

B- and D-meson leptonic decay constants and quark masses
from four-flavor lattice QCD

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We describe a recent lattice-QCD calculation of the leptonic decay constants of heavy-light pseudoscalar mesons containing charm and bottom quarks and of the masses of the up, down, strange, charm, and bottom quarks. Results for these quantities are of the highest precision to date. Calculations use 24 isospin-symmetric ensembles of gauge-field configurations with six different lattice spacings as small as approximately 0.03 fm and several values of the light quark masses down to physical values of the average up- and down-sea-quark masses. We use the highly-improved staggered quark (HISQ) formulation for valence and sea quarks, including the bottom quark. The analysis employs heavy-quark effective theory (HQET). A novel HQET method is used in the determination of the quark masses.

PRESENTED AT

Conference on Intersections of Particle and Nuclear Physics
Palm Springs, California, 29 May – 3 June, 2018

1 Introduction

A promising strategy for discovering new physics compares high-precision predictions of the standard model with high-precision experimental measurements, especially for rare higher-order weak processes. Precise values of the decay constants f_B and f_{B_s} are needed to provide accurate predictions for rare decays such as $B \rightarrow \tau\nu$, for neutral B mixing, and for the rare process $B_s \rightarrow \mu^+\mu^-$. They are also needed to probe the $V - A$ structure of the weak $W/u/b$ vertex and to resolve or sharpen the tension between inclusive and exclusive determinations of the CKM matrix element $|V_{ub}|$. Precise values of the quark masses are needed for precise Standard-Model predictions and to test Higgs origin of those masses. This talk describes results from two such high-precision *ab initio* studies using lattice quantum chromodynamics, one devoted to decay constants [1] and one, to quark masses [2, 3]. For details, please see those references.

Several improvements over previous calculations have allowed us to achieve high precision. We use gluon gauge-field configurations generated with the highly-improved staggered quark (HISQ) formulation for sea quarks, which reduces significantly light-quark lattice discretization errors [5]. We use the same quark formulation for all valence quarks, including b , following the HPQCD collaboration [6]. We combine three effective field theories (EFTs) to carry out extrapolations to the physical point, namely, heavy-quark effective theory (HQET) to treat heavy-quark discretization effects, heavy-meson rooted, all-staggered chiral perturbation theory (HMrASχPT) to treat the light-quark mass dependence, and Symanzik effective theory (SET) to treat light-quark and gluon discretization errors. For the quark-mass calculation, we use a new minimal-renormalon-subtraction (MRS) scheme to improve the HQET descriptions of the meson masses in terms of quark masses [7, 8]. Finally, we have accumulated a large simulation sample that includes 24 gauge-field ensembles with approximate lattice spacing ranging from 0.03 to 0.15 fm with four flavors of sea quarks, namely, mass-degenerate up and down quarks and physical-mass strange and charm quarks. Except for lattice spacing 0.03 fm, one of the ensembles for each lattice spacing has all physical-mass sea quarks. The others have varying ratios of the mass of the light sea quark and strange sea quark. For further details, please see [1].

2 Decay-constant methodology

Since the study varies the heavy and light valence quark masses, we use the notation “ H_x ” instead of “ B^+ ” or “ B_s ” for the heavy-light pseudoscalar meson. The heavy-light-meson decay constant is obtained from the amplitude $A_{\text{pt-pt}}$ of the point-to-point pseudoscalar density-density correlator at large Euclidean time t :

