Quantifying Equivocation for Finite Blocklength Wiretap Codes

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***Abstract*—This paper presents a new technique for providing the analysis and comparison of wiretap codes in the small blocklength regime over the binary erasure wiretap channel. A major result is the development of Monte Carlo strategies for quantifying a code's equivocation, which mirrors techniques used to analyze forward error correcting codes. For this paper, we limit our analysis to coset-based wiretap codes, and give preferred strategies for calculating and/or estimating the equivocation in order of preference. We also make several comparisons of different code families. Our results indicate that there are security advantages to using algebraic codes for applications that require small to medium blocklengths.**

I. Introduction

Due to the increased number of automated and wireless devices in use today, it appears that the internet of things (IoT) is slowly, but surely, becoming a reality. With the increased flexibility and convenience that the IoT promises to bring about, come also a plethora of security and privacy challenges. These challenges are unique to other applications for several reasons. For one, the IoT will be comprised of power-constrained devices; for two, these devices will likely need only short packets to communicate a large proportion of transmitted data; and for three, communications will need to have low latency to cope with small memory sizes on smaller connected devices [1]. The architectures currently deployed in communication systems are unsuited for this new environment, as they typically rely on large blocklength coding schemes for reliability, including interleaving techniques that bring about added latency, and power-hungry and complicated algorithms for secret key exchange and/or cryptography. Thus, there is a current need for low-power secrecy algorithms that can make security guarantees over short blocklengths.

One technique that may prove itself to be a nice match for many security and privacy issues in the IoT is that of wiretap (or secrecy) coding [2], [3] for physical-layer security [4], [5]. The general idea of such techniques is to code data in such a way that the channel over which an eavesdropper observes communications naturally secures the data transmission, while also allowing reliability over other communications channels for legitimate receivers.

If coding for secrecy is to prove itself adequate for solving the security issues inherent in the IoT, it must be better understood how these codes perform in the finite blocklength regime, particularly with very short blocklengths. Traditionally, wiretap codes are evaluated and analyzed as blocklengths approach infinity using information theoretic security measures. Let a message M be encoded into a length n codeword X^n for transmission across a communications channel. The eavesdropper observes a possibly noisy version of X^n denoted by Z^n. Data are transmitted with *weak secrecy* [4] if the leakage rate of information about the message goes to zero in the limit; that is,

$$\frac{1}{n}\mathbb{I}(M;Z^n) \to 0 \text{ as } n \to \infty. \tag{1}$$

Data are communicated with *strong secrecy* [6], [7] if the total amount of leaked information about the original message approaches zero as blocklength approaches infinity, or equivalently

$$\mathbb{I}(M;Z^n) \to 0 \text{ as } n \to \infty. \tag{2}$$

While a majority of secrecy coding structures (e.g., [2], [8], [9]) make use of these measures to classify their security achievements, we argue that a new approach in the finite blocklength regime, beginning with extremely short blocklength codes, would be of great value. Furthermore, we wish to actually quantify the total equivocation as a function of channel parameters in the eavesdropper's channel, rather than only analyzing codes in the asymptotic blocklength regime. In this paper, we analyze coset-based secrecy codes (as originally presented in [4], [8]) over finite blocklengths to quantify exactly (where possible) or estimate (using Monte Carlo techniques) the precise amount of information-theoretic security in terms of the equivocation

$$\Delta = \mathbb{H}(M|Z^n). \tag{3}$$

In essence, we are proposing that finite blocklength secrecy codes can be analyzed individually using simulation techniques similar to those that create bit error rate (BER) curves in generic error-control codes. When possible we can calculate the exact equivocation, or bound it as appropriate; but when

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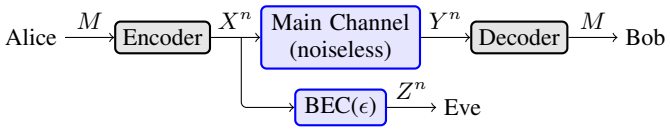

Fig. 1. The BEWC model has a noiseless main channel between Alice and Bob, and a BEC eavesdropper's channel.

these techniques fail, we can simply estimate the equivocation using Monte Carlo simulations.

