The Age of Updates in a Simple Relay Network

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***Abstract*—In this paper, we examine a system where status updates are generated by a source and are forwarded in a First-Come-First-Served (FCFS) manner to the monitor. We consider the case where the server has other tasks to fulfill referred to as vacations, a simple example being relaying the packets of another non age-sensitive stream. Due to the server's necessity to go on vacations, the age process of the stream of interest becomes complicated to evaluate. By leveraging specific queuing theory tools, we provide a closed form of the average age of the stream which enables us to optimize its packet generation rate and achieve the minimum possible average age. Numerical results are provided to corroborate the theoretical findings and highlight the interaction between the stream and the vacations in question.**

I. Introduction

The Age of Information (AoI) is a new concept that has been introduced in [1] and is considered of broad interest in communication systems. More particularly, ubiquitous connectivity and the cheap hardware cost have created new applications where sensors are able to send status updates to a certain receiver. The status can range from being as simple as the temperature of a room [2] to as complicated as a vehicle's position and velocity [3]. In these applications, a source generates time-stamped updates that are forwarded through a network to reach the intended monitor. These updates should therefore be as timely as possible, i.e., the receiver should have the freshest possible information about what is being monitored. Knowing that the transmission of the packets may include channel errors and back-off timers to mitigate interference from neighboring sensors, the characterization of this metric is far from being straightforward.

Although practical scenarios of interest can be quite complex, the investigation of this metric for even the simplest models was found to be challenging. In [1], the average age was formulated for the case of First-Come-First-Served (FCFS) disciplines: M/M/1, M/D/1 and D/M/1 where it was shown that an optimal packets generation rate can be found to minimize the average age. The case where multiple information sources share the same queue was also evaluated in [4].

In [5], it was shown that if the source had the ability to manage packets, the average age of the stream can be further reduced. Moreover, a related metric called the *average peak age* was introduced in [5]. With this metric being generally more tractable than the average age, the authors in [6] formulated the average peak age of information for multiple information sources under general service time distributions.

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With the AoI metric gaining more attention, a surge of papers have been recently published on the subject. The age of information for energy harvesting sources was intesively investigated in the literature (e.g. [7]). More recently, a shift of interest to multi-hop scenarios can be witnessed [8]. This can be justified by the fact that the AoI is of high interest in wireless sensor networks where, in general, sensors can be multiple hops away from the monitor. In [9], it was shown that when the transmission times of packets in the network are exponentially distributed across all nodes, the Last-Come-First-Served (LCFS) preemptive policy at the relaying nodes minimizes the average age of the stream. Although this work gives insights on the disciplines to be used, it does not provide ways to explicitly calculate the average age of each stream. More recently, an explicit calculation of the average age was calculated for a single information stream over a network of preemptive servers in [10]. In another line of work, the focus was primarily on the study of the average age of streams with different assigned priotories [11][12][13]. More specifically, due to the complexity of the average age calculations, an upper bound and lower bound were provided in [13] for the case where a server has two streams to serve: a FCFS regular stream and a LCFS with preemption priority stream.

In the present work, we overcome the complexity that was found in [13] by leveraging some key queuing theory tools. By doing so, the study of a system where the server has other tasks to fulfill (referred to as vacations) will be feasible as will be depicted in the sequel. More precisely, we are able to formulate a *closed form* of the average age of the stream of interest along with a discussion on the interaction between the stream of interest and the vacations in terms of both stability and average age. In fact, it will be shown in the sequel that the minimization of the age of the stream of interest admits a unique optimal packet generation rate and we will be able to answer a pivotal question: "which has the least effect on the age of the stream, short and numerous vacations, or long rare vacations?". This question will be answered in the setting where the vacation consists of relaying the packets of a non age-sensitive stream. This comes from the fact that some applications are more restricted by throughput than age. It is worth mentioning that our system model and conclusions are general and they are not bound to the relay scenario taken into account. It can be used for any type of scenarios where the server is allowed to be unavailable for a certain amount time. Some non-exclusive examples include the node transitioning to SLEEP mode, doing necessary offline processing or serving a higher priority age-sensitive stream.

The paper is organized as follows: Section II describes the system model. Section III presents the theoretical results on the average age of the stream. Section IV provides numerical results that corroborate the theoretical findings while Section V concludes the paper.

