Delay and Energy Efficiency Tradeoff for Information Pushing System

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***Abstract*—We study the optimum designs of a unicast push-based information delivery system, where information is sent from a server to a user by using server-initiated pushing actions. Information arrives at the server at random and the server adopts a "hold-then-serve" strategy, where new information is temporarily stored in a queue for a later one-time transmission. At any given time instant the server decides whether to stop waiting and push all information in the queue to the user, or to keep waiting based on the current queue status and the long-term design objective. A shorter waiting time can preserve the "freshness" or timeliness of the information, which can be measured by using various delay penalty functions. However, frequent pushing actions will increase the power consumption of user devices by frequently waking up the client. The objective is to identify the optimum stopping rule that can optimize the tradeoff between the delay penalty and energy consumption. Motivated by the fact that different applications have different delay requirements, we adopt three different types of delay penalty functions, including linear, exponential, and logarithmic penalty functions. Using optimum stopping theories, optimum stopping rules are developed to minimize a weighted combination of delay penalty and energy efficiency. Different Pareto-optimum tradeoffs between delay and energy efficiency can be achieved by tuning the weight coefficient in the objective function. In particular, if the delay penalty function is convex, it is proved that the one-step look ahead stopping rule is optimum.**

***Index Terms*—Optimum stopping rule, information pushing, scheduling.**

I. Introduction

THE EXPLOSIVE growth of mobile devices and mobile applications have enabled the ubiquitous and timely access to a large variety of information, such as news, stock prices, weather, and traffic, etc. It has transformed how we access and consume information. Mobile services are expected to make any information available anytime and anywhere. However, fulfilling these expectations is difficult at times due to limited resources on mobile devices and communication links. The growing demands for these types of applications have motivated various research works on how to efficiently deliver information to mobile users [1]. In any information delivery system, the information can be delivered to a client through either server-initiated information pushing, or client-initiated information pulling, or both. The push and pull models are compared in details in [2].

One of the main design objectives of information delivery systems is to preserve the timeliness or "freshness" of the information because the values of information depreciate rapidly as time elapses. The "freshness", or conversely, "staleness" of information can be measured by using the delay between information generation and consumption, or by using age-of-information (AoI) [3]–[8]. Under an energy harvesting setting, various status update policies are developed to minimize the AoI for systems with infinite [9], [10] or finite batteries [11]. The tradeoff between delay and energy efficiency is studied in [12] by using optimum stopping theory. All above results assume a linear penalty function. Non-linear penalty functions are introduced in [13] to capture the diverse information aging processes in practical applications.

In a traditional "request-then-serve" unicast scheme adopted by a push-based information delivery system, the delay can be minimized by pushing information to the user (mobile client) as soon as it arrives at the server. However, frequent pushing operations will also increase the power consumption of a client since the client needs to wake up every time new information arrives. Alternatively, the server can adopt a "hold-then-serve" strategy, in which the server temporarily holds the information in the queue for a later one-time transmission. This will reduce the frequency of information pushing, at the cost of a larger delay. Thus it is vital to design push scheduling schemes that can balance the tradeoff between delay and energy efficiency.

Optimum scheduling has been studied extensively in the literature. A network-centric scheduling mechanism is proposed in [1], where mobile push notifications are scheduled by sensing and predicting users' cellular network activities. In [14], a data push scheduling scheme is proposed for efficient mobile advertisement delivery by using a content preference model of the users. In [15], multicast scheduling strategies are proposed to optimize the tradeoff between energy efficiency and queuing delay, and the impacts of different delay constraints are considered during the design. The effect of transmission delay is not considered in [15] based on the assumption that the transmission rate is much larger than the information arrival rate. The optimum multicast scheduling for cache-enabled content-centric networks is studied in [16] and [17], where the cost

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functions are linear combinations of average delay, power, and/or fetching cost. The problems are solved by using Markov decision process (MDP), and it is shown that the optimum scheduling is of a threshold type.

In this paper, we study the optimum tradeoff between delay and energy efficiency by designing optimum scheduling strategies in a push-based unicast information delivery system. New information arrives at the server randomly and it is stored in a queue waiting for transmission. At any given time instant, the server decides whether to keep waiting or push all information in the queue to the user. This decision depends on the causal observations of the state of the queue, hence the time instants of the push action can be modeled as a stopping time [18]. So, the problem of optimum pushing scheduling can be cast in the framework of optimum stopping theory.

The optimum stopping theory is about the problem of deciding the time for a given action based on causal and sequential observations of system state information [18]. It has a wide range of applications in statistics, engineering, finance, etc [19]–[22]. The optimum stopping theory is used to develop optimum scheduling strategies for multicast in cellular networks [15] or opportunistic random access in ad hoc networks [23].

In this paper, the optimum stopping theory will be used to identify the optimum stopping rules regarding the condition under which the server should stop waiting and initiate the pushing action. Particularly, the optimum stopping rule will be designed to minimize an objective function expressed as a weighted combination of energy efficiency and delay penalty functions. During the analysis, it is assumed that the information arrival rate is much less than the information transmission rate, such that the transmission delay is less than one slot with a high probability. Consequently, the delay is mainly due to waiting time in the buffer. It will be shown that the achievable energy-delay tradeoff region is convex, and different Pareto-optimum tradeoff points between delay and energy can be achieved by tuning the weight coefficient in the objective function. Since it is hard to directly minimize the original objective function, a surrogate objective function is introduced and the equivalence between the original and surrogate objective function is analytically established. Using the surrogate objective function, we first identify the optimum stopping rule for a system with the general form of delay penalty functions. Then the optimum stopping rules for systems with linear and nonlinear delay penalty function are developed. It is shown that if the delay penalty function is convex, which includes both the linear and exponential delay penalty as special cases, then the optimum stopping rule can be formulated as a low complexity one-step look-ahead rule (LAR), where the server can achieve the optimum performance by comparing the current objective function with the expected objective function of the next time slot. For systems with linear delay penalty, we analytically identify the design parameters and the associated minimum objective function through statistical analysis of the random stopping time. For systems with non-linear delay penalty, the parameters of the optimum stopping rule can be obtained through a mixture of analytical and numerical methods, where the numerical method only needed to be performed once offline. The performances of the optimum stopping rules for systems with different types of delay penalty functions are compared with each other in terms of the optimum objective function and the energy efficiency-delay tradeoff.

A list of the notations used in this paper is summarized in Table I. The remainder of this paper is organized as follows. Section II presents the system model and problem formulation. The optimum stopping rule for a system with general delay penalty function is developed in Section III. Optimum stopping rules for systems with linear and nonlinear delay penalty functions are studied in Sections IV and V, respectively. Optimum stopping rules for an alternative problem formulated with time-normalized metrics are discussed in Section VI. Simulation results are presented in Section VII, and Section VIII concludes the paper.

TABLE I
LIST OF NOTATIONS

N_t	number of bits arriving at the start of t-th time slot
μ	average information arrival rate per slot (bits per slot)
$\mathcal{F}_t$	σ-algebra generated by $N^{1:t} = [N_1, N_2, \cdots, N_t]$
ϕ	stopping rule
D	hard deadline for pushing action (slots)
R	transmission rate (bits per second)
$\mathcal{S}$	set of all feasible stopping rules
T	stopping time, i.e., index of the time slot of pushing action
$f(\tau)$	delay penalty function
A_ϕ	bit-normalized average delay penalty
E_ϕ	bit-normalized average energy consumption
E_0	fixed amount of energy consumed in each pushing action
E_b	amount of energy consumed for receiving one bit of information
ω	parameter to adjust tradeoff between E_ϕ and A_ϕ

II. System Model and Problem Formulation

Consider a push-based information delivery system with the "hold-then-serve" strategy adopted at the server. We divide the time into slots and denote the amount of information arriving at the server at the beginning of the t-th slot as N_t bits, where N_t, for $t = 1, 2, \ldots$, are identically and independently distributed (i.i.d.) random variable (RV) with mean μ and variance σ_N^2. Let $\mathcal{F}_t = \sigma([N_1, \ldots, N_t])$ be the σ-algebra generated by $N^{1:t} \triangleq [N_1, \ldots, N_t]$.

We assume, without loss of generality, that the previous push action occurs at time slot 0. The server decides when to push all the queued information based on the current system status, that is, the total number of queued bits, and the long-term design objective. Define the action space of the server as $\mathcal{A} = \{0, 1\}$, where 1 represents the action of pushing all queued information to the user at the current slot, and 0 represents the action of waiting or not pushing. The stopping rule can then be represented as a mapping from $\mathcal{F}_t$ to the action space as

$$\phi : \mathcal{F}_t \to \mathcal{A}, \tag{1}$$

where $\phi(\mathcal{F}_t) = 1$ means that the server will push all the queued information bits to the user at time slot t, and $\phi(\mathcal{F}_t) = 0$ means the server will keep waiting and postpone pushing to future slots.

Define the set of all feasible stopping rules as

$$\mathcal{S} = \{\phi \mid \phi(\mathcal{F}_T) = 1, 1 \leq T \leq D\}, \tag{2}$$

where $D > 0$ is a hard deadline for the pushing action. The variable T is used to represent the index of the slot of the pushing action. It is clear that T is a stopping time as it depends only on the causal observations of the information $\mathcal{F}_t$ [18]. The stopping time T is a random variable, the distribution of which depends on the distribution of $N^{1:t}$ and the stopping rule ϕ.

