

Network Virtualization Resource Allocation and Economics Based on Prey–Predator Food Chain Model

Reginald A. Banez¹, Haitao Xu, *Member, IEEE*, Nguyen H. Tran², *Member, IEEE*,
Ju Bin Song, *Member, IEEE*, Choong Seon Hong³, *Senior Member, IEEE*,
and Zhu Han, *Fellow, IEEE*

Abstract—Network virtualization (NV) allows multiple heterogeneous virtual networks (VNs) to coexist and operate over the same physical network (PN) infrastructures. Some of the benefits of this advancement include flexibility in VN topologies, heterogeneity in VN technologies, and modularity of network operations. However, there are a few areas, such as resource allocation and economics, which challenge the implementation of NV. In this paper, we first introduce some NV parameters that influence the resource allocation and economics of an NV system. Next, we formulate an economic model for NV using the prey-predator food chain model. This model takes into account the dynamics in an NV system, such as the service, payoff, failure, and competition rates within each VN and PN. The solution point to this model represents the resource strategy of the service provider (SP) given the number of users trying to use its VN, as well as the resource strategy of the infrastructure provider (InP) given the strategy of the VN leasing its PN. In addition, we establish economic models that relate the capacities of the end users, the SP, and the InP. Finally, we provided simulations that show how the prey-predator food chain model fits well on an NV system.

Index Terms—Networks, economics, modeling, resource management.

I. INTRODUCTION

INTERNET has been successful in providing users access to information and communication through an assortment of applications. These applications include web browsing,

voice and data communication, video streaming, among others. The service providers (SPs) offer these applications through networks that vary in topology and architecture. The infrastructure providers (InPs), entities that manage a network of physical infrastructures, can support these multiple diverse networks through network virtualization. Network virtualization (NV) allows multiple heterogeneous virtual networks (VNs) to share common physical network (PN) infrastructures. In the case of the Internet, it will enable the support of multiple independent VNs which in turn offer various applications.

There are many advantages to implementing NV including flexibility, diversity, security, and manageability [1]. However, NV faces some challenges such as the allocation of network resources and the economic relationship between the SPs and the InPs. Hence, researchers have developed algorithms and models that solve NV resource allocation problems. Resource allocation in NV is performed by the InPs after receiving resource requests from the SPs [2]. Thus, the main focus of many works has been the dynamics between the SPs and the InPs. The lack of a unified framework for NV that involves all the levels of abstraction of NV (e.g. the end users, the SPs, and the InPs) has inspired us to develop an economic model that represents the dynamics within the entire NV system. The dynamics in an NV system include the failure, payoff, competition, and service rates at each level of abstraction. This model depicts not only the economic relationship of the InPs to the SPs, but also that of the SPs to their users.

In this paper, we present a novel approach to NV resource allocation and economics. NV is a dynamic system where the quantity of resource trading between the consumers and the producers is constantly changing. Moreover, there are multiple levels of interactions that are hard to be analyzed by the conventional game theoretical approaches. Thus, we adapt a prey-predator food chain model to integrate the dynamics and multiple level nature in an NV system to its economic model. The prey-predator food chain model mathematically describes the interactions among different agents in an environment. These interactions are exhibited by the transfer of energy from one agent to another. Consider an ecosystem with different species living together where a species is the food source of only one other species, and this species is the food source of only one other species and so on (i.e., a food chain). The interactions among many agents of the same

Manuscript received August 21, 2017; revised February 14, 2018 and May 17, 2018; accepted May 22, 2018. Date of publication June 4, 2018; date of current version October 16, 2018. The research is partially supported by US MURI, NSF CNS-1717454, CNS-1731424, CNS-1702850, CNS-1646607. The associate editor coordinating the review of this paper and approving it for publication was B. Liang. (*Corresponding author: Reginald A. Banez.*)

R. A. Banez is with the Department of Electrical and Computer Engineering, University of Houston, Houston, TX 77004 USA (e-mail: rabanez@uh.edu).

H. Xu is with the School of Computer and Communication Engineering, University of Science and Technology Beijing, Beijing 100083, China (e-mail: xuhaitao@ustb.edu.cn).

N. H. Tran is with the School of Information Technologies, The University of Sydney, Sydney, NSW 2006, Australia, and also with the Department of Computer Science and Engineering, Kyung Hee University, Yongin 17104, South Korea (e-mail: nguyen.tran@sydney.edu.au).

J. B. Song is with the Department of Electronic Engineering, Kyung Hee University, Yongin 17104, South Korea (e-mail: jsong@khu.ac.kr).

C. S. Hong is with the Department of Computer Science and Engineering, Kyung Hee University, Yongin 17104, South Korea (e-mail: cshong@khu.ac.kr).

Z. Han is with the University of Houston, Houston, TX 77004 USA, and also with the Department of Computer Science and Engineering, Kyung Hee University, Yongin 17104, South Korea (e-mail: zhan2@uh.edu).

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TCOMM.2018.2843367

species (i.e., competition) and the interactions between two agents of different species (i.e., consumption) can be modeled mathematically. In NV, the end users (EUs), the SPs, and the InPs are the agents, while the service payment (e.g. money) represents the transfer of energy within the system. Therefore, the model represents the economics in an NV system. The main contributions of this paper are as follows:

- We propose a unified mathematical framework based on the prey-predator food chain model that represents the economics in an NV system. We also provide an analysis of this model, as well as its properties and equilibrium point. This equilibrium point stands as the resource strategies at each level of the NV system given the economic dynamics of the system.
- We provide economic models that emulate the relationship between the total amount of demand for the network resources and the total amount of resources that needs to be supplied based on the dynamics in the NV system.
- We provide simulations that show the changes in the behavior of the NV system when the dynamics within the system have changed. We also provide simulations that illustrate the relationship of the demand and supply of network resources, between the EUs and the SPs, and between the SPs and the InPs.

This paper is organized as follows. Section II discusses some related research on resource allocation in NV. Section III provides a brief discussion of NV architecture and system parameters. Section IV presents the development of the system model and the definition of the problem. In Section V, we provide an analysis of the NV system model, as well as its properties. Then, in Section VI, we provide numerical simulations which manifest the properties and behavior of the NV system. Finally, we conclude the paper in Section VII.

II. RELATED WORKS

To the best of our knowledge, the prey-predator food chain model has not been used to model the resource allocation and economics in an NV system. However, several related research papers have inspired us to develop a new mathematical framework for NV. Some of these works utilized game theory, contract theory, and queuing theory. These frameworks have not only been applied to traditional NV but also to wireless NV and to network function virtualization (NFV).

Zhou *et al.* [3] proposed a non-cooperative game theoretic approach to bandwidth allocation. They developed an algorithm that iteratively solved for the Nash equilibrium of the game, and in turn, found the solution to the bandwidth allocation problem. Meanwhile, Seddiki and Frikha [4] proposed a two-stage non-cooperative game for bandwidth allocation. In the first stage, the bandwidth negotiation game, the SPs request bandwidth from multiple InPs. In the second stage, the bandwidth provisioning game, the SPs sharing the same physical link of an InP compete for bandwidth. Game theory was also used to improve the utilization of resources. Xie *et al.* [5] proposed a two-stage Stackelberg game-based mechanism that allows the InP to determine a reselling rate

that maximizes its revenue and enables the SP to calculate the amount of bandwidth that minimizes its expenses.

Virtualization in wireless networks, or wireless NV, allows multiple wireless networks operated by different SPs to dynamically share a common physical infrastructure operated by mobile network operators (MNO) [6]. Game theory was also applied to wireless NV, where the MNOs are responsible for dynamic wireless resource management, while the SPs aim at providing proper bandwidth allocation to their users. In [7], the interactions between the SPs and the NO were modeled as a stochastic game, since the demand of the users of the SPs varies with time. However, since the resulting stochastic game has strong dependency among SPs and on future actions, the authors utilized the Vickrey-Clarke-Groves (VCG) auction mechanism and conjectural pricing to transform the game into a series of independent games. Other papers that used game theory to formulate and solve an NV resource allocation problem include [8]–[10] and [11].

In NFV, software implementations of network functions are decoupled from the hardware [12]. Hence, multiple virtual network functions (VNFs) can be multiplexed in common physical servers. Yoon and Kamal [13] used mixed queuing model to allocate the resources of the servers to VNFs. Each user of the network requires different combinations of VNFs, or service chains. Thus, each VNF was modeled as a queue, and the model was designed to minimize the waiting time of service chains. Since VNFs are normally “chained” to each other, and thus may be utilized in succession by its users, effective placement and scheduling of these VNF chains in physical servers have been proposed in [14] to achieve high network resource utilization and low network response latency. The authors proposed a priority-driven weighted algorithm to maximize the average resource utilization, and a novel heuristic algorithm to minimize the average response latency. When the central offices (COs), where physical servers are housed, are geo-distributed, assigning VNFs to servers can be challenging. Consequently, to provide load balancing and cost efficiency, Fei *et al.* [15] proposed a framework that selects a set of COs that minimizes the communication cost within the set and then uses a shadow-routing approach that jointly optimizes VNF-CO and VNF-server assignment.

Many other approaches have been used to model the allocation of resources in NV. Nguyen *et al.* [16] proposed a parallel and distributed algorithm based on alternating direction method of multipliers (ADMM) for bandwidth allocation and routing problem that minimizes the traffic that the SP encounters. Information asymmetry in wireless NV, a situation where the NO has no access to private information about the SPs, was investigated by Nguyen *et al.* [17]. They proposed a solution to the resource allocation problem using contract theory, where the NO offers each SP a contract that allows each to maximize its own payoff. In [18], when the SPs could not report their traffic, a traffic predictor was integrated into the bandwidth allocation algorithm so that the InPs can supply accurate amount of resources to the SPs. Other approaches to resource allocation in NV include matching theory [19], [20], machine learning [21], [22], and fuzzy logic [23].

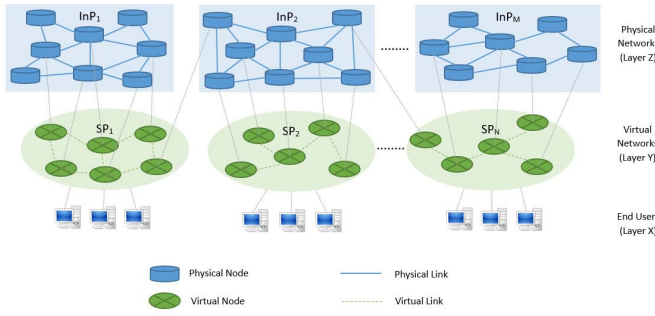


Fig. 1. Network virtualization architecture.

III. NETWORK VIRTUALIZATION

A. System Architecture

The NV architecture is shown in Fig. 1. It has three layers of abstraction: the InPs, the SPs, and the EUs levels. An InP provides and manages a physical network (PN) infrastructure consisting of physical resources (PRs) such as physical nodes and physical links. An SP operates and manages a virtual network (VN) which is composed of virtual resources (VRs) such as virtual nodes and virtual links. In order to build and expand its VN, an SP needs to lease PRs from InPs. Through NV, one or more SPs can lease the same PRs at the same time. In other words, multiple VRs may reside at a single PR. Also, an SP may only lease a subset of the PN according to its VN needs. The EUs can communicate with the network or to each other, as well as use web applications, through the services provided by an SP.

B. System Parameters

Before defining the system parameters used in the system model, let us first state that the focus of the system model presented in the next section is the distribution of bandwidth in an NV system. Thus, the primary network resource of interest in this manuscript is bandwidth, and we use the terms “network resource” and “bandwidth” interchangeably, unless otherwise stated. Other network resources such as CPU and memory are assumed to have satisfied their respective constraints, and therefore are not affecting the distribution of bandwidth in the system. As a consequence of these assumptions, the following system parameters are related to the amount of bandwidth demand by or supply from a particular entity (i.e., EUs, SPs, and InPs) in an NV system. This relation is explained further in this section.

Now, we introduce important frequencies or rates $r^{(l)}$ at each level l of the NV system where the main players at each level $l \in \{1, 2, 3\}$, respectively, are:

- the EUs in metropolitan areas (MAs);
- the SPs, each managing a VN; and
- the InPs, each operating a PN.

