

\mathcal{CP} Violation from String Theory

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We identify a natural way to embed \mathcal{CP} symmetry and its violation in string theory. The \mathcal{CP} symmetry of the low energy effective theory is broken by the presence of heavy string modes. \mathcal{CP} violation is the result of an interplay of \mathcal{CP} and flavor symmetry. \mathcal{CP} violating decays of the heavy modes could originate a cosmological matter-antimatter asymmetry.

I. INTRODUCTION

Aspects of \mathcal{CP} symmetry and its violation play a crucial role in several physics phenomena. This includes the question of \mathcal{CP} symmetry in strong interactions (the so-called strong \mathcal{CP} problem), the violation of \mathcal{CP} in the Yukawa sector of the standard model (SM) (with at least 3 families of quarks and leptons) and the desire for a source of \mathcal{CP} violation (CPV) in the process of a dynamical creation of the cosmological matter-antimatter asymmetry. We are thus confronted with the following questions: What is the origin of \mathcal{CP} symmetry and its violation? Is there a relation to the flavor symmetries in the SM of particle physics? Is there a “theory of \mathcal{CP} ” in the ultraviolet completion of the SM that explains both the origin of \mathcal{CP} symmetry and its breakdown?

In the present letter we try to address these questions about \mathcal{CP} and flavor symmetries in the framework of string theory. Our approach to a “theory of \mathcal{CP} ” is based on orbifold compactifications of heterotic string theory (the so-called MiniLandscape [1–4]) but should be valid qualitatively for a wide range of string theory constructions. From our exploration of these models the following general picture emerges:

- we find \mathcal{CP} candidates strongly connected to flavor symmetries, specifically \mathcal{CP} as an outer automorphism of the flavor group;
- the light (“massless”) string spectrum results in a low-energy effective field theory with a well-defined \mathcal{CP} transformation,

which can be conserved only in the absence of couplings to the heavy modes;

- the presence of heavy modes (here the winding modes of string theory) initiates a breakdown of \mathcal{CP} (similar to the picture of “explicit geometrical \mathcal{CP} violation”);
- \mathcal{CP} violating decays of the heavy (winding) modes could induce the cosmological matter-antimatter asymmetry. Other possible CPV effects can be induced through couplings of light fields to the heavy modes.

This provides us with a picture where the source of \mathcal{CP} breakdown is already included within the construction of the symmetry itself. It also shows that the breakdown of \mathcal{CP} requires a certain amount of complexity of the theory (reminiscent of the need of three families in the CKM case).

The origin of \mathcal{CP} violation in the context of string theory and extra dimensions has been discussed in many regards, see [5] for a review and references therein. Our approach is new in the following sense: While it has been known that extra dimensions provide an origin of discrete (flavor) symmetries [6–8], a more recent insight, based on the original idea of “explicit geometrical \mathcal{CP} violation” [9], is that a large class of discrete groups is generally incompatible with \mathcal{CP} [10]. This comes about because these groups do not allow for complex conjugation outer automorphisms which, however, correspond to physical \mathcal{CP} transformations in the most general sense [11, 12]. In these cases, \mathcal{CP} is explicitly violated by phases which are discrete and calculable because they originate from the complex Clebsch-Gordan coefficients of the respective flavor group. The main progress in this letter is to demonstrate that such a situation arises naturally in string theory.

As a specific example we consider a \mathbb{Z}_3 orbifold with flavor group $\Delta(54)$ that appears naturally in the MiniLandscape constructions [6]. In this

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case, \mathcal{CP} should be a subgroup of S_4 , the group of outer automorphisms of $\Delta(54)$; thus flavor group and \mathcal{CP} are intimately related. The irreducible representations of $\Delta(54)$ include singlets, doublets, triplets and anti-triplets. The massless spectrum of the theory, however, contains only singlets and triplets (as well as anti-triplets) of $\Delta(54)$. This allows for a \mathcal{CP} symmetric low-energy effective field theory of the massless states. The presence of the heavy winding modes that transform as doublets of $\Delta(54)$ leads to an obstruction for the definition of \mathcal{CP} symmetry thereby realizing the mechanism of “explicit geometrical \mathcal{CP} violation”. All \mathcal{CP} violating effects originate through couplings of the light states to at least three non-trivial doublets. \mathcal{CP} violating decays of the heavy doublets are a generic property of the scheme. Combined with baryon- and/or lepton-number violation this could lead to a cosmological baryon- and/or lepton asymmetry.

