Microscopic structure of coexisting 0⁺ states in ⁶⁸Ni probed via two-neutron transfer

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and amounts to an integral of 4.2(16)% relative to the ground state. The observed difference in feeding of the 0^+ states is explained by the transfer of neutrons, mainly in the *pf* shell below N = 40 for the ground state, and across N = 40 in the $g_{9/2}$ orbital for the 0^+_2 , based on second-order distorted-wave Born approximation calculations combined with state-of-the-art shell-model two-nucleon amplitudes. However, the direct feeding to the 2^+_1 state [29(3)%] is incompatible with these calculations.

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I. INTRODUCTION

As finite many-body quantum systems, atomic nuclei are unique in the way that single-particle and collective degrees of freedom span the same energy scale. The result is a subtle interplay that leads to different configurations coexisting in the spectra of nuclei. One manifestation of this phenomenon is the variety in the nature of 0^+ states in nuclei with an even number of protons and neutrons (even-even nuclei). Besides the ground state, for which the 0^+ spin parity is a direct consequence of pairing, excited 0^+ states of different character are often present in such systems at low excitation energy. In doubly-magic nuclei the energy necessary to promote (multiple) nucleon pairs through large shell gaps is compensated by the gain due to pairing correlations creating 0^+ states with a deformed character as the lowest excitation mode. This is the case of ¹⁶O and ⁴⁰Ca [1]. In singly-closed-shell

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nuclei, as a consequence of the additional correlations induced by the proton-neutron interactions, deformed 0⁺ states can come very close in energy to the ground state, resulting in shape coexistence phenomena like in the remarkable case of ¹⁸⁶Pb [2], or they may even become energetically favorable and become the ground state as observed in ³²Mg [3,4]. Spectacularly, if the configurations involved in these coexisting states are different enough, then they can even trigger shape isomerism as recently discovered in ⁶⁶Ni [5,6]. Since the shape-coexistence phenomenon is now identified in several known regions with closed-proton shell and midshell neutrons [1] and could induce sudden changes of low-lying states properties in unexplored regions, unraveling precisely the microscopic configurations involved in these states has become one of the main challenges of current nuclear physics studies.

In this respect, a lot of experimental and theoretical work on ⁶⁸Ni, with (in a first approach) its protons filling the Z = 28closed shell and its neutrons the N = 40 harmonic oscillator shell, has suggested this nucleus as one of the key objects of study in nuclear-structure. The low-energy spectrum of ⁶⁸Ni contains three known 0⁺ states. The second 0⁺ state (0⁺₂, with the ground state being 0⁺₁) is also the first excited state, with a half-life $t_{1/2} = 270(5)$ ns [7] and an excitation energy remeasured at 1604 keV using complementary probes [8–10]. The third 0⁺ state (0⁺₃) at 2511 keV was first observed in a β -decay experiment, resulting in a tentative spin and parity assignment [11] and confirmed later [12].

At present, a number of calculations predict the existence of the observed 0^+ (and the two 2^+) states and have to a certain extent reproduced their excitation energy [13-15]. It was suggested that the nature of the 0^+_2 state consists primarily of neutron two-particle two-hole (2p-2h) excitations from the *pf* shell to the neutron $g_{9/2}(\nu g_{9/2})$ orbital [13]. However, state-of-the-art large-scale shell-model calculations [14,15] point to significant mixing between the natural (0p-0h) and "intruder" (2p-2h) configurations, resulting in a remaining 0p-0h configuration content of only about 50% in the ground 0_1^+ state. The 0_3^+ state, on the other hand, is a good candidate for the proton-2p-2h excitation across the Z = 28 shell gap; the state was predicted at 2202 keV using neighboring 1p-2h and 2p-1h states in ⁶⁷Co and ⁶⁹Cu [16,17]. The shapes of these three 0⁺ states in ⁶⁸Ni were studied in Monte Carlo shellmodel (MCSM) calculations [15], predicting a coexistence of spherical (the 0_1^+ ground state), oblate (0_2^+) , and prolate (0_3^+) shapes. The possible presence of rotational bands built on top of the 0_2^+ and 0_3^+ states, suggested by calculated quadrupole moments [14,15,18], and by observed and calculated relative transition probabilities between the 2^+ and 0^+ states [8,19], support the shape-coexistence picture.