The remainder of this paper is organized as follows. Section II contains background information about the channel model used throughout the paper, coset coding in general, and a specific encoding and decoding algorithm. Section III demonstrates how to quantify equivocation when using coset coding techniques over binary erasure channels. Section IV introduces simulation techniques for analyzing the equivocation of short blocklength codes, including a new parameter to compare finite-length codes with the achievable secrecy limits under the infinite blocklength assumption. Finally, Sections V and VI present empirical results for different coset coding techniques and summarize the major findings of the paper, respectively.

II. Background

In this section, we discuss the channel model used for this paper, as well as existing techniques for wiretap coding over the binary erasure wiretap channel (BEWC).

A. Overview of Channel Model

The channel model assumed in this paper is a variant of Wyner's wiretap model [4] called the BEWC, which is depicted in Fig. 1. In this model, Alice wishes to securely transmit a binary message to Bob in the presence of Eve, an eavesdropper who has full knowledge of the coding scheme in use. Alice encodes a message M from the alphabet $\mathcal{M} = \{1, 2, \ldots, 2^k\}$ into a corresponding n-bit codeword X^n ($n \geq k$). Alice transmits X^n to Bob through the main channel of communication, and Bob receives Y^n at the output of the channel. From this observation, Bob decodes and forms his estimate of the original message, denoted M'. In the BEWC, the main channel is noiseless, and thus, $Y^n = X^n$ and $M' = M$. An eavesdropper named Eve observes Z^n through the eavesdropper's channel, which is a BEC with parameter ϵ. Each bit of X^n is erased by the channel with probability ϵ independent of all other bits, and erasures are denoted as '?' symbols. Our choice of the BEWC for this work is for two reasons: (1) the problem is made much simpler by considering only this case for now, and (2) good forward error correcting code designs over binary erasure channel models have tended to produce good codes over more practical channels [10], and the same may be true for secrecy codes.

B. Overview of Coset Coding

In general, data should be encoded to minimize the probability of error for Bob and to restrict the amount of information intercepted by Eve. Since the main channel is noiseless, however, we need not worry about Bob. Secrecy over the eavesdropper's channel can be achieved through the coset coding procedure described in [4], [11]. Let the messages, $M \in \mathcal{M}$, be chosen uniformly at random. An $(n, n-k)$ linear block code C_1 (also referred to as the base code) is chosen that contains 2^{n-k} n-bit codewords [12]. From C_1, 2^k cosets $(C_1, C_2, C_3, \ldots, C_{2^k})$ can be obtained. By Lagrange's Theorem of cosets [12], each coset will contain 2^{n-k} n-bit binary vectors. Each message is then assigned to a unique coset, forming a codebook that contains every binary vector in the n-bit space. To encode M, a codeword is chosen at random from its corresponding coset and is transmitted as X^n. Since the main channel is noiseless, Bob simply has to find Y^n in the codebook and map back to M. Eve also has access to the codebook and can obtain M provided Z^n allows her to rule out all but one coset. If Z^n contains erasures, it is possible to achieve a measure of security further explored in Section III. The following example depicts the encoding process and the possible security benefits of coset coding.

TABLE I
Codebook structure outlined in example 1.

M	Codewords			
1	0000	0110	1001	1111
2	0001	0111	1000	1110
3	0010	0100	1011	1101
4	0011	0101	1010	1100

Example 1. Let $k = 2$, and let the elements in $\mathcal{M} = \{1, 2, 3, 4\}$ be equally likely. We choose the base code C_1 to be the (4, 2) linear, block code containing the codewords $\{0000, 0110, 1001, 1111\}$. Cosets are formed, and each message is arbitrarily mapped to a corresponding coset resulting in the codebook seen in Table I. Suppose we wish to transmit m = 3. A codeword from the third coset is chosen at random, for example 1011, and is transmitted as x^n. Since the main channel is noiseless, Bob receives $y^n = x^n$ and he can map $y^n = 1011$ back to the message $m = 3$. Suppose that Eve observes $z^n = 10??$. Since each coset contains a codeword consistent with z^n, Eve cannot rule out any cosets. Therefore, $\mathbb{H}(M|Z^n = z^n) = 2$ bits, meaning Eve gains no information from the observation about m.