Consider a metropolitan area, where an SP is overlaying its VN and an InP has already established its PN. Suppose the EUs in the area, who want to connect to and use the VN, submit their requests, or user requests (URs), to the SP. For simplicity, assume that a single EU can only submit one UR

at a time. If we let χ be the number of EUs trying to access the VN at time τ , then it is also the number of URs generated simultaneously in the area. Since we assume that bandwidth is the network resource of interest, then χ also refers to the number of unit EU bandwidth requests generated in the area. For example, if $\chi = 10$ and a unit EU bandwidth is 10 Mbps, then there are 10 users requesting for 10 Mbps of bandwidth each. We can also refer to χ as the resource (i.e., bandwidth) demand at the EU level. The arrival rate $r_0^{(1)}$ of these URs is the theoretical maximum increase rate of the number of URs χ generated in the network coverage area per EU,

$$r_0^{(1)} = \frac{1}{\chi} \frac{d\chi}{d\tau}, \quad (1)$$

where τ denotes time. Assuming the number of URs approaches a maximum limit, $\kappa^{(1)}$, we can use the logistic model equation [24] where (1) becomes

$$r^{(1)} = \frac{\frac{1}{\chi} \frac{d\chi}{d\tau}}{1 - \frac{\chi}{\kappa^{(1)}}}, \quad (2)$$

where $\kappa^{(1)}$ is the maximum number of EUs in the area. The term $\frac{\chi}{\kappa^{(1)}}$ signifies the percentage of EUs trying to connect to and use the VN. The solution to (1) is called the logistic function as

$$\chi(\tau) = \frac{\kappa^{(1)}}{1 + \left(\frac{\kappa^{(1)}}{\chi_0} - 1\right)e^{-r^{(1)}\tau}}, \quad (3)$$

where χ_0 is the initial value of $\chi(\tau)$ at $\tau = \tau_0$, $\kappa^{(1)} > 0$ is the limiting value of $\chi(\tau)$, and $r^{(1)} > 0$ for a logistic growth model.

In order to accommodate the additional VN users and lease a PR, the SP has to submit a VR request (VRR) for a PR to the InP. If we let ψ be the number of VRRs generated simultaneously in the area at time τ , then ψ also refers to the number of unit VR bandwidth requests generated in the area. For example, if $\psi = 5$ and a unit VR bandwidth is 100 Mbps, then there are 5 requests for a VR of 100 Mbps each. Moreover, we can refer to ψ as the resource demand at the SP level. Also, let $\kappa^{(2)}$ as the total number of VRRs that can be generated in the area. Assume that the SP can submit only one VRR per VR at a time. Therefore, $\kappa^{(2)}$ also refers to the maximum number of VRs an SP can manage. Not all of these VRRs can successfully turn into a VR. Hence, we define the failure rate $r^{(2)}$ of VRRs as the theoretical maximum VRR failures per unit time per VR. Mathematically, it is written as

$$r^{(2)} = -\frac{1}{\psi} \frac{d\psi}{d\tau}. \quad (4)$$

For the InP, if we let ω be the number of active or operational PRs at time τ , then ω also refers to the number of unit PR bandwidth requests. For instance, if $\omega = 2$ and a unit PR bandwidth is 1 Gbps, then there are 2 operational PR each with capacity of 1 Gbps. Also, let $\kappa^{(3)}$ be the total number of PRs owned by the InP. Some of these active PRs may be suddenly unavailable for lease. Thus, the unavailability rate $r^{(3)}$ of PRs is the number of PRs z unavailable per unit time per PR. It can be shown mathematically as

$$r^{(3)} = -\frac{1}{\omega} \frac{d\omega}{d\tau}. \quad (5)$$

TABLE I
NETWORK VIRTUALIZATION PARAMETERS

Symbol	Definition
χ	number of user requests (URs)
ψ	number of virtual resource requests (VRRs)
ω	number of active physical resources (PRs)
τ	time
$r^{(1)}$	arrival rate of user requests (per unit time)
$r^{(2)}$	failure rate of virtual resource requests (per unit time)
$r^{(3)}$	unavailability rate of physical resources (per unit time)
$\kappa^{(1)}$	maximum number of user requests (URs)
$\kappa^{(2)}$	maximum number of virtual resource requests (VRRs)
$\kappa^{(3)}$	maximum number of physical resources (PRs)
s	service rate (per unit time per VRR (or PR))
p	payoff rate (per unit time per UR (or VRR))
q	congestion/competition rate (per unit time per VRR (or PR))
n	max. no. of EUs per VRRs (URs per VRR)
m	max. no. of VRs per PR (VRRs per PR)

Each UR, VRR, and PR corresponds to a request for a unit resource quantity at its respective level. That is, χ , ψ , and ω represent an amount of network resource (i.e., bandwidth). Thus, χ and ψ refer to the resource demand at the EUs and SP, respectively. On the other hand, ω represents the resource supply at the InP.

There are other important parameters in NV. The network *service availability* $s^{(l)}$ is the network user acceptance or service rate per VR (or PR); it measures how many users at level $l \in \{1, 2\}$ are being served by the network. The network *payoff rate* $p^{(l)}$ is regarded as the network-user-to-network-income conversion rate per network user; it quantifies the gain of network at level $l \in \{2, 3\}$ from rendering services. The parameters $s^{(l)}$ and $p^{(l)}$ are related: for the SP,

$$p^{(2)} = \frac{s^{(1)}}{n}, \quad (6)$$

where n is the maximum allowable number of EUs per VR; for the InP,

$$p^{(3)} = \frac{s^{(2)}}{m}, \quad (7)$$

where m is the maximum allowable number of VRs that can coexist per PR. Lastly, the network *congestion (or competition) rate* q is defined as the VR (or PR) competition rate per VR (or PR); accordingly, it is a measure of how much the network traffic degrades the network resources.

If we divide the service rate s by the unit cost of service c_s , we get the per unit cost service rate S . Likewise, dividing the payoff rate p by the unit income quantity c_p yields the per unit income payoff rate P . Also, getting the ratio of the congestion rate q to the unit penalty from congestion c_q gives the per unit penalty congestion rate Q . Mathematically, these quantities are expressed as:

$$S = \frac{s}{c_s}, \quad P = \frac{p}{c_p}, \quad \text{and} \quad Q = \frac{q}{c_q}. \quad (8)$$

Table I summarizes the parameters discussed in this section.

IV. SYSTEM MODEL AND PROBLEM FORMULATION

The main goal of this paper is to introduce a unified framework for the resource allocation and economics problem

in NV. In an NV system that consists of many users, SPs, and InPs, two problems exist. One of the problems is how much resources an SP must own and allocate to the EUs given the dynamics that exist in the NV system such as the amount of connection requests it receives from the EUs, the intensity of competition it experiences from other SPs, and the amount of resources it can request to and can be granted by an InP. Similarly, an InP faces the same problem of how much resources it must operate and allocate to the SPs that are trying to lease resources from it depending on factors such as the number of requests it receives from the SPs, and the intensity of competition it experiences from other InPs. In this paper, these two NV system problems are modeled together in order to provide a framework that covers the three levels of the NV system. The resulting system model, based on the prey-predator food chain model, takes as input the NV system parameters that characterize the dynamics in the system. Then, the resource strategies of the SPs and InPs are computed where a resource strategy refers to the bandwidth capacity that an SP or InP should own in order to support its users.

Three mathematical models, one for each configuration of a three-level NV system, are introduced and developed in this section. The first model is for a simple NV system that has one SP and one InP. The second model is for an NV system with two SPs and one InP. Then, a generalized model for NV systems with N-SPs and M-InPs is introduced. Each model has finite number of EUs. The EUs that are granted with network connection communicate with their respective SP through to its VN. Hence, an SP forms a star topology with its EUs. Since an SP requests only for virtual nodes that are needed to support the EUs, then we assume that its VN capacity is equal to the sum of the capacities of the virtual nodes. Meanwhile, the SPs that are granted with lease of physical resources communicate with their respective InP through its PN. Thus, an InP forms a star topology with the SPs that are leasing its PN. Since an InP operated and manages only physical nodes that are needed to support its client SPs, then we assume that its PN capacity is equal to the sum of the physical node capacities. In each model, the utilities of the SPs and InPs are first modeled as the net amount of cash per unit time they receive from their respective consumers. Since we choose to focus on the distribution of network resources (i.e., bandwidth) within the NV system, the cash flows are converted to the amount of bandwidth the consumers are demanding from their respective producers. These models can be implemented by a trusted central authority that supervises and regulates the NV system in the coverage area.

The purpose of each model is to calculate the equilibrium point of the NV system which signifies the resource strategy of each entity in the NV system and to simulate the economics of the NV system. The results of each simulation demonstrate how the resource strategy at each level and the relationship between demand and supply change with the network parameters. The concept and analysis of equilibrium is discussed in the next section.

This section is organized as follows. First, models for a one-SP, one-InP NV system and a two-SP, one-InP NV system are formulated. Then, the extension of this formulation to

an N -SP, M -InP NV system is developed in the following subsection. Finally, the key results are summarized in the last subsection.

A. One-SP, One-InP Network Virtualization System

Consider an NV system in a metropolitan area with multiple EUs, one SP, and one InP. Let χ be the number of EUs trying to access the VN of the SP by submitting URs to the SP. Assuming an EU submits only one UR at a time, the total number of URs submitted to the SP is also χ . Moreover, χ is also related to the demand at the EU level. Meanwhile, suppose ψ is the number of VRRs the SP submits to the InP. Hence, ψ also means the total number of VRRs needed by the SP to realize its VN and support the demand of the EUs. Lastly, let ω be the number of active PRs in the InP.

To formulate the mathematical model for the NV system, we will follow the flow of payment from the EUs to the SP, and from the SP to the InP. Let $c^{(l)}$ be the unit cost of resources at each level $l \in \{1, 2, 3\}$ of the NV system. Assign constants $c^{(1)}$ as the cost per EU bandwidth, $c^{(2)}$ as the cost per VR bandwidth, and $c^{(3)}$ as the net cost incurred by the InP per PR. Each EU with a successful VN connection pays the SP; hence, the SP earns an amount proportional to the number of successful URs. Meanwhile, the SP has to pay the InP an amount proportional to the number of VRs in its VN. The InP incurs a cost proportional to the number of active PRs in its PN. We can say that the EUs, the SP, and the InP interact through these transactions. Based on the transactions just described, we can formulate the rate of change of the net income at each level.

