dominated by the total number of PUs and SUs, which is $\mathcal{O}(MN)$. Therefore, the overall complexity of the propose D-FROST algorithm is $\mathcal{O}(M^5N^9)$.

5) PUs in D-FROST With Expected MIS: PUs in D-FROST with expected MIS make decisions of accessing/rejecting SUs, according to the promising/expected maximum independent set, which potentially yields the largest revenues, instead of currently observed MISs. For example, there are SU_1, SU_2, and SU_3 with unit evaluated valuation proposed to PU_A during the 1st round, and SU_1 and SU_2 have no mutual interferences, but both of them interfere with SU_3. However, according to the initial $\mathcal{G}^1$, SU_3 belongs to the maximum independent set,[8] say $\{\text{SU}_3, \text{SU}_4, \text{SU}_5\}$, and this set can bring PU_A the largest revenue, if SU_3, SU_4 and SU_5 all propose to PU_A and get accessed to PU_A's band. In this case, SU_1 and SU_2 will be accessed by PU_A during the 1st round in our proposed D-FROST with current MIS. However, in D-FROST with expected MIS, SU_3 will be accessed by PU_A during the 1st round, since PU_A expects to access $\{\text{SU}_3, \text{SU}_4, \text{SU}_5\}$ to achieve the largest revenue. With this objective, PU_A waits for SU_4 and SU_5 to join, and denies other SUs' proposals in the following rounds until the end of matching.

V. Performance Evaluation

A. Simulation Setup

We consider a spectrum trading market consisting of PUs and $|\mathcal{N}| = 20$ SUs, where 20 nodes are randomly deployed in a 1000×1000 m^2 area. Considering the AWGN channel, we assume the noise power σ^2 is 10^{-10} W at all transmitters and receivers. Moreover, suppose the path loss factor $\alpha = 4$, the antenna parameter $\gamma = 3.90625$, the receiver sensitivity $P_T = 100\sigma^2 = 10^{-8}$ W and the interference threshold $P_T = 6.25 \times 10^{-10}$ W. According to the illustration in Sec. II-B, we can calculate the transmission range R_T and the interference range R_I, which are equal to 250 m and 500 m, respectively. For illustrative purposes, we assume all the bands have different bandwidths, which are randomly selected from 10 MHz to 15MHz. We also assume transmission power of PU, SU and SU when PU coming back are 20×10^{-8} W, 15×10^{-8} W and 7×10^{-8} W, respectively. The distance between transmitter and receiver of SU is 20m. And the distances between PU and SU are randomly from 1m to 60m. The data transmission rates of SUs can be calculate by Eq. (6), where the probability values of PUs' coming back, i.e., β_k values, are randomly selected from 0 to 1. For simplicity's sake, every SU evaluated value is randomly picked from [\$1,\$10], and we set $Z = 10000$ as a large enough number for the MISs. The numerical simulations are conducted with a random topology, where 20 SUs transmission pairs are randomly deployed in a 1000×1000 m^{2}area.

B. Results and Analysis

We compare the proposed D-FROST with current MIS algorithm with another three algorithms: C-FROST, D-FROST with expected MIS and Gale-Shapley (GS) algorithms [31]. Here, by employing Z MISs found in multi-dimensional $\mathcal{G}$, C-FROST is the solution to the relaxed centralized optimization in Eq. (8). As a near-optimal solution to the centralized formulated spectrum trading optimization, C-FROST can be obtained by commercial solvers such as CPLEX [32], and serve as a benchmark for the performance comparison.

In Fig. 5 and Fig. 8, we show the total number of accessed SUs under the four algorithms above with $\mathcal{M} = 2$ and 5, respectively. Actually, the number of accessed SUs also represents the dynamic spectrum accessing opportunities created, which reflects the spectrum utilization. It is not surprising that GS algorithm has the worst performance, since GS has no consideration about frequency reuse and only allows each PU to trade the spectrum with one SU. Taking spatial reuse into account, the number of accessed SUs for both D-FROST with current MIS and D-FROST with expected MIS algorithms increases, when there are more SUs participate in the network. The proposed D-FROST with current MIS is superior to

[8]Maximum independent set is the largest MIS.