$$C_{\text{pt-pt}}(t) \rightarrow A_{\text{pt-pt}} \exp(-M_{H_x} t). \quad (1)$$

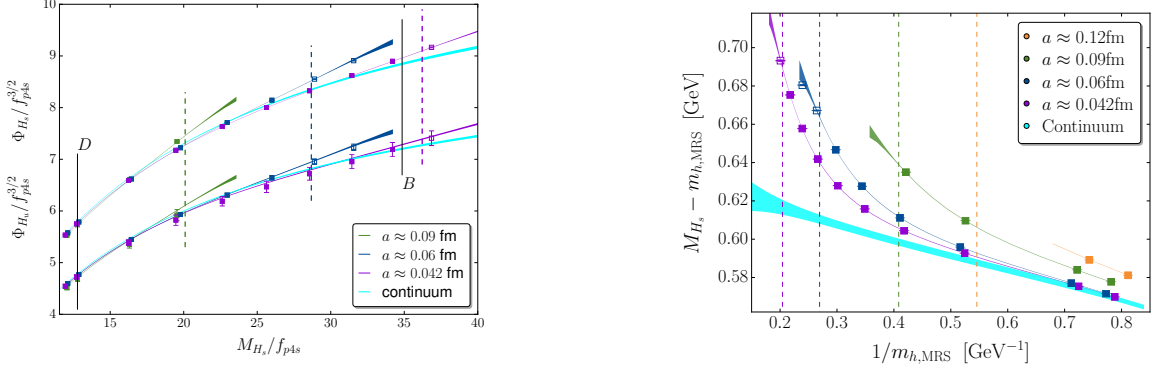


Figure 1: Left: snapshot of the base fit and lattice data for the decay constant with (lower curves) physical up quark mass and (upper curves) physical strange quark mass. The fit yielded $\chi^2/\text{d.o.f} = 466/432$ with p value=0.12. Right: snapshot of the base fit and lattice data for the difference of the heavy-strange meson mass and heavy-quark MRS mass *vs.* the inverse of that quark mass. The fit yielded $\chi^2/\text{d.o.f} = 320/307$, p value=0.3. For both panels the dashed lines indicate the $am_h = 0.9$ cutoff for the various lattice spacings and the cyan bands show the extrapolation to zero lattice spacing.

The decay constant F_{H_x} is obtained from its square root:

$$F_{H_x} = (m_h + m_x) \sqrt{\frac{3V A_{pt-pt}}{2M_{H_x}^3}}. \quad (2)$$

This calculation is done with bare light valence-quark masses $m_x \in [(m_u + m_d)/2, m_s]$ and bare heavy valence-quark masses $m_h \in [m_c, m_b]$. To avoid large heavy-quark discretization errors, we include results only with bare heavy-quark masses with approximately $m_h < 0.9/a$ in this study.

We set the lattice scale through the well-controlled intermediate quantity f_{p4s} , the decay constant of a fictitious pseudoscalar meson with both valence masses equal to $m_{p4s} \equiv 0.4m_s$. Its physical value is, in turn, determined from f_π . This strategy results in a precise determination of both the lattice spacing a and the quark mass am_{p4s} and in turn $m_s = 2.5m_{p4s}$. The values of f_{p4s} and quark mass ratio m_s/m_l are determined by analyzing light-light meson data from the same ensembles. Various systematic errors (such as finite volume and electromagnetic effects, continuum extrapolation *etc.*) in the estimate of f_{p4s} and tuned quark masses are incorporated to our estimate of uncertainties

We calculate the decay constant on each ensemble as a function of lattice spacing and the valence and sea-quark masses. We fit the result to a model that eventually permits an interpolation/extrapolation to the physical point at zero lattice spacing.

We use a cascade of EFTs to construct the fit functions. We start from the following schematic form for decay constants of H_x mesons

$$f_{H_x} \sqrt{M_{H_x}} \equiv \Phi_{H_x} = C (1 + \text{SET}) (1 + \text{HQET}) (1 + \text{HMrAS}\chi\text{PT}) \left(\frac{m'_c}{m_c}\right)^{3/27} \tilde{\Phi}_0. \quad (3)$$

The above terms correspond to different effective field theories: Symanzik effective theory,

$$\text{SET} = c_1 \alpha_s (a\Lambda_{\text{QCD}})^2 + \dots + c_3 \alpha_s (am_h)^2 + \dots,$$

Wilson coefficient,

$$C = \left[\alpha_s(M_{H_s})\right]^{-6/25} \left(1 + \mathcal{O}(\alpha_s)\right),$$

HQET (and integrating out sea-charm),

$$\text{HQET} = k_1 \frac{\Lambda_{\text{HQET}}}{M_{H_s}} + \dots + k'_1 \frac{m_c}{m'_c} \quad \text{and} \quad \frac{\Lambda_{\text{QCD}}^{(3)}(m'_c)}{\Lambda_{\text{QCD}}^{(3)}(m_c)} \approx \left(\frac{m'_c}{m_c}\right)^{2/27},$$

and heavy-meson rooted partially-quenched all-staggered chiral perturbation theory (HMrAS χ PT)[4]

$$\text{HMrAS}\chi\text{PT} = \text{NLO nonanalytic terms} + L_x m_x + L_s (2m'_l + m'_s) + L_a a^2.$$

(We use primes to distinguish simulation values from physical values where ambiguities may occur.) Chiral terms contain effects of taste splittings, hyperfine and flavor splittings, and finite lattice size.

