

# Broader analysis of scattering from a subwavelength dielectric sphere

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**Abstract**—Forward and backward scattering from subwavelength dielectric spheres is analyzed in a broad frequency range with the focus on the impact of phase changes of resonance oscillations at magnetic and electric Mie resonances. The effects of sphere permittivity on the specifics of scattering are discussed.

**Keywords** – dielectric resonator, directional scattering, Mie resonance, Kerker's conditions

## I. INTRODUCTION

As it has been recently demonstrated, arrays of low-loss dielectric resonators (DRs) can successfully compete with plasmonic structures in controlling the light. Especially interesting opportunities have been revealed at overlapping responses from magnetic and electric dipolar resonances (MDR and EDR). Analysis of resonance scattering has originally relied on Mie theory developed for dielectric spheres at plane wave incidence. This theory allows for calculating the spectra of Mie coefficients, which characterize electromagnetic responses at MDR, EDR, and respective quadrupolar resonances (QMR and QER). It was shown [1] that at specific relations between Mie coefficients representing comparable in strength resonances, it was possible to obtain either forward (FS) or backward (BS) directional scattering from spheres, i.e. the effects predicted earlier for magneto-dielectric particles at so-called Kerker's conditions [2]. Although these discoveries were actively discussed in literature ([3-5]), unanswered questions regarding the nature of the processes still remain.

In this work, we perform a broader analysis of scattering from dielectric spheres having various permittivity values that allows for clarifying the specifics of scattering processes and for resolving revealed ambiguities. Taking into account the possibility to scale resonance phenomena in DRs from optical range down to microwaves, the studies were conducted for spheres of 7 mm in diameter with relative permittivity of 10, 15, 25 and 35, i.e. with values achievable for silicon at optical frequencies. At scaling the dimensions of spheres, obtained results can be easily transferred to frequencies of photonic applications.

## II. SPECIFICS OF OBSERVED RESONANCE RESPONSES

Upper row in Fig. 1 presents the spectra of Mie coefficients for MDR ( $b_1$ ), EDR ( $a_1$ ), QMR ( $b_2$ ), and QER ( $a_2$ ). These data look similar to those in [2], however, comparison of spectra for spheres with

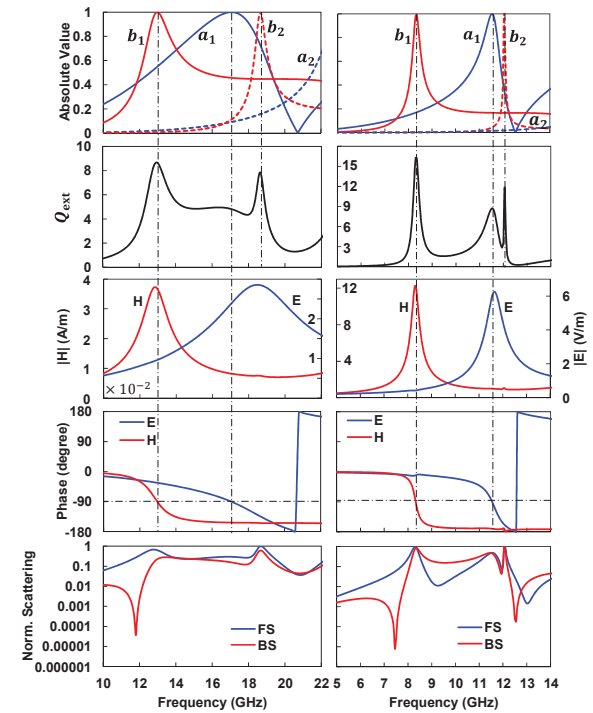


Fig. 1. Resonance responses of spheres with dielectric permittivity of 10 (left column) and 25 (right column). Rows: 1<sup>st</sup> – spectra of Mie coefficients, 2<sup>nd</sup> – extinction spectra, 3<sup>rd</sup> – H- and E-field probe signal spectra, 4<sup>th</sup> – phase spectra of the probe signals, 5<sup>th</sup> – FS and BS spectra.

different permittivity shows that increase of sphere permittivity leads to increase of Q-factors of all resonances. Increase of Q-factors is especially noticeable in extinction spectra (2<sup>nd</sup> row in Fig. 1), where resonance responses can be much better distinguished than in probe signal spectra (3<sup>rd</sup> row in Fig. 1), which demonstrate, at lower permittivity, blue shifts of E-field resonance peaks to the position of QMR. H-field peaks do not show similar shifts. 4<sup>th</sup> row in Fig. 1 presents the changes in phases of dipolar resonance oscillations with respect to the phase of