C. Practical Encoding and Decoding Algorithm

An explicit method for encoding and decoding data in the coset coding scheme was developed in [8]. The message M is mapped to k-bits and is now denoted as $M^k \in \{0, 1\}^k$. We first select an $(n, n-k)$ linear block code for C_1 with generator matrix G and parity check matrix H. The rows of H are denoted $h_1, h_2, \ldots, h_k$. We now choose k linearly independent n-bit vectors $(q_1, q_2, \ldots, q_k)$ that satisfy the following conditions

$$q_i \notin C_1 \text{ for } 1 \leq i \leq k, \tag{4}$$

$$q_i h_i^T = 1 \text{ for } 1 \leq i \leq k, \tag{5}$$

$$q_i h_j^T = 0 \text{ for } 1 \leq i, j \leq k, i \neq j. \tag{6}$$

The last two requirements ensure that the syndrome equals the message, which will be explained shortly. We will now create

a matrix, G', whose rows are $q_1, q_2, \ldots, q_k$. We also generate a random $(n-k)$ bit vector $v^{(n-k)}$ for each transmission. The encoding procedure is a simple matrix multiplication and is represented as

$$x^n = \begin{bmatrix} m^k & v^{n-k} \end{bmatrix} \begin{bmatrix} G' \\ G \end{bmatrix}. \tag{7}$$

Using this encoding procedure, m^k determines the coset while the particular codeword within the coset is determined by v^{n-k}. If it is assumed that y^n is received erasure-free, the receiver calculates the syndrome to obtain

$$\begin{aligned} s^k &= y^n H^T \\ &= x^n H^T \\ &= \begin{bmatrix} m^k & v^{n-k} \end{bmatrix} \begin{bmatrix} G' \\ G \end{bmatrix} H^T \\ &= m^k (G' H^T) + v^{n-k} (G H^T) \\ &= m^k, \end{aligned}$$

because (5) and (6) ensure that $G'H^T = I_k$, and $GH^T = 0$ by definition, where I_k is the $k \times k$ identity matrix. The authors of [8] further reduce the complications required by the encoder resulting in very efficient algorithms.

III. Equivocation Calculation over the BEC

Let us now calculate $\mathbb{H}(M|Z^n)$ for an eavesdropper in our system. Note that [13]

$$\begin{aligned} \mathbb{H}(M|Z^n) &= \sum_{z^n \in \mathcal{Z}^n} p(z^n) \mathbb{H}(M|Z^n = z^n), \\ &= \mathbb{E}\left[\mathbb{H}(M|Z^n = z_n)\right], \end{aligned} \tag{8}$$

where $\mathcal{Z}^n = \{0, 1, ?\}^n$.

The expression $\mathbb{H}(M|Z^n = z^n)$ measures Eve's level of uncertainty regarding the message conditioned upon a particular observation z_n from the eavesdropper's channel and is measured in units of bits. Our goal is to maximize Eve's equivocation using coset coding. The following theorem quantifies Eve's equivocation for a specific observation z^n given the number of erasures, the placement of erasures, and the generator matrix of C_1.

Theorem 1. *Assume M^k is chosen uniformly at random from $\{0,1\}^k$. Let the $(n, n-k)$ linear, block code C_1 be the base code to be used in the coset coding scheme. Let G, a binary $(n-k) \times n$ matrix, be the generator matrix for C_1. Consider an instance of an eavesdropper's observation $z^n \in \{0, 1, ?\}^n$. Let μ represent the number of revealed positions in observation z^n and let G_μ be a binary matrix with dimensions $(n-k) \times \mu$ whose columns correspond to the revealed column indices of G. Then,*

$$\mathbb{H}(M^k|Z^n = z^n) = k - \mu + \operatorname{rank}(G_\mu). \tag{9}$$

Proof. If G_μ has rank r, then there exist 2^r ways to fill in the revealed positions within the codewords of C_1. Due to the properties of cosets, there are also 2^r ways to fill in the the revealed positions within the codewords of any and all solitary cosets. With this in mind, there exist $2^{n-k}/2^r = 2^{n-k-r}$ possible codewords in each possible coset. There must exist $2^{n-\mu}$ total codewords consistent with z^n, therefore, $2^{n-\mu}/(2^{n-k-r})$ cosets are consistent with z^n. Since all cosets are equally likely,

$$\begin{aligned} \mathbb{H}(M^k|Z^n = z^n) &= \log_2(2^{k-\mu+r}) \\ &= k - \mu + \operatorname{rank}(G_\mu). \end{aligned} \tag{10}$$

□

It should be noted that this result is stronger than that given in Theorem 2 of [8], which was derived from results in [11], and a similar observation was made in [14]. The previous result from [8], [11] merely indicates that an observation z^n is secured by C_1 iff $\operatorname{rank}(G_\mu) = \mu$, which is a special case of our result.