To take into account SET, higher-order χ PT effects, and higher order HQET effects, we include analytic terms. They are typically polynomials in dimensionless, “natural” expansion parameters. They model

- Light-quark and gluon discretization (SET): $(a\Lambda_{\text{QCD}})^2$ with $\Lambda_{\text{QCD}} = 600$ MeV,
- Heavy-quark discretization effects (SET-HISQ): $(2am_h/\pi)^2$,
- Light valence and sea quark mass effects (χ PT): $B_0/(4\pi^2 f_\pi^2) m_q$,
- HQET: $\Lambda_{\text{HQET}}/M_{H_s}$.

The coefficients of the polynomials are fit parameters. They are expected to be $\mathcal{O}(1)$. Altogether, there are 60 fit parameters for 492 data points. Figure 1 gives a slice of the fit and data.

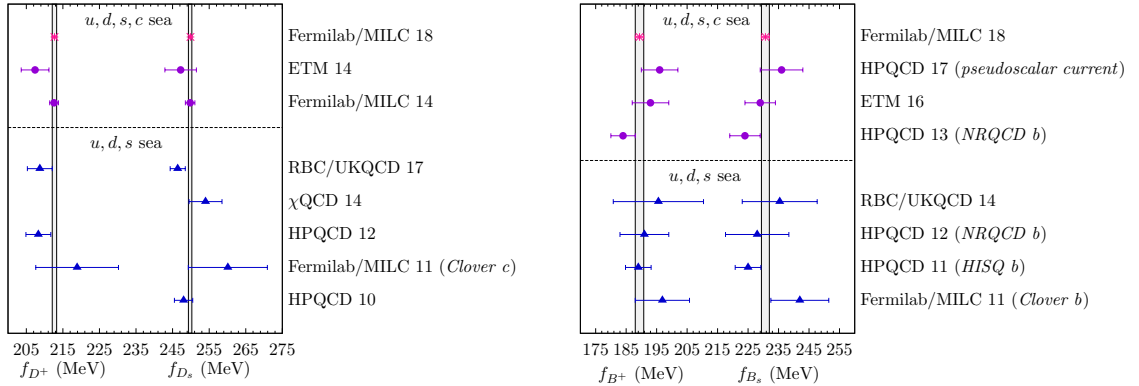


Figure 2: Comparison of our results (magenta) with previous three- and four-flavor lattice-QCD calculations. Labels are defined with citations in [1]. The gray bands indicate the total error.

3 Results for decay constants

We obtain

$$f_{D^0} = 211.6(0.3)_{\text{stat}}(0.5)_{\text{syst}}(0.2)_{f_{\pi,\text{PDG}}} [0.2]_{\text{EM-scheme}} \text{ MeV}, \quad (4)$$

$$f_{D^+} = 212.7(0.3)_{\text{stat}}(0.4)_{\text{syst}}(0.2)_{f_{\pi,\text{PDG}}} [0.2]_{\text{EM-scheme}} \text{ MeV}, \quad (5)$$

$$f_{D_s} = 249.9(0.3)_{\text{stat}}(0.2)_{\text{syst}}(0.2)_{f_{\pi,\text{PDG}}} [0.2]_{\text{EM-scheme}} \text{ MeV}, \quad (6)$$

$$f_{B^+} = 189.4(0.8)_{\text{stat}}(1.1)_{\text{syst}}(0.3)_{f_{\pi,\text{PDG}}} [0.1]_{\text{EM-scheme}} \text{ MeV}, \quad (7)$$

$$f_{B^0} = 190.5(0.8)_{\text{stat}}(1.0)_{\text{syst}}(0.3)_{f_{\pi,\text{PDG}}} [0.1]_{\text{EM-scheme}} \text{ MeV}, \quad (8)$$

$$f_{B_s} = 230.7(0.8)_{\text{stat}}(1.0)_{\text{syst}}(0.2)_{f_{\pi,\text{PDG}}} [0.2]_{\text{EM-scheme}} \text{ MeV}. \quad (9)$$

The systematic error includes uncertainties in the continuum extrapolation, finite volume correction, the electromagnetic contribution to meson masses used to fix the quark masses, and the adjustment for non-equilibration of topological charge.