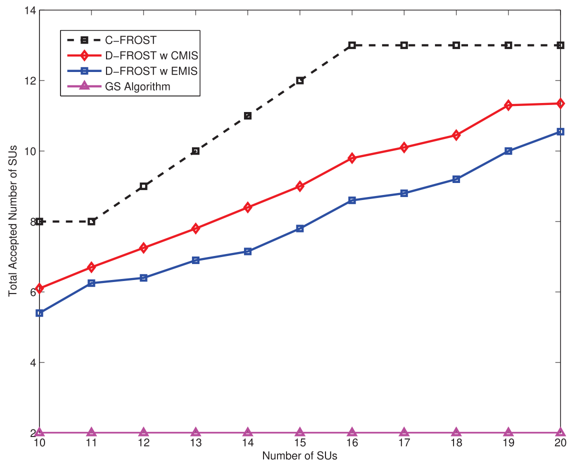


Fig. 5. The number of accessed SUs/Spectrum utilization, $M = 2$.

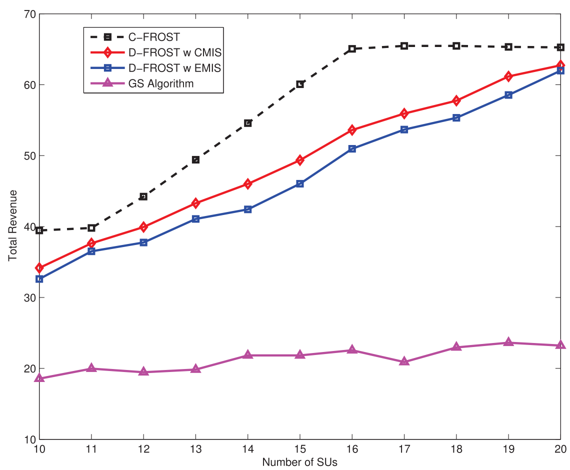


Fig. 6. Total revenues of PUs, $M = 2$.

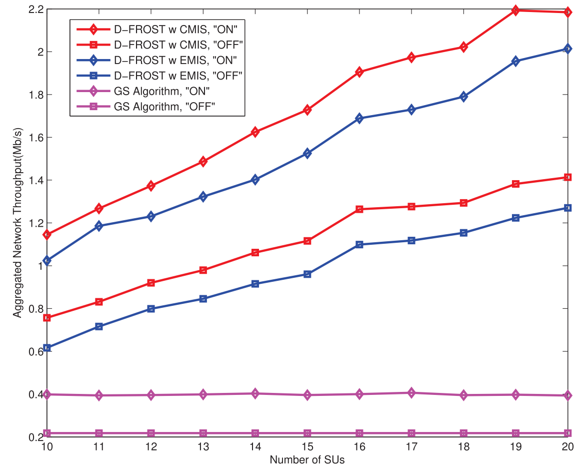


Fig. 7. Aggregated SU network throughput, $M = 2$.

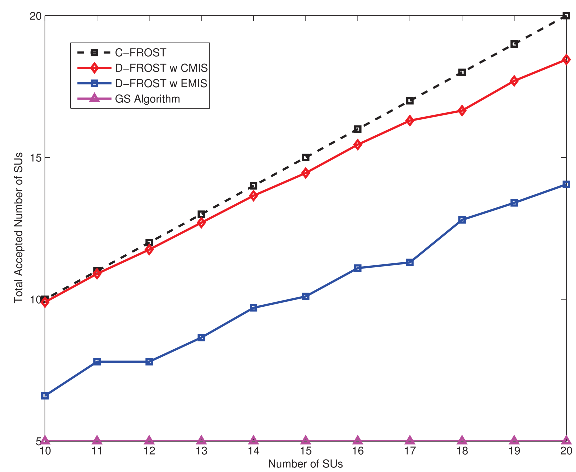


Fig. 8. The number of accessed SUs/Spectrum utilization, $M = 5$.

D-FROST with expected MIS in terms of spectrum utilization. That's because in the network which employs the D-FROST with expected MIS algorithm, allocated SUs are waiting for other non-interference SUs who may never propose to the same PUs.

Then, we compare total revenue of PUs under those algorithms in Fig. 6 and Fig. 9. Again, GS has worst performance in terms of PUs' revenues, since it ignores the frequency reuse. Note that the revenue increase of C-FROST generally stops when the number of SUs is beyond 16 as shown in Fig. 6. That is because there is only 2 PUs in the spectrum trading market, and there is still a cap for spectrum trading opportunities even though frequency reuse is considered. Moreover, the proposed D-FROST with current MIS is still much better than D-FROST with expected MIS. It is shown that, as number of SU increases, PUs' revenue of the D-FROST with current MIS algorithm increases more rapidly than D-FROST with expected MIS, and much closer to the sub-optimal C-FROST solution. The reason behind that is because D-FROST with current MIS adopts the MIS with highest revenue in current round, and evolves preference list during matching process. Despite the fact that D-FROST with expected MIS may have potential to obtain more revenue since the PU in this scheme targets at the maximum independent set with the biggest revenue, it cannot guarantee all SUs in that specific set will propose to the designated PU. Those SUs may be accepted by other PUs in previous rounds, according to their own preference lists.