II. System Model

Consider an information source sending status updates to a monitor. In this scenario, the instantaneous age of information at the receiver at time instant t is defined as:

$$\Delta(t) = t - U(t) \tag{1}$$

where $U(t)$ is the time stamp of the last successfully received packet by the monitor. Suppose that packet j is generated at time instant t_j and is received by the monitor at time t'_j, the evolution of the instantaneous age in this case can be depicted in Fig. 1 where A_j and T_j denote the inter-arrival and sojourn time of packet j respectively.

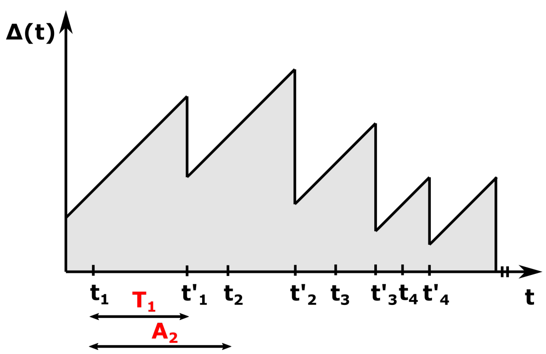

Fig. 1: The time evolution of the instantaneous age $\Delta(t)$

The main interest lays in the computation, and eventual minimization, of the average age of information in the aim of keeping the information as fresh as possible at the monitor. The average age is nothing but the saw-tooth area highlighted in Fig.1 and is defined as:

$$\Delta = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^{\tau} \Delta(t) dt \tag{2}$$

The computation of this metric is however challenging, even in the simplest scenarios [1]. With the metric being relatively new, giving insights on the computation and minimization of the average age in particular realistic scenarios is of paramount importance.

The scenario of interest that we investigate in the following consists of an information source generating packets, denoted as stream 1, and buffering them in a single queue (FCFS discipline) where the packet j inter-arrival time A_j is exponentially distributed with mean $\frac{1}{\lambda_1}$. In other words, λ_1 can be seen as generation rate of packets belonging to stream 1. The physical service time of the packet G_j is assumed to be exponential as well with mean $\frac{1}{\mu_1}$. However, we consider the case where the server is not always available and can take vacations. When a vacation takes place, the transmission of the packet of stream 1 is interrupted (with eventual resume) until the vacation finishes. The decision to go on vacation is dictated by an appropriate timer S that is assumed to be exponentially distributed with mean $\frac{1}{s}$. The vacation duration W is also considered to be exponentially distributed with mean $\frac{1}{w}$. What makes this model of broad interest is the fact that, in realistic scenarios, a device may have different functionality than just being occupied by forwarding its own generated packets. Some non-exclusive examples of vacations are:

- The device may transition into $SLEEP$ mode for a certain amount of time after which it wakes up. The motivation behind this transition is to prolong the battery's life
- The device may proceed to do necessary offline processing
- The device may relay packets of different streams other than its own
- The device may generate and serve higher priority age-sensitive packets

In the sequel, for clearer presentation, we focus on the case where the vacation is motivated by relaying packets of a different stream although the analysis is still valid for any of the aforementioned cases. Sensor 1 is considered as the sensor of interest that is generating age-sensitive data and forwarding them to a monitor that is only one hop away. However, at the same time, it keeps a separate queue for packets generated by sensor 2 in the aim of forwarding them to the monitor as seen in both Fig. 2. We focus on the case where the packets of the relayed stream require a desired throughput as will be discussed in the remainder of the paper.