II. $\Delta(54)$ FLAVOR SYMMETRY FROM STRING THEORY AND THE LIGHT SPECTRUM

In order to understand the origin of $\Delta(54)$ from strings it is sufficient to concentrate on the compactification of two extra dimensions on a $\mathbb{T}^2/\mathbb{Z}_3$ orbifold. For a full string model this $\mathbb{T}^2/\mathbb{Z}_3$ can easily be extended to a six-dimensional orbifold, e.g. $\mathbb{T}^6/\mathbb{Z}_3 \times \mathbb{Z}_3$.

Geometrically, a $\mathbb{T}^2/\mathbb{Z}_3$ orbifold can be defined in two steps: (i) one defines a torus \mathbb{T}^2 by specifying a lattice $\Lambda = \{n_1 e_1 + n_2 e_2 \mid n_i \in \mathbb{Z}\}$, spanned by the vectors e_1 and e_2 . We choose $|e_1| = |e_2|$ and the angle between e_1 and e_2 is set to 120° . (ii) one identifies points on \mathbb{T}^2 that differ by a 120° rotation generated by θ . The resulting orbifold has the shape of a triangular pillow, see FIG. 1 and 2.

Closed strings on the $\mathbb{T}^2/\mathbb{Z}_3$ orbifold come in three classes: (i) trivially closed strings, which are closed even in uncompactified space, (ii) winding strings with winding numbers n_1 and n_2 in the torus directions e_1 and e_2 , respectively, and (iii) twisted strings, which are closed only up to a θ^k rotation for $k = 1, 2$. For $k = 1$ or $k = 2$ they belong to the so-called first or second twisted sector, respectively. On the other hand, trivially closed strings and winding strings belong to the so-called untwisted sector and live in the bulk of the orbifold. In contrast, twisted strings are localized at the three corners (fixed points) of the $\mathbb{T}^2/\mathbb{Z}_3$ orbifold. For $k = 1, 2$, they are created by twisted

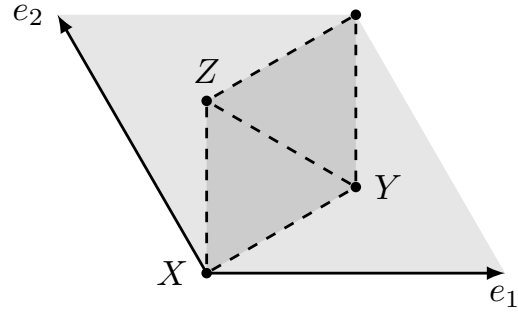


FIG. 1: $\mathbb{T}^2/\mathbb{Z}_3$ orbifold with fixed points X, Y, Z . The fundamental domain is shaded in dark gray.

vertex operators which we label as

$$\chi^{(\mathbf{3}_1)} \sim \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \quad \text{and} \quad \psi^{(\bar{\mathbf{3}}_1)} \sim \begin{pmatrix} \bar{X} \\ \bar{Y} \\ \bar{Z} \end{pmatrix}, \quad (1)$$

respectively (compared to Ref. [13] we set $X = \sigma_0^+$, $Y = \sigma_1^+$, $Z = \sigma_2^+$ and $\bar{X} = \sigma_0^-$, $\bar{Y} = \sigma_1^-$, $\bar{Z} = \sigma_2^-$).

Interactions of strings on orbifolds are restricted by selection rules. In the case of $\mathbb{T}^2/\mathbb{Z}_3$ the point group (PG) and space group (SG) selection rules result in a $\mathbb{Z}_3^{\text{PG}} \times \mathbb{Z}_3^{\text{SG}}$ symmetry [14]. Massless untwisted strings transform trivially, while twisted strings transform as

$$\chi^{(\mathbf{3}_1)} \xrightarrow{\mathbb{Z}_3^{\text{PG}}} \text{diag}(\omega, \omega, \omega) \chi^{(\mathbf{3}_1)}, \quad (2a)$$

$$\chi^{(\mathbf{3}_1)} \xrightarrow{\mathbb{Z}_3^{\text{SG}}} \text{diag}(1, \omega, \omega^2) \chi^{(\mathbf{3}_1)}, \quad (2b)$$