These results are obtained in a specific scheme that matches QCD+QED to pure QCD for the light and heavy meson masses. When using our results in a setting that does not take into account the subtleties of the EM scheme, one may wish to also include the estimates of scheme-dependence given in the last quantities, in brackets.

In Fig. 2 we compare some of these results with those of previous three and four-flavor calculations.

4 Quark-mass methodology

The quark masses are determined from the masses of the mesons that contain them. For the charm and bottom quarks we use a new method based on HQET to extract masses of quarks from masses of heavy-light mesons, starting from a decomposition of the mass:

$$M_H = m_h + \bar{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2(m_h)}{2m_h} + \mathcal{O}(1/m_h^2), \quad (10)$$

where the parameters are

- $\bar{\Lambda}$: energy of light quark and gluons inside the system
- $\mu_\pi^2/2m_h$: kinetic energy of the heavy quark inside the system
- $\mu_G^2(m_h)/2m_h$: hyperfine energy due to heavy quark's spin. (can be estimated from B^*-B splitting: $\mu_G^2(m_b) \approx 0.35 \text{ GeV}^2$)
- m_h , the pole mass of the heavy quark. The conventional pole mass is ambiguous because of the renormalon problem.

We use the new minimal renormalon-subtracted (MRS) mass introduced by [8]. It removes the leading infrared renormalon from the pole mass. It is a gauge- and scale-independent scheme, and admits a well-behaved perturbative expansion in α_s .

In the continuum, the quark mass in the $\overline{\text{MS}}$ scheme is mapped to the MRS mass using

$$m_{\text{MRS}} = \bar{m} \left(1 + \sum_{n=0}^{\infty} [r_n - R_n] \alpha_s^{n+1}(\bar{m}) + J_{\text{MRS}}(\bar{m}) \right), \quad (11)$$

where $\bar{m} \equiv m_{\overline{\text{MS}}}(m_{\overline{\text{MS}}})$ and $J_{\text{MRS}}(\bar{m})$ is known [8]. The small coefficients $[r_n - R_n]$ are the differences between the $\overline{\text{MS}}$ and MRS expansion. We introduce a ‘‘reference mass’’, $m_r = m_{p4s, \overline{\text{MS}}}(\mu)$ and choose $\mu = 2 \text{ GeV}$.

To construct the fit function we begin with the identity,

$$m_{h, \text{MRS}} \equiv m_{r, \overline{\text{MS}}}(\mu) \frac{m_{h, \text{MRS}}}{\bar{m}_h} \frac{\bar{m}_h}{m_{h, \overline{\text{MS}}}(\mu)} \frac{am_h}{am_r}. \quad (12)$$

The first factor is a fit parameter. The second comes from Eq. (11) above. The third comes from $\overline{\text{MS}}$ mass running;

$$\frac{\bar{m}_h}{m_{h, \overline{\text{MS}}}(\mu)} = \frac{C(\alpha_{\overline{\text{MS}}}(\bar{m}_h))}{C(\alpha_{\overline{\text{MS}}}(\mu))}. \quad (13)$$

Finally, the last factor comes from simulation parameters.

For lattice quantities, the continuum relation above must be modified to include discretization effects and the light-quark mass dependence. These terms are obtained through HMrASχPT

$$M_H = m_{h,\text{MRS}} + \bar{\Lambda}_{\text{MRS}} + \frac{\mu_\pi^2 - \mu_G^2(m_h)}{2m_{h,\text{MRS}}} + \text{HMrAS}\chi\text{PT} + \text{higher order HQET}. \quad (14)$$

As with the decay constants, all terms get corrections for light-quark- and gluon-discretization effects, based on polynomials in the natural expansion parameters. Polynomial coefficients are fit parameters. Altogether, there are 67 parameters (6 with external priors) for 384 data points.

The heavy quark masses are obtained by interpolation to the physical masses of the heavy-light mesons. Since electromagnetic effects have been omitted in the lattice calculation, the physical heavy-light meson masses must be adjusted before matching to the continuum extrapolation of the fit result. Light-quark masses and decay constants use light-quark rSχPT [10, 11].