Suppose $X = c^{(1)}\chi$ is the potential “revenue” at the EU level coming from all the URs, $Y = c^{(2)}\psi$ is the potential revenue of the SP coming from all the VRRs, and $Z = c^{(3)}\omega$ is the potential revenue of the InP for leasing and operating its PN. Also, consider the quantities

$$\begin{aligned} K^{(1)} &= c^{(1)}\kappa^{(1)}, \quad S^{(l)} = \frac{s^{(l)}}{c^{(l+1)}} \Big|_{l \in \{1,2\}}, \\ P^{(l)} &= \frac{p^{(l)}}{c^{(l-1)}} \Big|_{l \in \{2,3\}}, \end{aligned}$$

and

$$Q^{(l)} = \frac{q^{(l)}}{c^{(l)}} \Big|_{l \in \{2,3\}}, \quad (9)$$

where s, p, q, S, P , and Q are the network parameters defined in the previous section, with indices $l \in \{1, 2, 3\}$ indicating the EUs, the SP, and the InP levels, respectively. Referring to interaction between the EUs and the SP described in the previous paragraph, the revenue rate at EU level $\frac{1}{X} \frac{dX}{d\tau}$ is reduced by the service cost charged by the SP, $S^{(1)}Y$. Thus, (2) becomes an equation for net income stated as

$$\frac{1}{X} \frac{dX}{d\tau} = r^{(1)} \left(1 - \frac{X}{K^{(1)}} \right) - S^{(1)}Y. \quad (10)$$

Similarly, the net income rate $\frac{1}{Y} \frac{dY}{d\tau}$ of the SP is the sum of the failed VRR rate, $-r^{(2)}$, the payoff from the EUs, $+P^{(2)}X$, the cost of congestion among the VRRs in

the VN, $-Q^{(2)}Y$, and the cost of service availability of the InP, $-S^{(2)}Z$. Therefore, the net income rate equation (4) for the SP becomes

$$\frac{1}{Y} \frac{dY}{d\tau} = -r^{(2)} + P^{(2)}X - Q^{(2)}Y - S^{(2)}Z. \quad (11)$$

Lastly, the net income rate $\frac{1}{Z} \frac{dZ}{d\tau}$ of the InP becomes the sum of the unavailable PR rate, $-r^{(3)}$, the payoff from the SP, $+P^{(3)}Y$, and the cost of congestion among active PRs, $-Q^{(3)}Z$. Consequently, the net income equation in (5) for the InP is now

$$\frac{1}{Z} \frac{dZ}{d\tau} = -r^{(3)} + P^{(3)}Y - Q^{(3)}Z. \quad (12)$$

Dividing both sides of (10), (11), and (12) by their respective unit costs $c^{(1)}$, $c^{(2)}$, and $c^{(3)}$, and applying the substitutions in (9) yield the following system of nonlinear differential equations:

$$\begin{aligned} \frac{d\chi}{d\tau} &= \chi \left(r^{(1)} \left(1 - \frac{\chi}{\kappa^{(1)}} \right) - s^{(1)}\psi \right)', \\ \frac{d\psi}{d\tau} &= \psi(-r^{(2)} + p^{(2)}\chi - q^{(2)}\psi - s^{(2)}\omega), \\ \frac{d\omega}{d\tau} &= \omega(-r^{(3)} + p^{(3)}\psi - q^{(3)}\omega), \end{aligned} \quad (13)$$

where $s^{(1)}$ and $s^{(2)}$ are the SP and InP service availability rates, $p^{(2)} = \frac{s^{(1)}}{n}$ and $p^{(3)} = \frac{s^{(2)}}{m}$ are the SP and InP payoff rates, and $q^{(2)}$ and $q^{(3)}$ are the SP and InP congestion rates, respectively.

To make the analysis easier, reduce the number of parameters of system in (13) by introducing the following substitutions: $x = \frac{\chi}{\kappa^{(1)}}$, $y = \frac{\psi}{\kappa^{(2)}}$, $z = \frac{\omega}{\kappa^{(3)}}$, and $t = r^{(1)}\tau$. Thus, x, y, z and t represent the following quantities:

- x is the EU capacity demand;
- y is the VN capacity supply/demand;
- z is the PN capacity supply; and
- t is the dimensionless time.

The substitutions yield the following system of dimensionless equations representing the one-SP, one-InP NV system model:

$$\begin{aligned} \frac{dx}{dt} &= x(1 - x - w_1y) = xf(x, y, z), \\ \frac{dy}{dt} &= y(-w_2 + w_3x - w_4y - w_5z) = yg(x, y, z), \\ \frac{dz}{dt} &= z(-w_6 + w_7y - w_8z) = zh(x, y, z). \end{aligned} \quad (14)$$

For convenience, we assume that the SPs and InPs approve and grant only the connection requests they receive, and operate and manage only the nodes that are active. Thus, their VNs and PNs capacities, y and z , respectively, are the same as the total virtual/physical node capacities. That is, a network capacity is equal the sum of its node capacities.

The parameters of (14) are listed in Table II. Since it has been established in Section III that $p^{(2)} = \frac{s^{(1)}}{n}$ for the SP and that $p^{(3)} = \frac{s^{(2)}}{m}$ for the InP, then parameters w_3 and w_7 can also be expressed as

$$w_3 = \frac{w_1}{\eta^{(2)}} \quad \text{and} \quad w_7 = \frac{w_5}{\eta^{(3)}}, \quad (15)$$

TABLE II
ONE-SP, ONE-INP NV DIMENSIONLESS PARAMETERS

Parameter	Definition	Name
x	$\frac{\chi}{\kappa^{(1)}}$	EU capacity demand
y	$\frac{\psi}{\kappa^{(2)}}$	VN capacity supply/demand
z	$\frac{\omega}{\kappa^{(3)}}$	PN capacity supply
t	$r^{(1)}\tau$	dimensionless time
w_1	$\frac{s^{(1)}}{r^{(1)}/\kappa^{(2)}}$	SP service availability ratio
w_2	$\frac{r^{(2)}}{r^{(1)}}$	SP failure ratio
w_3	$\frac{p^{(2)}}{r^{(1)}/\kappa^{(1)}}$	SP payoff factor
w_4	$\frac{q^{(2)}}{r^{(1)}/\kappa^{(2)}}$	SP congestion factor
w_5	$\frac{s^{(2)}}{r^{(1)}/\kappa^{(3)}}$	InP service availability ratio
w_6	$\frac{r^{(3)}}{r^{(1)}}$	InP unavailability ratio
w_7	$\frac{p^{(3)}}{r^{(1)}/\kappa^{(2)}}$	InP payoff factor
w_8	$\frac{q^{(3)}}{r^{(1)}/\kappa^{(3)}}$	InP congestion factor

where

$$\eta^{(2)} = \frac{\kappa^{(2)}}{\kappa^{(1)}/n} \quad \text{and} \quad \eta^{(3)} = \frac{\kappa^{(3)}}{\kappa^{(2)}/m} \quad (16)$$

are the network coverage ratios of the SP and the InP, respectively. These two quantities represent the ratio of the maximum network resource supply to the maximum network resource demand.

The association between the demand x and supply y can be derived using the chain rule, $\frac{dv}{du} = \frac{dv}{dt} \cdot \frac{dt}{du}$. Hence, it follows that,

$$\frac{dy}{dx} = \frac{y(-w_2 + w_3x - w_4y - w_5z)}{x(1 - x - w_1y)} \quad (17)$$

Likewise, the relationship between demand y and supply z can be written as

$$\frac{dz}{dy} = \frac{z(-w_6 + w_7y - w_8z)}{y(-w_2 + w_3x - w_4y - w_5z)} \quad (18)$$

For the one-SP, one-InP system in (14), the resource strategy is the positive equilibrium point denoted by $E(x^*, y^*, z^*)$. Meanwhile, the demand-supply relationships can be found by solving for $y = f(x)$ and $z = g(y)$. $f(x)$ indicates the behavior of the VN capacity supply with respect to the EU capacity demand x ; whereas, the function $g(y)$ specifies the reaction of the PN capacity supply in terms of the VN capacity demand. Since (17) and (18) are both first-order nonlinear differential equations (NDE), a numerical method is used to graph $f(x)$ and $g(y)$.

B. Two-SP, One-InP Network Virtualization System

Based on the formulation presented in the previous subsection, the system model for an NV system serviced by two SPs ($N = 2$) and one InP ($M = 1$) is expressed by the

following system of equations:

$$\begin{aligned} \frac{d\chi}{d\tau} &= \chi \left(r^{(1)} \left(1 - \frac{\chi}{\kappa^{(1)}} \right) - s_1^{(1)}\psi_1 - s_2^{(1)}\psi_2 \right), \\ \frac{d\psi_1}{d\tau} &= \psi_1(-r_1^{(2)} + p_1^{(2)}\chi - q_{11}^{(2)}\psi_1 - q_{21}^{(2)}\psi_2 - s^{(2)}\omega), \\ \frac{d\psi_2}{d\tau} &= \psi_2(-r_2^{(2)} + p_2^{(2)}\chi - q_{12}^{(2)}\psi_1 - q_{22}^{(2)}\psi_2 - s^{(2)}\omega), \\ \frac{d\omega}{d\tau} &= \omega(-r^{(3)} + p^{(3)}(\psi_1 + \psi_2) - q^{(3)}\omega). \end{aligned} \quad (19)$$

where $s_j^{(1)}$, $j \in \{1, 2\}$ is the SP_{*j*}-to-EU service availability rate, $s^{(2)}$ is the InP-to-SP service availability rate, $p_j^{(2)} = \frac{s_j^{(1)}}{m_j}$ is the EU-to-SP_{*j*} payoff rate, $p^{(3)} = \frac{s^{(2)}}{n}$ is the SP-to-InP payoff rate, $q_{jj}^{(2)}$ is the SP_{*j*} congestion rate, $q_{Jj}^{(2)}$, $j \neq J$ is the SP_{*J*}-to-SP_{*j*} competition rate, and $q^{(3)}$ is the InP congestion rate.

For analysis convenience, the dimensionless form of (19) for a two-SP, one-InP NV system is given by

$$\begin{aligned} \frac{dx}{dt} &= x(1 - x - w_1y_1 - w_2y_2) = xf(x, y_1, y_2, z), \\ \frac{dy_1}{dt} &= y_1(-w_3 + w_4x - w_5y_1 - w_6y_2 - w_7z) \\ &= y_1g_1(x, y_1, y_2, z), \\ \frac{dy_2}{dt} &= y_2(-w_8 + w_9x - w_{10}y_1 - w_{11}y_2 - w_7z) \\ &= y_2g_2(x, y_1, y_2, z), \\ \frac{dz}{dt} &= z(-w_{12} + w_{13}y_1 + w_{14}y_2 - w_{15}z) = zh(x, y, z), \end{aligned} \quad (20)$$

where

- x is the EU capacity demand;
- y_j , $j \in \{1, 2\}$ is the VN_{*j*} capacity supply/demand;
- z is the PN capacity supply; and
- t is the dimensionless time.

In this manuscript, we assume that the SPs and InPs accept and allow only the connection requests they receive, and operate and manage only the nodes that are active. Thus, their VNs and PNs capacities, y_j and z , respectively, are the same as the total virtual/physical node capacities. That is, a network capacity is equal the sum of its node capacities.

The parameters of (20) are listed in Table III. Since it has been established in Section III that $p_j^{(2)} = \frac{s_j^{(1)}}{m_j}$ for SP_{*j*} and that $p^{(3)} = \frac{s^{(2)}}{n}$ for the sole InP, we can express

$$w_4 = \frac{w_1}{\eta_1^{(2)}}, \quad w_9 = \frac{w_2}{\eta_2^{(2)}}, \quad w_{13} = \frac{w_7}{\eta_1^{(3)}}, \quad \text{and} \quad w_{14} = \frac{w_7}{\eta_2^{(3)}}, \quad (21)$$

where the quantities

$$\eta_j^{(2)} = \frac{\kappa_j^{(2)}}{\kappa^{(1)}/n_j} \quad \text{and} \quad \eta_j^{(3)} = \frac{\kappa^{(3)}}{\kappa_j^{(2)}/m} \quad (22)$$

are the SP and InP network coverage ratios, respectively.