This macroscopic picture, similar to the notable case of ¹⁸⁶Pb [2], has found in ⁶⁸Ni a microscopic interpretation thanks to shell-model calculations, which are still out of reach for the lead region. The advances in the production of energetic radioactive ion beams makes it now possible to probe in detail this underlying microscopic structure in the nickel region with the selectivity offered by direct reactions. Here we report on the first experimental study of the low-lying states in ⁶⁸Ni through the two-neutron transfer reaction ⁶⁶Ni(*t*, *p*) (*Q* = 5.118(3) MeV [20]) in inverse kinematics to

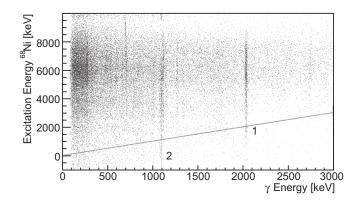


FIG. 1. Excitation energy of ⁶⁸Ni versus γ -ray energy in prompt coincidence with protons. The gray line is an indication for possible ground-state transitions. The numbers indicate (1) the ground-state transition from the first excited 2⁺ state and (2) random events from the Doppler-broadened background line of 1039 keV arising from the β decay of ⁶⁶Cu.

probe 2p-2h excitations across the N = 40 subshell closure. The technique used is similar to the ${}^{30}Mg(t, p){}^{32}Mg$ transfer reaction experiment [4], where neutron excitations across the N = 20 harmonic oscillator shell were identified.

II. EXPERIMENT

The ⁶⁶Ni beam (purity >99%) was produced at the ISOLDE facility in CERN [21], using the RILIS ion source [22] and was postaccelerated by REX [23,24] to 2.6 MeV/nucleon, which resulted in a center-of-mass energy of $E_{\rm c.m.} = 7.5$ MeV. The beam with an average intensity of $2.4(3) \times 10^6$ particles per second (pps) was guided onto a tritium-loaded titanium foil [4]. The light charged recoils were detected using the T-REX silicon particle detector array [25] and the γ rays using the Miniball detection array [26]. The T-REX setup consists of a double-sided segmented annular strip detector, the CD detector [27] that covers the laboratory angles 152° to 172° , and eight position-sensitive silicon-strip detectors (the "barrel") covering the angles 27° to 78° in the forward and 103° to 152° in the backward directions. The excitation energy resolution, derived from the detected proton energy and angle, averaged to 1.3 MeV and 0.8 MeV in the forward and backward barrel detectors, respectively, and 0.22 MeV in the backward CD detector. This energy resolution originates mainly from the beam straggling in the target and variations of the target thickness over the area where the beam impinged.

III. RESULTS

Figure 1 shows the measured excitation energy of ⁶⁸Ni versus the detected γ energy in prompt coincidence. The gray line in the figure indicates the region for events in which a populated excited state deexcites with a γ -ray transition directly to the ground state. Only the ground-state transition from the first excited 2_1^+ state at 2033 keV was observed (marked as 1 in Fig. 1). The line marked as 2 in Fig. 1 denotes a background transition from the β decay of ⁶⁶Cu, the β -decay daughter of ⁶⁶Ni nuclei that were partially implanted in the

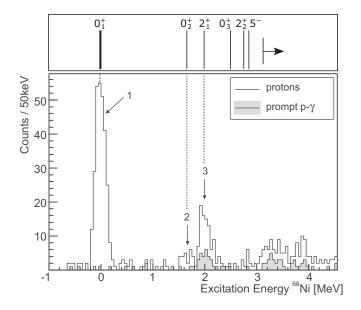


FIG. 2. Excitation energy spectrum of ⁶⁸Ni deduced from protons detected in the CD detector. The numbers indicate feeding to (1) the ground state, (2) the second 0⁺ state, and (3) the first excited 2⁺ state. The nonshaded area of the figure shows all detected protons, and the light gray area shows the protons that were detected in prompt coincidence with a γ ray. Known levels in ⁶⁸Ni are indicated in the panel above the figure. The pointing arrow indicates the excitation energy above which states have been omitted.

detection chamber. Since this β -delayed γ ray is emitted at rest, the line appears broadened and shifted due to the applied Doppler correction.