Figure 1 gives a slice of the fit and data. After extrapolating to continuum and matching to measured M_{D_s} and M_{B_s} masses *with EM effects removed*, we determine the strange-quark mass, the charm- and bottom-quark mass ratios, and their masses:

$$m_{s,\overline{\text{MS}}}(2 \text{ GeV}) = 92.47(39)_{\text{stat}}(18)_{\text{syst}}(52)_{\alpha_s}(11)_{f_\pi,\text{PDG}} \text{ MeV}, \quad (15)$$

$$m_c/m_s = 11.783(11)_{\text{stat}}(21)_{\text{syst}}(00)_{\alpha_s}(08)_{f_\pi,\text{PDG}}, \quad (16)$$

$$m_b/m_s = 53.94(6)_{\text{stat}}(10)_{\text{syst}}(1)_{\alpha_s}(5)_{f_\pi,\text{PDG}}, \quad (17)$$

$$m_b/m_c = 4.578(5)_{\text{stat}}(6)_{\text{syst}}(0)_{\alpha_s}(1)_{f_\pi,\text{PDG}}, \quad (18)$$

$$\bar{m}_c^{(n_f=4)} = 1273(4)_{\text{stat}}(1)_{\text{syst}}(10)_{\alpha_s}(1)_{f_\pi,\text{PDG}} \text{ MeV}, \quad (19)$$

$$\bar{m}_b^{(n_f=5)} = 4195(12)_{\text{stat}}(1)_{\text{syst}}(8)_{\alpha_s}(1)_{f_\pi,\text{PDG}} \text{ MeV}. \quad (20)$$

where $\bar{m}_h = m_{h,\overline{\text{MS}}}(m_{h,\overline{\text{MS}}})$.

The systematic error includes uncertainties in the determination of scale setting quantities, quark mass tuning, continuum extrapolation, finite volume, estimating electromagnetic effects. Shown separately is the uncertainty in the strong coupling constant, which we take to be $\alpha_s^{\overline{\text{MS}}}(5 \text{ GeV}; n_f = 4) = 0.2128(25)$ [9], and the uncertainty in the value of f_π .

Light quark masses are determined from a study of light-light pseudoscalar meson masses [3]:

$$m_{u,\overline{\text{MS}}}(2 \text{ GeV}) = 2.130(18)_{\text{stat}}(35)_{\text{syst}}(12)_{\alpha_s}(03)_{f_\pi,\text{PDG}} \text{ MeV}, \quad (21)$$

$$m_{d,\overline{\text{MS}}}(2 \text{ GeV}) = 4.675(30)_{\text{stat}}(39)_{\text{syst}}(26)_{\alpha_s}(06)_{f_\pi,\text{PDG}} \text{ MeV}. \quad (22)$$

Our results for the bottom, charm, strange, and the average of the up and down quark masses are compared with results of other groups in Figs. 3 and 4.

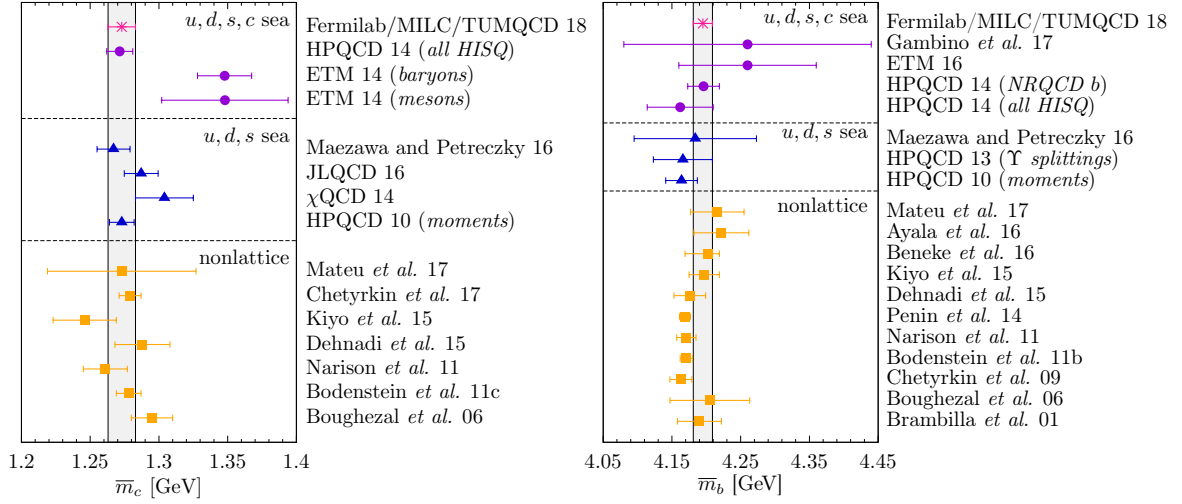


Figure 3: A comparison of our results (in magenta) with those of other groups. Labels are defined with citations in [2]. The gray bands indicate the total error.