The relationship between the demand x and its supplies y_1 and y_2 , as well as the demands y_1 and y_2 and their supply z ,

TABLE III
TWO-SP, ONE-INP NV DIMENSIONLESS PARAMETERS

Parameter	Definition	Name
x	$\frac{\chi}{\kappa^{(1)}}$	EU capacity demand
y_j	$\frac{\psi_j}{\kappa_j^{(2)}}$	VN _j capacity supply/demand
z	$\frac{\omega}{\kappa^{(3)}}$	PN capacity supply
t	$r^{(1)}\tau$	dimensionless time
w_1	$\frac{s_1^{(1)}}{r^{(1)}/\kappa_1^{(2)}}$	SP ₁ -to-EU service availability factor
w_2	$\frac{s_2^{(1)}}{r^{(1)}/\kappa_2^{(2)}}$	SP ₂ -to-EU service availability factor
w_3	$\frac{r_1^{(2)}}{r^{(1)}}$	SP ₁ failure ratio
w_4	$\frac{p_1^{(2)}}{r^{(1)}/\kappa^{(1)}}$	EU-to-SP ₁ payoff factor
w_5	$\frac{q_1^{(2)}}{r^{(1)}/\kappa_1^{(2)}}$	SP ₁ congestion factor
w_6	$\frac{q_2^{(2)}}{r^{(1)}/\kappa_2^{(2)}}$	SP ₂ -to-SP ₁ competition factor
w_7	$\frac{s^{(2)}}{r^{(1)}/\kappa^{(3)}}$	InP-to-SP service availability factor
w_8	$\frac{r_2^{(2)}}{r^{(1)}}$	SP ₂ failure ratio
w_9	$\frac{p_2^{(2)}}{r^{(1)}/\kappa^{(1)}}$	EU-to-SP ₂ payoff factor
w_{10}	$\frac{q_1^{(2)}}{r^{(1)}/\kappa_1^{(2)}}$	SP ₁ -to-SP ₂ competition factor
w_{11}	$\frac{q_2^{(2)}}{r^{(1)}/\kappa_2^{(2)}}$	SP ₂ congestion factor
w_{12}	$\frac{r^{(3)}}{r^{(1)}}$	InP unavailability factor
w_{13}	$\frac{p^{(3)}}{r^{(1)}/\kappa_1^{(2)}}$	SP ₁ -to-InP payoff factor
w_{14}	$\frac{p^{(3)}}{r^{(1)}/\kappa_2^{(2)}}$	SP ₂ -to-InP payoff factor
w_{15}	$\frac{q^{(3)}}{r^{(1)}/\kappa^{(3)}}$	InP congestion factor

are implicitly defined by the following equations:

$$\frac{dy_1}{dx} = \frac{y_1(-w_3 + w_4x - w_5y_1 - w_6y_2 - w_7z)}{x(1 - x - w_1y_1 - w_2y_2)}, \quad (23)$$

$$\frac{dy_2}{dx} = \frac{y_2(-w_8 + w_9x - w_{10}y_1 - w_{11}y_2 - w_7z)}{x(1 - x - w_1y_1 - w_2y_2)}, \quad (24)$$

$$\frac{dz}{dy_1} = \frac{z(-w_{12} + w_{13}y_1 + w_{14}y_2 - w_{15}z)}{y_1(-w_3 + w_4x - w_5y_1 - w_6y_2 - w_7z)}, \quad (25)$$

$$\frac{dz}{dy_2} = \frac{z(-w_{12} + w_{13}y_1 + w_{14}y_2 - w_{15}z)}{y_2(-w_8 + w_9x - w_{10}y_1 - w_{11}y_2 - w_7z)}. \quad (26)$$

C. N-SP, M-InP Network Virtualization System

In this subsection, we generalize the system model to an N-SP, M-InP NV system. Consider an NV system in a metropolitan area with multiple EUs, N SPs, and M InPs. Let χ_j be the number of EUs in the area that submits URs to SP_j, $j \in \{1, 2, \dots, N\}$. Assuming an EU submits only one UR at a time, the total number of URs in the area is $\chi = \sum_{j=1}^N \chi_j$. Meanwhile, suppose ψ_{jk} is the number of VRRs in SP_j to any InP_k, $k \in \{1, 2, \dots, M\}$. Thus, the total number of VRRs needed by SP_j to realize its VN is $\psi_j = \sum_{k=1}^M \psi_{jk}$. Lastly, let ω_k be the number of active PRs in InP_k. As in the previous

subsection, we will follow the flow of payment from the EUs to the SPs, and from the SPs to the InP.

Suppose $X = \sum_{j=1}^N c_j^{(1)} \chi_j$ is the potential “revenue” at the area from all the URs to any SP_j, $Y_j = \sum_{k=1}^M c_k^{(2)} \psi_{jk}$ is the potential revenue of SP_j from all the VRRs to any InP_k, and $Z_k = c_k^{(3)} \omega_k$ is the potential revenue of the InP_k for leasing and operating its PN. The revenue at the area is reduced by the service cost charged by every SP_j to the EUs, $S_{11}^{(1)} Y_1 + S_{12}^{(1)} Y_2 + \dots + S_{1N}^{(1)} Y_N$, where the first subscript “1” refers to the metropolitan area (i.e., $i = 1$) where the NV system is. Consequently, the net “income” rate $\frac{1}{X} \frac{dX}{d\tau}$ in (2) can now be stated as

$$\frac{1}{X} \frac{dX}{d\tau} = r^{(1)} \left(1 - \frac{X}{K^{(1)}} \right) - \sum_{j=1}^N S_{1j}^{(1)} Y_j. \quad (27)$$

Meanwhile, the net income rate $\frac{1}{Y_j} \frac{dY_j}{d\tau}$ of SP_j is now the sum of the failed VRR rate, $r_j^{(2)}$, the payoff from the EUs at the service area, $P_j^{(2)} X$, the cost of *congestion* within SP_j, $Q_{jj} Y_j$, the cost of *competition* from every other SP_J, $J \in \{1, 2, \dots, N\} \setminus j$, $Q_{j1}^{(2)} Y_1 + Q_{j2}^{(2)} Y_2 + \dots + Q_{jN}^{(2)} Y_N$, and the cost service availability from every InP_k, $S_{j1}^{(2)} Z_1 + S_{j2}^{(2)} Z_2 + \dots + S_{jM}^{(2)} Z_M$. Therefore, the net income rate equation (4) for SP_j becomes

$$\frac{1}{Y_j} \frac{dY_j}{d\tau} = -r_j^{(2)} + P_j^{(2)} X - \sum_{J=1}^N Q_{jJ}^{(2)} Y_J - \sum_{k=1}^M S_{jk}^{(2)} Z_k. \quad (28)$$

Lastly, the net income rate $\frac{1}{Z_k} \frac{dZ_k}{d\tau}$ of InP_k becomes the sum of the unavailable PR rate, $r_k^{(3)}$, the payoff from every SP_j, $P_{k1}^{(3)} Y_1 + P_{k2}^{(3)} Y_2 + \dots + P_{kN}^{(3)} Y_N$, the cost of *congestion* within InP_k, $Q_{kk} Z_k$, and the cost of *competition* from every other InP_K, $K \in \{1, 2, \dots, M\} \setminus k$, $Q_{k1}^{(3)} Z_1 + Q_{k2}^{(3)} Z_2 + \dots + Q_{kM}^{(3)} Z_M$. Hence, (5) for InP_k is now

$$\frac{1}{Z_k} \frac{dZ_k}{d\tau} = -r_k^{(3)} + \sum_{j=1}^N P_{kj}^{(3)} Y_j - \sum_{K=1}^M Q_{kK}^{(3)} Z_K. \quad (29)$$

Using the average VR bandwidth cost, $c^{(2)} = (\sum_{k=1}^M c_k^{(2)})/M$, we can approximate $Y_j \approx c^{(2)} \psi_j$. Substituting $K^{(1)} = c^{(1)} \kappa^{(1)}$, $P_{\{j,k\}}^{(l)} = p_{\{j,k\}}^{(l)}/c^{(l-1)}$, $Q_{\{jJ,kK\}}^{(l)} = q_{\{jJ,kK\}}^{(l)}/c^{(l)}$, and $S_{\{1j,jk\}}^{(l)} = s_{\{1j,jk\}}^{(l)}/c^{(l+1)}$ to (27), (28), and (29) yields the following system of equations:

$$\begin{aligned} \frac{d\chi}{d\tau} &= \chi \left(r^{(1)} \left(1 - \frac{\chi}{\kappa^{(1)}} \right) - \sum_{j=1}^N s_{1j}^{(1)} \psi_j \right), \\ \frac{d\psi_j}{d\tau} &= \psi_j \left(-r_j^{(2)} + p_j^{(2)} \chi - \sum_{J=1}^N q_{jJ}^{(2)} \psi_J - \sum_{k=1}^M s_{jk}^{(2)} \omega_k \right), \\ \frac{d\omega_k}{d\tau} &= \omega_k \left(-r_k^{(3)} + \sum_{j=1}^N p_{kj}^{(3)} \psi_j - \sum_{K=1}^M q_{kK}^{(3)} \omega_K \right). \end{aligned} \quad (30)$$

The parameters in an N-SP, M-InP NV system in (30) are listed in Table IV. The dimensionless form

TABLE IV
N-SP, M-INP NV SYSTEM PARAMETERS

Symbol	Definition
χ	number of user requests (URs) at service area
ψ_j	number of virtual resource requests (VRRs) at SP_j
ω_k	number of active physical resources at InP_k
τ	time
$r^{(1)}$	arrival rate of URs at service area
$r_j^{(2)}$	failure rate of VRRs at SP_j
$r_k^{(3)}$	unavailability rate of PRs at InP_k
$s_{1j}^{(1)}$	SP_j -to-EU service rate
$p_j^{(2)}$	EU-to- SP_j payoff rate
$q_{jJ}^{(2)}$	SP_j -to- SP_j congestion/competition rate
$s_{jk}^{(2)}$	InP_k -to- SP_j service rate
$p_{kj}^{(3)}$	SP_j -to- InP_k payoff rate
$q_{kK}^{(3)}$	InP_K -to- InP_k congestion/competition rate

TABLE V
N-SP, M-INP NV SYSTEM DIMENSIONLESS PARAMETERS

Symbol	Definition	Name
x	$\frac{\chi \tau}{\kappa_j^{(1)}}$	EU demand
y_j	$\frac{\psi_j \tau}{\kappa_j^{(2)}}$	VN_j network demand
z_k	$\frac{\omega_k \tau}{\kappa_k^{(3)}}$	PN_k network supply
t	$r^{(1)} \tau$	time
w_{r_j}	$\frac{r_j^{(2)}}{r^{(1)}}$	failure rate of VRRs at SP_j
w_{r_k}	$\frac{r_k^{(3)}}{r^{(1)}}$	unavailability rate of PRs at InP_k
$w_{s_{1j}}$	$\frac{s_{1j}^{(1)}}{r^{(1)}/\kappa_j^{(2)}}$	SP_j -to-EU service rate
w_{p_j}	$\frac{p_j^{(2)}}{r^{(1)}/\kappa_j^{(1)}}$	EU-to- SP_j payoff rate
$w_{q_{jJ}}$	$\frac{q_{jJ}^{(2)}}{r^{(1)}/\kappa_j^{(2)}}$	SP_j -to- SP_j congestion/competition rate
$w_{s_{jk}}$	$\frac{s_{jk}^{(2)}}{r^{(1)}/\kappa_k^{(3)}}$	InP_k -to- SP_j service rate
$w_{p_{kj}}$	$\frac{p_{kj}^{(3)}}{r^{(1)}/\kappa_j^{(2)}}$	SP_j -to- InP_k payoff rate
$w_{q_{kK}}$	$\frac{q_{kK}^{(3)}}{r^{(1)}/\kappa_k^{(3)}}$	InP_K -to- InP_k congestion/competition rate

of (30) is

$$\begin{aligned}
 \frac{dx}{dt} &= x \left(1 - x - \sum_{j=1}^N w_{s_{1j}} y_j \right), \\
 \frac{dy_j}{dt} &= y_j \left(-w_{r_j} + w_{p_j} x - \sum_{J=1}^N w_{q_{jJ}} y_J - \sum_{k=1}^M w_{s_{jk}} z_k \right), \\
 \frac{dz_k}{dt} &= z_k \left(-w_{r_k} + \sum_{j=1}^N w_{p_{kj}} y_j - \sum_{K=1}^M w_{q_{kK}} z_K \right), \quad (31)
 \end{aligned}$$

where the corresponding parameters are defined in Table V.

D. Network Economics

The relationship between a capacity demand and a capacity supply in an NV system can be found by combining their time rate equations through chain rule and solving the resulting differential equation numerically. Specifically, if we are

interested in the relationship between the capacity demand x at the EU level and the capacity supply y_j of SP_j , then we find $\frac{dy_j}{dx}$ and solve it numerically. Similarly, if we are interested in how the capacity supply z_k of InP_k behaves with respect to the capacity demand y_j of SP_j , then we determine $\frac{dz_k}{dy_j}$ and solve it using a numerical method. In this paper, the numerical method used to solve the resulting differential equation is the 4th-order Runge-Kutta method [33].

V. PROPERTIES AND ANALYSES

The following section provides the properties of NV systems modeled as a prey-predator food chain system. In addition, this section presents the derivation of the solution point of an NV system. In a prey-predator food chain system, this solution point, also known as the equilibrium point, refers to the state at which the number of agents at each level of the chain does not change [25]. Equivalently, in NV, this equilibrium point refers to the resource strategy (i.e., amount of resources) at each level of the NV system based on the system dynamics, such as service, payoff, failure, and congestion/competition rates. In other words, if we can quantify the dynamics of the NV system, we can compute the resource strategy of each entity in an NV system based on these dynamics using the prey-predator food chain model.