$$\psi^{(\bar{\mathbf{3}}_1)} \xrightarrow{\mathbb{Z}_3^{\text{PG}}} \text{diag}(\omega^2, \omega^2, \omega^2) \psi^{(\bar{\mathbf{3}}_1)}, \quad (2c)$$

$$\psi^{(\bar{\mathbf{3}}_1)} \xrightarrow{\mathbb{Z}_3^{\text{SG}}} \text{diag}(1, \omega^2, \omega) \psi^{(\bar{\mathbf{3}}_1)}, \quad (2d)$$

where $\omega := e^{2\pi i/3}$. In the absence of non-trivial backgrounds on $\mathbb{T}^2/\mathbb{Z}_3$ there is in addition an S_3 symmetry corresponding to all permutations of the three twisted strings. Combining this symmetry with the PG and SG symmetries, one obtains a $\Delta(54)$ flavor symmetry, see Appendix A. Massless untwisted strings transform as trivial singlets $\mathbf{1}_0$, while the twisted strings $\chi^{(\mathbf{3}_1)}$ and $\psi^{(\bar{\mathbf{3}}_1)}$ transform as $\mathbf{3}_1$ and $\bar{\mathbf{3}}_1$ of $\Delta(54)$, respectively [6, 8].

III. $\Delta(54)$ AND EXPLICIT GEOMETRICAL \mathcal{CP} VIOLATION

Let us discuss some details of $\Delta(54)$ and how this group can lead to the phenomenon of explicit

geometrical \mathcal{CP} violation. The non-trivial irreps of $\Delta(54)$ are the real $\mathbf{1}_1$, a quadruplet of real doublets $\mathbf{2}_{k=1,2,3,4}$ as well as the faithful complex triplets $\mathbf{3}_1$, $\mathbf{3}_2$ and their respective complex conjugates $\bar{\mathbf{3}}_1$ and $\bar{\mathbf{3}}_2$. Tensor products relevant to this work are

$$\mathbf{3}_i \otimes \bar{\mathbf{3}}_i = \mathbf{1}_0 \oplus \mathbf{2}_1 \oplus \mathbf{2}_2 \oplus \mathbf{2}_3 \oplus \mathbf{2}_4, \quad (3a)$$

$$\mathbf{2}_k \otimes \mathbf{2}_k = \mathbf{1}_0 \oplus \mathbf{1}_1 \oplus \mathbf{2}_k. \quad (3b)$$

The outer automorphism group (Out) of $\Delta(54)$ is

$$\text{Out}[\Delta(54)] \cong S_4, \quad (4)$$

the permutation group of four elements. On the four doublets, S_4 acts as all possible permutations. On the triplets, odd permutations in S_4 act as complex conjugation, while even permutations in S_4 map the triplets to themselves [12]. In addition, all these transformations are typically endowed with matrices that act on the representations internally.

A physical \mathcal{CP} transformation maps all fields of a theory in some irreps \mathbf{r} to their respective complex conjugate fields, which transform in \mathbf{r}^* [12]. Therefore, a physical \mathcal{CP} transformation should be an outer automorphism of $\Delta(54)$ which maps all occurring irreps to their respective complex conjugates [10, 11]. However, depending on the specific group, such outer automorphisms do not need to exist, and groups which do not have them are called “type I” [10].

It turns out that $\Delta(54)$ is a group of type I. This becomes manifest by the fact that in the presence of triplets it is *only* possible to find a physical \mathcal{CP} transformation as subset of $\text{Out}[\Delta(54)]$, *if* a given theory contains fields in *no more than two* distinct doublet representations. By contrast, *if* a given theory with triplets contains *more than two* doublets, physical \mathcal{CP} is violated by complex Clebsch-Gordan coefficients of $\Delta(54)$ and this is called explicit geometrical \mathcal{CP} violation.

Coming back to our model, we recall that the light (from a string perspective massless) spectrum consists only of $\Delta(54)$ triplets and singlets. Consequently, there exist outer automorphisms of $\Delta(54)$ which correspond to physical \mathcal{CP} transformations for the light spectrum. One may thus be led to the conclusion that \mathcal{CP} can be conserved in this model even though $\Delta(54)$ is a group of type I. However, this conclusion is premature because it disregards heavy string modes. We will take them into account in the next section.