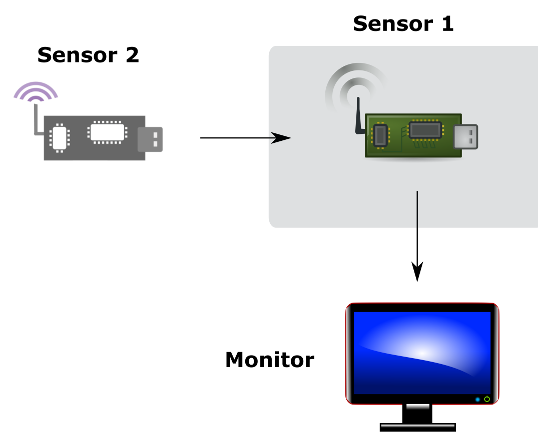

Fig. 2: A simple relay network

We first start by providing the general mathematical formula used to calculate the age of the stream of interest (stream 1). When the system is in steady state, both the inter-arrival and the sojourn times of the packets of stream 1 are stochastically identical, i.e., $A_j \overset{\text{st}}{=} A_{j-1} \overset{\text{st}}{=} A$ and $T_j \overset{\text{st}}{=} T_{j-1} \overset{\text{st}}{=} T$. We therefore lose the packet's index j in the sequel. In this case, the average age of stream 1 is evaluated as [1]:

$$\Delta_1 = \lambda_1(\mathbb{E}(AT) + 1/\lambda_1^2) \tag{3}$$

The main difficulty in finding a closed form of the average age of a certain stream lays in the term $\mathbb{E}(AT)$. In fact, the two variables are not independent (a large inter-arrival time allows the system to be emptied and T to be smaller) and the evaluation of this term, even in the simplest scenarios, can be challenging. In the following, we provide a closed form expression of the average age of the stream of interest. Using the formulated expression, insights on the effect of the vacation is highlighted along with finding the optimal packets generation rate λ_1^* to minimize the average age of stream 1 is provided. To do so, we first model the system using a $2D$ Markov chain. We then proceed to leverage the notion of probability generating functions and Little's distributional law. After appropriate analysis, the closed form is formulated along with different discussions on practical scenarios considerations.

III. Age Calculation

Let us consider the 2D continuous time stochastic process $\{(N(t), M(t)) : t \geq 0\}$ where $N(t) \in \mathbb{N}$ and $M(t) \in \{V, B\}$ denote the number of packets of stream 1 inside the system and the server status at time instant t respectively. $M(t)$ can only take two values: 1) V when the server is on vacation and 2) B when the server is not on vacation and is serving or is ready to serve the stream of interest. By taking into account the model's assumptions, $\{(N(t), M(t)) : t \geq 0\}$ becomes a Markovian process and the evolution of the system can be represented by a 2D continuous time Markov chain as seen in Fig. 3 where the states are all the possible combinations of (N, M).

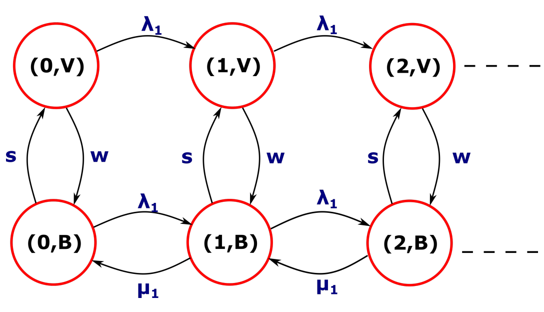

Fig. 3: Markov Chain

Let $\pi_B(i)$ be the stationary distribution for the state where the server is not on vacation while i packets of stream 1 are in the system. On the counterpart, let $\pi_V(i)$ be the case where the server is on vacation with i packets of stream 1 in the system. Based on the preceding, we can define the Probability Generating Function (**p.g.f**) of the random variable N as: $P(x) = \mathbb{E}(x^N) = \sum_{i=0}^{\infty} \pi(i)x^i = \sum_{i=0}^{\infty} \pi_V(i)x^i + \sum_{i=0}^{\infty} \pi_B(i)x^i = P_V(x) + P_B(x)$.

Theorem 1. *In the aforementioned system, the p.g.f of the number of packets of stream* 1 *in the system is given by:*

$$P(x) = \frac{\mu_1 \pi_B(0)(\lambda_1 + w + s - \lambda_1 x)}{\lambda_1^2 x^2 - (\lambda_1 w + \lambda_1^2 + \lambda_1 s + \lambda_1 \mu_1)x + \lambda_1 \mu_1 + w\mu_1} \tag{4}$$

where[1]*:*

$$\pi_B(0) = \frac{\mu_1 - \lambda_1 - \lambda_1 \frac{s}{w}}{\mu_1(1 + \frac{s}{w})} \tag{5}$$

Proof: The proof can be found in [14, Appendix A]. ∎

To be able to evaluate the difficult term $\mathbb{E}(AT)$, we provide in the following theorem the distribution of the sojourn time T of the packets of stream 1.