This section also introduces the conditions at which the equilibrium point is stable. A stable equilibrium point means that the state of the system goes back to its equilibrium after a small perturbation in the number of agents. In NV, this means that the resource strategy remains the same even after the introduction of a small change in the number of bandwidth requests from the EUs the SPs or allocation from the InPs. For more comprehensive analysis on the prey-predator food chain model, the reader may refer to [26]–[28].

This section is organized as follows. First, the properties and analysis of a one-SP, one-InP NV system is presented. Then, we analyze and extend these properties to a two-SP, one-InP NV system. Next, we present pointers on analyzing an N -SP, M -InP NV system. Finally, a summary of this section is provided.

A. One-SP, One-InP Network Virtualization System

This subsection is devoted to the properties and analysis of the one-SP, one-InP NV system in (14). These properties support the use of the prey-predator food chain model to represent an NV system. The network parameters w_1 through w_8 described and derived in the previous section dictate the behavior and properties of a virtualized network. Moreover, an analysis to compute the solution point to the system in (14) is also discussed. Theoretically, this solution (i.e., the positive equilibrium point) signifies the values of x , y , and z for which the system is stable indefinitely with respect to time given that the system parameters remain the same. Thus, in NV, this solution refers to the resource strategy at each level given the economic dynamics of the NV system.

The prey-predator food chain model exhibits properties that an NV system also possess. The following lemma states the effect of SP service availability to the entire system.

Lemma 1: If $w_1 = 0$, and consequently, $w_3 = 0$, then $\lim_{t \rightarrow \infty} x(t) = 1$ and $\lim_{t \rightarrow \infty} y(t) = \lim_{t \rightarrow \infty} z(t) = 0$.

Proof: If we substitute $w_1 = w_3 = 0$ to (14), its first equation becomes $\frac{dx}{dt} = x(1 - x)$. The solution of this differential equation is $x(t) = \frac{x_0}{x_0 + (1 - x_0)e^{-t}}$, where $x_0 = x(t = 0)$. Evaluating its limit yields $\lim_{t \rightarrow \infty} x(t) = 1$. Doing the same to the second and third equations of (14) brings $\lim_{t \rightarrow \infty} y(t) = \lim_{t \rightarrow \infty} z(t) = 0$. ■

That is, if the SP is unavailable, the EU demand still reaches maximum while the VN supply/demand, and consequently, the PN supply eventually becomes zero.

Likewise, we arrive at the following statement regarding the availability of the InP.

Proposition 1: If $w_5 = 0$, and hence, $w_7 = 0$, then it follows that $\lim_{t \rightarrow \infty} x(t) = \frac{w_1 w_2 + w_4}{w_1 w_3 + w_4} = \bar{x} > 0$, $\lim_{t \rightarrow \infty} y(t) = \frac{-w_2 + w_3 \bar{x}}{w_4} = \bar{y} > 0$, and $\lim_{t \rightarrow \infty} z(t) = 0$.

The following lemma states that if the payoff of the SP is high enough, it can continue to demand from the InP, and hence, can continue to support the EUs.

Lemma 2: If $w_3 > \frac{w_2 + w_5}{x} z$, then $y > 0$.

Proof: The solution to the second equation of (14) is $y(t) = \frac{b_1 y_0}{b_2 y_0 + (b_1 - b_2 y_0)e^{-b_1 t}}$, where $y_0 = y(t = 0)$, $b_1 = -w_2 + w_3 x - w_5 z$ and $b_2 = w_4$. Consequently, $\lim_{t \rightarrow \infty} y(t) = \frac{b_1}{b_2} > 0$, where it follows that $w_3 > \frac{w_2 + w_5}{x} z$. ■

A similar statement can be expressed for the InP as well.

Proposition 2: If $w_7 > \frac{w_6}{y}$, then $z > 0$.

The analysis of the system in (14) starts with finding its equilibrium point. The following statement expresses the definition of an equilibrium point.

Definition 1: The equilibrium points of a system $U(t) = [x(t) y(t) z(t)]^T$ are the solutions to $\frac{dU}{dt} = 0$.

Consequently, the system defined by (14) has four equilibrium points:

- $E_0(0, 0, 0)$ indicates the absence of all the system levels,
- $E_1(1, 0, 0)$ shows that the EU will still reach its maximum capacity even in the absence of VN and PN networks,
- $E_2(\bar{x}, \bar{y}, 0)$ denotes the absence of the PN, and
- $E_3(x^*, y^*, z^*)$ specifies the strategies of the SP and the InP given the capacity demand at the EUs.

The three equilibrium points are irrelevant to our study of a fully functional NV environment. Therefore, we will only focus on the positive equilibrium point $E_3(x^*, y^*, z^*)$ or simply $E(x^*, y^*, z^*)$. The existence of this point is defined as follows.

Definition 2: The positive equilibrium point $E(x^*, y^*, z^*)$ exists if there is a positive solution $\{(x^*, y^*, z^*) | x^* > 0, y^* > 0, z^* > 0\}$ to the following equalities:

$$\frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = 0. \quad (32)$$

Solving (32) simultaneously gives the respective resource strategy of the SP and the InP:

$$y^* = \frac{1 - x^*}{w_1}, \quad (33)$$

$$z^* = \frac{-w_6 + w_7 y^*}{w_8}. \quad (34)$$

Since x^* , y^* , and $z^* > 0$, it follows that

$$1 - w_1 y^* > 0, \quad w_3 w_8 + w_5 w_6 > w_2 w_8, \quad \text{and } w_7 y^* > w_6. \quad (35)$$

In order for the system in (14) to be stable near or at its equilibrium point $E(x^*, y^*, z^*)$, it must satisfy certain conditions. According to [29], a locally asymptotically stable system is defined as follows:

Definition 3: A system $U(t) = [x(t) y(t) z(t)]^T$ is locally asymptotically stable near or at an equilibrium point U_E if there is a constant $K > 0$ such that $\|U(0) - U_E\| \leq K$. Thus, $U(t) \rightarrow U_E$ as $t \rightarrow \infty$.

To determine the behavior of the system near or at its equilibrium points, its Jacobian matrix must be computed first. For a system of the form

$$\frac{dx}{dt} = x f(x, y, z), \quad \frac{dy}{dt} = y g(x, y, z), \quad \frac{dz}{dt} = z h(x, y, z), \quad (36)$$

the Jacobian matrix is defined by

$$J(x^*, y^*, z^*) = \begin{bmatrix} x \frac{\partial f}{\partial x} + f & x \frac{\partial f}{\partial y} & x \frac{\partial f}{\partial z} \\ y \frac{\partial g}{\partial x} & y \frac{\partial g}{\partial y} + g & y \frac{\partial g}{\partial z} \\ z \frac{\partial h}{\partial x} & z \frac{\partial h}{\partial y} & z \frac{\partial h}{\partial z} + h \end{bmatrix}. \quad (37)$$

The characteristic polynomial of $J(x^*, y^*, z^*)$ is

$$p_J(\lambda) = \lambda^3 + A_2 \lambda^2 + A_1 \lambda + A_0 \quad (38)$$

where $A_0 = -J_{11} J_{22} J_{33} + J_{11} J_{23} J_{32} + J_{12} J_{21} J_{33}$, $A_1 = J_{11} J_{22} + J_{11} J_{33} + J_{22} J_{33} - J_{23} J_{32} - J_{12} J_{21}$, $A_2 = -(J_{11} + J_{22} + J_{33})$.

The Routh-Hurwitz stability criterion [30], [31] provides the necessary and sufficient condition for the stability of linear time-invariant (LTI) systems. According to this criterion, a third-order system characterized by its third-order characteristic polynomial (38) is locally asymptotically stable near or at its equilibrium point if $A_2 > 0$, $A_0 > 0$, and $A_2 A_1 > A_0$.

Lemma 3: If $J_{11} < 0$, $J_{22} < 0$, and $J_{33} < 0$, then the system in (14) is locally asymptotically stable near or at the equilibrium point $E(x^*, y^*, z^*)$. That is, $J_{11} = 1 - 2x^* - w_1 y^* < 0$, $J_{22} = -w_2 + w_3 x^* - 2w_4 y^* - w_5 z^* < 0$, and $J_{33} = -w_6 + w_7 y^* - 2w_8 z^* < 0$.

Proof: All the terms of A_0 are greater than zero if and only if J_{11} , J_{22} , and J_{33} are all less than zero. Similarly, $A_2 > 0$ if and only if all of its terms are greater than zero. Consequently, the condition $A_2 A_1 > A_0$ is also satisfied. ■

B. Two-SP, One-InP Network Virtualization System

The properties of the system in (14) can be extended for the system in (20). From Lemma 1 about the SP availability, if $w_1 = w_2 = 0$, and consequently, $w_4 = w_9 = 0$, then $\lim_{t \rightarrow \infty} x(t) = 1$ and $\lim_{t \rightarrow \infty} y_1(t) = \lim_{t \rightarrow \infty} y_2(t) = \lim_{t \rightarrow \infty} z(t) = 0$. Regarding the InP availability, if $w_7 = 0$, and consequently, $w_{13} = w_{14} = 0$, then, $\lim_{t \rightarrow \infty} x(t) = \bar{x} > 0$, $\lim_{t \rightarrow \infty} y_1(t) = \bar{y}_1 > 0$, $\lim_{t \rightarrow \infty} y_2(t) = \bar{y}_2 > 0$, and

$\lim_{t \rightarrow \infty} z(t) = 0$. Extending Lemma 2 to (20), the respective payoff of each SP must be high enough to continue supporting the URs of the EUs. That is,

$$w_4 > \frac{w_3 + w_6 y_2 + w_7 z}{x} \quad \text{and} \quad w_9 > \frac{w_8 + w_{10} y_1 + w_7 z}{x}. \quad (39)$$

The positive equilibrium point $E(x^*, y_1^*, y_2^*, z^*)$ of (20) is found by letting

$$\frac{dx}{dt} = \frac{dy_1}{dt} = \frac{dy_2}{dt} = \frac{dz}{dt} = 0. \quad (40)$$

Hence, $E(x^*, y_1^*, y_2^*, z^*)$ is the solution to the following system of equations:

$$\begin{aligned} 1 - x - w_1 y_1 - w_2 y_2 &= 0, \\ -w_3 + w_4 x - w_5 y_1 - w_6 y_2 - w_7 z &= 0, \\ -w_8 + w_9 x - w_{10} y_1 - w_{11} y_2 - w_7 z &= 0, \\ -w_{12} + w_{13} y_1 + w_{14} y_2 - w_{15} z &= 0. \end{aligned} \quad (41)$$

Solving (41) for x^* , y_1^* , y_2^* , and z^* and setting x^* , y_1^* , y_2^* , and $z^* > 0$ provide the following conditions to be satisfied for the existence of the positive equilibrium point $E(x^*, y_1^*, y_2^*, z^*)$:

$$\begin{aligned} w_1 y_1^* + w_2 y_2^* &< 1, \\ &\times \left(w_2 w_9 + w_{11} + \frac{w_7 w_{14}}{w_{15}} \right) \left(-w_3 + w_4 + \frac{w_7 w_{12}}{w_{15}} \right) \\ &> \left(w_2 w_4 + w_6 + \frac{w_7 w_{14}}{w_{15}} \right) \left(-w_8 + w_9 + \frac{w_7 w_{12}}{w_{15}} \right), \\ &\times \left(w_1 w_4 + w_5 + \frac{w_7 w_{13}}{w_{15}} \right) \left(-w_8 + w_9 + \frac{w_7 w_{12}}{w_{15}} \right) \\ &> \left(w_1 w_9 + w_{10} + \frac{w_7 w_{13}}{w_{15}} \right) \left(-w_3 + w_4 + \frac{w_7 w_{12}}{w_{15}} \right), \\ &\times w_{13} y_1^* + w_{14} y_2^* > w_{12}. \end{aligned} \quad (42)$$

The behavior of the system in (20) near or at $E(x^*, y_1^*, y_2^*, z^*)$ can be analyzed by computing its Jacobian matrix J at $E(x^*, y_1^*, y_2^*, z^*)$.

