

# Employing GRIN PC- inspired approach for building invisibility cloak media from photonic crystals

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**Abstract**—Employing photonic crystals in transformation media requires realizing prescribed anisotropic spatial dispersions of refractive index components. We show that in invisibility cloaks, anisotropy can be provided by using crystals with rectangular lattices, while inspired by GRIN PCs approach can be employed to decrease scattering cross-section.

**Keywords**—transformation optics, invisibility cloak, anisotropic photonic crystals, graded index

## I. INTRODUCTION

Dielectric photonic crystals (PCs) provide a low-loss platform for designing various photonic devices. One of perspective applications for PCs is their employment in transformation media, in particular, in invisibility cloaks [1]. It is known that Transformation Optics (TO) requests materials with properties, which cannot be found in nature. In our recent work [2] it was demonstrated that in cylindrical cloaks formed from 2D PCs with square lattices, waves could move with superluminal phase velocities. Later in [3] we have shown that at employment of PC with rectangular lattices, anisotropic dispersions of refractive index components could be achieved in the cloak media, in addition to superluminal wave propagation.

In this work, in order to decrease scattering from cloaked objects, we investigate an opportunity of using an approach, inspired by GRIN PCs [4, 5], to form the cloak medium with prescribed index dispersions. As known, GRIN PCs are formed by appropriate gradual changes of PC lattice constants in one direction. For the cloak medium, we had to solve a more complicated task of providing gradual modification of PC parameters in two directions to control radial dispersions of azimuthal and radial index components. Obtained results demonstrate that employed approach allows for substantial decreasing the thickness of the cloak without deteriorating expected according to TO predictions wave front reconstruction beyond cloaked target. Decreased thickness of the cloak leads to essentially less scattering from cloaked object compared to scattering from bare target thus providing more efficient cloaking effect.

Full-wave simulations in this work have been performed by using COMSOL Multiphysics software, while MPB package was employed for calculating dispersion diagrams of PCs.

## II. TO PRESCRIPTIONS FOR CLOAK MEDIUM BUILT FROM ANISOTROPIC PCS

To provide proper performance of cylindrical invisibility cloak, TO requires specific radial dispersions for azimuthal index component  $n_\theta$  and for radial index component  $n_r$ . The value of  $n_\theta$  has to grow up from zero at inner boundary of the cloak towards 1 at outer boundary of the cloak, while the value of  $n_r$  has to descend from bigger than 1 values at inner boundary of the cloak to 1 at the outer cloak's boundary [2]. PCs with rectangular lattices were found capable of providing anisotropy of refractive indices along two normal to each other crystallographic axes of crystals [3]. However, modifications of rectangular unit cells did not allow for ensuring such high degree of index anisotropy, which was prescribed by TO for cloak layers located close to inner boundary of the medium. As described in [3], the possibility to overcome this discrepancy was found in replacing TO-prescribed dispersion law for  $n_r$  by reduced dependence incorporating achievable in PC values of  $n_r$  near the cloaked object and higher than TO-prescribed values of  $n_r$  in outer layers of the cloak. Careful balancing of these changes has allowed for observing the TO-predicted cloaking effect in cloak models with reduced index dispersion that justified an employment of proposed reduction. Fig. 1 presents TO-prescribed dispersion for  $n_\theta$  and the reduced dispersion for  $n_r$ . The figure also shows an approximation of the dispersion curves by step-functions made of six gradual

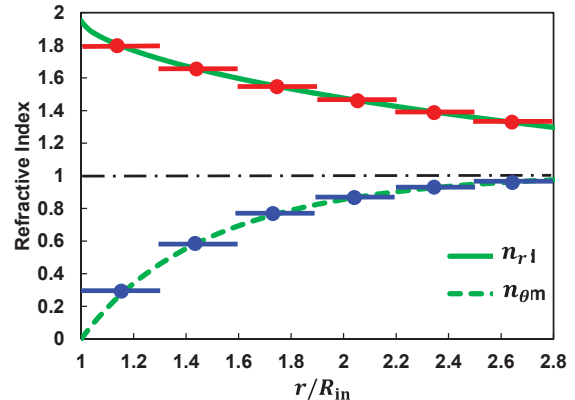


Fig. 1. Green curves—dispersions of index components chosen for building a cloak with  $R_{out}/R_{in} = 2.8$ ; red and blue steps—parts of step-functions used for approximating chosen dispersions.

steps. These steps were used to determine the values of index components for circular arrays, which represented fragments of PCs necessary to form the cloak.

Fig. 2 demonstrates frequency dependencies of index components calculated for two orthogonal directions  $X$  and  $Y$  in six infinite PCs composed of dielectric rods with different sets of lattice constants  $a_x$  and  $a_y$ . Presented dependencies have been obtained from the 2<sup>nd</sup> transmission branches in dispersion diagrams of respective PCs [2]. It is well seen in the figure that at the frequency of 11.2 GHz chosen as operating frequency, indices controlling wave propagation along  $X$ -axis are spread in the range from 0.3 up to 1 and, so, are capable of supporting “superluminal” phase velocity of waves, while indices controlling  $Y$ -direction are in the range between 1.25 and 1.8, and so, can provide reduced dispersion of  $n_r$ .