Theorem 2. *The sojourn time T of packets belonging to stream* 1 *in the system has the following distribution:*

$$f_T(t) = C_1 \exp(\alpha_1 t) + C_2 \exp(\alpha_2 t) \tag{6}$$

where α_1 and α_2 are the roots of the second degree equation:

$$z^2 + z(w + s + \mu_1 - \lambda_1) + w\mu_1 - \lambda_1 w - \lambda_1 s = 0 \tag{7}$$

Moreover, C_1 and C_2 are given by:

$$C_1 = \frac{(\mu_1 - \lambda_1 - \lambda_1 \frac{s}{w})(w + s + \alpha_1)}{(1 + \frac{s}{w})(\alpha_1 - \alpha_2)} \tag{8}$$

$$C_2 = \frac{(\mu_1 - \lambda_1 - \lambda_1 \frac{s}{w})(w + s + \alpha_2)}{(1 + \frac{s}{w})(\alpha_2 - \alpha_1)} \tag{9}$$

Proof: The proof can be found in [14, Appendix B]. ∎

A. Application 1: Relay of non age sensitive data

Suppose that the packets that are being relayed are not age sensitive and our aim is to have a desired average throughput for stream 2. The average throughput experienced by the relayed stream is equal to the portion of time the server spends on vacation[2].

Corollary 1. *The average amount of time the server spends on vacation is:*

$$\pi_V = \frac{\frac{s}{w}}{\frac{s}{w} + 1} \tag{10}$$

Proof: The proof can be found in [14, Appendix C]. ∎

We can see from the results of this corollary that the average throughput of stream 2 depends on the ratio of $\frac{s}{w}$ and not individually on them. These results are of paramount interest: In fact, investigating the average age of stream 1, for a *fixed* $\frac{s}{w}$, answers the important question; which has the least effect on the age of stream 1, short and numerous vacations, or long rare vacations? If it turns out to be the first, then packets of stream 2 are better off being served in fragments by assigning short mean vacation times. If it is the latter, then packets of stream 2 are better served as a *batch* (i.e assigning long mean vacation

[1] The system is stable if and only if $\pi_B(0) > 0$ which translates into having $\mu_1 > \lambda_1 + \lambda_1 \frac{s}{w}$

[2] It is assumed that the queue corresponding to stream 2 is always backlogged, i.e., whenever the server goes on vacation, it finds a packet to relay. If not, dummy packets are supposed to be sent which is a natural assumption to evaluate the average throughput performance [15][16][17, p.8]

time which would result in numerous packets being served in a single vacation). In both cases, the average throughput of stream 2 is the same as the frequency of the vacations is different. To be able to answer this question, we provide a closed form of the average age of stream 1 in the following.

Proposition 1. *The average age of the stream of interest is given by:*

$$\Delta_1(\lambda_1, \mu_1, s, w) = \frac{\lambda_1^2 C_1}{\alpha_1^2(\alpha_1 - \lambda_1)^2} + \frac{\lambda_1^2 C_2}{\alpha_2^2(\alpha_2 - \lambda_1)^2} + \frac{1}{\lambda_1} + \frac{\mu_1 s(\mu_1 - \lambda_1 - \lambda_1 \frac{s}{w}) + (s+w)(\lambda_1 + w)(\mu_1(1+\frac{s}{w}))}{(w\mu_1)(\lambda_1 + w)(\mu_1(1+\frac{s}{w}))} \tag{11}$$

Proof: The proof can be found in [14, Appendix D]. ■

The next step revolves around finding, for a fixed setting (μ_1, s, w), the optimal packet generation rate λ_1^* to minimize the average age of stream 1. Insights on this optimization is provided in the following proposition.