The characteristic polynomial of $J(x^*, y_1^*, y_2^*, z^*)$ is

$$p_J(\lambda) = \lambda^4 + A_3 \lambda^3 + A_2 \lambda^2 + A_1 \lambda + A_0 \quad (43)$$

where the leading coefficients are found by solving for the determinant

$$p_J(\lambda) = \det(J - \lambda I). \quad (44)$$

For a fourth-degree polynomial $p_J(\lambda)$, the conditions for stability are $A_3 > 0$, $A_2 > 0$, $A_3 A_2 > A_1 > 0$, and $(A_3 A_2 A_1 - A_1^2)/A_3^2 > A_0 > 0$ [32].

Lemma 4: If $J_{11} < 0$, $J_{22} < 0$, $J_{33} < 0$, and $J_{44} < 0$, then the system in (20) is locally asymptotically stable near or at the equilibrium point $E(x^*, y_1^*, y_2^*, z^*)$. That is, $J_{11} = 1 - 2x^* - w_1 y_1^* - w_2 y_2^* < 0$, $J_{22} = -w_3 + w_4 x^* - 2w_5 y_1^* - w_6 y_2^* - w_7 z^* < 0$, $J_{33} = -w_8 + w_9 x^* - w_{10} y_1^* - 2w_{11} y_2^* - w_7 z^*$, and $J_{44} = -w_{12} + w_{13} y_1^* + w_{14} y_2^* - 2w_{15} z^* < 0$.

Proof: Based on (41), J_{11} , J_{22} , J_{33} , and $J_{44} < 0$ when x^* , y_1^* , y_2^* , and $z^* > 0$. ■

C. N-SP, M-InP Network Virtualization System

Referring to the previous subsection, the properties and analysis of (14) and (20) can be generalized to a more general case of a N -SP, M -InP NV system. For instance, for an NV system over one metropolitan area, the positive equilibrium point $E^*(x^*, y_1^*, \dots, y_N^*, z_1^*, \dots, z_M^*)$ of (31) is the solution to the following system of equations:

$$\frac{dx}{dt} = \frac{dy_1}{dt} = \dots = \frac{dy_N}{dt} = \frac{dz_1}{dt} = \dots = \frac{dz_M}{dt} = 0. \quad (45)$$

Combining (31) and (45), the system has only one positive equilibrium point where $x, y_j, z_k > 0$ since the resulting $N + M + 1$ simultaneous equations

$$\begin{aligned} 1 - x - \sum_{j=1}^N w_{s_{1j}} y_j &= 0, \\ -w_{r_j} + w_{p_j} x - \sum_{J=1}^N w_{q_{jJ}} y_J - \sum_{k=1}^M w_{s_{jk}} z_k &= 0, \\ -w_{r_k} + \sum_{j=1}^N w_{p_{kj}} y_j - \sum_{K=1}^M w_{q_{kK}} z_K &= 0, \end{aligned} \quad (46)$$

are all linear in variables x , y_j , and z_k .

The stability analysis of the NV system involves computing for its $(N + M + 1) \times (N + M + 1)$ Jacobian matrix at $E^*(x^*, y_1^*, \dots, y_N^*, z_1^*, \dots, z_M^*)$, as well as conforming with the Routh-Hurwitz stability criteria for a $(N + M + 1)$ -degree characteristic polynomial.

D. Network Resource Strategies

This section has focused on solving for the equilibrium point or resource strategies in an NV system. From the first equation of (46), the strategy of SP_j is

$$y_j^* = \frac{1}{w_{s_{1j}}} \left(1 - x^* - \sum_{\substack{J=1 \\ J \neq j}}^N w_{s_{1J}} y_J^* \right). \quad (47)$$

Meanwhile, the strategy of InP_k can be derived from the third equation of (46),

$$z_k^* = \frac{1}{w_{q_{kk}}} \left(-w_{r_k} + \sum_{j=1}^N w_{p_{kj}} y_j^* - \sum_{\substack{K=1 \\ K \neq k}}^M w_{q_{kK}} z_K^* \right). \quad (48)$$

Eqs. (47) and (48) signify the optimal strategies of the SPs and the InPs given the dynamics of the NV system. These are helpful in analyzing how the network parameters affect the network resource strategies of the SPs and the InPs.

VI. NUMERICAL SIMULATIONS

In this section, we perform numerical simulations that demonstrate the prey-predator food chain model as a mathematically framework for NV. For simplicity, we assume that the VN/PN network capacities are the same as the total virtual/physical node capacities. That is, a network capacity is equal the sum of its node capacities.

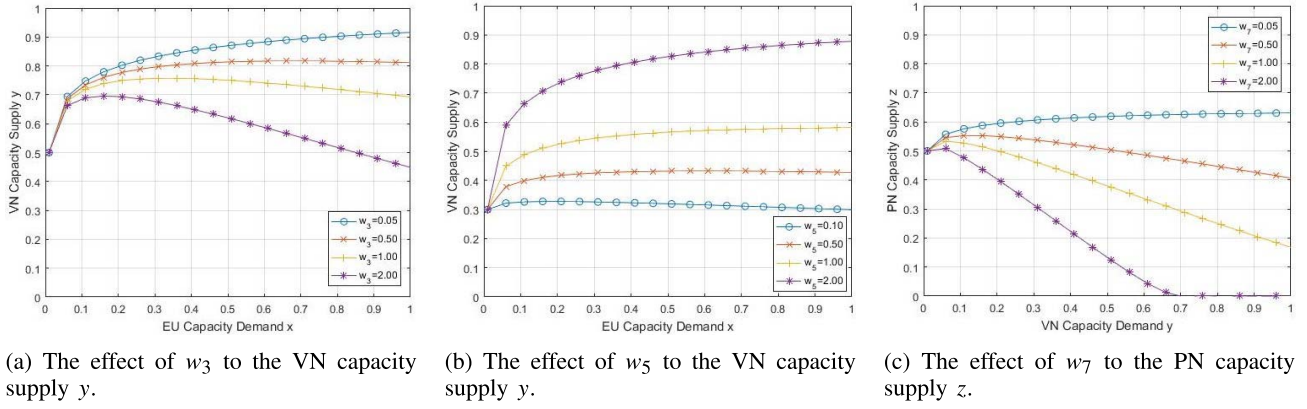


Fig. 2. Network economics of a one-SP, one-InP NV system.

TABLE VI
PARAMETER VALUES FOR ONE-SP, ONE-INP NV SYSTEM

EU		SP		InP	
Parameter	Value	Parameter	Value	Parameter	Value
$\kappa^{(1)}$	1000	$\kappa^{(2)}$	50	$\kappa^{(3)}$	50
		n	25	m	4
		w_1	0.50	w_5	0.50
		w_2	0.10	w_6	0.10
		w_3	0.50	w_7	0.50
		w_4	0.10	w_8	0.10
x_0	0.50	y_0	0.50	z_0	0.50

A. One-SP, One-InP Network Virtualization System

In this subsection, we investigate the economics of a simple one-SP, one-InP NV system. Assume the following setup. Suppose that the number of URs (or EUs) in the serviced area is limited to 1000. The sole SP can expand its VN up to a maximum of 50 virtual nodes and allow 25 EUs per VR. In addition, the sole InP can scale its PN up to 50 physical nodes and allow 4 VRs to share a single PR. Let each virtual and physical node have a capacity of 25 and 100 Mbps, respectively. In each simulation, we provide values for w_1 through $w_8 > 0$ that satisfy Lemma 3 and (35). Table VI lists the values used in this subsection, unless otherwise stated.

1) *Network Resource Strategies*: The following simulations shows the effect of NV parameters to the resource strategy $E(x^*, y^*, z^*)$ by solving the system defined by (14) using (32).

First, we see the influence of the VN service availability w_1 to the SP resource strategy y^* . Hence, we set four different values to w_1 : 0.10, 0.50, 1.00, and 2.00. From (15) and (16), increasing w_1 , while keeping other parameters the same, indicates that the SP has either increased n or expanded $k^{(2)}$. The computed value for w_7 is 0.125. The results of the simulations are listed in Table VII. When w_1 is increased from 0.10 to 2.00, while keeping other parameters constant, especially the VN payoff $w_3 = \frac{w_1}{\eta_2}$, the SP adapts by decreasing its VN capacity demand strategy y^* from 0.70, or equivalently, $0.70 \cdot 50 \text{ VR} \cdot 25 \frac{\text{Mbps}}{\text{VR}} = 875 \text{ Mbps}$ to 0.31 or 388 Mbps. The reason is that the SP has no incentive to increase its demand strategy from the InP when its payoff from the EU is not improving. Likewise, the InP reduces its supply strategy z^* in response to the lower VN demand.

Next, we investigate the effect of the PN payoff factor w_7 to the InP resource strategy z^* . Thus, we allow w_7 to have

TABLE VII
EFFECT OF w_1 AND w_7 ON $E(x^*, y^*, z^*)$

E	w_1				w_7			
	0.10	0.50	1.00	2.00	0.10	0.15	0.20	0.25
x^*	0.93	0.72	0.56	0.37	0.73	0.79	0.83	0.86
y^*	0.70	0.55	0.44	0.31	0.55	0.42	0.34	0.28
z^*	0.77	0.59	0.45	0.29	0.45	0.53	0.58	0.61

the following values: 0.10, 0.15, 0.20, and 0.25. From (15) and (16), the InP has to either reduce m or scale down $k^{(3)}$ to increase w_7 while keeping the other parameters constant. The computed value w_3 is 0.40. The results are listed in Table VII. We assume that the SP can only lease one portion of a PR. It follows that, there are $20 \frac{\text{Mbps}}{\text{PR}}$. When $w_7 = 0.10$, z approaches $z^* = 0.45 \cdot 50 \text{ PR} \cdot 25 \frac{\text{Mbps}}{\text{PR}} = 563 \text{ Mbps}$; however, when $w_7 = 0.25$, $z^* = 0.61$ or 763 Mbps. That is, when the payoff w_7 increases, especially with PN service availability $w_5 = \frac{w_7}{\eta_3}$ constant, the InP can afford to supply a higher additional capacity z^* to the SP. On the other hand, the resource strategy y^* of the SP is to reduce its demand from the InP because as w_7 becomes higher, n or k_3 is reduced. Consequently, the high payoff that the InP acquires will come from the larger share of resources, and hence cost, the SP has to pay in return.

2) *Network Economics*: The relationship between the EU capacity demand and the VN capacity supply is explored through solving (17) by implementing a numerical method called the 4th-order Runge-Kutta method [33].

First, we discover the effect of the VN payoff factor w_3 to the VN capacity supply. Consider four different values for w_3 : 0.05, 0.50, 1.00, and 2.00. Also, set $z = 0.50$ and $w_1 = 5.00$. The results of this case is illustrated in Fig. 2a. As w_3 increases from 0.05 to 2.00 at 50% EU activity (i.e., $x = 0.50$), the available VN supply decreases from $y = 0.87$ to 0.62. This happens since an increase in w_3 means that $\eta^{(2)}$ has been decreased, which is caused by a small n or $k^{(2)}$. Hence, when a VR can accommodate lesser EUs or when the SP can only manage a smaller VN, the SP adapts by decreasing its VN demand from the InP, and thus, its VN supply to the EUs.