IV. $\Delta(54)$ DOUBLETS FROM HEAVY WINDING STATES

An untwisted string on $\mathbb{T}^2/\mathbb{Z}_3$ is generally massless only if it does not wind around the torus \mathbb{T}^2 and does not carry Kaluza-Klein (KK) momentum. For this reason, winding strings often have been ignored in the literature. In this respect, one of our main results is that general untwisted strings on $\mathbb{T}^2/\mathbb{Z}_3$ can transform as doublets of $\Delta(54)$, as we show in the following.

A general untwisted string with vertex operator $V^{p,w}$ is characterized by its winding on the lattice, $w \in \Lambda$, and momentum on the dual lattice, $p \in \Lambda^*$. It can be constructed by joining two strings: one string from the first twisted sector X , Y , Z combines with another string from the second twisted sector \bar{X} , \bar{Y} , \bar{Z} . Such processes are described by the corresponding operator product expansions (OPEs), which are given explicitly for $\mathbb{T}^2/\mathbb{Z}_3$ in Ref. [13]. Solving these OPEs for the untwisted strings we get

$$\mathbf{1}_0: \quad V^{(00)} = \frac{1}{3} (X\bar{X} + Y\bar{Y} + Z\bar{Z}), \quad (5a)$$

$$\mathbf{2}_1: \quad \begin{pmatrix} V^{(02)} \\ V^{(01)} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} Y\bar{Z} + Z\bar{X} + X\bar{Y} \\ Z\bar{Y} + X\bar{Z} + Y\bar{X} \end{pmatrix}, \quad (5b)$$

$$\mathbf{2}_2: \quad \begin{pmatrix} V^{(10)} \\ V^{(20)} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} X\bar{X} + \omega Y\bar{Y} + \omega^2 Z\bar{Z} \\ X\bar{X} + \omega^2 Y\bar{Y} + \omega Z\bar{Z} \end{pmatrix}, \quad (5c)$$

$$\mathbf{2}_3: \quad \begin{pmatrix} V^{(12)} \\ V^{(21)} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} Y\bar{Z} + \omega Z\bar{X} + \omega^2 X\bar{Y} \\ Z\bar{Y} + \omega^2 X\bar{Z} + \omega Y\bar{X} \end{pmatrix}, \quad (5d)$$

$$\mathbf{2}_4: \quad \begin{pmatrix} V^{(11)} \\ V^{(22)} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} Z\bar{Y} + \omega X\bar{Z} + \omega^2 Y\bar{X} \\ Y\bar{Z} + \omega^2 Z\bar{X} + \omega X\bar{Y} \end{pmatrix}. \quad (5e)$$

Here we have introduced classes of untwisted strings

$$V^{(MN)} := \sum_{\substack{p \in \Lambda_M^* \\ w \in \Lambda_N}} C_{p,w} V^{p,w} \quad \text{for } M, N = 0, 1, 2, \quad (6)$$

where Λ_N is a sublattice of Λ with winding number $N := n_1 + n_2 \bmod 3$ and Λ_M^* the sublattice of Λ^* with KK number $M := -m_1 + m_2 \bmod 3$ (Note the difference to [13] due to a basis change of the torus lattice Λ). Furthermore, the coefficients $C_{p,w}$ are defined in Ref. [13] and they tend to zero, $C_{p,w} \rightarrow 0$, for several limits: for higher windings $|w| \rightarrow \infty$, for higher momenta $|p| \rightarrow \infty$ or, keeping p and w fixed, for larger torus radii $|e_1| = |e_2| \rightarrow \infty$.

Comparing Eqs. (5) to the $\Delta(54)$ tensor product $\mathbf{3}_1 \otimes \bar{\mathbf{3}}_1$, we identify the multiplets of winding strings as $\mathbf{1}_0$, $\mathbf{2}_1$, $\mathbf{2}_2$, $\mathbf{2}_3$ and $\mathbf{2}_4$, respectively, see Eq. (3a) and Eq. (A3). The $\Delta(54)$ doublets $\mathbf{2}_k$ of