Proposition 2. *For a fixed service rate $\mu_1 > 0$, mean vacation time $\frac{1}{s} > 0$ and mean vacation duration $\frac{1}{w} > 0$, the following optimization problem admits a unique minimizer λ_1^*:*

$$\begin{aligned} &\underset{\lambda_1}{\text{minimize}} \quad \Delta_1(\lambda_1) \\ &\text{subject to} \quad 0 < \lambda_1 < \frac{\mu_1}{1+\frac{s}{w}} \end{aligned} \tag{12}$$

Proof: The proof can be found in [14, Appendix E]. ■

By using the results of Proposition 2, we can answer the question previously stated as for the nature of vacations that have the least effect on the average age of stream 1. As we have shown in the proof in [14, Appendix E], the optimal solution of the problem (12) in the interval $]0, \frac{\mu_1}{1+\frac{s}{w}}[$ can be achieved by finding λ_1^* such that $\frac{d\Delta_1(\lambda_1)}{d\lambda_1}|_{\lambda_1=\lambda_1^*} = 0$ (please refer to [14, Appendix E] for more details). After formulation of λ_1^*, we can compare the minimal achievable average age $\Delta_1(\lambda_1^*, \mu_1, s_1, w_1)$ and $\Delta_1(\lambda_1'^*, \mu_1, s_2, w_2)$ such that $s_1 > s_2$ and $w_1 > w_2$ while preserving $\frac{s_1}{w_1} = \frac{s_2}{w_2}$ to find the answer to our question. It turns out, as will be seen in the numerical results section, that it is better to have numerous short vacations rather than long rare vacations to reduce the effect of the vacations on the average age of stream 1.

B. Application 2: age sensitive priority stream

A different application to the vacation model that was provided previously in our paper is when the vacation itself consists of serving the packets of another higher priority stream (denoted by stream 2) generated by the same sensor 1 with the prioritized stream being served in a LCFS with preemption manner. In this case, the vacation can be thought to be an arrival of a packet of stream 2 that preempts any packet being transmitted, either from stream 1 or stream 2. Therefore, the priority stream see the system as an M/M/1/1 queue and its average age is therefore [4]:

$$\Delta_2 = \frac{1}{\mu_2} + \frac{1}{\lambda_2} \tag{13}$$

where λ_2 and μ_2 are the arrival and service rate of stream 2 respectively. This type of scenarios was investigated in [13] where a lower and upper bound on the average age of stream 1 was provided using the detour flow graph tool. Unlike the work in [13], we develop a *closed form* of the average age of stream 1 and give insights on choosing the optimum packets generation rate λ_1 to minimize the age of stream 1.

Proposition 3. *The average age of stream 1 is given by replacing $s = \lambda_2$ and $w = \mu_2$ in eq. (11)*

$$\Delta_1(\lambda_1, \mu_1, \lambda_2, \mu_2) = \frac{\lambda_1^2 C_1}{\alpha_1^2(\alpha_1 - \lambda_1)^2} + \frac{\lambda_1^2 C_2}{\alpha_2^2(\alpha_2 - \lambda_1)^2} + \frac{1}{\lambda_1} + \frac{\mu_1 \lambda_2(\mu_1 - \lambda_1 - \lambda_1 \frac{\lambda_2}{\mu_2}) + (\mu_2+\lambda_2)(\lambda_1 + \mu_2)(\mu_1(1+\frac{\lambda_2}{\mu_2}))}{(\mu_2\mu_1)(\lambda_1 + \mu_2)(\mu_1(1+\frac{\lambda_2}{\mu_2}))} \tag{14}$$

Proof: The analysis is similar to that of [14, Appendix D]. In this scenario, the vacation starts when a packet from stream 2 arrives to the system and preempts the service of packets of stream 1. The vacation finishes once the packet of stream 2 that is being served finishes. The same Markov chain of Fig. 3 holds, by replacing $s = \lambda_2$ and $w = \mu_2$. However, the difference between the two scenarios is the fact that new packets of stream 2 can preempt the previous stream 2 packet in service. In other words, the vacation duration timer can be reset due to a new arrival from stream 2. Due to the memoryless property of the exponential vacation timer, this reset of the vacation timer does not affect stream 1 and the same sojourn and virtual service time for packets of stream 1 as in Application 1 is to anticipated and the same proof of [14, Appendix D] holds. ■

IV. Numerical Results

This section provides numerical results to give insights on the theoretical conclusions previously stated.