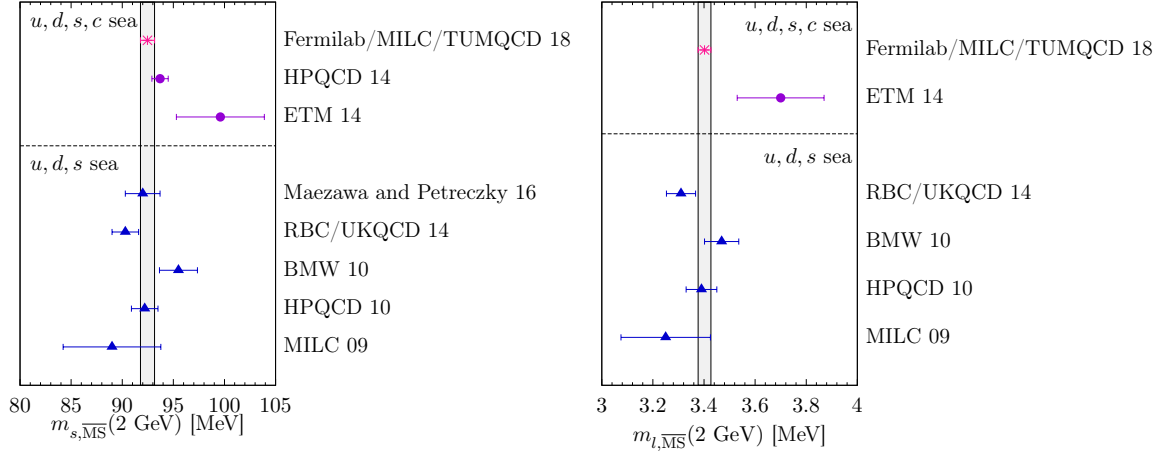


Figure 4: A comparison of our results (in magenta) with those of other other lattice QCD calculations. Labels are defined with citations in [2]. The gray bands indicate the total error.

5 Conclusion

In this lattice QCD study of the decay constants of heavy-light mesons, we used a combination of effective field theories in a correlated, multidimensional fit to lattice data at multiple lattice spacings. This approach reduces statistical errors and provides control of the systematic errors of extrapolation. We presented results for decay constants f_{D^+} , f_{D_s} , f_{B^+} , and f_{B_s} . For quark masses, we developed a method based on heavy-quark effective theory and the minimal renormalon subtraction scheme to extract quark masses from masses of the heavy-light mesons. Again, we used a combination of effective field theories to fit those heavy-light meson masses. We presented results for all quark masses and their ratios.

ACKNOWLEDGEMENTS

Computations for this work were carried out with resources provided by the USQCD Collaboration, the National Energy Research Scientific Computing Center, the Argonne Leadership Computing Facility, the Blue Waters sustained-petascale computing project, the National Institute for Computational Science, the National Center for Atmospheric Research, the Texas Advanced Computing Center, and Big Red II+ at Indiana University. USQCD resources are acquired and operated thanks to funding from the Office of Science of the U.S. Department of Energy.

This work was supported in part by the U.S. Department of Energy under grants No. DE-FG02-91ER40628 (C.B., N.B.), No. DE-FC02-12ER41879 (C.D.), No. DE-SC0010120 (S.G.), No. DE-FG02-91ER40661 (S.G.), No. DE-FG02-13ER42001 (A.X.K.), No. DE-SC0015655 (A.X.K.), No. DE-SC0010005 (E.T.N.), No. DE-FG02-13ER41976 (D.T.); by the U.S. National Science Foundation under grants PHY14-14614 and PHY17-19626 (C.D.), PHY14-17805 (J.L.), and PHY13-16748 and PHY16-20625 (R.S.); by the MINECO (Spain) under grants FPA2013-47836-C-1-P and FPA2016-78220-C3-3-P (E.G.); by the Junta de Andalucía (Spain) under grant No. FQM-101 (E.G.); by the German Excellence Initiative and the European Union Seventh Framework Program under grant agreement No. 291763 as well as the European Union's Marie Curie COFUND program (J.K., A.S.K.), and by the UK Science and Technology Facilities Council (J.K.).

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