IV. Monte Carlo Channel Simulation Techniques

It is true that the choice of C_1 plays an important role in the equivocation of Eve. Although for small codes $\mathbb{H}(M|Z^n)$ may be calculated exactly by cycling through all possible $z^n \in \mathcal{Z}^n$ in (8) and using Theorem 1, this becomes computationally infeasible as blocklength grows to even moderate lengths. To estimate the security performance of any base code in the coset coding scheme, a Monte Carlo simulation can be performed.

A. Methodology

Let G be the binary generator matrix with dimensions $(n-k) \times n$ for C_1. Recall that the eavesdropper's channel has probability of erasure ϵ. The equivocation of a particular observation can be calculated using Theorem 1. This process is repeated for a predetermined number of iterations, resulting in an estimate of the average equivocation.

Lemma 1. *The expected value of*

$$\hat{H} = \frac{1}{N} \sum_{i=1}^{N} \mathbb{H}(M|Z^n = z_i^n), \tag{11}$$

where N is the number of iterations in a Monte Carlo simulation, is the true equivocation. Therefore, $\hat{H}$ is an unbiased estimator of $\Delta = \mathbb{H}(M|Z^n)$.

Proof. The expected value of $\hat{H}$ is

$$\begin{aligned} \mathbb{E}[\hat{H}] &= \mathbb{E}\left[\frac{1}{N} \sum_{i=1}^{N} \mathbb{H}(M|Z^n = z_i^n)\right] \\ &= \frac{1}{N} \sum_{i=1}^{N} \mathbb{E}\left[\mathbb{H}(M|Z^n = z_i^n)\right] \\ &= \mathbb{H}(M|Z^n), \end{aligned} \tag{12}$$

where the final line in the proof comes from (8). □

Although $\hat{H}$ is an unbiased estimator, we will also be interested in the confidence intervals of the simulated results as a statistical guarantee that N is large enough to differentiate between the equivocations of different codes. We will display these intervals as error bars in later sections. Using the estimator $\hat{H}$, the security performance of any coset code

can be thoroughly characterized by simulating across a range of L evenly spaced ϵ values. Similar types of Monte Carlo simulations have been used to characterize bit error rates (BER) of forward error correcting codes [12], [15], so it should not surprise us that simulation can be used to evaluate wiretap codes.

B. Achievability Gap

Suppose that the simulation techniques from the previous section are implements with large N and L requirements to return a high fidelity result. Suppose also that C_1 has a blocklength that is large enough such that the rank calculation in (9) is too costly to complete the required simulation in an acceptable amount of time. In this section, we present a single point simulation parameter ($L = 1$) that can be used to evaluate the equivocation rate at a particularly meaningful ϵ value.

Let C_1 be the $(n, n-k)$ linear, block code that will be used as the base code in the coset coding scheme as before. In the worst case scenario where z^n contains zero erasures, all the information is leaked to the eavesdropper, and $\mathbb{H}(M|Z^n) = 0$. In the best case scenario, z^n contains sufficient erasures such that $\mathbb{H}(M|Z^n) = k = \mathbb{H}(M)$. It now makes sense to present equivocation on a normalized scale, and we note that

$$\frac{\Delta}{n} = \frac{\mathbb{H}(M|Z^n)}{n} \tag{13}$$

is usually called the equivocation rate. Notice that this quantity can be bounded as

$$0 \leq \frac{\mathbb{H}(M|Z^n)}{n} \leq \frac{\mathbb{H}(M)}{n} = R, \tag{14}$$

