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High-Power Frequency Combs from Periodic Waveforms in Kerr Microresonators

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Abstract: We show that nonlinear frequency pulling shifts the stability region of periodic waveforms in strongly driven Kerr microresonators to shorter periods. Consequently, optimized narrow-band combs are obtained by judiciously adjusting the period.
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1. Introduction

In recent years, frequency comb sources based on Kerr microresonators have become ubiquitous. While considerable effort has been dedicated [1] to generation of bright solitons in cavities with anomalous dispersion, several other types of comb-producing waveforms have been generated and studied in Kerr resonators, including dark solitons [2], soliton crystals, and periodic patterns [3, 4].

In this work we present a theoretical study of periodic waveforms in Kerr cavities, also known as Turing rolls [3] and cnoidal waves [4], from the point of view of pattern formation. These are arguably the simplest comb-forming resonator waveforms, that arise spontaneously when external driving is increased beyond the modulational instability threshold, and as such become a stepping stone toward the generation of other waveforms like solitons.

Moreover, even though the spectral bandwidth of periodic waveforms is typically smaller than that of solitons, they are promising sources for applications such as the nonlinear generation of new frequencies, quantum networking, and astrocombs. For such applications, periodic waveforms offer the distinct advantages of easy access, very good power efficiency, and low noise levels.

In particular, for some applications it is desirable to increase the comb power by raising the external drive power, but it has been observed [3] that for strong drive the periodic waveforms are subject to instabilities leading to chaos. However, these studies were conducted for a fixed period, typically the one corresponding to the most unstable wavenumber of the underlying continuous wave (cw). For near-threshold powers waveforms with these periods are indeed the most stable, but as we show here theoretically, nonlinear frequency pulling shifts the period stability window strongly to shorter periods.

As a consequence, the period of the strongest stable comb that can be obtained for a given drive detuning is typically significantly shorter than the central modulational instability period, and the respective maximal comb power is typically several times larger, see Fig. 1 (left). These results are derived numerically from the stationary Lugiato-Lefever equation (LLE) [3] with free-length periodic boundary conditions.

2. Periodic waveforms

In the standard normalization the LLE takes the form

$$\frac{\partial \psi}{\partial t} = -(1+i\alpha)\psi \pm \frac{i}{2}\frac{\partial^2 \psi}{\partial z^2} + i|\psi|^2\psi + f \ , \qquad (1)$$

where ψ is the slow electric field envelope, depending on resonator coordinate z and time t, positive and negative signs in front of the second derivative stand for anomalous and normal dispersion (respectively), and α and $f > 0$ are the detuning and amplitude of the drive (respectively).

The uniform cw solutions of (1) become unstable for $f > f_c(\alpha) = \sqrt{2-\alpha(2-\alpha)}$, ($\alpha < 7/4$), and periodic solutions bifurcate. The bifurcation is supercritical for $\alpha < 41/30$, with central wavenumber $k_c = \sqrt{2(2-\alpha)}$. Weakly nonlinear analysis shows that for $f = f_c + \varepsilon$, ε small, the amplitude of the periodic waveform is proportional to $\sqrt{\varepsilon}$.

For f far above threshold, we use standard numerical methods to seek stationary solutions of (1) subject to $\psi_p(z) = \psi_p(z+z_p)$ with the period z_p as a free parameter. For a given detuning, we find that the band of allowed periods, whose width near threshold scales as $\sqrt{\varepsilon}$, continues to widen when f is increased far above f_c, giving rise to arbitrarily strong periodic solutions; when driving is very strong, however, these solutions become unstable at some point.