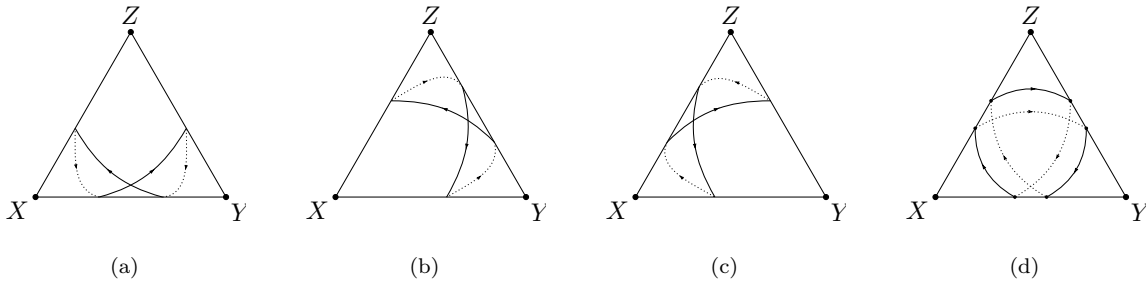


FIG. 2: Illustration of the geometrical winding strings. For example, (a), (b), and (c) depict the winding modes from XY , YZ , and XZ which all have winding number $N = 2$.

winding strings are generally massive (where the mass terms are $\Delta(54)$ invariant, see Eq. (3b)) and their masses are in general different.

There is a simple geometric intuition for these findings, revealing a remarkable difference between the winding modes in irreps $\mathbf{2}_{1,3,4}$, as compared to the modes in $\mathbf{2}_2$ and $\mathbf{1}_0$. The classes of untwisted strings $V^{(MN)}$ form $\Delta(54)$ covariant combinations of certain “geometric” winding modes, e.g. XY . For $\mathbf{2}_{1,3,4}$ these modes wind once around one fixed point and in a different orientation around another, see FIG. 2a-2c. The two different components of the doublets wind in opposing directions. By contrast, the doublet $\mathbf{2}_2$ is formed by a geometrical winding mode that has net zero winding number around all fixed points, see FIG. 2d.

V. \mathcal{CP} VIOLATION FROM HEAVY WINDING MODES

The effective operators in the holomorphic superpotential are given by $\Delta(54)$ invariants. For example, the simplest direct couplings between representations $\mathbf{3}_1$ and $\bar{\mathbf{3}}_1$, from the first and second twisted sector, respectively, to heavy winding modes in representations $\mathbf{2}_k$ are given by contractions of the form

$$\mathcal{W} \supset \sum_k (c_k)^{mab} \phi_m^{(\mathbf{2}_k)} \chi_a^{(\mathbf{3}_1)} \psi_b^{(\bar{\mathbf{3}}_1)}. \quad (7)$$

Here we have introduced exemplary fields $\phi^{(\mathbf{2}_k)} \equiv \phi_k$, $\chi^{(\mathbf{3}_1)}$, and $\psi^{(\bar{\mathbf{3}}_1)}$ in the according representations. The sum over the internal $\Delta(54)$ doublet and triplet components, denoted by $a, b = 1, 2, 3$ and $m = 1, 2$, is implicit. The coupling tensors to different winding modes $(c_k)^{mab}$ ($k = 1, 2, 3, 4$) are fixed by the requirement of $\Delta(54)$ invariance up to a global normalization $|c_k|$, corresponding to the

overall coupling strength which is determined by the T modulus (e.g. by the size of the $\mathbb{T}^2/\mathbb{Z}_3$ orbifold). The explicit form of the coupling tensors c_k is obtained from the Clebsch-Gordan coefficients (cf. Eq. (A3) in Appendix A) in a straightforward way. The presence of these couplings, i.e. $c_k \neq 0 \forall k$, is inconsistent with *any* physical \mathcal{CP} transformation

$$\phi_k \mapsto U_k \phi_k^*, \quad \chi \mapsto U_\chi \chi^*, \quad \text{and} \quad \psi \mapsto U_\psi \psi^*, \quad (8)$$

where we have allowed for the most general form of this transformation with arbitrary unitary matrices $U_{k,\chi,\psi}$ while we have suppressed the transformation of the space-time argument. Therefore, \mathcal{CP} is explicitly violated by the couplings Eq. (7).

As argued in Section III, this can readily be understood directly from group theory. One may also check here that there is no basis in which all coupling tensors $c_{k=1,2,3,4}$ are simultaneously real (which, however, is not sufficient to claim \mathcal{CP} , as complex couplings can co-exist with \mathcal{CP} conservation if there are conserved higher order \mathcal{CP} transformations [10][27]).