where $R = k/n$ is called the secret information rate, using the standard inequality rule of conditional entropy [13]. Further note that the secrecy capacity C_s, defined as the supremum of rates such that weak or strong secrecy can be achieved while also maintaining reliable communications over the main channel, is equal to ϵ for the BEWC [8]. Thus, it is also true that

$$\frac{\mathbb{H}(M|Z^n)}{n} \leq \epsilon. \tag{15}$$

Combining (14) and (15) results in the overall bound of

$$0 \leq \frac{\mathbb{H}(M|Z^n)}{n} \leq \min(\epsilon, R), \tag{16}$$

which is depicted in Fig. 2. Let us judge how closely a finite blocklength code gets to approaching the asymptotic secrecy supremum by considering the gap between $\mathbb{H}(M|Z^n)$ and R at $\epsilon = R$. Thus, we now define the *achievability gap*, A_g, as

$$A_g = R - \frac{\mathbb{H}(M|Z^n)}{n}\Big|_{\epsilon=R}. \tag{17}$$

Using Monte Carlo techniques, individual choices of C_1 in a coset coding scheme can now be compared side by side using their entire equivocation rate curves, or using a single metric A_g. Both of these are depicted in Fig. 2. As A_g gets smaller, the equivocation rate curve also approaches the bound in (16), which is best possible, even for infinite length codes. Therefore, good secrecy codes and codes with larger blocklengths will tend to have smaller A_g values. Due to this, code comparison should be done using A_g only at identical blocklengths for the codes to be compared.

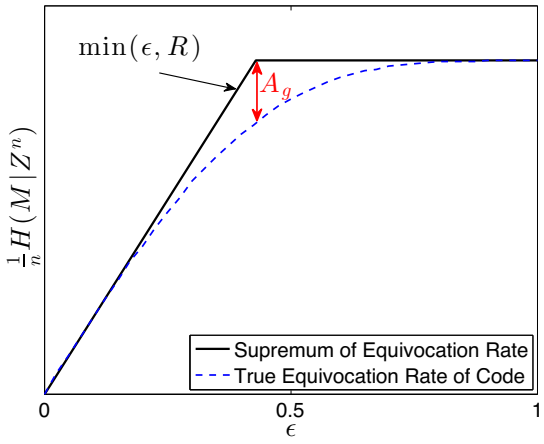

Fig. 2. The achievability gap is the difference between the true equivocation rate of a code and it's supremum in the limit evaluated at $\epsilon = R$.

Lemma 2. *The achievability gap measures the largest difference between the equivocation rate bound in (16) and a code's true equivocation rate; i.e.,*

$$\text{argmax}_\epsilon \left(\min(R, \epsilon) - \frac{1}{n}\mathbb{H}(M|Z^n) \right) = R. \tag{18}$$

Proof. A code's equivocation rate is always a concave function of ϵ [13]. For $0 \leq \epsilon \leq R$ the bound in (16) is a linear function of ϵ. Therefore, the difference between the bound and the code's true equivocation rate will continue to grow along this interval (from left to right). Along the interval $R \leq \epsilon \leq 1$ the bound in (16) is a horizontal line, and as a result, the difference between the bound and the code's true equivocation rate will shrink along this interval (also from left to right). Thus, the largest difference between the bound and the code's true equivocation must occur at $\epsilon = R$, precisely where the achievability gap is evaluated. □

While this is certainly a nice property, one should avoid making claims as to the superior nature of certain codes over others using only the achievability gap. Essentially this metric is a single point Monte Carlo simulation of the equivocation curve, and it will be seen in the following example that this is not sufficient to rank code performance for all ϵ.

Example 2. C_1 is chosen to be the (7,4) Hamming code with secret information rate $R \approx 0.4286$. Since the blocklength is reasonably small, the equivocation rate can be calculated exactly, as can all the equivocation rate curves for every linear block code with $n = 7$ and $k = 4$. The results of this experiment can be seen in Fig. 3. Notice that $\frac{1}{n}\mathbb{H}(M|Z^n)\Big|_{\epsilon=0.4286} < R$, as expected for finite length codes. For this code, $A_g \approx 0.0812$ bits. By inspection, it is easy to see that the largest difference between the bound and the true equivocation rate occurs at $\epsilon = R$. Careful inspection of the