Fig.7 and Fig.10 give some insights on the aggregated SU network throughput. The comparison is conducted under two modes, "ON" and "OFF", respectively. Here, "ON" mode means SUs still work but decrease their power below "interference temperature" when PUs come back, while "OFF" mode means SUs absolutely shut down when PUs return. Obviously, the performance of "ON" mode is better than that of "OFF" mode for all algorithms. Here, the performance of GS under "ON" mode is not a constant because it depends on the SU-PU matchings, and the matched SUs' transmission powers (thus their data rates) are affected by their distance from PUs' transmitters. Similar to the analysis for the spectrum utilization and PUs' revenues, the proposed D-FROST with current MIS outperforms D-FROST with expected MIS under both "ON" and "OFF" modes for the aggregated SU network throughput. The last but not the least, we employ Fig. 11 and Fig. 12 to

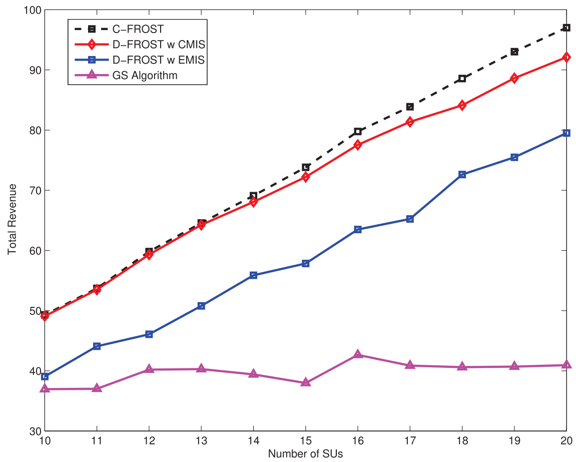


Fig. 9. Total revenues of PUs, $M = 5$.

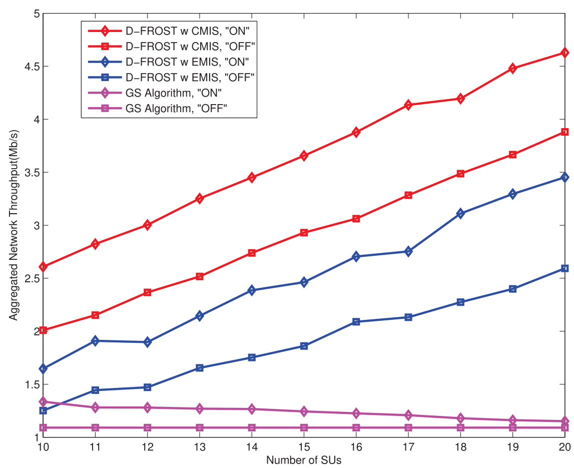


Fig. 10. Aggregated SU network throughput, $M = 5$.

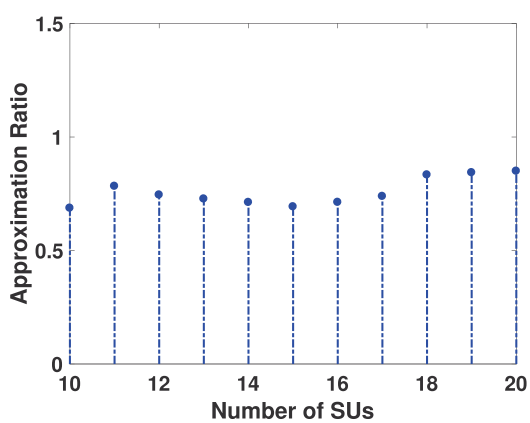


Fig. 11. Approximation ratio in terms of accepted SUs numbers, $|M| = 2$.

present the approximation ratio of C-FROST to the D-FROST algorithm under different number of SUs by simulations. All these statistical results indicate that the solutions found by the proposed D-FROST algorithm is very close to the optimal solution.

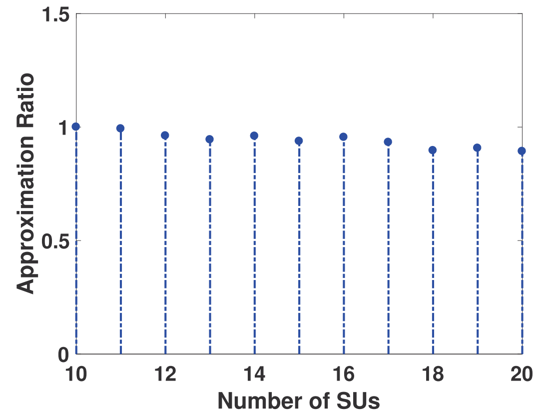


Fig. 12. Approximation ratio in terms of accepted SUs numbers, $|M| = 5$.