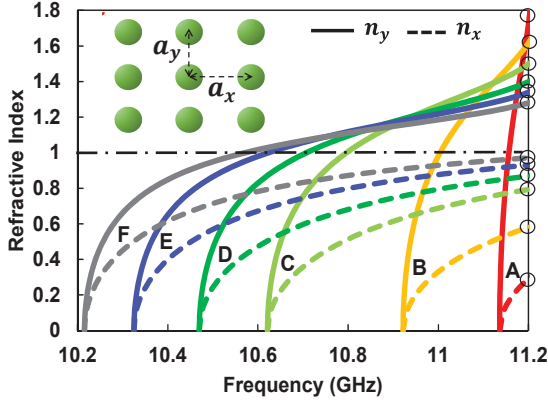
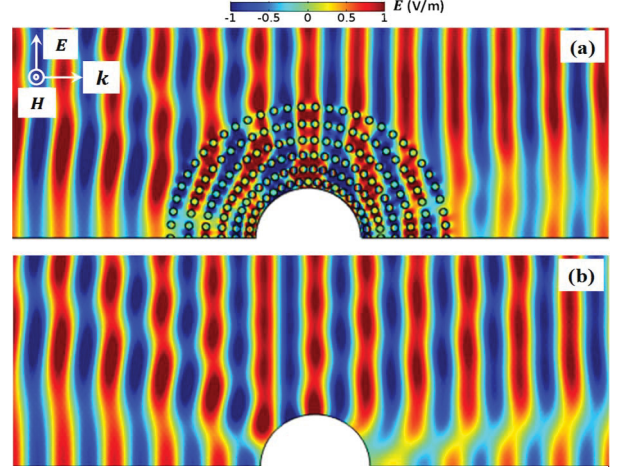


Fig. 2. Frequency dependencies of directional index components (in the 2<sup>nd</sup> transmission band at TM wave incidence) for composed of dielectric rods ( $\epsilon = 35$ ,  $R = 2$  mm) PCs with lattice constants  $a_x$  and  $a_y$  (in mm): A ( $4.95 \times 6.85$ ), B ( $5.2 \times 7.37$ ), C ( $5.75 \times 8.26$ ), D ( $6.2 \times 8.77$ ), E ( $6.8 \times 9.32$ ), and F ( $7.45 \times 9.75$ ).

Therefore, using properly rolled-up concentric fragments of PCs from series A-F in Fig. 2 provided an opportunity to build the cloak medium with dispersions presented by step-functions in Fig. 1.

### III. VERIFYING GRIN PC- INSPIRED APPROACH TO DESIGNING PC- BASED CLOAK MEDIUM

To verify the applicability of the GRIN PC- inspired approach, we represented fragments of PCs from series A-F (Fig. 2) in the cloak medium by single circular arrays. Thus we have built the cloak medium from six circular arrays. From captions to Fig. 2 it is seen that transition from one circular array to another is accompanied by gradual changes of two lattice constants (in azimuthal and radial directions). Although such changes should cause some distortions of the shapes of unit cells, we expected that these distortions would not significantly affect the index values found for respective lattice constants. Field patterns simulated at TM wave incidence on cloaked object and on bare object are presented in Figs. 3a and 3b, respectively. From



Snap-shots of TM wave incidence on (a) metallic object covered by PC-based cloak and (b) bare object at frequency of 11.49 GHz.

comparison of two figures it could be concluded that GRIN PC-inspired approach to designing the cloak medium provided an opportunity to realize predicted by TO wave front changes at wave movement around hidden object and wave front reconstruction beyond the object. Calculations of the total scattering cross-width (TSCW) for two cases have shown that TSCW of cloaked object appears to be 40% less than the TSCW of the bare object.

### IV. CONCLUSION

Obtained results confirm an opportunity to use the GRIN PC-inspired approach to designing the cloak medium from fragments of PCs with rectangular lattices. This allows for realizing close to TO-prescribed spatial dispersions of radial and azimuthal index components, which control bending the wave paths around the cloaked object and speeding up waves along curvilinear paths. The proposed approach to designing cylindrical cloaks can be scaled to higher frequencies, including optical range, by scaling dimensions of dielectric rods.

### REFERENCES

- [1] Y. A. Urzhumov and D. R. Smith, “Transformation optics with photonic band gap media,” *Physical Review Letters*, 105, 163901, 2010.
- [2] E. Semouchkina, R. Duan, N. P. Gandji, S. Jamilan, G. Semouchkin, and R. Pandey, “Superluminal media formed by photonic crystals for transformation optics-based invisibility cloaks,” *Journal of Optics*, 18, 044007, Apr. 2016.
- [3] S. Jamilan, G. Semouchkin, N. P. Gandji, and E. Semouchkina, “Spatial dispersion of index components required for building invisibility cloak medium from photonic crystals,” *Journal of Optics*, 20, 045102, 2018.
- [4] B. Vasic and R. Gajic, “Self-focusing media using graded photonic crystals: focusing, fourier transforming and imaging, directive emission, and directional Cloaking,” *Journal of Applied Physics*, 110, 053103, 2011.
- [5] Q. Zhu, L. Jin, and Y. Fu, “Graded index photonic crystals: a review,” *Annalen der Physik*, (Berlin) 527, No. 3–4, 205–218, 2015.