Finally, it is always possible to state physical \mathcal{CP} violation in a basis independent way. \mathcal{CP} -odd basis invariants can readily be constructed by contracting coupling tensors in such a way that (unitary) basis transformations cancel amongst the various index contractions. To produce a complex-valued (thus, \mathcal{CP} -odd) basis invariant it follows from our previous discussion that *at least three different types* of doublets (e.g. take $\mathbf{2}_{1,3,4}$) must be involved in such a contraction. This is confirmed by a scan over all possible basis invariant contractions of coupling tensors. The lowest order \mathcal{CP} -odd invariants arise at the four-loop level, with an example being displayed in FIG. 3. Explicitly, this basis invariant is given by

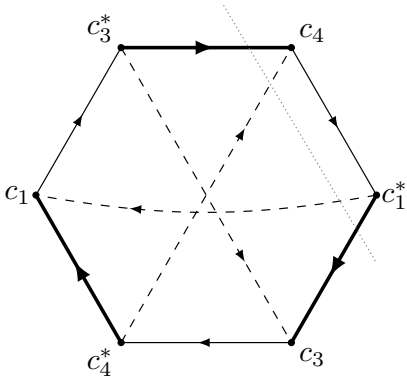


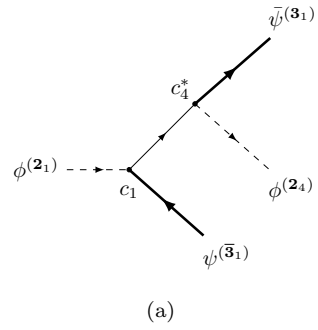
FIG. 3: The lowest-order complex (\mathcal{CP} -odd) basis invariant that can be formed out of the $\Delta(54)$ invariant coupling tensors $c_{k=1,3,4}$ of Eq. (7). Dashed lines correspond to $\Delta(54)$ doublets ϕ , thin solid lines correspond to the $\Delta(54)$ triplets χ and thick solid lines correspond to the $\Delta(54)$ anti-triplets ψ . Arrows here denote the $\Delta(54)$ charge flow. The dotted gray line denotes a cut that gives rise to the diagrams in FIG. 4.

$$\begin{aligned} \mathcal{I}_1 &= (c_1)^{mab} (c_4^*)^{ncb} (c_3)^{pcd} (c_1^*)^{med} (c_4)^{nef} (c_3^*)^{paf} \\ &= \frac{1 + 3e^{4\pi i/3}}{36} |c_1|^2 |c_3|^2 |c_4|^2. \end{aligned} \quad (9)$$

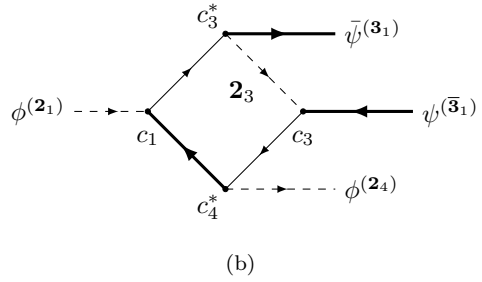
Here the summation over repeated indices is understood and we have used $|c_k|^2$ to denote the moduli of the coupling tensors. The fixed complex phase of the invariant is a group theoretically predicted parameter-independent \mathcal{CP} violating (weak) phase. Another \mathcal{CP} -odd invariant of the same order can be obtained from \mathcal{I}_1 by hermitean conjugation [or changing the order of indices of the coupling tensors according to $(143) \rightarrow (134)$]. Analogous invariants exist for all other sets of three distinct doublets. Further \mathcal{CP} -odd invariants exist for couplings of the type $\mathbf{2}_k \otimes \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3}$, where again, at least three different doublets have to be involved in a given invariant in order to generate \mathcal{CP} -odd contributions.

Diagrams corresponding to \mathcal{CP} violating physical processes such as oscillations and/or decays can be obtained from invariants such as the one in FIG. 3 by cutting edges appropriately. For example, the cut indicated by the dotted gray line in FIG. 3 gives rise to the pair of diagrams in FIG. 4 whose interference generates a \mathcal{CP} asymmetry in a decay $\mathbf{2}_1 \rightarrow \bar{\mathbf{3}}_1 \mathbf{3}_1 \mathbf{2}_4$ [28].