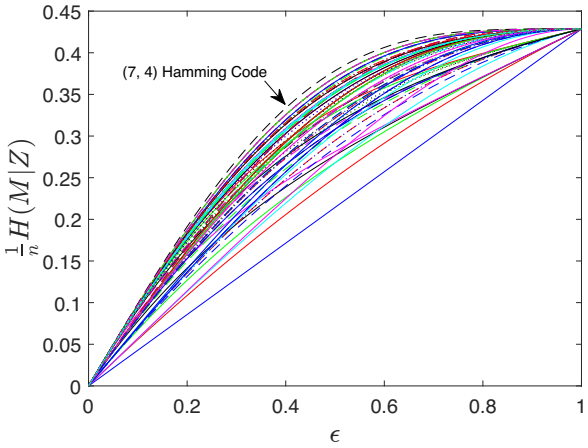

Fig. 3. Equivocation rate curves for all (7,4) linear block codes. The Hamming code achieves the highest equivocation for all codes of this size.

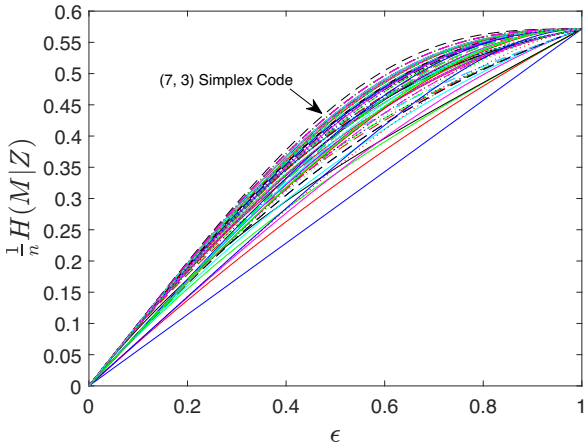

Fig. 4. Equivocation rate curves for all (7,3) linear block codes. The simplex code achieves the highest equivocation for all codes of this size.

figure reveals that A_g is actually minimized for (7,4) linear block codes in the choice of C_1 as the Hamming code, but it is also easy to see several equivocation curves that cross each other somewhere within the range of $\epsilon \in [0, 1]$, indicating that evaluation of code performance at a single point is not sufficient to rank codes for all ϵ.

V. Characterizing Algebraic and Random Codes with Small to Medium Blocklengths

In this section, we present recommendations on how to characterize the security performance of small to medium blocklength codes. Ideally, the true equivocation rate of a code should be calculated through (8). From a computational resource standpoint, this is only feasible for codes with very small blocklengths. For slightly larger codes, the logical next step would be to place bounds on the true equivocation rate. Steps toward bounding the true equivocation rate are presenented in [16], however these bounds tend to be loose for codes of small blocklength, and are not guaranteed to provide any reasonable comparison between codes. Thus, when (8) becomes too computationally costly, performing a Monte Carlo simulation over L evenly spaced ϵ values, as described in Section IV, provides a valid method to estimate the security performance of a code. When this likewise becomes too costly to execute, the single-point Monte Carlo estimate ($L = 1$) given by the achievability gap is a reasonable approach since it measures the maximum difference between the code's theoretical maximum equivocation rate and its true equivocation rate. We explore some of these ideas in the following subsections.

A. Calculating True Equivocation for Small Blocklengths

By directly calculating the equivocation rate (8), we have observed that Hamming and simplex codes are the best performing codes for their respective information rates when blocklength is equal to seven. Figure 3 shows the equivocation rate curves for every (7,4) linear block code in a coset coding scheme, while Fig. 4 shows the curves for every (7,3) linear block code. We note that the Hamming code wins among the (7,4) codes, and its dual, the simplex code, wins among the (7,3) codes for every value of ϵ. We also note in both figures that some codes perform better than their counterparts at larger values of ϵ but perform worse than their counterparts at smaller values of ϵ and vice versa. This makes it difficult to rank the codes in relation to one another (with the exception of the Hamming and simplex codes). Noting that these algebraic structures are quite interesting in a secrecy coding context, in the next section we investigate larger Hamming and simplex codes, and compare their equivocation rate curves and achievability gaps to those of randomly generated codes.

TABLE II
Achievability Gaps for Hamming and Simplex Codes.