Now, we find out the influence of the PN service availability w_5 to the VN capacity supply. Assume $w_1 = 10.00$, which is quite high just to accentuate the difference in the four curves to be plotted, $y_0 = 0.30$, and $z = 0.50$. Allow w_5 to have

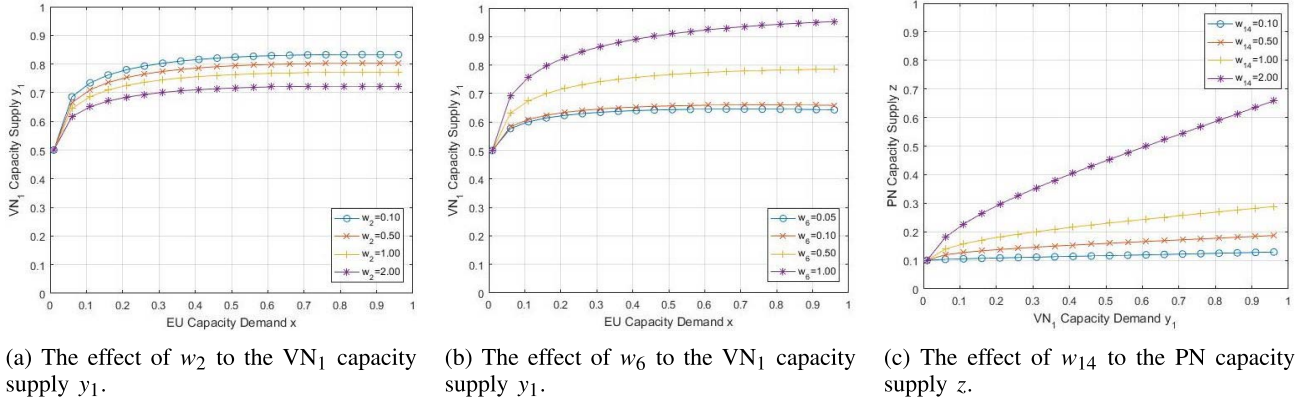


Fig. 3. Network economics of a two-SP, one-InP NV system.

TABLE VIII
PARAMETER VALUES FOR TWO-SP, ONE-INP NV SYSTEM

EU		SP ₁		SP ₂		InP	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
$\kappa^{(1)}$	1000	$\kappa_1^{(2)}$	50	$\kappa_2^{(2)}$	40	$\kappa^{(3)}$	50
		n_1	25	n_2	20	m	4
		w_1	0.50	w_2	+	w_7	0.20
		w_3	0.01	w_8	0.01	w_{12}	0.01
		w_4	0.40*	w_9	+	w_{13}	0.05*
		w_5	0.10	w_{11}	0.10	w_{14}	0.04*
		w_6	0.01	w_{10}	0.01	w_{15}	0.10
x_0	0.50	$y_{1(0)}$	0.50	$y_{2(0)}$	0.50	z_0	0.50

these values: 0.10, 0.50, 1.00, and 2.0. Fig. 2b shows the results of the simulation. We can conclude that higher w_5 allows the SP to increase its VN demand from the InP, and hence, its VN supply to the EU, and accommodate more EUs. For example, at $x = 0.50$, the VN capacity supply goes from $y = 0.32$ to 0.82 when w_5 varies from 0.10 to 2.00.

Lastly, we consider the connection between the VN capacity demand and the PN capacity supply through solving (18) using the Runge-Kutta method [33]. To investigate the consequence of the PN payoff factor to its own capacity, vary w_7 to these values: 0.05, 0.50, 1.00, and 2.00. Then, set $x = 0.50$, $w_3 = 0.05$, and $w_5 = 2.00$. The result is plotted in the Fig. 2c. At $y = 0.50$, the PN supply decreases from 0.62 to 0.13 as w_7 is increased from 0.05 to 2.00. It can be deduced from the definition of w_7 that it increases when m or $k^{(3)}$ is reduced. Thus, when w_7 is high, the PN decreases its supply because n is low.

B. Two-SP, One-InP Network Virtualization System

In this subsection, we focus on the effect of the existence of an additional SP in the economics of an NV system. In the following simulations, we use the values in Table VIII, unless otherwise stated. Entries labeled with * are computed values using (21), while those with + are varied. These values are chosen to satisfy Lemma 4 and (42).

1) *Network Resource Strategies*: The network strategy for a two-SP, one InP NV system of (20) is found by solving (41). Table IX shows the effect of the SP₂ service availability w_2 to the network strategies. The values for w_2 are 0.20, 0.25, 0.30, and 0.40, while the corresponding computed values for w_9 are 0.25, 0.31, 0.38, and 0.50. As w_2 is increased from 0.20 to 0.40, the strategy of SP₂ increases from 0.15 to 0.91

TABLE IX
EFFECT OF w_2 AND w_9 ON $E(x^*, y_1^*, y_2^*, z^*)$

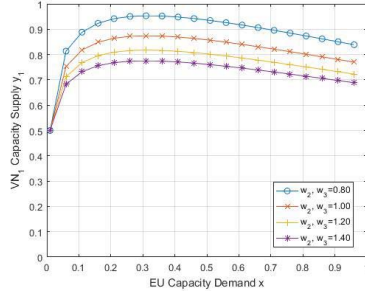
E	w_2				w_9			
	0.20	0.25	0.30	0.40	0.25	0.30	0.40	0.50
x^*	0.49	0.48	0.47	0.41	0.49	0.49	0.46	0.41
y_1^*	0.96	0.85	0.72	0.46	0.96	0.87	0.66	0.46
y_2^*	0.15	0.38	0.59	0.91	0.15	0.33	0.66	0.91
z^*	0.44	0.47	0.49	0.49	0.44	0.47	0.50	0.49

(i.e., from 150 to 910 Mbps) due to higher demand from EUs. However, the strategy of SP₁ has diminished from 0.96 to 0.46 (i.e., from 960 to 460 Mbps) since EUs have an alternative SP.

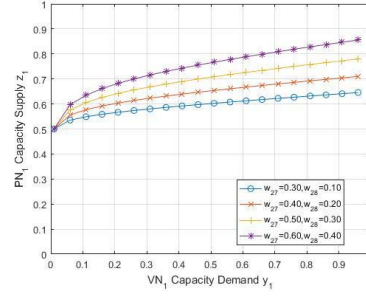
Next, w_9 assumes values of 0.25, 0.30, 0.40, and 0.50, and the computed values for w_2 are 0.20, 0.24, 0.32, and 0.40. When SP₂ has an increasing payoff w_9 from 0.25 to 0.50, its strategy also increases from 0.15 to 0.91 (i.e., from 150 to 910 Mbps). However, SP₁, whose payoff w_4 remains constant at 0.40, decreases its strategy from 0.96 to 0.46 (i.e., from 960 to 460 Mbps). This conflicting behavior between the two SPs allows the EUs to still get service even if one of the SPs is decreasing its strategy.

2) *Network Economics*: Fig. 3a illustrates the effect of SP₂ service availability w_2 to the VN capacity of SP₁ as a result of solving (23) using the Runge-Kutta method. At $x = 0.5$ and as w_2 is increased, the VN capacity of SP₁ y_1 decreases from 0.83 to 0.72. That is, a more reachable competing SP can accommodate more users, causing the demand for other SPs to decrease.

Likewise, from numerically solving (23), the influence of the SP₂ competition w_6 to the VN capacity of SP₁ is plotted in Fig. 3b. It shows that SP₁ increases its VN capacity when the effect of competition from SP₂ w_6 is greater. For instance,



(a) The effect of w_2 and w_3 to the VN_1 capacity supply y_1 .



(b) The effect of w_{27} and w_{38} to the PN_1 capacity supply z_1 .

Fig. 4. Network economics of a three-SP, two-InP NV system.

TABLE X
EFFECT OF w_2 AND w_3 ON $E^*(x, y_1^*, y_2^*, y_3^*, z_1^*, z_2^*)$

E	w_2, w_3			
	0.50	0.55	0.60	0.65
x^*	0.7773	0.7621	0.7475	0.7332
y_1^*	0.0589	0.0577	0.0565	0.0553
y_2^*	0.1485	0.1459	0.1434	0.1411
y_3^*	0.2380	0.2341	0.2304	0.2286
z_1^*	1.0331	1.0125	0.9924	0.9731
z_2^*	1.8463	1.8099	1.7747	1.7406

at $x = 0.5$, y_1 goes from 0.65 to 0.92. In other words, the SP_1 will try to increase its capacity to satisfies its users.

Lastly, the PN capacity as a function of the VN_1 capacity is found by solving (25). Fig. 3c shows the changes to the PN capacity with respect to the SP_2 -to-InP payoff w_{14} , with all initial capacities lowered to 0.10. At $y_1 = 0.5$, as the payoff w_{14} of SP_2 is increased, the PN capacity supply z increases from 0.12 to 0.42 to keep up with the increasing demand from SP_2 .

C. Three-SP, Two-InP Network Virtualization System

We conclude this section by showing the economics of a three-SP, two-InP NV system. From (31), this configuration includes 36 network parameters. Consequently, for brevity, we only show the values of parameters that vary in each setup.

1) *Network Resource Strategies*: To show the effect of the service availabilities w_2 and w_3 of SP_2 and SP_3 to the equilibrium point $E^*(x, y_1^*, y_2^*, y_3^*, z_1^*, z_2^*)$, we set w_2 and w_3 to 0.50, 0.55, 0.60, and 0.65 while keeping all the other parameters constant. The results are tabulated in X. We can infer that when the service availabilities of SP_2 and SP_3 increase, the resource strategy y_1^* of SP_1 decreases. Moreover, the resource strategies y_2^* of SP_2 and y_3^* of SP_3 also decrease.

2) *Network Economics*: The effect of service availabilities w_2 and w_3 of SP_2 and SP_3 , respectively, to the VN capacity supply of SP_1 is shown in Fig. 4a. It is evident that as the other two SPs become more available to the EUs, with w_2 and w_3 taking values 0.80, 1.00, 1.20, and 1.40, the capacity supply of SP_1 decreases. This may be attributed to the behavior of the EUs having more choices for the SPs. However, the trend for VN_1 capacity supply y_1 stays the same where it decreases with increasing EUs demand.

In Fig. 4b, the effect of SP_2 -to-InP₁ payoff w_{27} and SP_3 -to-InP₁ payoff w_{28} to the PN capacity supply of InP₁ z_1 is shown.

As w_{27} and w_{28} increases, z_1 increases with the demand from SP_1 as well. That is, InP₁ can afford to increase its supply as long as its payoff for doing so increases, no matter from which SP the increase in payoff comes from. Finally, for networks with higher number of SP and InP, they can be analyzed and simulated in a similar way.

VII. CONCLUSION

In this paper, we have focused on a novel approach to NV resource allocation and economics based on prey-predator food chain model. We have formulated models for a one-SP, one-InP and two-SP, one-InP NV systems, as well as generalized it to N -SP, M -InP NV system. Properties and analyses of these configurations have also been presented. The derivation of the equilibrium point of these models, as well as the conditions for its existence and stability, have also been discussed. In addition, models that associate the EU demand to the SP supply and the SP demand to the InP supply have been proposed. Numerical simulations have been performed to demonstrate the characteristics of these models. We have learned that the SP and the InP resource strategies depend on NV parameters such as payoff, failure, service, and congestion factors. Therefore, the SP and InP can control their resource strategies by changing the relevant network parameters. Moreover, the capacity supplies of the SP and the InP as a function of their respective demand also change with these network parameters.

REFERENCES

- [1] N. M. M. K. Chowdhury and R. Boutaba, "Network virtualization: State of the art and research challenges," *IEEE Commun. Mag.*, vol. 47, no. 7, pp. 20–26, Jul. 2009.
- [2] A. Belbekkouche, M. M. Hasan, and A. Karmouch, "Resource discovery and allocation in network virtualization," *IEEE Commun. Surveys Tuts.*, vol. 14, no. 4, pp. 1114–1128, 4th Quart., 2012.
- [3] Y. Zhou, Y. Li, G. Sun, D. Jin, L. Su, and L. Zeng, "Game theory based bandwidth allocation scheme for network virtualization," in *Proc. IEEE Global Telecommun. Conf.*, Miami, FL, USA, Dec. 2010, pp. 1–5.
- [4] M. S. Seddiki and M. Frikha, "A non-cooperative game theory model for bandwidth allocation in network virtualization," in *Proc. 15th Int. Telecommun. Netw. Strategy Planning Symp.*, Rome, Italy, Oct. 2012, pp. 1–6.
- [5] W. Xie, J. Zhu, C. Huang, M. Luo, and W. Chou, "Network virtualization with dynamic resource pooling and trading mechanism," in *Proc. IEEE Global Commun. Conf.*, Austin, TX, USA, Dec. 2014, pp. 1829–1835.
- [6] C. Liang and F. R. Yu, "Wireless network virtualization: A survey, some research issues and challenges," *IEEE Commun. Surveys Tuts.*, vol. 17, no. 1, pp. 358–380, 1st Quart., 2015.