Most of the feeding in the two-neutron transfer reaction goes to high-energy states in ⁶⁸Ni between 5 and 9 MeV (see Fig. 1). Next to feeding to these high-energy states, strong direct feeding in the ⁶⁶Ni(t, p) reaction to the ground state and to the first excited 2⁺ state at 2033 keV is observed. This can be seen in Fig. 2, which shows the deduced excitation energy of ⁶⁸Ni for protons detected in the CD detector. Figure 2 also shows a small direct-feeding component to a state at 1621(28) keV (label 2), which is identified as the 0^+_2 state observed at 1604 keV [8–10]. In contrast to the feeding to the 2^+_1 state (label 3 in Fig. 2) no prompt γ rays were detected following the population of the 0_2^+ state. Indeed, the 0_2^+ state can only decay via an E0 transition to the 0_1^+ ground state, which corresponds to 1.56-MeV conversion electrons with a 55% probability and to pair creation with 45% probability. The latter gives rise to 511-keV γ -ray radiation. However, this radiation was not observed because the 270(5)-ns half-life of the 0_2^+ state [7] implies that most of the ⁶⁸Ni recoiling ions, moving at a velocity of about 2 cm ns⁻¹, decay far upstream from the reaction chamber. The few proton- γ events that can be seen underneath this state in Fig. 2 are due to random coincidences (as visible in Fig. 1). The population of the first excited 0^+ and 2^+ states was measured to be respectively 4.2(16)% and 29.3(29)%, relative to 100% ground-state feeding, for the protons detected in the CD detector, which spans the most forward center-of-mass angles ($\theta_{c.m.} \simeq 4^{\circ}$ to 16°). As can be seen in Figs. 1 and 2, direct feeding to levels between

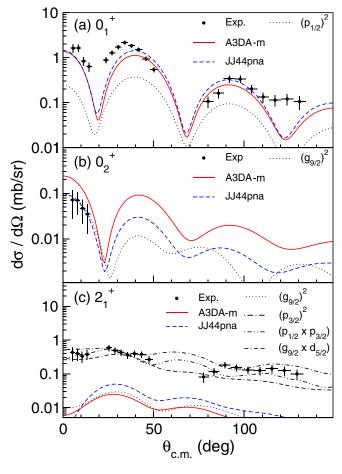


FIG. 3. Measured angular distributions together with DWBA calculations including direct and sequential transfer to the (a) ground state, (b) 0_2^+ , and (c) 2_1^+ state in ⁶⁸Ni. Global optical model parameters are taken from Ref. [28–30]. DWBA calculations use different two-nucleon amplitudes, either from shell-model calculations with the A3DA-m interaction [15] (solid red line) or the JJ44pna interaction [31] (dashed blue line) or assuming pure two-neutron configurations (black lines). See text for details.

2.5 and 3.0 MeV is limited. The three states 0_3^+ at 2511 keV, 2_2^+ at 2743 keV, and 5⁻ at 2847 keV were treated together because they lie close in energy with respect to the proton energy resolution. An upper limit for their combined feeding of < 2.3% within a 1- σ confidence level, for the angular range of the CD detector, was determined.

IV. ANALYSIS AND DISCUSSION

Angular distributions were measured for the ground state and first excited 0⁺ and 2⁺ states and are shown in Fig. 3 together with two-step distorted-wave Born approximation (DWBA) calculations performed with the FRESCO code [32] including both direct and sequential transfer. Solid red lines correspond to differential cross sections calculated using two-nucleon amplitudes (TNAs) from MCSM calculations in a model space including the full *pf* shell plus the 0*g*_{9/2} and 1*d*_{5/2} orbitals without any truncation for both neutrons and protons (thus taking ⁴⁰Ca as a core) with the A3DA-m