A. Information Delay

In the hold-then-serve model, the information delay is contributed by two factors: the queuing delay τ_{Q} and the transmission delay τ_{Tx}. The queuing delay is the time duration that a bit needs to spend in the buffer before transmission. The transmission delay is the actual transmission time of the information in buffer. Assume the transmission rate of the system is R bits per second, then the transmission delay associated with stopping time T is

$$\tau_{\text{Tx}} = \frac{\sum_{t=1}^{T} N_t}{RT_0} \qquad \text{slots}, \tag{3}$$

where T_0 is the duration of one slot. Since N_t and T are random, τ_{Tx} is a random variable. Based on the Wald equation [18, Ch. 3.3] and the fact that $T \leq D$, it can be easily shown that the average transmission delay satisfies

$$\mathbb{E}[\tau_{\text{Tx}}] = \frac{\mathbb{E}[T]\mu}{RT_0} \leq \frac{D\mu}{RT_0}. \tag{4}$$

To ensure queue stability, the average arrival rate must be less than or equal to the average transmission rate [24], that is $\mu \leq RT_0$. When $\mu = RT_0$ or very close to RT_0, the scheduling problem is trivial, because in this case the data needs to be transmitted in almost every slot. For almost all practical mobile pushing-based applications such as news, weather, stock prices, etc, we have $\mu \ll RT_0$. Specifically, under the assumption $\frac{\mu}{RT_0} \ll 1$, we pick the hard deadline D based on the following constraint

$$\frac{D\mu}{RT_0} < 1 - \Delta, \tag{5}$$

where $\Delta \in [0, 1]$ is an adjustable parameter. Given $\frac{\mu}{RT_0} \ll 1$, the constraint in (5) can be easily met by picking the appropriate hard deadline D and parameter Δ.

From (4), we can see that the constraint in (5) implies $\mathbb{E}[\tau_{\text{Tx}}] < 1 - \Delta$, that is, the average transmission delay is less than 1 slot for $\Delta \in [0, 1]$. The following Lemma bounds the probability that $\tau_{\text{Tx}} > 1$.

Lemma 1: Assume that N_t follows i.i.d. Poisson distribution with mean μ. Under the constraint (5), the probability that the transmission delay is longer than one slot is bounded by

$$\Pr(\tau_{\text{Tx}} > 1) < \frac{\mu^2}{R^2 T_0^2 \Delta^2}\left(\frac{D}{\mu} + D^2\right). \tag{6}$$

Proof: The proof is in Appendix A. ■

Under the assumption that $\frac{\mu}{RT_0} \ll 1$ and the constraint in (5), it can be seen from (6) that the probability that the transmission delay is greater than 1 slot is negligible for appropriate choices of D and Δ. In addition, the constraint in (5) ensures that the average transmission delay is less than 1 slot. Therefore in this paper, we assume that the transmission delay is bounded by one slot. Since the time domain considered in the system model is discrete, we can assume that for a given stopping time T in a particular transmission, the information bits will reach the user at the $(T + 1)$-th time slot. Correspondingly, if a bit arrives at the server at the t-th slot, then the total delay experienced by the bit is $\tau = \tau_{\text{Q}} + \tau_{\text{Tx}} = T - t + 1$.

The impact of delay is measured by using a penalty function $f(\tau)$, where $\tau \geq 0$ is the summation of waiting time for the information in the queue and the aforementioned constant transmission delay. The delay penalty function $f(\tau)$ satisfies the following two conditions: 1) $f(\tau)$ is a monotonically increasing function; and 2) $f(0) = 0$. As τ goes to infinity, $f(\tau)$ can be either bounded or unbounded. In this paper, we will consider three different delay penalty functions [13],

1) *Linear Delay:* $f(\tau) = \alpha\tau$,
2) *Exponential Delay:* $f(\tau) = \exp(\alpha\tau) - 1$,
3) *Logarithmic Delay:* $f(\tau) = \log(\alpha\tau + 1)$,

where $\alpha > 0$ is a tuning parameter. It is easy to verify that all three functions satisfy the two conditions required for delay penalty functions.

The three penalty functions corresponding to applications with different tolerances on information delays. The exponential delay penalty function can be used for applications with information that is highly sensitive to delays, thus a small delay will incur a big penalty. Examples of such applications are instant messages and stock prices, because the values of these information depreciate rapidly as time elapses. The linear delay penalty can be assigned to applications like news or weather updates, where a small delay in information delivery is acceptable. The logarithmic penalty function is more appropriate for applications that can tolerate longer delays, such as advertisements or entertainment.

B. Delay-Energy Tradeoff

Suppose that for a given stopping rule ϕ, the pushing operations are carried out L times, and denote $T_1, T_2, \ldots, T_L$ as the corresponding stopping times. The average delay penalty normalized by the amount of delivered information can then be calculated as

$$A_\phi = \frac{\sum_{l=1}^{L} \sum_{t=1}^{T_l} N_{l,t} f(T_l - t + 1)}{\sum_{l=1}^{L} \sum_{t=1}^{T_l} N_{l,t}}, \tag{7}$$

where $N_{l,t}$ is the amount of information arriving at the start of the t-th time slot associated with the l-th pushing operation. It must be noted that in the delay penalty function a constant transmission delay of one time-slot is considered as previously explained. The system model is illustrated in the Fig. 1.

Denote the amount of energy consumed for receiving one bit of information at the user as E_b. Moreover, each pushing operation will wake up the client, and this will consume a fixed

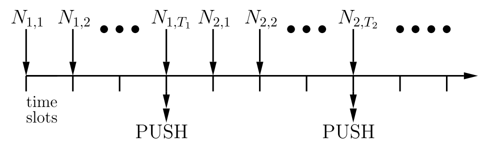


Fig. 1. Sample realizations of queued information bits followed by pushing operations.

amount of energy E_0. The energy efficiency can be measured by using the average normalized energy consumption, which can be calculated as

$$E_\phi = \frac{\sum_{l=1}^{L}\left[E_0 + E_b \sum_{t=1}^{T_l} N_{l,t}\right]}{\sum_{l=1}^{L}\sum_{t=1}^{T_l} N_{l,t}}. \tag{8}$$

If both the numerators and denominators of (7) and (8) are divided by L, then by the law of large numbers, as $L \to \infty$, we have

$$A_\phi \to \frac{\mathbb{E}\left[\sum_{t=1}^{T} N_t f(T-t+1)\right]}{\mathbb{E}\left[\sum_{t=1}^{T} N_t\right]}, \qquad a.s., \tag{9}$$

$$E_\phi \to \frac{\mathbb{E}\left[E_0 + E_b \sum_{t=1}^{T} N_t\right]}{\mathbb{E}\left[\sum_{t=1}^{T} N_t\right]}, \qquad a.s. \tag{10}$$

Here the expectation operator $\mathbb{E}()$ is performed with respect to both T and $N^{1:T}$. The distribution of the stopping time T is determined by the stopping rule ϕ and the distribution of $N^{1:t}$.

The energy efficiency can be improved by increasing T, because a longer T means less pushing actions in a unit time, thus less power consumption due to E_0. On the other hand, a longer T will increase the normalized delay penalty A_ϕ. Thus we need to find a stopping rule that can balance the tradeoff between E_ϕ and A_ϕ.

Define the achievable E_ϕ-A_ϕ region $\mathcal{R}$ as,

$$\mathcal{R} = \{(E_\phi, A_\phi), \phi \in \mathcal{S}\}. \tag{11}$$

It will be shown in the following lemma that the achievable region $\mathcal{R}$ is convex.

Lemma 2: The achievable E_ϕ-A_ϕ region $\mathcal{R}$ defined in (11) is convex.

Proof: The proof is in Appendix B. ■

Due to the convexity of the achievable region, the optimum tradeoff between E_ϕ and A_ϕ can be achieved on the Pareto-optimum boundary as shown in Fig. 2. The Pareto-optimum boundary corresponds to the region that A_ϕ and E_ϕ cannot be increased simultaneously, and it is between the two points with stopping rules corresponding to $\min E_\phi$ and $\min A_\phi$, respectively. Any point on the Pareto-optimum boundary characterizes a different tradeoff between E_ϕ and A_ϕ. We can identify each point on the Pareto-optimum boundary by identifying a linear function, $E_\phi + \omega A_\phi$, that is tangent to the boundary of $\mathcal{R}$ as shown in Fig. 2, where $\omega > 0$ is a tuning parameter used to adjust the tradeoff between E_ϕ and A_ϕ. Different values of ω correspond to different points on the Pareto-optimum boundary.

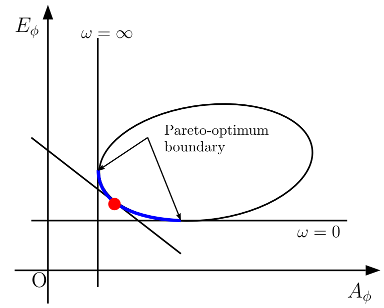


Fig. 2. Optimum tradeoff between E_ϕ and A_ϕ.

The problem can thus be formulated as

$$\text{(P1)} \qquad \text{minimize}_{\phi \in \mathcal{S}} \qquad E_\phi + \omega A_\phi.$$

Problem (P1) is a stochastic optimization problem and is usually difficult to solve. Given the fact that T is a stopping time, we propose to solve the problem by applying the stopping time theory [18]. However, since the expectation operator is on both numerator and denominator of the objective function, the stopping time theory cannot be readily applied to (P1). To facilitate the solution to the problem, we define a surrogate objective function as follows,

$$V_T(\lambda) = E_0 + E_b \sum_{t=1}^{T} N_t + \omega \sum_{t=1}^{T} N_t f(T-t+1) - \lambda \sum_{t=1}^{T} N_t, \tag{12}$$

where λ is a parameter to be solved later.