The discussion becomes exceedingly model dependent at this point. For example, the questions of whether a given decay is (kinematically) allowed, whether or not significant \mathcal{CP} asymmetry is generated (individual amplitudes can van-



(a)



(b)

FIG. 4: Example tree-level and one-loop diagrams whose interference can give rise to \mathcal{CP} violation in a decay $\mathbf{2}_1 \rightarrow \bar{\mathbf{3}}_1 \mathbf{3}_1 \mathbf{2}_4$.

ish or cancel against one another), or whether or not other quantum numbers such as lepton and/or baryon number are violated can only be answered in a concrete model. In existing, semi-realistic string theory models with $\Delta(54)$ flavor symmetry, SM quarks and leptons as well as flavons transform as $\Delta(54)$ triplets and anti-triplets [15, 16]. For instance, inspecting Tab. 1 of [15] one finds right-handed neutrinos in $\bar{\mathbf{3}}_1$ as well as SM neutral flavons in $\mathbf{3}_1$. The \mathcal{CP} and lepton number violating decay of a heavy doublet to final states which include these modes, hence, could generate a lepton asymmetry [29]. This illustrates how baryon and/or lepton number asymmetries can be generated by our mechanism. We do not further detail this discussion for the scope of the present letter since we have achieved to establish our main point which is the general existence of group theoretical \mathcal{CP} violation in string theory.

VI. DISCUSSION

We have seen that the mechanism of explicit geometrical \mathcal{CP} violation has a natural embedding in string theory. \mathcal{CP} violation here is for a symmetry reason, enforced by a discrete (flavor) symmetry of “type I” which prohibits any physical \mathcal{CP}

transformation and dictates discrete \mathcal{CP} violating phases.

In our example, the low-energy effective theory allows for a \mathcal{CP} transformation, but \mathcal{CP} is broken in the presence of heavy string modes. A generic property of this scheme is the \mathcal{CP} violating decay of heavy modes that could originate a cosmological baryon/lepton asymmetry. The discussion of other \mathcal{CP} violating effects such as θ_{QCD} and the CKM phases is strongly model dependent. As explicit model building is very complicated in string theory, one would be encouraged to tackle these more refined questions in bottom-up constructions of the scheme in detail. This could then help to identify those models that are of phenomenological interest, small θ_{QCD} and realistic CKM phases. One would hope to reproduce these models in a top-down construction and understand these specific properties from the geometry of extra dimensions and the geographical location of strings in compactified space. The complexity of \mathcal{CP} violation in the standard model (with 3 families of quarks and leptons) could then be related to the complexity to the spectrum and couplings of heavy string states (here at least 3 doublets of string winding modes).

From the more theoretical (top-down) point of view there remain a few question that have to be analyzed in detail. The work of Ref. [13, 17] [that lead to the selection rules in Eq. (5)] studied the role of T-duality in the framework of compactified string theory. In a UV-complete picture one now would have to understand the connection between the flavor group $[\Delta(54)]$ in our example] and T-duality. Especially, whether this does lead to a symmetry enhancement of the flavor group and how T-duality correlates with the outer automorphisms of $\Delta(54)$ (and thus \mathcal{CP}).

Among other things this would then allow us to decide whether \mathcal{CP} violation in the UV-complete theory is explicit or spontaneous, and whether \mathcal{CP} should be interpreted as a discrete gauge symmetry [18, 19]. Our discussion up to now did not address this question as we have only considered the low-energy effective theory where the appearing flavor symmetry is incompatible with \mathcal{CP} , which hence appears to be violated explicitly. Nevertheless, the fact that \mathcal{CP} violation of this type can originate spontaneously, even though in a subtle manner, has been previously demonstrated [20]. We would have to consider a UV-complete theory with additional symmetries (like T-duality) before we could hope for a definite answer here. A UV-complete theory might as well give further insight into the phenomenological properties of the scheme, as e.g. the suppression of θ_{QCD} or how

the remaining flavor symmetry is ultimately broken to a realistic pattern. We hope to report on progress towards the UV-complete picture in the near future.

As a final remark, we stress that many phenomenologically viable string compactifications feature discrete groups of type I, see e.g. [16, 21]. If all possible irreducible representations of these symmetry groups actually appear in the spectrum (fulfilling some kind of stringy “completeness conjecture”) all these models have the presented mechanism of \mathcal{CP} violation built in.