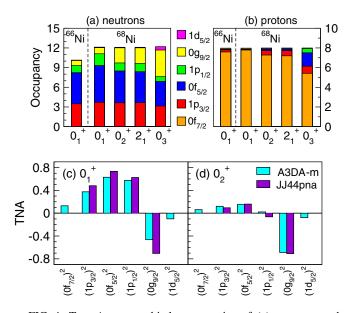


FIG. 4. Top: Average orbital occupancies of (a) neutrons and (b) protons for $0^+_{1,2,3}$ and 2^+_1 states in ⁶⁸Ni calculated using the A3DA-m interaction [15] (see text for details). Bottom: Two-nucleon amplitudes between the ⁶⁶Ni ground state and the $0^+_{1,2}$ states of ⁶⁸Ni [(c) and (d)] calculated using two different shell-model interactions (A3DA-m [15] and JJ44pna [31]). All of the TNA contributions in panel (c) add coherently to the two-neutron transfer cross section.

effective interaction as described in Ref. [15]. These MCSM calculations reproduce well energies of low-lying states and predict a triple-shape coexistence situation originating from strong changes of shell structure within the same nucleus driven largely by proton-neutron tensor interaction [15]. Dashed-blue lines on Fig. 3 result from the same cross-section calculations but using TNAs obtained from a shell-model calculation in the restricted neutron $0f_{5/2}$, $1p_{3/2}$, $1p_{1/2}$, $0g_{9/2}$ model space (taking ⁵⁶Ni as a core) with the JJ44pna effective interaction [31] and the NUSHELLX code [33].

In line with our main experimental observations for the $0_{1,2}^+$ states, all the calculations predict a transfer cross section to the 0_2^+ state significantly smaller compared to the ground state (21% and 9% using the A3DA-m or JJ44pna interaction, respectively). The origin of this reduced 0^+_2 population can be interpreted starting from the calculated average nucleon occupancies displayed in Figs. 4(a) and 4(b) together with the detailed components of the TNAs between the ⁶⁶Ni ground state and the $0^+_{1,2}$ states of ⁶⁸Ni [Figs. 4(c) and 4(d)]. Indeed, the average neutron occupancies displayed in Fig. 4(a) indicate that the 0^+_2 contains a strong contribution from configurations where neutrons are excited from the pf shell to the $g_{9/2}$ above the N = 40 gap (resulting in an average occupancy of 2.3 neutrons), whereas the 0^+_1 ground state is dominated by neutrons in the pf shell with a much lower weight of configurations with neutrons in the $g_{9/2}$ orbital (0.9 neutrons in average). With respect to the ⁶⁶Ni ground state, the additional neutrons in ⁶⁸Ni are thus mainly occupying the pf orbitals (+1.8 neutrons in average) for the ground state and the $g_{9/2}$ orbital (+1.5) for the 0^+_2 state. This structural difference between the two states, reflected in their TNAs, is

enhanced in the two-neutron transfer cross sections due to the rather different matching between the pair transfer on the $g_{9/2}$ and the $p_{3/2}$, $p_{1/2}$, $f_{5/2}$ orbitals.

This is illustrated in Figs. 3(a) and 3(b), where we show with dotted lines the calculations assuming a pure $(p_{1/2})^2$ and $(g_{9/2})^2$ two-neutron transfer to the 0_1^+ and 0_2^+ (TNA = 1), respectively. Quantitatively, the pure $(g_{9/2})^2$ pair transfer above N = 40 is unfavored compared to the $(p_{1/2})^2$ by a factor of about 2 in the $[4^\circ-16]^\circ$ angular range $(12 \ \mu b/21 \ \mu b)$ and by a factor of about 16 in the full angular range $(64 \ \mu b/1)$ mb). Due to this mismatch and the canceling of most of the TNA components for the 0_2^+ state apart from the $(g_{9/2})^2$ one, an overall hindrance of the cross section to the 0_2^+ is predicted and compatible with experiment.