For a given stopping rule $\phi \in \mathcal{S}$, define

$$V_\phi(\lambda) = \mathbb{E}[V_T(\lambda)] = C_\phi - \lambda D_\phi, \tag{13}$$

where

$$C_\phi = \mathbb{E}\left[E_0 + E_b \sum_{t=1}^{T} N_t + \omega \sum_{t=1}^{T} N_t f(T-t+1)\right], \tag{14}$$

$$D_\phi = \mathbb{E}\left[\sum_{t=1}^{T} N_t\right]. \tag{15}$$

We define a new optimization problem

$$\text{(P2)} \qquad \text{minimize}_{\phi \in \mathcal{S}} \qquad V_\phi(\lambda).$$

Denote the optimum solution to the above problem as

$$V^*(\lambda) = \inf_{\phi \in \mathcal{S}} V_\phi(\lambda), \tag{16}$$

where the infimum operation is performed over all feasible stopping rules in $\mathcal{S}$.

Let λ^* be the solution to $V^*(\lambda) = 0$. Then we have the following lemma regarding the relationship between (P1) and (P2).

Lemma 3: The optimum stopping rule for (P1) is the same as the optimum stopping rule for (P2) when $\lambda = \lambda^*$. In addition, the optimum objective function of (P1) is

$$\lambda^* = \inf_{\phi \in \mathcal{S}} E_\phi + \omega A_\phi. \tag{17}$$

Proof: The proof is in Appendix C. ■

Algorithm 1 Two-Step Solution to (P1)

1: Step 1: Solve (P2) for arbitrary λ, identify $V^*(\lambda)$, and denote the optimum stopping rule with respect to (P2) as $\phi^*(\lambda)$.
2: Step 2: Identify the value of λ^* by solving $V^*(\lambda) = 0$.
3: Output: The optimum stopping rule for (P1) is hence $\phi^*(\lambda^*)$.

Based on these results, (P1) can be solved by using the following two-step approach.

The solution to (P2) in Step 1 will be discussed in the next Section. In order to ensure that there is always a solution to $V^*(\lambda) = 0$ in Step 2, we have the following Lemma regarding the property of $V^*(\lambda)$.

Lemma 4: The optimum objective function of (P2), $V^*(\lambda)$, is decreasing and concave.

Proof: The proof is in Appendix D. ■

When $\lambda = 0$, it can be easily seen from (13) that $V_\phi(0) > 0$ and $V^*(0) = \inf_{\phi\in\mathcal{S}} V_\phi(0) > 0$. Based on Lemma 4, we must have $\lim_{\lambda\to\infty} V^*(\lambda) < 0$, otherwise it will violate the concavity of the function. Since $V^*(0) = \inf_{\phi\in\mathcal{S}}(0) > 0$ and $\lim_{\lambda\to\infty} V^*(\lambda) < 0$ and $V_\phi(\lambda)$ is continuous in λ, there must exists $\lambda^* > 0$ such that $V^*(\lambda^*) = 0$. Thus the existence of λ^* and the optimum stopping rule is guaranteed.

III. Optimum Stopping Rule for (P2) With Arbitrary Delay Penalty Function

The optimum stopping rule with respect to (P2) for systems with the general form of delay penalty function is studied in this section. The optimum stopping rule that can solve (P1) with linear or non-linear delay penalty functions will be discussed in subsequent sections.

A. Optimum Stopping Rule

Theorem 1: The stopping rule that can minimize $V_\phi(\lambda)$ in (P2) can be written as a threshold test as

$$T^*(\lambda) = \inf\left\{1 \le T \le D : \min_{1\le k\le D-T} U_T^k(\lambda) \ge 0\right\}, \quad (18)$$

where

$$U_T^k(\lambda) = \omega\sum_{t=1}^{T} N_t[f(T+k-t+1) - f(T-t+1)] + (E_b - \lambda)k\mu + \omega\mu\sum_{i=1}^{k} f(i). \quad (19)$$

Proof: The proof is in Appendix E. ■

The stopping rule given in Theorem 1 is a threshold test. The server will stop waiting and push all information at slot T if the condition $\min_{1\le k\le D-T} U_T^k(\lambda) \ge 0$ is satisfied. From (19), the value of $U_T^k(\lambda)$ can be updated in every time slot as new values of N_t come in. For the optimum stopping rule given in Theorem 1, at each slot T we need to look ahead k-steps, that is, calculate the expected objective function, $\mathbb{E}[V_{T+k}(\lambda)|\mathcal{F}_T]$, for $1 \le k \le D–T$. This is the k-step look ahead rule (LAR) as defined in (62) in Appendix E. In the following subsection, it will be shown that the k-step LAR is equivalent to the one-step LAR if the delay penalty function is convex.

B. Optimum Stopping Rule for (P2) With Convex Delay Penalty Function

In this subsection, we will study the optimum stopping rule of (P2) for systems with convex delay penalty functions. Specifically, we will show that for systems with convex delay penalty function, the k-step LAR in Theorem 1 is equivalent to the one-step LAR.

Theorem 2: For a convex delay penalty function, the optimum stopping rule for (P2) is

$$T^*(\lambda) = \inf\left\{1 \le T \le D : U_T^1(\lambda) \ge 0\right\}, \quad (20)$$

where

$$U_T^1(\lambda) = \omega\sum_{t=1}^{T} N_t[f(T-t+2) - f(T-t+1)] + (E_b - \lambda)\mu + \omega\mu f(1). \quad (21)$$

Proof: The proof is in Appendix F. ■

The significance of Theorem 2 is that, for a system with convex delay penalty function, we only need to look one step ahead in the threshold test, and this will considerably reduce the computation complexity.

For the one-step LAR, at any time slot T, for $1 \le T \le D$, we only need to evaluate $U_T^1(\lambda)$ as shown in Theorem 2, and this has a fixed complexity. Thus the total complexity of one-step LAR scales linearly with D as $O(D)$.

For the k-step LAR, at any time slot T, for $1 \le T \le D$, we need to evaluate $U_T^k(\lambda)$, for $1 \le k \le D–T$ as shown in Theorem 1. Thus the total complexity of k-step LAR scales with $\sum_{T=1}^{D}(D-T) = \frac{D(D-1)}{2}$, or $O(D^2)$.

Therefore, k-step LAR has quadratic complexity, and one-step LAR has linear complexity.

In the following two sections, we will develop the optimum stopping rules that can solve (P1) for systems with linear or nonlinear delay penalty functions, respectively.

IV. Optimum Stopping Rule for Linear Delay Penalty Function

The optimum stopping rule that can solve (P1) for systems with linear delay penalty function is studied in this section.

For a system with a linear delay penalty function, the optimum stopping rule that can solve (P2) can be further simplified, and the result is given in the following Corollary.

Corollary 1: For a linear delay penalty function, $f(\tau) = \alpha\tau$, where $\alpha > 0$ is a constant, the optimum stopping rule of (P2) is

$$T^*(\lambda) = \min\left\{1 \le T \le D : \sum_{t=1}^{T} N_t \ge \frac{(\lambda - E_b)\mu}{\omega\alpha} - \mu\right\}. \quad (22)$$

Proof: The proof is in Appendix G. ■

For the optimum stopping rule given in (22), at each slot T, the server compares the accumulated number of information in the queue to a constant threshold and it will stop waiting and start pushing as soon as the accumulated number of information in the queue exceeds the threshold. The stopping time $T^*(\lambda)$ is a random variable, the distribution of which depends on the stopping rule (22) and the distribution of $N^{1:t}$.

If N_t follows i.i.d. Poisson distribution [25]–[27] with mean μ, we can obtain the probability mass function (PMF) of the optimum stopping time $T^*(\lambda)$ for the stopping rule given in Corollary 1.

Corollary 2: Assume N_t follows i.i.d. Poisson distribution with mean μ. For a linear delay penalty function $f(\tau) = \alpha\tau$ and stopping rule given in Corollary 1, the PMF of the optimum stopping time $T^*(\lambda)$ is

$$\begin{aligned} \Pr(T^*(\lambda) = 1) &= \Lambda(C, \mu), \\ \Pr(T^*(\lambda) = T) &= \Lambda(C, \mu T) - \Lambda(C, \mu(T-1)), \\ &\quad 1 < T < D, \\ \Pr(T^*(\lambda) = D) &= 1 - \Lambda(C, \mu(D-1)), \end{aligned}$$

where $C = \frac{(\lambda - E_b)\mu}{\omega\alpha} - \mu$, and

$$\Lambda(x,\ \mu) = \begin{cases} 1 - e^{-\mu} \sum_{i=0}^{\lfloor x \rfloor} \frac{\mu^i}{i!}, & x \geq 0 \\ 1, & x < 0 \end{cases} \tag{23}$$

is the complementary cumulative distribution function (CCDF) of the Poisson distribution with parameter μ.

Proof: The proof is in Appendix H. ∎

By using the results in Corollary 2, we can calculate the optimum surrogate objective function $V^*(\lambda)$, and the result is given in the following theorem.

Theorem 3: Assume N_t follows i.i.d. Poisson distribution with mean μ. For a linear delay penalty function $f(\tau) = \alpha\tau$ and stopping rule given in Corollary 1, the minimum surrogate objective function is

$$V^*(\lambda) = E_0 + \left[(E_b - \lambda)\mu + \frac{\omega\mu\alpha}{2}\right] m_1(\lambda) + \frac{\omega\mu\alpha}{2} m_2(\lambda), \tag{24}$$

where

$$m_1(\lambda) = D - \sum_{T=1}^{D-1} \Lambda\left(\frac{(\lambda - E_b)\mu}{\omega\alpha} - \mu, \mu T\right), \tag{25}$$

$$m_2(\lambda) = D^2 - \sum_{T=1}^{D-1} (2T+1) \Lambda\left(\frac{(\lambda - E_b)\mu}{\omega\alpha} - \mu, \mu T\right). \tag{26}$$

Proof: The proof is in Appendix I. ∎

Theorem 3 gives the minimum objective function associated with (P2), which is expressed as a function of λ. In order to solve (P1), we first solve $V^*(\lambda) = 0$ by using the result in Theorem 3, and denote the result as λ^*. With the analytical expression of $V^*(\lambda)$ given in Theorem 3, the value of λ^* can be easily solved by using numerical methods such as Bisection search or Newton's method, etc. Then the optimum stopping rule for (P1) can be obtained by replacing λ with λ^* in (22). The minimum objective function associated with (P1) is λ^*.