A. Application 1

In this scenario, we have a fixed average service rate of stream 1 and is set to $\mu_1 = 1$. Moreover, we have a certain average throughput requirement for the relayed stream to ensure its queuing stability. This requirement is supposed to be equal to $\frac{1}{2}$ in the sequel, i.e., the portion of time where the server should be relaying packets from stream 2 is equal to $\frac{1}{2}$. One question arises: knowing that the throughput depends on the ratio of $\frac{s}{w}$ rather than individually on them, how could we calibrate these two parameters in a way to have the least impact on the average age of stream 1?

For this purpose, we simulate different scenarios of s and w such that we keep achieving the same required average throughput ($\frac{s}{w} = 1$). As seen in Fig. 4, for the same ratio $\frac{s}{w} = 1$, it is better to opt for high values of s and w. In other words, as previously mentioned, it is better to go often on vacation but for short amount of time. Consequently, packets of stream 2 are better served as fragments by making w be as high as practically possible while providing a vacation rate s high enough to keep the same ratio of $\frac{s}{w} = 1$. Moreover, there is a certain optimal rate λ_1^* that achieves the lowest possible average age of stream 1 for a fixed setting. Discussions on this matter will be provided in the next subsection.

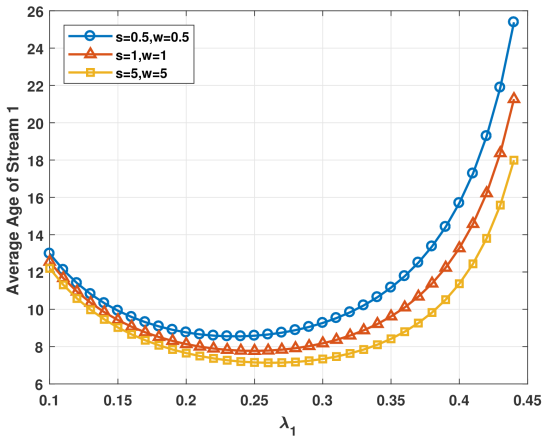

Fig. 4: Illustration of the effect of s and w on the average age of stream 1

B. Application 2

In this scenario, we have a fixed average service rate of $\mu_1 = 1$ and fixed vacation timer $w = \mu_2 = 4$ which is dictated by the size of stream 2 packets. In this case $\Delta_2 = \frac{1}{4} + \frac{1}{\lambda_2}$, therefore once can clearly that increasing $s = \lambda_2$ reduces the average age of stream 2. Our goal becomes to find the packets generation rate that minimizes the age of stream 1 for a certain fixed packet generation rate λ_2. This can be thought as an optimal packets generation rate λ_1^* that mitigate the effect of serving stream 2 as much as possible. In fact, one can see that as the arrival rate λ_2 of stream 2 grows larger, the optimum λ_1^* grows smaller and the achieved minimal average age is larger. This is due to the server being unavailable for a higher duration of time and therefore, delays are anticipated by waiting for vacations to finish which results in a necessity to reduce the packets generation rate λ_1 to mitigate this delay and therefore reduce the average age.

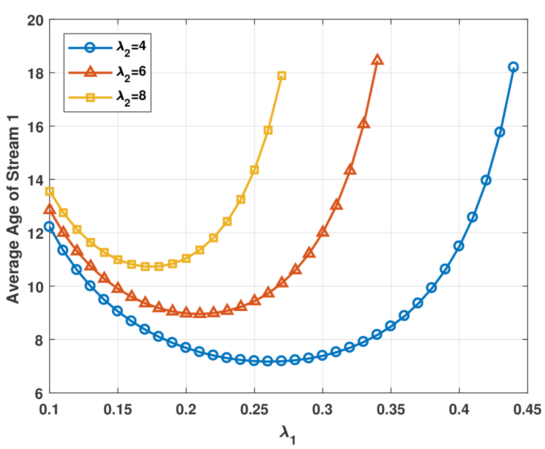

Fig. 5: Illustration of the optimal packets generation rate

V. Conclusion

In this paper, we have investigated the age of information in the case where the server is not always available and is allowed to go on vacation. This setting is of broad interest as it incorporates several practical scenarios, such as the relay of packets from different streams. By leveraging several key queuing theory concepts, we were able to provide a closed form expression of the average age of the stream of interest which allowed us to provide insights on the interaction between its age process and the vacations in question. Numerical results that corroborate the theoretical findings were provided to further understand the interaction between the age of the stream of interest and the vacations considered.

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