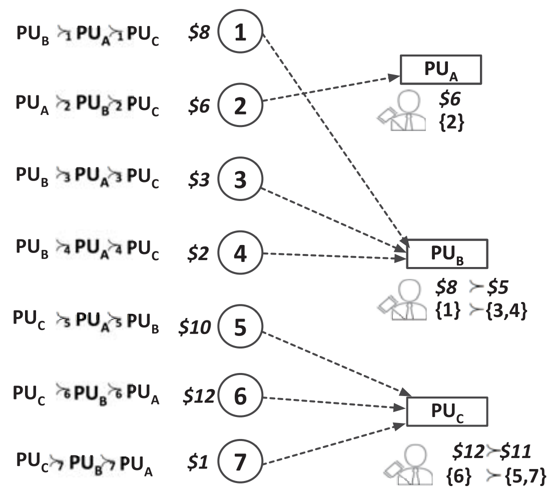


Fig. 13. Example of PU with several.

VI. Conclusion and Discussions

In this paper, we have proposed a novel distributed frequency reuse based opportunistic spectrum trading scheme (D-FROST) with consideration for reusing spectrum and trading the spectrum in distributed manners. In D-FROST, we have employed conflict graph to characterize the SUs co-channel interference and radio interference. Based on the conflict graph, we have formulated centralized FROST optimization. Due to the NP-completeness to solve this problem, we have developed the D-FROST algorithm based on matching with evolving preferences, proved its pairwise stability and analyzed its complexity. Through simulations, we have shown that the proposed D-FROST algorithm is better than other distributed spectrum trading algorithms, which is able to yield sub-optimal solutions and effectively improve PUs' revenues, the aggregated SU network throughput, and spectrum utilization. Moreover, we would like to make the following discussions on the combination between our work and VCG-like incentive mechanism incorporated with the bidding process, strategy-proof auction.

On the perspective of each PU, it is possible to integrate VCG-like incentive mechanism to our matching scheme. In existing literature, there are some spectrum auction designs, which have attempted to achieve auction truthfulness in spectrum auction [11], [33]. Considering spatial reuse, Zhou *et al.* in [7] proposed a spectrum auction framework and VERITAS algorithm to support dynamic single-sided buyer network. As shown in Fig. 13, in each matching round of our work, each PU can be considered as a singe-sided buyer employing VCG auction. It does not impact the pairwise-stable characteristic since the preference lists of SUs and PUs and the matching procedure remain the same. The SUs propose to PUs according to their preference list/ highest capacity, and PUs will choose the MIS which has the highest revenue. However, the total received revenue may decrease if we integrate strategy-proof auction since different pricing mechanisms (The similar analysis also applies to VCG and VERITAS auction). In summary, it is possible to integrate VCG-like mechanism to our matching based spectrum trading, but it reduces the received revenue of PUs.

There are two major differences between the double auction and the matching based spectrum trading with VCG auction. First, double auction based spectrum trading is a centralized scheme (It needs a centralized auctioneer and has scalability issue), and it needs an entity to play the role of auctioneer in the network (However, it is not clear which entity in the network should play this role); by contrast, the matching based spectrum trading with VCG-like mechanism is a distributed approach. Specifically, in the double auction, the SUs submit their bid and PUs simultaneously submit their asking price to an auctioneer, and then the auctioneer chooses a price p that clears the market: all the PUs who asked less than p sell and all buyers who bid more than p buy at the price p. After double auctions, the bidding price for all SUs is p. However, in the double auction mechanism, the involved "auctioneer" is a centralized entity which is aware of all the bid price and asking price information from SUs and PUs. It indicates that the double auction is a centralized spectrum trading design. The major concern with centralized design is the failure of the central traders. Besides, the centralized spectrum trading design may also have salability issues, when the network size becomes large. Therefore, the main contribution in our paper is considers spectrums special feature, spatial reuse, and allows spectrum trading between PUs and SUs in distributed manners. Second, the clearing price in double auction makes PUs charge all the SUs at the same price. That will downgrade the spectrum trading into our previous design [19], where the SUs have same bidding values. In this paper, we can allow different SUs to have different bidding values, and SUs will be charged differently, even if we integrate VCG-like mechanism into the matching based spectrum trading.

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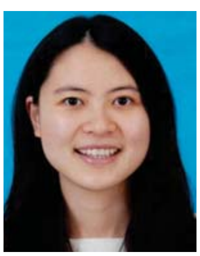

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