The results in Corollary 2 and Theorem 3 are developed based on the assumption of Poisson information arrival. For more general distributions on information arrival, we can always solve the problem by using the method described in the next section.

V. Optimum Stopping Rule for Nonlinear Delay Penalty Functions

In this section, we study optimum stopping rules for systems with general delay penalty functions, including both the exponential and logarithmic delay penalty functions. The stopping rule developed in this section can also be applied to the general forms of delay penalty functions, including the linear penalty function as a special case.

In order to solve (P1), we need to identify the optimum objective function $V^*(\lambda) = \mathbb{E}[V_{T^*(\lambda)}(\lambda)]$ and solve $V^*(\lambda^*) = 0$, where the expectation is performed over both $T^*(\lambda)$ and $N^{1:T}$. It is in general difficult to obtain the analytical expression of $V^*(\lambda)$ for systems with nonlinear delay penalty functions. Thus we need to exploit the properties of $V^*(\lambda)$ and solve λ^* numerically.

Based on Lemma 4, $V^*(\lambda)$ is decreasing and concave in λ. We will utilize this property to develop an iterative numeric method to solve λ^*, which is described in details as follows.

For a given stopping time T, the conditional expected objective function is

$$\mathbb{E}_{N^{1:T}}[V_T(\lambda) | T] = E_0 + \omega\mu \sum_{t=1}^{T} f(T - t + 1) + (E_b - \lambda)\mu T. \tag{27}$$

For a given $\lambda = \lambda_n$, denote the optimum stopping rule as $\phi^*(\lambda_n)$, and the corresponding optimum stopping time as $T^*(\lambda_n)$. The optimum objective function can then be calculated as

$$\begin{aligned} V_n^*(\lambda) &= \mathbb{E}\Big[V_{T^*(\lambda_n)}(\lambda)\Big] \\ &= \sum_{T=1}^{D} \mathbb{E}_{N^{1:T}}[V_T(\lambda) | T] \Pr(T^*(\lambda_n) = T). \end{aligned} \tag{28}$$

Combining (27) and (28) yields

$$\begin{aligned} V_n^*(\lambda) = &\sum_{T=1}^{D} \left[E_b \mu T + \omega\mu \sum_{t=1}^{T} f(T - t + 1)\right] \\ &\times \Pr(T^*(\lambda_n) = T) \\ &+ E_0 - \lambda\mu \sum_{T=1}^{D} T \Pr(T^*(\lambda_n) = T). \end{aligned} \tag{29}$$

We propose to identify λ^* by using the Newton's method [18, Ch. 6.1] as

$$\begin{aligned} \lambda_{n+1} &= \lambda_n - \frac{V_n^*(\lambda_n)}{V_n'^*(\lambda_n)} \\ &= \frac{E_0 + \sum_{T=1}^{D} \Big[E_b \mu T + \omega\mu \sum_{t=1}^{T} f(T-t+1)\Big] \Pr(T^*(\lambda_n) = T)}{\mu \sum_{T=1}^{D} T \Pr(T^*(\lambda_n) = T)}, \end{aligned} \tag{30}$$

where $V_n'^*(\lambda)$ is the first derivative of $V_n^*(\lambda)$ with respect to λ. The above iteration in Newton's method can be initiated with any $\lambda_1 > 0$, and the iteration has quadratic convergence to the solution λ^*. Simulation results demonstrate that the iterative algorithm converges in about 2 iterations under most system configurations.

In order to implement Newton's iterative procedure in (30), we need to find the PMF of the optimum stopping time $T^*(\lambda_n)$, which is difficult to evaluate analytically. Thus we propose to evaluate the PMF $\Pr(T^*(\lambda_n) = T)$, for $T = 1, \ldots, D$, by using numerical simulations of the threshold test described in Theorem 1. For each λ_n, we will perform simulation of the threshold test in Theorem 1 to obtain $\Pr(T^*(\lambda_n) = T)$, and the result is then used in (30) to get λ_{n+1}. It should be noted that the simulations only need to be performed once offline for the calculation of λ^*. Once λ^* is identified, we can directly replace λ in (18) with λ^* to obtain the optimum stopping rule for (P1).

Algorithm 2 summarizes the algorithm for computing λ^*, i.e., the minimum objective associated with (P1). Using the value of λ^*, the optimum stopping rule for (P1) can be obtained by replacing λ with λ^* in Theorem 1.

The above algorithm is applicable to systems with arbitrary delay penalty functions, including both the exponential and logarithmic delay penalty functions as special cases. Specifically, the exponential delay penalty function is convex, thus the stopping rule can be further simplified.

Corollary 3: For an exponential delay penalty function, $f(\tau) = \exp(\alpha\tau) - 1$, where $\alpha > 0$ is a constant, the optimum stopping rule of (P2) is

$$T^*(\lambda) = \min\left\{1 \le T \le D\colon \sum_{t=1}^{T} N_t e^{\alpha(T+1-t)} \ge \frac{(\lambda - E_b)\mu}{\omega(e^{\alpha}-1)} - \mu\right\}. \tag{31}$$

Proof: The proof is in Appendix J. ∎

Due to the complex nature of the threshold test in (31), it is hard to get the PMF of the optimum stopping time $T^*(\lambda)$ analytically. So, we need to numerically evaluate $T^*(\lambda)$ and search for λ^* using Algorithm 2. The optimum stopping rule for (P1) can then be obtained by replacing λ with λ^* in (31).

Algorithm 2 Algorithm for Computing λ^*

1: Initialize the parameters $\lambda_1 > 0$, error tolerance $\kappa > 0$, and a large integer $L > 0$.
2: **for** $i = 1$ to L **do**
3:   Generate a random number of information bits N_t for $t = 1, 2, \ldots, D$.
4:   Find $T^*(\lambda_n)$ based on $[N_1, N_2, \ldots, N_D]$ using (18).
5: **end for**
6: Compute the numerical PMF of $T^*(\lambda_n)$, i.e., $\Pr(T^*(\lambda_n) = T)$ for $T = 1, 2, \ldots, D$.
7: Compute λ_{n+1} by using (30).
8: **if** $\frac{|\lambda_{n+1} - \lambda_n|}{\lambda_n} > \kappa$ **then**
9:   $n = n + 1$.
10:   Repeat steps 2-7.
11: **else**
12:   $\lambda^* = \lambda_{n+1}$.
13:   Return.
14: **end if**

VI. Optimum Stopping Rule for Time-Normalized Metrics

In all previous discussions, the metrics E_ϕ and A_ϕ are obtained by normalizing with respect to the average number of bits. In this section, we consider the design with time-normalized metrics. It will be shown that replacing bit-normalized metrics with time-normalized ones will not affect the energy-delay tradeoff.

Define the new metrics normalized with respect to the average time as

$$A'_\phi = \frac{\mathbb{E}\left[\sum_{t=1}^{T} N_t f(T-t+1)\right]}{\mathbb{E}[T]}, \tag{32}$$

$$E'_\phi = \frac{\mathbb{E}\left[E_0 + E_b \sum_{t=1}^{T} N_t\right]}{\mathbb{E}[T]}, \tag{33}$$

where T is the stopping time obtained by the stopping rule ϕ.

Based on the Wald equation [18, Ch. 3.3], we have $\mathbb{E}[\sum_{t=1}^{T} N_t] = \mathbb{E}[T]\mu$. Thus it can be easily shown that $A'_\phi = \mu A_\phi$ and $E'_\phi = \mu E_\phi$. This result indicates that the metrics normalized with respect to time are just scaled versions of the metrics normalized with respect to the number of bits. Since the scaling factors are the same for both A'_ϕ and E'_ϕ, the tradeoff relationship between (E'_ϕ, A'_ϕ) are exactly the same as that for (E_ϕ, A_ϕ) under any feasible stopping rule $\phi \in \mathcal{S}$.

The alternative optimization problem with the new metrics can be formulated as

$$\text{(P3)} \qquad \text{minimize}_{\phi \in \mathcal{S}} \qquad E'_\phi + \omega A'_\phi,$$

and the surrogate objective function can be expressed as

$$V'_T(\lambda) = E_0 + E_b \sum_{t=1}^{T} N_t + \omega \sum_{t=1}^{T} N_t f(T-t+1) - \lambda T. \tag{34}$$

Comparing (34) with (12) reveals that the only difference is the last term, where $\lambda \sum_{t=1}^{T} N_t$ is replaced by λT. Following similar procedures as for the bit-normalized metrics, we can show that the optimum stopping rules for (P3) are quite similar to those with respect to (P1) and (P2), with the only difference being that the $\lambda\mu$ terms in the original stopping rules are replaced by λ. This change is applicable to almost all the results, including Lemmas 2–4, Theorems 1–3, and Corollaries 1, 2, and 3.