3. Stability

According to the Floquet-Bloch theory, the linear stability normal modes of the LLE are quasi-periodic $\Psi_{n,q}(z) = \Phi(n,q)(z)e^{2\pi iqz/z_p}$, $|q| \leq 1$, where $\Phi_{n,q}$ is periodic, q/z_p is the lattice wavenumber, and n a discrete index. Ψ and Φ are two-component vectors since for the linear stability analysis ψ and its complex conjugate need to be treated as independent quantities. The corresponding eigenvalues $\lambda_{n,q}$ form continuous bands as q is changed from zero to one (reflection symmetry implies that $\lambda_{n,-q} = \lambda_{n,q}$). The periodic waveform is stable if $\mathrm{Re}\,\lambda_{n,q} < 0$ for all n, q.

We solved the linear stability problem by discretizion of the linear differential operator on a uniform grid, obtaining a q-dependent matrix whose eigenvalues are approximations to $\lambda_{n,q}$. For a given α we find that near threshold waveforms with periods in the central part of the allowed window are stable, while those with period in the flanks of the window are Eckhaus-unstable, with instability onset at $q = 0$, see Fig. 1 (right). This behavior persists for driving far above threshold, yielding a band of Eckhaus-st
nonlinear, instability sets in at $q = 1$ for suffici
delimit a finite area region of the z_p-f paramete

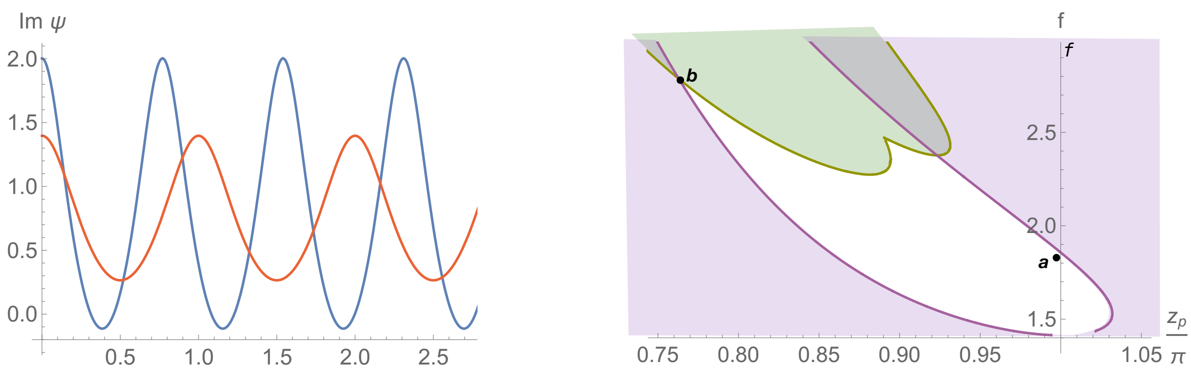

Fig. 1. Left: Stable periodic solutions of the LLE with anomalous dispersion, showing the imaginary part of the field envelope $\mathrm{Im}\,\psi$ as a function of cavity position z, for detuning $\alpha = 0$, drive $f = 2.7$ and period $z_p = 0.77\pi$ (blue) and $\alpha = 0$, $f = 1.8$, and $z_p\pi$ (red), corresponding to points a and b in the right panel. The respective comb powers are 0.54 and 0.17, showing that comb power can be raised significantly by adjusting the period. Right: The unshaded area is the stability region of periodic waveforms in the space of periods z_p and drives f for $\alpha = 0$. Solutions in the green-shaded area are Eckhaus-unstable to perturbations with zero lattice wavenumber, and those in the purple area to are unstable to z_p-lattice wavenumber perturbations.

4. Generation of periodic waveforms

For a typical resonator with a large dimensionless length Z, the stable periodic solutions are defined by the intersection of the stable region (unshaded) of Fig. 1 (right) with a sequence of vertical lines $z_p = Z/n$. A desired solution (the one marked on the figure by b, say) can be obtained by seeding a perturbation with the appropriate wavelength. A second route is to choose an initial α such that the modulational instability sets in at the appropriate z_p, and to adjust α and f together adiabatically toward the desired values.

References

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