Code	Blocklength	R	A_g (bits)
Hamming	7	0.4286	0.0812
Hamming	15	0.2667	0.0723
Hamming	31	0.1613	0.0311
Hamming	63	0.0952	0.0181
simplex	7	0.5714	0.0779
simplex	15	0.7333	0.0526
simplex	31	0.8387	0.0305
simplex	63	0.9048	0.0179

B. Estimating Equivocation Rates for Small to Medium Codes

Using the Monte Carlo simulation technique described earlier, experimental values of A_g for Hamming and simplex codes with larger blocklengths were obtained and are given in Table II. Here we note a general trend that the achievability gap A_g shrinks as blocklength grows. This makes sense, because A_g measures the difference between a code's equivocation rate and the supremum of achievable equivocation rates, which is to be understood in the limit as $n \to \infty$. Note, however, that A_g/R actually maintains a constant ratio for all tested Hamming codes; thus we see the need to avoid using A_g to directly compare codes of different blocklength.

We now increase the blocklength and generate codes randomly so as to compare with these highly structured algebraic codes. The random codes that we consider have a single parameter α, and generators for these codes are constructed

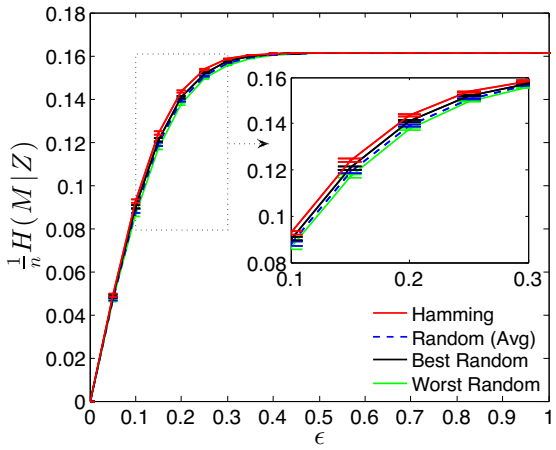

Fig. 5. Equivocation rate curves for (63,57) random codes (average case, best case, and worst case) and the (63,57) Hamming code.

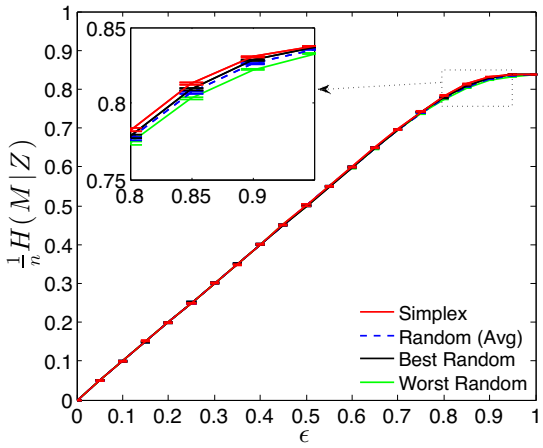

Fig. 6. Equivocation rate curves for (31,5) random codes (average case, best case, and worst case) and the (31,5) simplex code.

such that each bit in the generator matrix is equal to one with probability α, independent from all other bits. For blocklengths slightly larger than ten, we are no longer capable of calculating equivocation exactly in any reasonable amount of time. Thus, we employ the Monte Carlo techniques developed herein, and find that simulations show random codes with $\alpha \approx 0.5$, tend to have smaller A_g values. Simulations also show that the (31,26) Hamming code slightly outperforms (31,26) random codes with $\alpha \approx 0.5$. To test this, ten (31,26) random codes with $\alpha \approx 0.5$ were created and tested using the Monte Carlo simulation techniques. The average security performance of these random codes compared to the (31,26) Hamming code's performance is shown in Fig. 5 with 95% confidence intervals. Simulations further indicate that the (31,5) simplex code outperforms (31,5) random codes with $\alpha \approx 0.5$. Again, ten (31,5) random codes with $\alpha \approx 0.5$ were created and tested using the Monte Carlo simulation techniques, and the results can be viewed in Fig. 6. We expect the difference between algebraic codes and randomly chosen codes to further shrink for yet larger blocklengths, indicating that codes generated somewhat randomly may be expected to perform within some small difference to more optimized structures.