The only exception to the above simple replacement is the numerical iterative calculation of λ^* for system with general delay penalties. For a given stopping time T, the conditional expected objective function becomes

$$\mathbb{E}_{N^{1:T}}\left[V'_T(\lambda) | T\right] = E_0 + E_b\mu + \omega\mu \sum_{t=1}^{T} f(T-t+1) - \lambda T. \tag{35}$$

For a given $\lambda = \lambda_n$, denote the optimum stopping rule as $\phi^*(\lambda_n)$, and the corresponding optimum stopping time as $T^*(\lambda_n)$. The optimum objective function can then be calculated as

$$\begin{aligned} V_n^*(\lambda) &= \mathbb{E}\Big[V_{T^*(\lambda_n)}(\lambda)\Big] \\ &= \sum_{T=1}^{D} \mathbb{E}_{N^{1:T}}[V_T'(\lambda)|T]\Pr(T^*(\lambda_n) = T) \\ &= \sum_{T=1}^{D}\left[E_b\mu T + \omega\mu\sum_{t=1}^{T} f(T-t+1)\right] \\ &\quad \times \Pr(T^*(\lambda_n) = T) \\ &\quad + E_0 - \lambda\sum_{T=1}^{D} T\Pr(T^*(\lambda_n) = T). \end{aligned} \tag{36}$$

Using the iterative approach mentioned in Section V yields

$$\lambda_{n+1} = \frac{E_0 + \sum_{T=1}^{D}\Big[E_b\mu T + \omega\mu\sum_{t=1}^{T} f(T-t+1)\Big]\Pr(T^*(\lambda_n) = T)}{\sum_{T=1}^{D} T\Pr(T^*(\lambda_n) = T)}, \tag{37}$$

where λ_n converges to λ^*, as $n \to \infty$. The difference between the iterative solutions of λ^* for the original problem and the new problem is the μ term in the denominator as observed from (30) and (37).

Once λ^* is obtained, following the same two-step solution approach as mentioned in Section II, new stopping rules can be developed regarding the new metrics. Simulation results verify that the two different groups of metrics yield the same energy-delay tradeoff.

VII. Simulation Results

Simulation results are presented in this section to demonstrate the performance of the optimum stopping rule for information pushing systems. In the simulation, we set the duration of each time slot $T_0 = 1$ ms, $R = 1$ Mbps, $E_b = 1$ μJ and $E_0 = 10$ μJ [28], [29]. The average information arrival rate is set at $\mu/T_0 = 1$ kbps, the delay penalty parameter is $\alpha = 0.1$, and the relative tolerance value used in the iterative solution of λ^* is $\kappa = 10^{-3}$. With the above configuration, we have $\frac{\mu}{RT_0} = 0.001 \ll 1$. In addition, we consider various values of the hard deadline $D \leq 25$, which always satisfy the mean transmission delay constraint in (5) with $\Delta = 0.975$. Based on Lemma 1, we have $\Pr(\tau_{\text{Tx}} > 1) < 6.84 \times 10^{-4}$, thus the event $\{\tau_{\text{Tx}} > 1\}$ has negligible probability. All simulation results are obtained by averaging over 10,000 Monte-Carlo trials.

Fig. 3 shows the optimum objective function as a function of the hard deadline D, under the values of $\omega = 0.5 \times 10^{-6}$ and $\omega = 1.5 \times 10^{-6}$, respectively. Since the delay penalties do not have any unit, the objective functions are unitless. Under all system configurations, the optimum objective functions converge to constant values when D is sufficiently large. The convergence is due to the fact that when D is sufficiently large, the pushing action is in general executed before the hard deadline, thus the performance is no longer affected by D. For a given value of D, the optimum objective function increases with ω. It is worth mentioning that a smaller optimum objective function does not necessarily mean a better E_{ϕ^*}-A_{ϕ^*} tradeoff due to the impacts of ω. Under the same value of ω, the system with the exponential delay penalty function has the largest objective function, followed by the linear and logarithmic penalties, respectively. In addition, it is beneficial to use a larger hard deadline for systems with a smaller ω. For example, in the case of linear delay penalty function, when $\omega = 1.5 \times 10^{-6}$ and 0.5×10^{-6}, the optimum objective functions reach their minimum values at $D = 11$ and 17, respectively. This is due to the fact that a smaller ω will put less weight on delay, hence favoring longer delays.

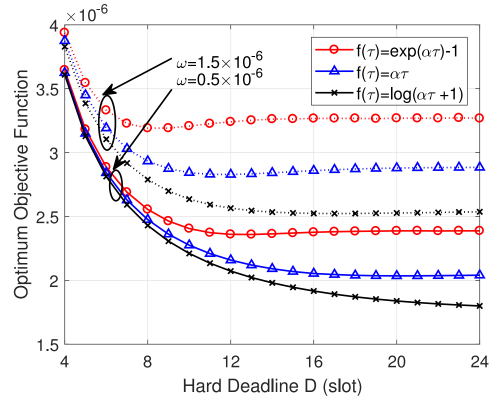


Fig. 3. Optimum objective function as a function of hard deadline D.

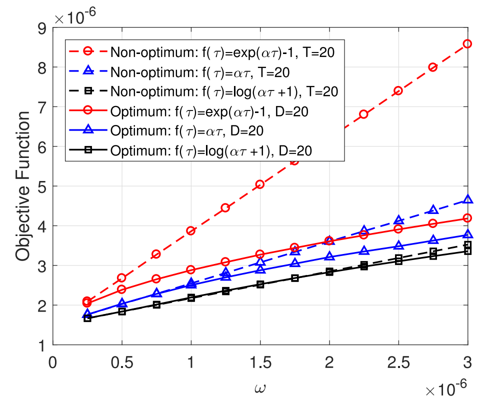


Fig. 4. Objective function as a function of the tuning parameter ω.

Fig. 4 shows the optimum objective function as a function of the tuning parameter ω, under various values of D and delay penalties. For comparison, we also plot the objective functions obtained from a non-optimum approach that always stops at a deterministic value $T = 20$ regardless of the system status. Since the stopping time of the non-optimum approach is deterministic, no computation is needed for finding the stopping rule or stopping time. In the simulations, the hard deadline for the optimum rule is set as $D = 20$. As expected, the objective functions obtained from the non-optimum approaches are worse compared to their optimum counterpart. The non-optimum objective functions grow linearly with ω, yet the optimum ones grow in a sub-linear fashion.

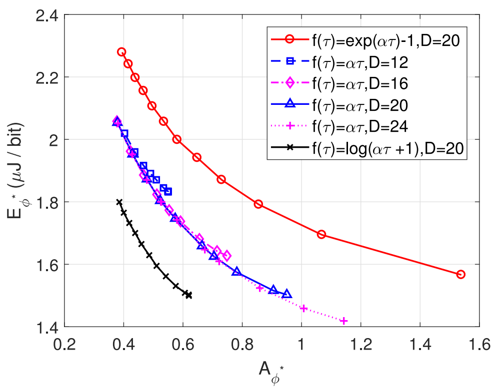
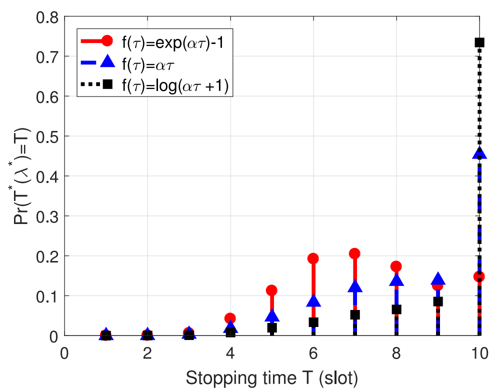
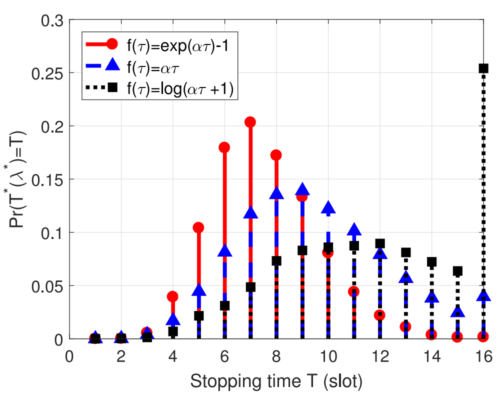


Fig. 5. Energy efficiency vs delay penalty tradeoff. The tuning parameter $\omega \in (0.2 \times 10^{-6}, 2.6 \times 10^{-6})$.



(a) $D = 10$



(b) $D = 16$

Fig. 6. PMF for different deadlines D and penalty functions. The tuning parameter $\omega = 2.5 \times 10^{-6}$.

Fig. 5 shows the optimum E_{ϕ^*} as a function of A_{ϕ^*}. Different tradeoff points between E_{ϕ^*} and A_{ϕ^*} on a single curve are obtained by adjusting the tuning parameter ω. As expected, E_{ϕ^*} decreases monotonically with A_{ϕ^*} due to their tradeoff relationship. As ω increases, the optimum A_{ϕ^*} decreases under the same E_{ϕ^*}, because more weight is put on the delay during the optimization process. Under the same E_{ϕ^*}, the system with the logarithmic delay penalty function has the smallest A_{ϕ^*}, followed by the systems with linear and exponential penalties, respectively. We also obtained the tradeoff curves of E'_{ϕ^*} and A'_{ϕ^*} for systems with time-normalized metrics. The curves overlap with those from systems with bit-normalized metrics as predicted in Section VI, and they are not shown here for clarity of presentation.

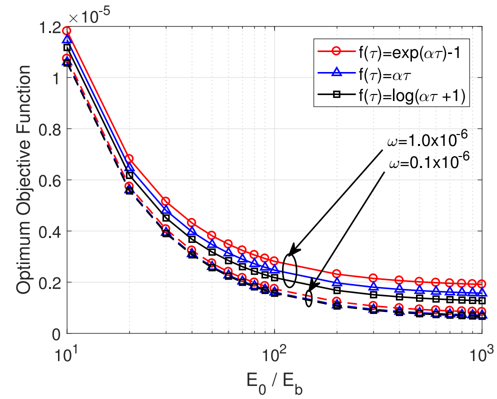


Fig. 7. Minimum objective vs $\frac{E_0}{E_b}$. The hard deadline $D = 20$.