Acknowledgments

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Appendix A: Conventions for $\Delta(54)$

We follow the conventions of Ref. [12] where more details about $\Delta(54)$ can be found (cf. also [22, 23] but mind the different notational conventions). A minimal generating set of matrices for the three-dimensional representation $\mathbf{3}_1$ of $\Delta(54)$ is

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (\text{A1})$$

The \mathbb{Z}_3 PG and SG symmetries acting in Eq. (2) are the subgroups generated by A^2B^2AB and B , respectively. The two-dimensional representations $\mathbf{2}_1$, $\mathbf{2}_2$, $\mathbf{2}_3$ and $\mathbf{2}_4$ are generated by the matrices

$$\mathbb{1}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \Omega_2 = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}, S_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (\text{A2})$$

according to the assignments in Table I. In this basis the Clebsch-Gordan coefficients relevant to

TABLE I: Explicit matrices for the doublet representations of $\Delta(54)$, see (A2) for a definition of the matrices Ω_2 and S_2 .

	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$	$\mathbf{2}_4$
A_{2_i}	$\mathbb{1}_2$	Ω_2	Ω_2	Ω_2
B_{2_i}	Ω_2	$\mathbb{1}_2$	Ω_2	Ω_2^*
C_{2_i}	S_2	S_2	S_2	S_2

this work are given by

$$(x_{2_i} \otimes y_{2_i})_{\mathbf{1}_0} = \frac{1}{\sqrt{2}}(x_1 y_2 + x_2 y_1), \quad (\text{A3a})$$

$$(x_{3_i} \otimes y_{\bar{3}_i})_{\mathbf{1}_0} = \frac{1}{\sqrt{3}}(x_1 \bar{y}_1 + x_2 \bar{y}_2 + x_3 \bar{y}_3), \quad (\text{A3b})$$

$$(x_{3_i} \otimes y_{\bar{3}_i})_{\mathbf{2}_1} = \frac{1}{\sqrt{3}} \begin{pmatrix} x_1 \bar{y}_2 + x_3 \bar{y}_1 + x_2 \bar{y}_3 \\ x_2 \bar{y}_1 + x_1 \bar{y}_3 + x_3 \bar{y}_2 \end{pmatrix}, \quad (\text{A3c})$$

$$(x_{3_i} \otimes y_{\bar{3}_i})_{\mathbf{2}_2} = \frac{1}{\sqrt{3}} \begin{pmatrix} x_1 \bar{y}_1 + \omega x_2 \bar{y}_2 + \omega^2 x_3 \bar{y}_3 \\ x_1 \bar{y}_1 + \omega^2 x_2 \bar{y}_2 + \omega x_3 \bar{y}_3 \end{pmatrix}, \quad (\text{A3d})$$

$$(x_{3_i} \otimes y_{\bar{3}_i})_{\mathbf{2}_3} = \frac{1}{\sqrt{3}} \begin{pmatrix} x_2 \bar{y}_3 + \omega x_3 \bar{y}_1 + \omega^2 x_1 \bar{y}_2 \\ \omega x_2 \bar{y}_1 + x_3 \bar{y}_2 + \omega^2 x_1 \bar{y}_3 \end{pmatrix}, \quad (\text{A3e})$$

$$(x_{3_i} \otimes y_{\bar{3}_i})_{\mathbf{2}_4} = \frac{1}{\sqrt{3}} \begin{pmatrix} \omega^2 x_2 \bar{y}_1 + x_3 \bar{y}_2 + \omega x_1 \bar{y}_3 \\ x_2 \bar{y}_3 + \omega^2 x_3 \bar{y}_1 + \omega x_1 \bar{y}_2 \end{pmatrix}. \quad (\text{A3f})$$

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 - [27] Other examples of higher order CP transformations with irreducible complex coupling coefficients exist in the context of multi-Higgs doublet models [24] and in the context of extra dimensions [25, 26] (we have to stress, however, that our notion of geometrical CPV is substantially different from the same-named phenomenon in the latter references).
 - [28] For this decay to be effective it is required that $m_{\mathbf{2}_1} > m_{\mathbf{2}_3}, m_{\mathbf{2}_4}$. Similar decays exist for all possible mass orderings of the doublets.
 - [29] The decay in FIG. 4 does not allow to generate a lepton number asymmetry among the final-state triplets. However, other cuts of the invariant \mathcal{I}_1 and of the other CP-odd invariants allow for such processes.