VI. Conclusion

In conclusion, we have presented a list of ordered techniques in terms of preference for quantifying the equivocation rate of small blocklength secrecy codes over the BEWC. The best solution, when possible, is to calculate exactly the equivocation. When this becomes infeasible due to large blocklength codes, Monte Carlo techniques can provide quantification of the equivocation of codes. For cases where the blocklength is large enough to restrict the number of ϵ parameters over which the equivocation rate can be simulated, we propose the achievability gap is a meaningful single-point estimate of the equivocation rate as it marks the maximum deviation from established bounds.

References

[1] G. Durisi, T. Koch, and P. Popovski, "Toward massive, ultrareliable, and low-latency wireless communication with short packets," *Proceedings of the IEEE*, vol. 104, no. 9, pp. 1711–1726, Sept 2016.

[2] W. K. Harrison, J. Almeida, M. R. Bloch, S. W. McLaughlin, and J. Barros, "Coding for secrecy: An overview of error-control coding techniques for physical-layer security," *IEEE Signal Processing Magazine*, vol. 30, no. 5, pp. 41–50, Sep. 2013.

[3] M. Bloch, M. Hayashi, and A. Thangaraj, "Error-control coding for physical-layer secrecy," *Proceedings of the IEEE*, vol. 103, no. 10, pp. 1725–1746, Oct. 2015.

[4] A. D. Wyner, "The wire-tap channel," *Bell Syst. Tech. J.*, vol. 54, no. 8, pp. 1355–1387, Oct. 1975.

[5] M. Bloch and J. Barros, *Physical-Layer Security: From Information Theory to Security Engineering*. Cambridge, UK: Cambridge University Press, 2011.

[6] U. M. Maurer, *The Strong Secret Key Rate of Discrete Random Triples*. Boston, MA: Springer US, 1994, pp. 271–285. [Online]. Available: http://dx.doi.org/10.1007/978-1-4615-2694-0_27

[7] U. Maurer and S. Wolf, "Information-theoretic key agreement: From weak to strong secrecy for free," in *Advances in Cryptology — EUROCRYPT 2000*, ser. Lecture Notes in Computer Science, B. Preneel, Ed., vol. 1807. Springer-Verlag, May 2000, pp. 351–368.

[8] A. Thangaraj, S. Dihidar, A. R. Calderbank, S. W. McLaughlin, and J.-M. Merolla, "Applications of LDPC codes to the wiretap channel," *IEEE Trans. Inf. Theory*, vol. 53, no. 8, pp. 2933–2945, Aug. 2007.

[9] A. Subramanian, A. T. Suresh, S. Raj, A. Thangaraj, M. Bloch, and S. McLaughlin, "Strong and weak secrecy in wiretap channels," in *Proc. 6th Int. Symp. Turbo Codes Iterative Information Processing (ISTC)*, Sep. 2010, pp. 30–34.

[10] T. Richardson and R. Urbanke, *Modern Coding Theory*. New York, NY: Cambridge University Press, 2008.

[11] L. H. Ozarow and A. D. Wyner, "Wire-tap channel II," *Bell Syst. Tech. J.*, vol. 63, no. 10, pp. 2135–2157, Dec. 1984.

[12] T. K. Moon, *Error Correction Coding: Mathematical Methods and Algorithms*. Hoboken, New Jersey: John Wiley & Sons, Inc., 2005.

[13] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Hoboken, NJ: John Wiley & Sons, Inc., 2006.

[14] I. L. Anuradha Wickramasooriya and R. Subramanian, "Comparison of equivocation rate of finite-length codes for the wiretap channel," in *IEEE International Conference on Services Computing (SCC)*, Munich, Germany, Jan. 2013, pp. 1–6.

[15] S. Lin and D. J. C. Jr., *Error Control Coding*, 2nd ed. Upper Saddle River, New Jersey: Pearson Prentice Hall, 2004.

[16] C. W. Wong, T. F. Wong, and J. M. Shea, "LDPC code design for the BPSK-constrained Gaussian wiretap channel," in *Proc. IEEE Global Telecommunications Conf. (GLOBECOM) Workshops*, Dec 2011, pp. 898–902.