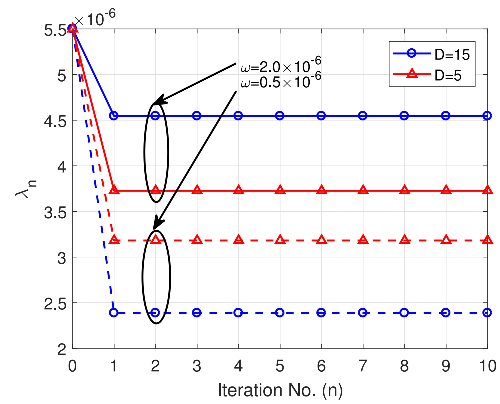


Fig. 8. Convergence of λ_n for a system with exponential delay penalty.

To demonstrate the impacts of D on the energy-delay tradeoff, we also show the results for systems with linear penalty under different values of D. Increasing D results in better energy-delay tradeoff. The benefits of using a larger D gradually diminishes as D is sufficiently large. The energy-delay tradeoff for systems with $D = 20$ and 24 are very close to each other. Similar results are observed for systems with exponential and logarithmic delay penalties.

To better understand the behaviors of the optimum stopping rule, Fig. 6 shows the PMF of the stopping time T for different deadlines D and delay penalty functions. As can be seen in the figure, when T is much smaller than the hard deadline D (e.g., $T = 8$ and $D = 16$), for a given T, the system with the exponential delay penalty has the largest probability of stopping, followed by the systems with linear and logarithmic penalties, respectively. This trend is reversed when T is closer to D. Specifically, the system with the logarithmic penalty always has the highest probability of stopping at $T = D$. This is because the logarithmic penalty grows sub-linearly with T, thus the system can tolerate a longer delay. On the other hand, for systems with the exponential penalty, the probability of stopping at larger T is very small.

Fig. 7 shows the optimum objective function as a function E_0/E_b. It can be seen from the results that the minimum objective is a monotonic decreasing function in E_0/E_b, and it converges to a constant value when E_0/E_b is large.

The convergence of the numerical iterative algorithm described in Algorithm 2 is demonstrated in Fig. 8 for a system with the exponential delay penalty function. The algorithm is used to identify λ^* by iteratively updating λ_n with (30). The numerical algorithm converges in 2 iterations for all cases considered in this example. Thus the iterative algorithm is very efficient.

VIII. Conclusion

We have studied the optimum stopping rules for information pushing systems with different types of delay penalty functions. The analysis was performed under the assumption that the information arrival rate is much less than the information transmission rate, such that the transmission delay is less than one slot with high probability. The design was performed to identify the Pareto-optimum tradeoff between energy and delay penalty. For systems with convex delay penalty functions (such as linear and exponential functions), the optimum delay-energy tradeoff can be achieved by using the one-step look ahead stopping rule. For systems with linear delay penalty function, the optimum objective function has been analytically derived by studying the statistical properties of the optimum stopping time, and the results have been used to identify the operation parameters of the optimum stopping rule. For systems with arbitrary delay penalty functions, the optimum objective function and the operation parameter of the stopping rule are evaluated numerically using an iterative approach. The developed algorithms have enabled flexible control between the energy efficiency and delay under various system configurations.

Appendix A
Proof of Lemma 1

To bound the probability of τ_{Tx}, we will first derive the variance of τ_{Tx}, which can be calculated by evaluating the following second moment.

$$\mathbb{E}\left[\left(\sum_{t=1}^{T} N_t\right)^2\right] = \sum_{n=1}^{D}\sum_{i=1}^{T}\sum_{j=1}^{T}\mathbb{E}\big[\mathbf{1}(T=n)N_iN_j\big] \quad (38a)$$

$$= \sum_{i=1}^{D}\sum_{j=1}^{D}\sum_{n=\max(i,j)}^{D}\mathbb{E}\big[\mathbf{1}(T=n)N_iN_j\big] \quad (38b)$$

$$= \sum_{i=1}^{D}\sum_{j=1}^{D}\Pr(T\ge \max(i,j))\mathbb{E}\big[N_iN_j\big] \quad (38c)$$

$$= \sum_{i=1}^{D}\Pr(T\ge i)\mathbb{E}\left[N_i^2\right] + \sum_{\substack{i,j=1\\ i\neq j}}^{D}\Pr(T\ge \max(i,j))\mathbb{E}[N_i]\mathbb{E}\big[N_j\big] \quad (38d)$$

$$= \left(\sigma_N^2+\mu^2\right)\mathbb{E}[T] + \mu^2\sum_{\substack{i,j=1\\ i\neq j}}^{D}\Pr(T\ge \max(i,j)), \quad (38e)$$

where $\mathbf{1}(\mathcal{E})$ is the indicator function with $\mathbf{1}(\mathcal{E})=1$ if the event $\mathcal{E}$ is true and 0 otherwise, (38c) is based on the fact that T is a stopping time, thus $T\ge\max(i,j)$ is independent of N_i or N_j, (38e) is based on the fact that $\sum_{i=1}^{D}\Pr(T\ge i) = \sum_{i=1}^{D}\sum_{k=i}^{D}\Pr(T=k) = \sum_{k=1}^{D}\sum_{i=1}^{k}\Pr(T=k) = \mathbb{E}[T]$.

Next, we evaluate $\sum_{\substack{i,j=1\\ i\neq j}}^{D}\Pr(T\ge\max(i,j))$, which can be alternatively written as

$$\sum_{\substack{i,j=1\\ i\neq j}}^{D}\Pr(T\ge\max(i,j)) = \sum_{i=1}^{D}\sum_{j=1}^{i-1}\Pr(T\ge i) + \sum_{i=1}^{D}\sum_{j=i+1}^{D}\Pr(T\ge j) \quad (39a)$$

$$= \sum_{i=1}^{D}(i-1)\sum_{k=i}^{D}\Pr(T=k) + \sum_{j=1}^{D}\sum_{i=1}^{j-1}\Pr(T\ge j) \quad (39b)$$

$$= \sum_{k=1}^{D}\Pr(T=k)\sum_{i=1}^{k}(i-1) + \sum_{k=1}^{D}\Pr(T=k)\sum_{j=1}^{k}(j-1) \quad (39c)$$

$$= \mathbb{E}[T(T-1)]. \quad (39d)$$

Thus $\mathbb{E}[(\sum_{t=1}^{T}N_t)^2] = \sigma_N^2\mathbb{E}[T] + \mu^2\mathbb{E}[T^2]$. As a result, we get

$$\text{Var}\left(\sum_{t=1}^{T}N_t\right) = \sigma_N^2\mathbb{E}[T] + \mu^2\left(\mathbb{E}[T^2]-\mathbb{E}^2[T]\right) = \sigma_N^2\mathbb{E}[T]+\mu^2\sigma_T^2, \quad (40)$$

where σ_T^2 is the variance of T. Since T is bounded between 0 and D, $\sigma_T^2\le D^2$.

Under the constraint (5), we have $\mathbb{E}[\tau_{\text{Tx}}]\le\frac{D\mu}{RT_0}<1-\Delta$, thus the event $\{\tau_{\text{Tx}}>1\}$ implies $\{\tau_{\text{Tx}}>\mathbb{E}[\tau_{\text{Tx}}]+\Delta\}$. Consequently,

$$\Pr(\tau_{\text{Tx}}>1) < \Pr(\tau_{\text{Tx}}-\mathbb{E}[\tau_{\text{Tx}}]>\Delta). \quad (41)$$

Based on Chebyshev's inequality, we have

$$\Pr(\tau_{\text{Tx}}-\mathbb{E}[\tau_{\text{Tx}}]>\Delta) \le \frac{\text{Var}(\tau_{\text{Tx}})}{\Delta^2} \le \frac{\sigma_N^2 D+\mu^2D^2}{R^2T_0^2\Delta^2}. \quad (42)$$

Specifically, if N_t follows Poisson distribution, then $\sigma_N^2=\mu$. This completes the proof.

Appendix B
Proof of Lemma 2

Let (E_{ϕ_1}, A_{ϕ_1}) and (E_{ϕ_2}, A_{ϕ_2}) denote two achievable energy-delay pairs, which are achieved by stopping rules ϕ_1 and ϕ_2, respectively. In the following, we prove that for an arbitrary $0\le\theta\le1$, the energy-delay pair $\theta(E_{\phi_1}, A_{\phi_1})+(1-\theta)(E_{\phi_2}, A_{\phi_2})$ is also achievable by a certain stopping rule.

First, we examine $\theta E_{\phi_1}+(1-\theta)E_{\phi_2}$. Assume that we perform $L=L_1+L_2$ independent trials with the two stopping rules. Among them, stopping rule ϕ_1 is applied L_1 times, and stopping rule ϕ_2 is applied L_2 times. Denote T_{ln} as the stopping time associated with stopping rule ϕ_n during the l-th trial, and $N_{l,t}$ as the number of bits arriving at the t-th slot during the l-th trial, for $l = 1, \ldots, L_n$, and $n = 1$ or 2.