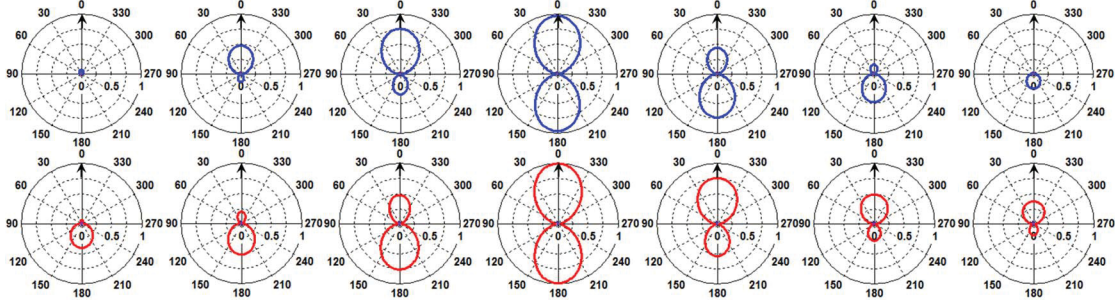


Fig. 2. 2D radiation patterns,  $W/m^2$  for a sphere with  $\epsilon=25$ : (1<sup>st</sup> row) near MDR in YZ plane, normalized by maximum power value ( $1.96 \times 10^{-7} W/m^2$ ) scattered at main lobe of MDR radiation at 8.35 GHz; (2<sup>nd</sup> row) near EDR in XZ plane, normalized by maximum power value ( $1.06 \times 10^{-7} W/m^2$ ) scattered at main lobe of EDR radiation at 11.45 GHz. Frequencies, GHz: (1<sup>st</sup> row) 7.45, 8.1, 8.19, 8.35, 8.47, 8.6, 9; (2<sup>nd</sup> row) 9.71, 10.45, 10.99, 11.45, 11.67, 11.77, 11.81. Arrows show  $k$ -vector direction, H-field is along Y, E-field – along X.

incident wave at passing MDR and EDR regions. It can be seen that, if at high permittivity these changes look as a jump or switch, at  $\epsilon=10$  they occur in a wide frequency range approaching 4 GHz at MDR and up to 10 GHz at EDR that roughly corresponds to the half-widths of resonance peaks in the spectra of Mie coefficients. The observed changes in phases at two resonances cause the phase difference between magnetic and electric dipolar oscillations by  $180^\circ$  in the frequency range between MDR and EDR. This difference should be accounted at the analysis of Kerker's conditions. 5<sup>th</sup> row in Fig. 1 presents the spectra of FS and BS. It can be seen that at both permittivity values, deep drops appear in BS spectra below MDR. These drops are usually considered to be related to satisfying the 1<sup>st</sup> Kerker's condition at crossing spectral dependencies for  $a_1$  and  $b_1$ . At  $\epsilon=10$ , no other meaningful drops are observed. At  $\epsilon=25$ , scattering spectra demonstrate a drop in FS at  $f > f_{MDR}$ , which, in literature, is considered as defined by satisfying the 2<sup>nd</sup> Kerker's condition at one more crossing of  $a_1$  and  $b_1$  spectra. However, since similar crossing at  $\epsilon=10$  is not accompanied by a drop of FS, there could be additional factors affecting the realization of the 2<sup>nd</sup> Kerker's condition. In this relation, accounting for differences in phase changes of dipolar oscillations at  $\epsilon=10$  and  $\epsilon=25$  at considering Kerker's balances of overlapping fields could play an important role. It is worth noting that, although appearance of additional drops of BS and FS in the scattering spectra of sphere with  $\epsilon=25$  at  $f > f_{EDR}$  (i.e. when the difference between phases of resonance oscillations at MDR and EDR decreases) should be still considered in frames of approaches used at formulating the 1<sup>st</sup> Kerker's condition [2], the contribution from QMR in the field balance should be taken into account, similar to how it was done in [5] at operating with QER.

Studies of radiation patterns were further used to characterize changes in scattering directivity expected at overturns of magnetic and electric dipoles following the changes in phases of resonance oscillations at

MDR and EDR. Fig. 2 presents 2D far-field patterns of scattered power from sphere with  $\epsilon=25$  in the range  $f_{1^{st} \text{kerker}} < f < f_{QMR}$ . As seen in the figure, at  $f_{1^{st} \text{kerker}} < f < f_{MDR}$ , FS dominates, however BS grows stronger at approaching  $f_{MDR}$  that provides expected parity between FS and BS at MDR (when 3D radiation pattern of bagel type is observed). At  $f > f_{MDR}$ , phase switching at magnetic resonance provides dominance of BS over FS, while both BS and FS decrease. Since FS decreases stronger, it defines the observed fall of FS at 2<sup>nd</sup> crossing of  $a_1$  and  $b_1$  spectra. At further increase of frequency, radiation patterns become affected by approaching EDR, when characteristics of far-field patterns can be seen in XZ cross-section (2<sup>nd</sup> row in Fig. 2). Again, both BS and FS grow up, and, although the dominance of BS is conserved up to  $f_{EDR}$ , a stronger growth of FS at approaching  $f_{EDR}$  leads to restoration of parity between BS and FS and to observation of bagel-type 3D radiation pattern. At  $f > f_{EDR}$  along with switching the phase of dipolar oscillations, an enhanced decrease of BS compared to that of FS is observed, which leads to dominating FS in radiation pattern up to QMR frequency.

It follows from the presented results that changes of phases of dipolar oscillations at MDR and EDR should be accounted for at the analysis of inference phenomena defining wave scattering from spheres.

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