Although this selective population mechanism seems describable schematically considering these two states as simple 0p-0h and 2p-2h configurations above the N = 40 gap, our study shows that the spread of TNAs over multiple components resulting from large-scale shell-model calculations is important to reach a more quantitative description of the cross section in general. This is especially clear for the ground state in Fig. 3(a) where the coherent combination of the full pfg TNA components from the shell model allows us to describe about 70% of the measured differential cross section while considering a pure $(p_{1/2})^2$ two-neutron transfer to the 0^+_1 (TNA = 1) leads only to 11%. It is true that a simpler two-state mixing approach, between the $(p_{1/2})^2$ and the $(p_{1/2})^{-2}(g_{9/2})^2$ configurations (somewhat similar to the one followed in Ref. [34]), yields two sets of mixing amplitudes, reproducing the ratio of integrated cross section between the 0_2^+ and 0_1^+ state in the covered angular range, but neither of these solutions simultaneously reproduce the absolute amplitude and shape of both angular distributions. Finally, differential cross sections calculated using the TNAs obtained with the JJ44pna interaction in a restricted model space reach a similar level of agreement with experiment than the one calculated with the A3DA-m interaction for the $0^+_{1,2}$ confirming that proton excitations above the Z = 28 and neutron excitation above N = 50 in the $\nu d_{5/2}$ orbital seem to play a reasonably minor role in the structure of these two states.

For the 2^+_1 state, different shell-model calculations with the A3DA-m [15], JJ44pna [31], and the LNPS [14] interaction link it with the 0^+_2 state and thus predict its configuration as based on neutron excitations in the $g_{9/2}$ orbital. As a result, the calculated angular distributions shown in Fig. 3(c) are similarly dominated by the $(g_{9/2})^2$ TNA component but are about an order of magnitude smaller than the experimental one. Within the current calculation framework, only a major increase of the $(p_{3/2})^2$, $(p_{1/2} \otimes p_{3/2})$, or $(g_{9/2} \otimes d_{5/2})$ TNA components could enhance the cross section enough to reproduce the magnitude of the measured cross section to the 2_1^+ state. This is illustrated in Fig. 3(c), where we plotted the calculations assuming pure configurations of this kind using dashed black lines. This result seems somewhat discrepant with the fact that measured $B(E2, 2_1^+ \rightarrow 0_2^+)$ transition strengths are well reproduced by these calculations using the same interactions [19]. On the reaction mechanism side, one could think of a coupling with another reaction channel, for example, via the excitation of ⁶⁶Ni to its 2_1^+ state, but it should be very strong and mainly affect the transfer to the 2_1^+ state. To now, no indication for such a strong coupling effect exists, suggesting that the structure of this state is not entirely well described. A systematic coupled-channels study including (t, p) reaction data to neighboring isotopes [35,36], beyond the scope of this article, would be valuable to confirm it.

Finally, the measured upper limit for the population of the 0_3^+ state is consistent with the A3DA-m calculations predicting that this state would include important components of proton excitations above Z = 28 [see Fig. 4(b)]. The TNAs between the ⁶⁶Ni ground state and this 0_3^+ state in ⁶⁸Ni are consequently very small (all $< 5 \times 10^{-2}$), leading to a calculated cross section of 7 μ b only. Intuitively, one could think that two-proton transfer is more suited to probe the structure of this state but the fact that it may involve simultaneously at least four neutrons excited above N = 40 (see Fig. 4(a) and Ref. [37]) could also suppress the corresponding TNAs and cross section.

V. CONCLUSIONS

The ⁶⁶Ni(*t*, *p*) reaction in inverse kinematics has been studied for the first time and used to assess directly the active neutrons orbitals responsible for shape coexistence in ⁶⁸Ni. The feeding of the 0⁺ states is explained by the transfer of neutrons mainly filling the N = 40 subshell closure for the ground state and across N = 40 for the 0^+_2 state, while the low

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upper limit for the population of the 0_3^+ state is consistent with the prediction that this state involves also considerable proton excitations above Z = 28. With the recently achieved energy upgrade of HIE-ISOLDE [38], a superconducting extension of the REX postaccelerator, these studies can now be extended to higher masses to firmly characterize the microscopic origin of shape coexistence in the lead region [1,2].

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