The total energy consumption due to stopping rule ϕ_n is thus

$$\Phi_n = \sum_{l=1}^{L_n} \left[E_0 + E_b \sum_{t=1}^{T_{ln}} \right], \quad n = 1, 2. \tag{43}$$

The total number of bits transmitted with stopping rule ϕ_n is

$$\Pi_n = \sum_{l=1}^{L_1} \sum_{t=1}^{T_{l1}} N_{l,t} \ , \quad n = 1, 2. \tag{44}$$

We have

$$E_{\phi n} = \lim_{L_n \to \infty} \frac{\Phi_n}{\Pi_n}, \quad n = 1, 2. \tag{45}$$

The average energy consumption per bit due to the mixture of stopping rules ϕ_1 and ϕ_2 can then be calculated as

$$E_\phi = \frac{\Phi_1 + \Phi_2}{\Pi_1 + \Pi_2} = \frac{\Pi_1}{\Pi_1 + \Pi_2} \frac{\Phi_1}{\Pi_1} + \frac{\Pi_1}{\Pi_1 + \Pi_2} \frac{\Phi_2}{\Pi_2}. \tag{46}$$

As both $L_1 \to \infty$ and $L_2 \to \infty$, we have

$$E_\phi = \left(\lim_{L_1, L_2 \to \infty} \frac{\Pi_1}{\Pi_1 + \Pi_2} \right) E_{\phi_1} + \left(\lim_{L_1, L_2 \to \infty} \frac{\Pi_2}{\Pi_1 + \Pi_2} \right) E_{\phi_2}. \tag{47}$$

If we let

$$\theta = \lim_{L_1, L_2 \to \infty} \frac{\Pi_1}{\Pi_1 + \Pi_2} = \lim_{L_1, L_2 \to \infty} \frac{\sum_{l=1}^{L_1} \sum_{t=1}^{T_{l1}} N_{l,t}}{\sum_{l=1}^{L_1} \sum_{t=1}^{T_{l1}} N_{l,t} + \sum_{l=1}^{L_2} \sum_{t=1}^{T_{l2}} N_{l,t}}, \tag{48}$$

then

$$E_\phi = \theta E_{\phi_1} + (1 - \theta) E_{\phi_2} \tag{49}$$

is achievable, and it can be achieved by a new stopping strategy ϕ that uses $\frac{L_1}{L_1 + L_2} \times 100\%$ of ϕ_1 and $\frac{L_2}{L_1 + L_2} \times 100\%$ of ϕ_2 among all trials.

Specifically, define $\beta = \frac{L_1}{L_1 + L_2}$ and keep it as a constant as we change L_1 and L_2, then (48) can be alternatively written as

$$\theta = \lim_{L_1, L_2 \to \infty} \frac{\beta \frac{1}{L_1} \sum_{l=1}^{L_1} \sum_{t=1}^{T_{l1}} N_{l,t}}{\beta \frac{1}{L_1} \sum_{l=1}^{L_1} \sum_{t=1}^{T_{l1}} N_{l,t} + (1 - \beta) \frac{1}{L_2} \sum_{l=1}^{L_2} \sum_{t=1}^{T_{l2}} N_{l,t}}. \tag{50}$$

Based on the law of large numbers, we can write that

$$\theta = \frac{\beta \mathbb{E}\left[\sum_{t=1}^{T_1} N_t \right]}{\beta \mathbb{E}\left[\sum_{t=1}^{T_1} N_t \right] + (1 - \beta) \mathbb{E}\left[\sum_{t=1}^{T_2} N_t \right]}, \tag{51}$$

which implies

$$\beta = \frac{\theta \mathbb{E}\left[\sum_{t=1}^{T_2} N_t \right]}{(1 - \theta) \mathbb{E}\left[\sum_{t=1}^{T_1} N_t \right] + \theta \mathbb{E}\left[\sum_{t=1}^{T_2} N_t \right]}. \tag{52}$$

Thus for any $0 \le \theta \le 1$, we can choose L_1 and L_2 such that $\beta = \frac{L_1}{L_1 + L_2}$ satisfies (52). We can obtain a new stopping rule by mixing $\beta \times 100\%$ of θ_1 with $(1 - \beta) \times 100\%$ of θ_2. The normalized energy of the new stopping rule is $E_\phi = \theta E_{\phi_1} + (1 - \theta) E_{\phi_2}$, which is achievable. Thus E_ϕ is convex.

The convexity of A_ϕ can be proved in a similar manner. As a result, for any $(E_{\phi_n}, A_{\phi_n}) \in \mathcal{R}$ with $n = 1$, 2, we have $\theta(E_{\phi_1}, A_{\phi_1}) + (1 - \theta)(E_{\phi_2}, A_{\phi_2}) \in \mathcal{R}$ for any $0 \le \theta \le 1$. Thus $\mathcal{R}$ is convex.

Appendix C
Proof of Lemma 3

Denote the optimum stopping rule for (P1) as ϕ_1, and the optimum stopping rule for (P2) when $\lambda = \lambda^*$ as ϕ_2. We will show that $\phi_1 = \phi_2$.

Proof by contradiction. Assume $\phi_1 \neq \phi_2$. Since ϕ_2 is the solution to (P2), we have

$$0 = V_{\phi_2}(\lambda^*) \le V_{\phi_1}(\lambda^*). \tag{53}$$

Denote the solution to the equation $V_{\phi_1}(\lambda) = 0$ as $\lambda^\dagger$. Then

$$V_{\phi_1}(\lambda^*) \ge 0 = V_{\phi_1}\left(\lambda^\dagger\right). \tag{54}$$

For any $\phi \in \mathcal{S}$, it is apparent that $V_\phi(\lambda)$ is a decreasing function in λ. Thus from (54), we have

$$\lambda^* \le \lambda^\dagger. \tag{55}$$

Based on the fact that $V_{\phi_2}(\lambda^*) = 0$ and $V_{\phi_1}(\lambda^\dagger) = 0$, we have

$$\lambda^* = \frac{C_{\phi_2}}{D_{\phi_2}} = E_{\phi_2} + \omega A_{\phi_2}, \tag{56}$$

$$\lambda^\dagger = \frac{C_{\phi_1}}{D_{\phi_1}} = E_{\phi_1} + \omega A_{\phi_1}. \tag{57}$$

Combining (55) with (57) yields

$$E_{\phi_2} + \omega A_{\phi_2} \le E_{\phi_1} + \omega A_{\phi_1}. \tag{58}$$

This contradicts with the fact that ϕ_1 is the optimum solution to (P1) thus it minimizes $E_\phi + \omega A_\phi$.

Thus ϕ_1 is the same as ϕ_2, and (17) follows from (57).

Appendix D
Proof of Lemma 4

Denote the optimum stopping rule with respect to (P2) as $\phi^*(\lambda)$. Then using the definitions in (13) and (16) we can write

$$V^*(\lambda) = \inf_{\phi \in \mathcal{S}} V_\phi(\lambda) = C_{\phi^*(\lambda)} - \lambda D_{\phi^*(\lambda)}.$$

Let $\lambda_1 < \lambda_2$. Then,

$$\begin{aligned} V^*(\lambda_1) &= C_{\phi^*(\lambda_1)} - \lambda_1 D_{\phi^*(\lambda_1)} \\ &> C_{\phi^*(\lambda_1)} - \lambda_2 D_{\phi^*(\lambda_1)} \\ &\ge C_{\phi^*(\lambda_2)} - \lambda_2 D_{\phi^*(\lambda_2)} = V^*(\lambda_2). \end{aligned}$$

So, $V^*(\lambda)$ is decreasing in λ.

To show concavity, let $0 < \theta < 1$ and $\lambda = \theta\lambda_1 + (1-\theta)\lambda_2$. Then,

$$\begin{aligned} V^*(\lambda) &= C_{\phi^*(\lambda)} - (\theta\lambda_1 + (1-\theta)\lambda_2) D_{\phi^*(\lambda)} \\ &= \theta\Big(C_{\phi^*(\lambda)} - \lambda_1 D_{\phi^*(\lambda)}\Big) \\ &\quad + (1-\theta)\Big(C_{\phi^*(\lambda)} - \lambda_2 D_{\phi^*(\lambda)}\Big) \\ &\geq \theta V^*(\lambda_1) + (1-\theta) V^*(\lambda_2). \end{aligned}$$

This completes the proof.

Appendix E
Proof of Theorem 1

The problem has a finite horizon due to the hard deadline *D*, that is, we must stop at time slot *D*. A finite horizon problem can be solved by using backward recursion, that is, since we must stop at slot *D*, we can first find the optimum stopping rule at slot *D*–1. Then, knowing the optimum rule at slot *D*–1, we can find the optimum rule at slot $D-2$, and so on back to the initial stage 1.

1) $T = D$. Since we must stop at slot *D*, the optimum stopping rule at $T = D$ is thus $\phi_D(\mathcal{F}_D) = 1$, and the optimum objective function associated with stopping at $T = D$ is $V_D^*(\lambda) = V_D(\lambda)$ as defined in (12).