- [7] F. Fu and U. C. Kozat, "Stochastic game for wireless network virtualization," *IEEE/ACM Trans. Netw.*, vol. 21, no. 1, pp. 84–97, Feb. 2013.
- [8] B. Liu and H. Tian, "A bankruptcy game-based resource allocation approach among virtual mobile operators," *IEEE Commun. Lett.*, vol. 17, no. 7, pp. 1420–1423, Jul. 2013.
- [9] J. Guo, F. Liu, J. C. S. Lui, and H. Jin, "Fair network bandwidth allocation in iaas datacenters via a cooperative game approach," *IEEE/ACM Trans. Netw.*, vol. 24, no. 2, pp. 873–886, Apr. 2016.
- [10] F. Fu and U. C. Kozat, "Wireless network virtualization as a sequential auction game," in *Proc. IEEE INFOCOM*, San Diego, CA, USA, Mar. 2010, pp. 1–9.
- [11] X. Lv, A. Xiong, S. Zhang, and X. Qiu, "VCG-based bandwidth allocation scheme for network virtualization," in *Proc. IEEE Symp. Comput. Commun.*, Jul. 2012, pp. 744–749.
- [12] B. Han, V. Gopalakrishnan, L. Ji, and S. Lee, "Network function virtualization: Challenges and opportunities for innovations," *IEEE Commun. Mag.*, vol. 53, no. 2, pp. 90–97, Feb. 2015.
- [13] M. S. Yoon and A. E. Kamal, "NFV resource allocation using mixed queueing network model," in *Proc. IEEE Global Commun. Conf.*, Washington, DC, USA, Dec. 2016, pp. 1–6.
- [14] Q. Zhang, Y. Xiao, F. Liu, J. C. S. Lui, J. Guo, and T. Wang, "Joint optimization of chain placement and request scheduling for network function virtualization," in *Proc. IEEE Int. 37th Conf. Distrib. Comput. Syst.*, Atlanta, GA, USA, Jun. 2017, pp. 731–741.
- [15] X. Fei, F. Liu, H. Xu, and H. Jin, "Towards load-balanced VNF assignment in geo-distributed NFV infrastructure," in *Proc. IEEE/ACM Int. Symp. Qual. Service*, Vilanova i la Geltrú, Spain, Jun. 2017, pp. 1–10.
- [16] H. K. Nguyen, Y. Zhang, Z. Chang, and Z. Han, "Parallel and distributed resource allocation with minimum traffic disruption for network virtualization," *IEEE Trans. Commun.*, vol. 65, no. 3, pp. 1162–1175, Mar. 2017.
- [17] D. H. N. Nguyen, Y. Zhang, and Z. Han, "A contract-theoretic approach to spectrum resource allocation in wireless virtualization," in *Proc. IEEE Global Commun. Conf.*, Washington, DC, USA, Dec. 2016, pp. 1–6.
- [18] Y. Wei, J. Wang, C. Wang, and X. Hu, "Bandwidth allocation in virtual network based on traffic prediction," in *Proc. 6th Int. Conf. Wireless Commun. Netw. Mobile Comput.*, Chengdu, China, Sep. 2010, pp. 1–4.
- [19] E. Datsika, A. Antonopoulos, N. Zorba, and C. Verikoukis, "Matching game based virtualization in shared LTE-A networks," in *Proc. IEEE Global Commun. Conf.*, Washington, DC, USA, Dec. 2016, pp. 1–6.
- [20] Y. Gu, W. Saad, M. Bennis, M. Debbah, and Z. Han, "Matching theory for future wireless networks: Fundamentals and applications," *IEEE Commun. Mag.*, vol. 53, no. 5, pp. 52–59, May 2015.
- [21] R. Mijumbi, J.-L. Gorricho, J. Serrat, M. Claeys, F. D. Turck, and S. Latre, "Design and evaluation of learning algorithms for dynamic resource management in virtual networks," in *Proc. IEEE Netw. Oper. Manage. Symp.*, Krakow, Poland, May 2014, pp. 1–9.
- [22] R. Shi *et al.*, "MDP and machine learning-based cost-optimization of dynamic resource allocation for network function virtualization," in *Proc. IEEE Int. Conf. Services Comput.*, New York, NY, USA, Jul. 2015, pp. 65–73.
- [23] J. Asmuss and G. Lauks, "A fuzzy logic based approach to bandwidth allocation in network virtualization," in *Proc. Sci. Inf. Conf.*, London, U.K., Oct. 2013, pp. 507–513.
- [24] E. Weisstein, *Logistic Equation*. Accessed: Apr. 2017. [Online]. Available: <http://mathworld.wolfram.com/LogisticEquation.html>
- [25] A. Hastings, *Population Biology: Concepts and Models*. New York, NY, USA: Springer, 1997.
- [26] H. Freedman and P. Waltman, "Mathematical analysis of some three-species food-chain models," *Math. Biosci.*, vol. 33, nos. 3–4, pp. 257–276, 1997.
- [27] R. K. Naji and A. T. Balasim, "Dynamical behavior of a three species food chain model with Beddington–DeAngelis functional response," *Chaos, Solitons Fractals*, vol. 32, no. 5, pp. 1853–1866, 2005.
- [28] P. Y. H. Pang and M. Wang, "Strategy and stationary pattern in a three-species predator–prey model," *J. Differ. Equ.*, vol. 200, no. 2, pp. 245–273, 2004.
- [29] S. Boyd, *Basic Lyapunov Theory*. Accessed: Apr. 2017. [Online]. Available: <https://stanford.edu/class/ee363/lectures/lyap.pdf>
- [30] E. J. Routh, *A Treatise on the Stability of a Given State of Motion: Particularly Steady Motion*. New York, NY, USA: Macmillan, 1877.
- [31] H. Bergmann, "On the conditions under which an equation has roots with negative real parts," in *Selected Papers on Mathematical Trends in Control Theory*. New York, NY, USA: 1964, pp. 70–82.

- [32] E. Weisstein, *Stable Polynomial*. Accessed: Apr. 2017. [Online]. Available: <http://mathworld.wolfram.com/StablePolynomial.html>
- [33] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes in FORTRAN: The Art of Scientific Computing*, 2nd ed. Cambridge, U.K.: Cambridge Univ. Press, 1992, pp. 704–716.



Reginald A. Banez received the B.S. degree in electronics and communications engineering from the Mapua Institute of Technology, Manila, Philippines, in 2008, and the M.S. degree in electrical engineering from the University of Houston, Houston, TX, USA, in 2015, where he is currently pursuing the Ph.D. degree in electrical engineering. His research interests include communication networks, game theory, and machine learning.



Haitao Xu (M'16) received the Ph.D. degree from the University of Science and Technology Beijing (USTB) in 2014. He held a visiting professor position at the Electrical and Computer Engineering Department, University of Houston, from 2016 to 2017. He is currently an Associate Professor with USTB. He has published 50 papers and one book for cyber security. His research interests include wireless communication, game theory, secure communications, cognitive radio, and mobile edge computing.



Nguyen H. Tran (S'10–M'11) received the B.S. degree in electrical and computer engineering from the Ho Chi Minh City University of Technology in 2005 and the Ph.D. degree in electrical and computer engineering from Kyung Hee University in 2011. He was an Assistant Professor with the Department of Computer Science and Engineering, Kyung Hee University, from 2012 to 2017. Since 2018, he has been with the School of Information Technologies, The University of Sydney, where he is currently a Senior Lecturer. His research interest is applying analytic techniques of optimization, game theory, and machine learning to cutting-edge applications, such as cloud and mobile-edge computing, datacenters, resource allocation for 5G networks, and Internet of Things. He received the Best KHU Thesis Award in engineering in 2011 and several best paper awards, including IEEE ICC 2016, APNOMS 2016, and IEEE ICCS 2016. He receives the Korea NRF Funding for Basic Science and Research from 2016 to 2023. He has been an Editor of the IEEE TRANSACTIONS ON GREEN COMMUNICATIONS AND NETWORKING since 2016.



Ju Bin Song (S'99–A'00–M'02) received the B.Sc. and M.Sc. degrees from the Department of Electronic Engineering, Kyung Hee University, Seoul, South Korea, in 1987 and 1989, respectively, and the Ph.D. degree from the Department of Electronic and Electrical Engineering, University College London (UCL), London, U.K., in 2001. From 1992 to 1997, he was a Senior Researcher with the Electronics and Telecommunications Research Institute, South Korea. He was a Research Fellow with the Department of Electronic and Electrical Engineering, UCL, in 2001. From 2002 to 2003, he was an Assistant Professor with the School of Information and Computer Engineering, Hanbat National University, South Korea. He has been a Professor with the Department of Electronic Engineering, Kyung Hee University, since 2003. He was the Vice Dean of the School of Electronics and Information, Kyung Hee University, from 2015 to 2016. From 2009 to 2010, he was a Visiting Professor with the Department of Electrical Engineering and Computer Science, University of Houston, TX, USA. His current research interests include resource allocation in communication systems and networks, cognitive networking, cooperative communications, game theory, optimization, cognitive radio networks, and smart grids. He has served as a member of the Technical Program Committee for the IEEE Vehicular Technology Conference, the IEEE Wireless Communications and Networking Conference, and the IEEE GLOBECOM. He was a recipient of the Kyung Hee University Best Teaching Award in 2004 and 2012, respectively. He has served as the General Chair for the 8th EAI International Conference on Game Theory for Networks. He has served a Guest Editor for *Mobile Networks and Application* journal.



Choong Seon Hong (S'95–M'97–SM'11) received the B.S. and M.S. degrees in electronic engineering from Kyung Hee University, Seoul, South Korea, in 1983 and 1985, respectively, and the Ph.D. degree from Keio University, Minato, Japan, in 1997. In 1988, he joined Korea Telecom (KT), where he was involved in broadband networks as a Member of Technical Staff. In 1993, he was with Keio University. He was with the Telecommunications Network Laboratory, KT, as a Senior Member of Technical Staff and the Director of the Networking Research Team until 1999. Since 1999, he has been a Professor with the Department of Computer Science and Engineering, Kyung Hee University. His research interests include future Internet, IoT networks, network management, and network security. He is a member of the ACM, IEICE, IPSJ, KIISE, KICS, KIPS, and OSIA. He has served as the General Chair, a TPC Chair/Member, or an Organizing Committee Member for international conferences, such as NOMS, IM, APNOMS, E2EMON, CCNC, ADSN, ICPP, DIM, WISA, BcN, TINA, SAINT, and ICOIN. He is currently an Associate Editor of the *International Journal of Network Management* and the *Journal of Communications and Networks* and an Associate Technical Editor of the *IEEE Communications Magazine*.



Zhu Han (S'01–M'04–SM'09–F'14) received the B.S. degree in electronic engineering from Tsinghua University in 1997 and the M.S. and Ph.D. degrees in electrical and computer engineering from the University of Maryland, College Park, in 1999 and 2003, respectively.

From 2000 to 2002, he was a Research and Development Engineer of JDSU, Germantown, MD, USA. From 2003 to 2006, he was a Research Associate with the University of Maryland. From 2006 to 2008, he was an Assistant Professor with Boise State University, ID, USA. He is currently a John and Rebecca Moores Professor with the Electrical and Computer Engineering Department and the Computer Science Department, University of Houston, TX, USA. His research interests include wireless resource allocation and management, wireless communications and networking, game theory, big data analysis, security, and smart grid. He received the NSF Career Award in 2010, the Fred W. Ellersick Prize of the IEEE Communication Society in 2011, the EURASIP Best Paper Award for the *Journal on Advances in Signal Processing* in 2015, the IEEE Leonard G. Abraham Prize in the field of communications systems (the Best Paper Award in the IEEE JSAC) in 2016, and several best paper awards in IEEE conferences. He is currently an IEEE Communications Society Distinguished Lecturer. He is a 1% Highly Cited Researcher 2017 according to Web of Science.