2) $1 \leq T < D$. Assume the optimum stopping rule at $t = T+1$ is known, and we can thus find the corresponding optimum objective function $V_{T+1}^*(\lambda)$. We will find the optimum stopping rule at $t = T$. When $t = T$, if we stop at the current slot, the corresponding objective function will be $V_T(\lambda)$ as defined in (12), given that $N^{1:T}$ is known. If we do not stop at the current slot, we will execute the optimum stopping rule at $t =$ T+1, and the corresponding objective function will be $\mathbb{E}[V_{T+1}^*(\lambda)|\mathcal{F}_T]$. Thus the optimum stopping rule at $t = T$ is

$$\phi_T(\lambda) = \begin{cases} 1, \text{ if } V_T(\lambda) \leq \mathbb{E}\Big[V_{T+1}^*(\lambda)|\mathcal{F}_T\Big] \\ 0, \text{ if } V_T(\lambda) > \mathbb{E}\Big[V_{T+1}^*(\lambda)|\mathcal{F}_T\Big] \end{cases}, \ 1 \leq T < D. \tag{59}$$

The corresponding optimum objective function is

$$V_T^*(\lambda) = \min\big\{V_T(\lambda), \mathbb{E}\big[V_{T+1}^*(\lambda)|\mathcal{F}_T\big]\big\}. \tag{60}$$

Eqn. (60) gives a recursive definition of the optimum objective function at each slot $1 \leq T < D$, with $V_D^*(\lambda) = V_D(\lambda)$. Based on backward recursion, (60) can be alternatively written as

$$\begin{aligned} V_T^*(\lambda) &= \min\big\{V_T(\lambda), \mathbb{E}\big[V_{T+1}^*(\lambda)|\mathcal{F}_T\big]\big\} \\ &= \min\big\{V_T(\lambda), \mathbb{E}\big[\min\big\{V_{T+1}(\lambda), \mathbb{E}\big[V_{T+2}^*(\lambda)|\mathcal{F}_T\big]\big\}|\mathcal{F}_T\big]\big\} \\ &= \min\{V_T(\lambda), \mathbb{E}[V_{T+1}(\lambda)|\mathcal{F}_T], \ldots, \mathbb{E}[V_D(\lambda)|\mathcal{F}_T]\}. \end{aligned} \tag{61}$$

Combining (59) with (61) yields

$$\phi_T(\lambda) = \begin{cases} 1, & \text{if } V_T(\lambda) \leq \min_{1\leq k\leq D-T} \mathbb{E}[V_{T+k}(\lambda)|\mathcal{F}_T] \\ 0, & \text{if } V_T(\lambda) > \min_{1\leq k\leq D-T} \mathbb{E}[V_{T+k}(\lambda)|\mathcal{F}_T] \end{cases}, \tag{62}$$

for $1 \leq T < D$.

The optimum stopping rule in (62) is called the *k*-step look ahead rule (LAR). Conditioned on the knowledge of current system status at time slot *T*, the *k*-step LAR makes a decision by comparing the current objective function with the expected objective function of the next *k* steps, for $1 \leq k \leq D-T$. It will stop only if the current objective function is no greater than the expected objective functions of all future slots.

With the definition of $V_T(\lambda)$ in (12), we have

$$\begin{aligned} &\mathbb{E}[V_{T+k}(\lambda)|\mathcal{F}_T] \\ &\quad = E_0 + E_b \sum_{t=1}^{T} N_t + \omega \sum_{t=1}^{T} N_t f(T+k+1-t) - \lambda \sum_{t=1}^{T} N_t \\ &\qquad + \mathbb{E}\Bigg[(E_b - \lambda) \sum_{t=T+1}^{T+k} N_t \\ &\qquad\quad + \omega \sum_{t=T+1}^{T+k} N_t f(T+k+1-t) \Bigg| \mathcal{F}_T\Bigg] \\ &\quad = V_T(\lambda) + U_T^k(\lambda), \end{aligned} \tag{63}$$

where $U_T^k(\lambda) = \mathbb{E}[V_{T+k}(\lambda)|\mathcal{F}_T] - V_T(\lambda)$. Combining (12) with (63) yields the expression of $U_T^k(\lambda)$ in (19).

Based on the *k*-step LAR, we will stop at slot *T* if $\min_{1\leq k\leq D-T} U_T^k(\lambda) \geq 0$, or $\sum_{t=1}^{T} N_t \geq R$, and continue to the next slot otherwise. This completes the proof.

Appendix F
Proof of Theorem 2

Setting $k = 1$ in (19) yields (21). The difference between $U_T^{k+1}(\lambda)$ and $U_T^k(\lambda)$ can be calculated as

$$\begin{aligned} U_T^{k+1}(\lambda) - U_T^k(\lambda) &= (E_b - \lambda)\mu + \omega\mu f(k) \\ &\quad + \omega \sum_{t=1}^{T} N_t [f(T+k+2-t) - f(T+k+1-t)]. \end{aligned} \tag{64}$$

Combining (21) and (64) results in

$$\begin{aligned} U_T^{k+1}(\lambda) - U_T^k(\lambda) - U_T^1(\lambda) &= \omega\mu f(k) \\ &\quad + \omega \sum_{t=1}^{T} N_t [f(T+k+2-t) - f(T+k+1-t) \\ &\quad - \{f(T+2-t) - f(T+1-t)\}]. \end{aligned} \tag{65}$$

Now for any given value of *T*, *t*, and *k*, it is evident that $T+1-t < T+2-t \leq T+k+1-t < T+k+2-t$. Since the delay penalty function $f(\tau)$ is a convex and monotonically increasing function in τ, and both the first and second derivatives of such function is non-negative, it can be easily verified that

$$f(T+k+2-t) - f(T+k+1-t) \geq f(T+2-t) - f(T+1-t).$$

Consequently from (65) we get,

$$U_T^{k+1}(\lambda) - U_T^k(\lambda) - U_T^1(\lambda) > 0. \tag{66}$$

Next, we will prove that $U_T^{k+1}(\lambda) > (k+1) U_T^1(\lambda)$, for $k = 1, \ldots, D-T-1$, through mathematical induction.

When $k = 1$, from (66), we have $U_T^2(\lambda) - U_T^1(\lambda) - U_T^1(\lambda) > 0$, or, $U_T^2(\lambda) > 2\,U_T^1(\lambda)$.

Assume $U_T^{k+1}(\lambda) > (k+1)\,U_T^1(\lambda)$. At $k+2$, from (66), we have $U_T^{k+2}(\lambda) > U_T^{k+1}(\lambda) + U_T^1(\lambda) > (k+2)\,U_T^1(\lambda)$.

Thus $U_T^{k+1}(\lambda) > (k+1)\,U_T^1(\lambda)$. If $U_T^1(\lambda) \geq 0$, then $U_T^k(\lambda)$ is monotonically increasing in *k*, and

$$\min_{1 \leq k \leq D-T} U_T^k(\lambda) = U_T^1(\lambda). \tag{67}$$

On the other hand, if $U_T^1(\lambda) < 0$, then $\min_{1 \leq k \leq D-T} U_T^k(\lambda) < 0$, which does not satisfy the stopping condition described in Theorem 1. As a result, the server will not initiate the push operation at slot *T*. So, we only need to look one step ahead in the stopping rule in Theorem 1. Combining (18) with (67) completes the proof.

Appendix G
Proof of Corollary 1

Since a linear function is convex, the optimum stopping rule for (P2) can be derived by using the one-step LAR as in Theorem 2. For a linear delay penalty function, $f(\tau) = \alpha\tau$, (21) can be simplified to

$$U_T^1(\lambda) = (E_b - \lambda)\mu + \omega\alpha \sum_{t=1}^{T} N_t + \omega\alpha\mu. \tag{68}$$

From (68), the threshold test $U_T^1(\lambda) \geq 0$ is equivalent to that in (22).

Appendix H
Proof of Corollary 2

Define $Z_T = \sum_{t=1}^{T} N_t$. Since N_t are i.i.d. Poisson random variables with parameter μ, Z_T is a Poisson random variable with parameter $\mu\, T$. The probability that we stop at slot *T* or earlier, where $T < D$, is therefore

$$\Pr(T^*(\lambda) \leq T) = \Pr(Z_T > C) = \Lambda(C, \mu T),$$

where $C = \frac{(\lambda - E_b)\mu}{\omega\alpha} - \mu$. Since *D* is the hard deadline, we have $\Pr(T^*(\lambda) \leq D) = 1$. Combining the above results with $\Pr(T^*(\lambda) = T) = \Pr(T^*(\lambda) \leq T) - \Pr(T^*(\lambda) \leq T-1)$ completes the proof.

Appendix I
Proof of Theorem 3

If we stop at slot *T*, the conditional expected objective function can be calculated as

$$\mathbb{E}_{N^{1:T}}[V_T(\lambda)|T] = E_0 + (E_b - \lambda)\mu T + \omega\mu\alpha\frac{T(T+1)}{2}, \tag{69}$$

where the expectation is performed over $N^{1:T}$ for a fixed *T*. With the optimum stopping rule in Theorem 1, the minimum surrogate objective function can then be calculated as

$$V^*(\lambda) = \mathbb{E}\Big[V_{T^*(\lambda)}(\lambda)\Big] = \sum_{T=1}^{D} \mathbb{E}_{N^{1:T}}[V_T(\lambda)|T]\Pr(T^*(\lambda) = T). \tag{70}$$

Combining (69) and (70) yields (24), where

$$m_1(\lambda) = \mathbb{E}[T^*(\lambda)] = \sum_{T=1}^{D} T \Pr(T^*(\lambda) = T), \tag{71}$$

$$m_2(\lambda) = \mathbb{E}\Big[\{T^*(\lambda)\}^2\Big] = \sum_{T=1}^{D} T^2 \Pr(T^*(\lambda) = T), \tag{72}$$

are the first and second moments of the optimum stopping time $T^*(\lambda)$, respectively. The results in (25) and (26) is obtained by combining Corollary 2 with (71) and (72), respectively.

Appendix J
Proof of Corollary 3

Since the exponential function is convex, the optimum stopping rule for (P2) can be derived from the one-step LAR in Theorem 2. For an exponential delay penalty function, $f(\tau) = \exp(\alpha\tau) - 1$, (21) can be written as

$$U_T^1(\lambda) = \omega \sum_{t=1}^{T} N_t \Big[e^{\alpha(T+2-t)} - e^{\alpha(T+1-t)}\Big] + (E_b - \lambda)\mu + \omega\mu(e^{\alpha} - 1). \tag{73}$$

Substituting the expression of $U_T^1(\lambda)$ in (20) completes the proof.

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